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Efficient Algorithms for Drawing Planar Graphs(平面グラフを描画する効率的アルゴリズム)

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論 文 内 容 要 旨

1 Introduction

In this chapter we first provide the necessary background and motivation for this study on graph drawing, and then summarize our new results.

A graph is an abstract structure that is used to model information. Graphs may be used to represent any information that can be modeled as objects and connections between those objects. Thus graph drawing addresses the problem of constructing geometric representations of conceptual structures that are modeled by graphs. Automatic graph drawings have important applications to key computer technologies such as computer networks, circuit schematics, software engineering, entity-relationship diagrams, information systems, etc. The field of graph drawing has been flourished very much in the last decade, and a number of drawing styles and corresponding drawing algorithms have come out.

An *orthogonal drawing* is a drawing of a plane graph in which each vertex is drawn as a point and each edge is drawn as a chain of horizontal and vertical line segments. (See Fig. 1(a).) The point at which an edge changes its direction in an orthogonal drawing is called a *bend*. Orthogonal drawings have attracted much attention due to their numerous applications in circuit layouts, database diagrams, entity-relationship diagrams, etc [Tam87]. Clearly a graph having a vertex of degree 5 or more has no orthogonal drawing, because at most four edges can be incident to a vertex in an orthogonal drawing.

A *box-orthogonal drawing* of a graph is a drawing such that each vertex is drawn as a rectangle, called a *box*, and each edge is drawn as a sequence of alternate horizontal and vertical line segments [PT98]. (See Fig. 1(b).) Every plane graph has a box-orthogonal drawing.

An orthogonal drawing is called a *rectangular drawing* if it has no bend and each face is drawn as a rectangle. Fig. 1(c) shows a rectangular drawing of a graph. Rectangular drawings have practical applications in VLSI floorplanning.

A *convex drawing* of a planar graph is a representation of the graph on the plane such that all edges are drawn by straight line segments without any crossing and that all the face boundaries are convex polygons.

A drawing of a graph in which vertices and bends are located at grid points of an integer grid is called a *grid drawing*. The *size* of a grid drawing is measured by the size of the smallest rectangle on the grid which

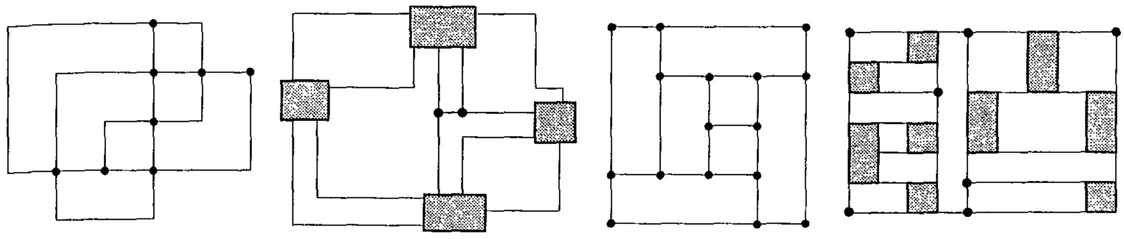

Figure 1: (a) An orthogonal drawing, (b)a box-orthogonal drawing, (c) a rectangular drawing, and (d) a box-rectangular drawing.

encloses the drawing. The *width* of the grid drawing is the width of the rectangle and the *height* of the grid drawing is the height of the rectangle.

In this thesis we study rectangular drawings, orthogonal drawings, convex drawings and our newly defined box-rectangular drawings. We give a linear-time algorithm for finding a rectangular drawing of a plane graph if it exists, and obtain the best possible upper bound on grid size for a rectangular drawing. Minimizing the number of bends in an orthogonal drawing is a challenging problem. We give a linear-time algorithm for finding an orthogonal drawing of a triconnected cubic plane graph with the minimum number of bends. We introduce a new drawing style called a box-rectangular drawing and obtain a necessary and sufficient condition for a plane graph to have a box-rectangular drawing. We also present the corresponding linear-time algorithm. We newly define 4-canonical decomposition and give a linear-time algorithm for finding a 4-canonical decomposition of a 4-connected planar graph. Our 4-canonical decomposition has applications in finding a convex drawing of a 4-connected planar graph.

2 Preliminaries

In this chapter we define some basic terms of graph theory.

A *graph* G is a structure (V, E) which consists of a finite set of *vertices* V and a finite set of *edges* E; each eadge is an unordered pair of distinct vertices. We denote the set of vertices of G by $V(G)$ and the set of edges by $E(G)$. We denote an edge between two vertices u and v of G by (u, v) or simply by uv. The *degree* of a vertex v in G is the number of edges incident to v. The *connectivity* $\kappa(G)$ of a graph G is the minimum number of vertices whose removal results in a disconnected graph or a single-vertex graph K_1. We say that G is *k-connected* if $\kappa(G) \geq k$.

A graph is *planar* if it can be embedded in the plane so that no two edges intersect geometrically except at a vertex to which they are both incident. A *plane graph* is a planar graph with a fixed embedding. A plane graph divides the plane into connected regions called *faces*. We regard the contour of a face as a clockwise cycle formed by the edges on the boundary of the face. We denote the contour of the outer face of graph G by $C_o(G)$.

Let $\{v_1, v_2, \cdots, v_{p-1}, v_p\}$ be a set of three or more consecutive vertices on $C_o(G)$ such that the degrees of the first vertex v_1 and the last one v_p are at least three and the degrees of all intermediate vertices $v_2, v_3, \cdots, v_{p-1}$ are two. Then we call the set $\{v_2, v_3, \cdots, v_{p-1}\}$ an *outer chain* of G.

An edge which is incident to exactly one vertex of a simple cycle C and located outside of C is called a *leg* of C. A simple cycle C in G is called a *k-legged cycle* of G if C has exactly k legs.

3 Rectangular drawings

In this chapter we study rectangular drawings of plane graphs.

Thomassen obtained a necessary and sufficient condition for a plane graph to have a rectangular drawing [Tho84]. His proof does not look to lead to an efficient algorithm for rectangular drawing; a straightforward implementation of his method requires at least $O(n^3)$ time.

In this chapter we first give a new constructive proof of Thomassen's characterization, and then derive from the proof a simple algorithm to find a rectangular grid drawing of a plane graph in linear time. The following theorem is the main result of this chapter.

Theorem 3.1 *Let G be a connected plane graph such that all vertices have degree three except four vertices of degree two on $C_o(G)$. Then G has a rectangular drawing if and only if G has none of the following three types of simple cycles [Tho84]:*

(r1) 1-legged cycles;

(r2) 2-legged cycles which contain at most one vertex of degree two; and

(r3) 3-legged cycles which contain no vertex of degree two.

Furthermore one can check in linear time whether G satisfies the condition above, and if G does then one can find a rectangular drawing of G in linear time [RNN98]. The sum of the width and the height of the produced rectangular grid drawing is bounded by $\frac{n}{2}$, where n is the number of vertices in G [RNN98].

4 Orthogonal drawings

In this chapter we consider orthogonal drawings of plane graphs with the minimum number of bends.

For a given planar graph G, if it is allowed to choose its planar embedding, then finding an orthogonal drawing of G with the minimum number of bends is NP-complete [GT95]. However, Tamassia [Tam87] and Garg and Tamassia [GT97] presented algorithms which find an orthogonal drawing of a given plane graph G with the minimum number of bends in $O(n^2 \log n)$ and $O(n^{7/4}\sqrt{\log n})$ time respectively unless it is allowed to choose its planar embedding, where n is the number of vertices in G.

In this chapter we give a linear-time algorithm **Minimum-Bend** to find an orthogonal drawing of a 3-connected cubic plane graph with the minimum number of bends. An outline of **Minimum-Bend** is illustrated in Fig. 2. Given a plane graph as shown in Fig. 2(a), we first put four dummy vertices a, b, c and d of degree two on the outer boundary of G, and let G' be the resulting graph. Fig. 2(c) illustrates G', where the four dummy vertices are drawn by white circles. We then contract each of some cycles $C_1, C_2, \cdots$ and their insides (shaded in Fig. 2(c)) into a single vertex as shown in Fig. 2(d) so that the resulting graph G'' has a rectangular drawing as shown in Fig. 2(e). We also find orthogonal drawings of those cycles $C_1, C_2, \cdots$ and their insides recursively (see Figs. 2(d) and (e)). Patching the obtained drawings, we get an orthogonal drawing of G' as shown in Fig. 2(f). Replacing the dummy vertices a, b, c and d in the drawing of G' with bends, we finally obtain an orthogonal drawing of G as shown in Fig. 2(b).

The following theorem is the main result of this chapter.

Theorem 4.1 *Algorithm* **Minimum-Bend** *produces an orthogonal drawing of a 3-connected cubic plane graph G with the minimum number of bends in linear time.*

5 Box-rectangular drawings

In this chapter we introduce a new drawing style of a plane graph, called a "box-rectangular drawing."

A *box-rectangular drawing* of a plane graph G is a drawing of G on an integer grid such that each vertex is drawn as a (possibly degenerated) rectangle, called a *box*, and the contour of each face is drawn as a rectangle, as illustrated in Fig. 1(d). If G has multiple edges or a vertex with degree 5 or more, then G has no rectangular drawing but may have a box-rectangular drawing. Unfortunately not every plane graph

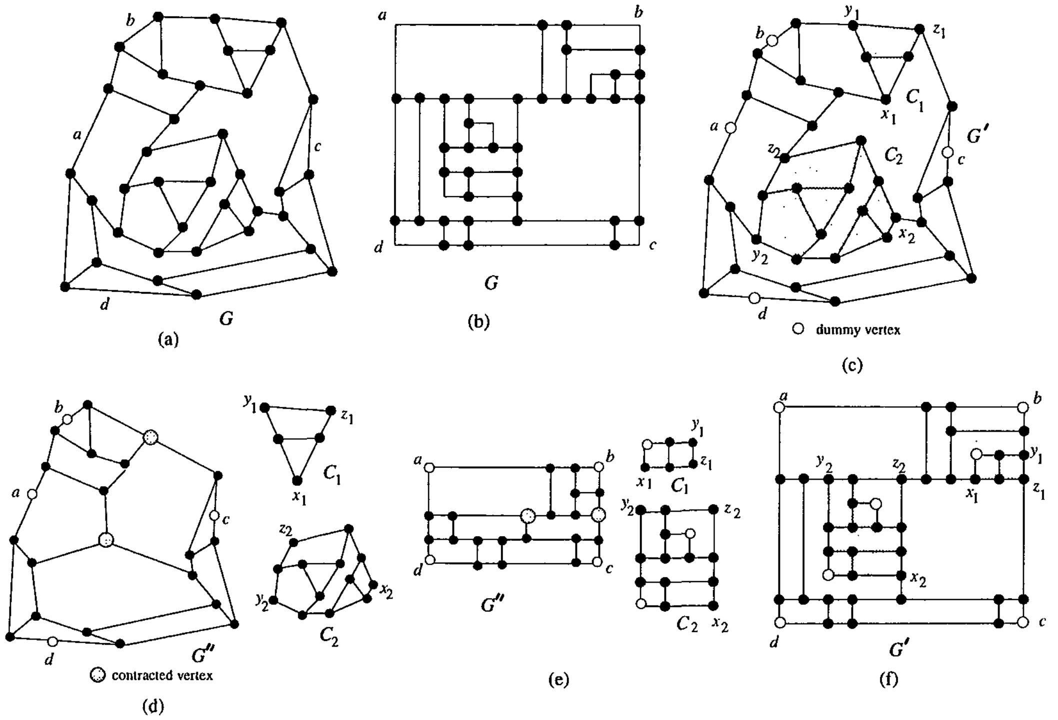

Figure 2: Illustration of the algorithm for an orthogonal drawing.

has a box-rectangular drawing. Box-rectangular drawings have beautiful applications in floorplanning of MultiChip Modules (MCM) and in architectural floorplanning.

The following theorem is the main result of this chapter.

Theorem 5.1 *Given a plane graph with m edges, one can determine in $O(m)$ time whether G has a box-rectangular drawing or not, and if G has, one can find a box-rectangular drawing of G in $O(m)$ time. The sum of the width and the height of a produced box-rectangular grid drawing is bounded by $m+2$.*

6 4-Canonical Decomposition

In this chapter we newly define a "4-canonical decomposition" and show that every 4-connected planar graph has a "4-canonical decomposition."

Assume that $G=(V,E)$ is a 4-connected plane graph with four designated distinct vertices u_1, u_2, u_3, u_4 on the same face of G. We may assume that u_1, u_2, u_3, u_4 lie on the contour $C_o(G)$ of G. We may furthermore assume that the four vertices u_1, u_2, u_3, u_4 appear on $C_o(G)$ of G in this order. Moreover we may assume that $(u_1, u_2), (u_3, u_4) \in E$. For a set $W_1, W_2, \cdots, W_i$ of pairwise disjoint subsets of V, we denote by G_i the subgraph of G induced by $W_1 \cup W_2 \cup \cdots \cup W_i$, and by $\overline{G_i}$ the subgraph of G induced by $V - W_1 \cup W_2 \cup \cdots \cup W_i$, that is, $\overline{G_i} = G - W_1 \cup W_2 \cup \cdots \cup W_i$. We say that a partition $\Pi = W_1, W_2, \cdots, W_l$ of V is a *4-canonical decomposition* of G if the following three conditions (co1)–(co3) are satisfied:

(co1) W_1 is the set of vertices on the inner face containing edge (u_1, u_2), and W_l is the set of vertices on the inner face containing edge (u_3, u_4);

(co2) for each i, $1 \le i < l$, both G_i and $\overline{G_i}$ are biconnected; and (co3) for each i, $1 < i < l$, either W_i consists of exactly one vertex on both $C_o(G_i)$ and $C_o(\overline{G_{i-1}})$ or W_i is an outer chain of G_i or $\overline{G_{i-1}}$.

We have the following theorem.

Theorem 6.1 *Let $G = (V, E)$ be a 4-connected plane graph with four designated distinct vertices u_1, u_2, u_3, u_4 appearing on $C_o(G)$ in this order. Then G has a 4-canonical decomposition $\Pi = W_1, W_2, \cdots, W_l$. Furthermore Π can be found in linear time.*

Using a 4-canonical decomposition, recently Miura *et al.* [MNN97] gave a linear-time algorithm for finding a convex grid drawing of a 4-connected planar graph G such that the sum of the width and the height of a produced drawing is at most n, where n is the number of vertices in G. We also successfully use the 4-canonical decomposition in "4-partitioning 4-connected planar graphs" [NRN97].

7 Conclusions

This thesis deals with algorithms for automatic drawings of plane graphs. We have presented efficient algorithms for rectangular drawings and orthogonal drawings. We have introduced a new drawing style called a box-rectangular drawing and obtained a necessary and sufficient condition for a plane multigraph to have a box-rectangular drawing. We have newly defined 4-canonical decomposition and effectively used it in solving not only some graph drawing problems but also other graph theoretical problems.

References

[GT95] A. Garg and R. Tamassia, *On the computational complexity of upward and rectilinear planarity testing*, Proc. of Graph Drawing'94, Lect. Notes in Computer Science, 894 (1995), pp. 286-297.

[GT97] A. Garg and R. Tamassia, *A new minimum cost flow algorithm with applications to graph drawing*, Proc. of Graph Drawing'96, Lect. Notes in Computer Science, 1190 (1997), pp. 201-226.

[MNN97] K. Miura, S. Nakano and T. Nishizeki, *Convex grid drawings of four-connected planar graphs*, Technical Report of IEICE, COMP97-45 (1997), pp. 1-8.

[NRN97] S. Nakano, M. S. Rahman and T. Nishizeki, *A linear-time algorithm for four-partitioning four-connected planar graphs*, Information Processing Letters, 62 (1997), pp. 315-322.

[PT98] A. Papakostas and I. G. Tollis, *Orthogonal drawings of high degree graphs with small area and few bends*, Proc. of 5th Workshop on Algorithms and Data Structures, Lect. Notes in Computer Science, 1272 (1998), pp. 354-367.

[RNN98] M. S. Rahman, S. Nakano and T. Nishizeki, *Rectangular grid drawings of plane graphs*, Comp. Geom. Theory Appl., 10 (1998), pp. 203-220.

[Tam87] R. Tamassia, *On embedding a graph in the grid with the minimum number of bends,* SIAM J. Computing, 16(3) (1987), pp. 421-444.

[Tho84] C. Thomassen, *Plane representations of graphs*, (Eds.) J.A. Bondy and U.S.R. Murty, Progress in Graph Theory, Academic Press Canada, (1984), pp. 43-69.

論文審査の結果の要旨

VLSIフロアプラン，建築設計，ビジュアル計算機言語等の様々な分野でグラフを自動描画することの必要性が益々増大している．著者は平面グラフの3種類の描画を求める線形時間アルゴリズムを設計するとともに，グラフの凸描画問題に応用することができるグラフの分解法を与えた．本論文はこれらの成果をとりまとめたものであり，全編7章からなる．

第1章は序論である．

第2章では，グラフの基本的な概念および平面グラフやアルゴリズムの一般的な性質を概観している．

第3章では，平面グラフが与えられたときに，その矩形描画，即ち各面が矩形であるような描画が存在するかどうかを判定し，存在するときには矩形描画を求めるアルゴリズムを与えている．このアルゴリズムの計算時間はグラフの点数に関し線形であり，きわめて高速である．

第4章では，平面グラフが与えられたときに，その直交描画，即ち各辺を水平線分と垂直線分からなる折れ線で描画したものを求める線形時間アルゴリズムを与えている．なお，求まる直交描画では辺の折れ曲がりの総数が最小であり，このアルゴリズムは実用上もきわめて重要である．

第5章では，点を箱形に描画してもよい矩形描画，即ち箱矩形描画を新しく定式化するとともに，平面グラフの箱矩形描画を求める線形時間アルゴリズムを与えている．この成果は高く評価できる．

第6章では，4連結平面グラフの4正規分解を新たに定義するとともに，それを見つける線形時間アルゴリズムを与えている．さらに，4正規分解を用いて平面グラフの4分割問題を解く線形時間アルゴリズムを求めている．4正規分解は，凸描画問題等の様々な問題に応用することができ，本章の成果はきわめて重要である．

第7章は結論である．

以上要するに本論文は，平面グラフの矩形描画および直交描画を求めるアルゴリズムを与え，箱矩形描画および4正規分解を新たに定義し，それらを求めるアルゴリズムや4分割問題を解くアルゴリズムを与え，それらのアルゴリズムの計算時間を理論的に解析したものであり，計算機科学，特にアルゴリズム理論の発展に寄与するところが少なくない．

よって，本論文は博士(情報科学)の学位論文として合格と認める．