One and Many
in
Plato's *Laws*

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PREFACE

Near the end of his presentation of the secondary grade curriculum of mathematics, the Athenian stranger refers to a problem dealing with the essential nature of interrelationship of the commensurable and the incommensurable (τὰ τῶν μετρητῶν τε καὶ ἀμέτρων πρὸς ἄλληλα 820c). The Athenian's reference appears immediately after his comments on the necessary studies (ἀναγκαῖα μαθήματα)[1] for anyone who is freeborn and intends to understand any single one of the fairest sciences (818d)[2].

What matter, then, did Plato exactly have in his mind, by referring to the essential nature of interrelationship at issue?

G. R. Morrow said that the introduction of the problem of incommensurable magnitudes to the curriculum was 'an innovation to which Plato attaches peculiar importance.'[3] He said also that whereas the belief that all magnitudes are somehow commensurable with one another is natural to us, by understanding that this belief is in fact false, one would rise above his human nature.[4]

Whereas Morrow seems to hit the mark principally, he did not fully explain, in strict conformity with the contexts of Platonic dialogues, why Plato's introduction to the learning of incommensurable magnitudes was "an innovation" with a "peculiar importance" and how the subject-

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matter of the essential nature of interrelationship of the commensurable and the incommensurable could 'emancipate the student from his instinctive sense-bound beliefs and thus, ……raise him above his human nature.'[5]

On the other hand, however, it is worth listening to his other words, which were uttered by him without referring to the incommensurable, nonetheless, which may be regarded as a penetration to the significance of mathematical thinking in the context of legislation of laws. Morrow said that 'the law, as we learn in the *Philebus*, results from the application of the Limit (πέρας) to an indefinitely varying qualitative continuum (ἄπειρον), and the Limit is conceived of as analogous to mathematical order.'[6]

The saying, however, it seems to me, fits more sufficiently with the context where the topics concerning the mathematical incommensurable make its appearance.[7]
In what follows I will take up the Athenian stranger's apparently negative evaluation of the subject of the incommensurable at 818e and investigate into its relevance to Plato's metaphysical principles, especially to his life long concern for the "turning mind from the Many to the One."

I

I insist that Plato, emphasizing the peculiar importance of the study of the essential nature of the interrelationship between the incommensurable and the commensurable, intends to give an inkling of the case that a person who is capable to discern in full the nature of the interrelationship between them is a "godlike"[8] and "a few"[9] person of a dialectic turn of mind.[10]

The person is naturally, in the contexts of the *Laws*, to be a prospective guardian and hence also to be a prospective member of the Nocturnal Council who, according to the Athenian stranger, 'must not only be able to pay regard to the many, but also to press *towards* the One so as to discern it properly and, on discerning it, to survey and organize all the rest *with a single eye to it*.' (965b)

In other words Plato at the above mentioned passage is making use of the method, by which the nature of the incommensurable magnitudes

can be elucidated, as a significant model in order to 'hold very tight and not to let go' (965d) until we can adequately explain the essential nature of the object existing as a unity.

Plato's reference to the problem of the incommensurable in the *Laws* is at first sight negative or at the best ambiguous. After his labor to explain the necessity of a subject concerning the incommensurable, the Athenian stranger says that, it is to be laid down *provisionally* only (820e).

However, the utterance is paradoxical in the same way as that his introduction of the Nocturnal Council in the closing books of the Laws appears paradoxically to be an appendix, whereas in reality the whole constitution of the state Magnesia was to be fundamentally rooted in the Council. The both facts correspond to one another and in effect put Plato's significant remark on the metaphysical facet of the incommensurability out of our sight.

In focusing the implications of metaphysical dimensions of the incommensurable in the *Laws*, I will try to bring them into the relevance to Plato's thought in his earlier dialogues, where Plato's deepest ideas on the incommensurable or the cognate ideas make their appearance vividly.

II

My thesis, however, may be taken as a problematic one, because of its apparently flimsy ground. The interconnection between (A) the incommensurable-commensurable relation and (B) the One-Many relation may appear to be a deceptive one. However, it is not the case.

At the outset, let us turn our eyes to a *tertium comparationis* between these relations and identify this as (C) the ἄρρητος (or ἄλογος)-ῥητός (or λόγος) relation.[11] Then, the following formula will be obtained: (A): (C)=(C):(B). So that (A) runs parallel to (B).[12] Thus, in virtue of this *tertium comparationis* (C), (A) and (B) could be tightly interconnected one another.

However, can we ascertain the case in *the Laws'* context itself? I believe we can, only if we pay a due attention to the following facts. The Athenian

stranger at 818e expresses his own uneasiness about the legislation of a course of the incommensurable and speaks as follows:

(a) 'But to give legal enactment to this program of ours is difficult (χαλεπὸν).

It is a notable fact that this utterance exactly corresponds to Athenian's another utterance at 968d:

(b) 'When and for how long they ought to receive instruction in each subject, it were idle (μάταιον) to lay down written regulation.'

The expression (a) is uttered by the Athenian stranger immediately before the introduction to the study of the incommensurable in contradistinction to the expression (b) which is uttered immediately after a summary of ἀκριβεστέραν παιδείαν for prospective members of the Nocturnal Council.

Concerning the expression (b) scholars should immediately remind its close resemblance to the *Seventh Letter* 341c; where the writer explains the inadequacy of word-expression about the cardinal tenet of philosophy. The writer says that 'it is not something that can be put into words like other branches of learning.'[13] Without any doubt, the subject matter here is nothing other than the contemplation of the One.

Contrasting to this, Plato's way of reference to the subject of the incommensurable may be taken as his negative or at the most ambivalent evaluation of it. As far as we are stickers for the meanings of ἄρρητος or ἄλογος and their immediate cognates[14], this must be somehow an indelible impression. We must pursue another path of contextual interpretation.

After his exhortation for studying the incommensurable magnitudes, the Athenian stranger at 820e says that the subject at issue is to be laid down as a necessary subject of instruction so as to be there no gap (διάκενα) in the code of laws, 'like pledges capable of redemption, apart from the rest of our constitution, in case they fail to satisfy either us

who enact them or you for whom they are enacted.'

The utterance may give us a negative impression, so that we might construe the Athenian's proposal as if Plato's sanction of neglecting the subject, even for the prospective member of the Nocturnal Council. However, it is not the case. On the contrary, we should pay attention to the Athenian's carefully worded speech, especially to his word "διάκενα." Plato's thoughtful remark suggests, in a paradoxical way, the absence of the most important requirement for the prospective member of the Nocturnal Council. The case runs parallel to the Athenian's stance concerning the legal enactment of the Council. At 960b-c, the Athenian stranger says as follows:

'Our task of legislation has nearly come to an end. But in every case, the full end does not consist in the doing, establishing or founding something: rather our view should be that it is only when we have discovered a means of salvation, endless and complete, for our creation, that we are at length justified in believing that we have done all that ought to be done: until then, we must believe, the whole of our creation is incomplete (ἀτελές).'

Thus, it is obvious that the both apparently negative words "διάκενα" and "ἀτελές" point to the most important conditions for the foundation of Magnesia.

Without the Nocturnal council, the state Magnesia could not be completed. And without its members who are necessitated to fill the "διάκενα" at issue and to master the ἀκριβέστερα παιδεία[15] for "a few" (τινας ὀλίγους 818a)[16], the Nocturnal Council could not attain its aim.

III

We are now in a position to look closely into the nature of isomorphic structure underlying the above mentioned two relations (A) and (B).[17] However, before setting about making inquiries of this, I have to point out another aspect of the two relations, which are connected with an aspect of the isomorphic structure in question.

In fact, they are so prominently isomorphic one another that Plato seems sometimes to have amused himself with his metaphorical manipulation of their relations.[18]

Near the end of the *Laws*, the Athenian stranger shows us such a metaphorical word-play, which one cannot get a taste of its subtleties unless s/he is familiar with this isomorphic structure of two relations.

Namely, at 968b-c, against Cleinias' request for "the right method" to enact a detailed program for the highest education of the member of Nocturnal Council, the Athenian stranger, as if he were the writer himself of the *Seventh Letter*, tells Cleinias that the subjects at issue need 'διδαχὴ μετὰ συνουσία πολλῆς,'[19] since 'it is neither easy to discover them for oneself nor is it easy to find another person who has made the discovery and learn from him.' In addition to this he says that 'although it would be wrong to term all these matters "indescribable [or forbidden] (ἀπόρρητα),"they should be termed somehow "imprescribable (ἀπρόρρητα)," seeinlg that 'the prescribing of them beforehand does nothing to elucidate the question under discussion.'[20]

Thus using a couplet ἀπόρρητα-ἀπρόρρητα Plato might have had in his mind a distinction between " ῥητόν " and "ἄρρητον" which are in this case mathematical terms corresponding to the "the expressible [diagonal]" and "the inexpressible [diagonal]" respectively. Plato seems to use the term ἀπρόρρητα, in contradistinction to ἀπόρρητα, as an approximate substitute for the ῥητόν [diagonal], which in turn, as *Republic* 546c4-5 tells us, signifies "expressible" or "rational" magnitude like that of the approximate value of the length of a diagonal.[21]

My assertion in the above has to do with the reading of "τοιοῦτον" at 968c: Οὐκέτι νόμους ······ περὶ τῶν τοιούτων δυνατόν ἐστι νομοθετεῖν, πρὶν ἂν κοσμηθῇ······ τότε δὲ κυρίους ὧν δεῖ γίγνεσθαι νομοθετεῖν.

What are, then, the referents of τοιοῦτον? With due regard to the Athenian's utterance at 968d-e[22] I take its referents as the conceivable arrangements of the highest subjects which belong to the range of " ἀκριβεστέρα σκέψις θέα " (965c).

However Plato here, as the case of *Republic* 531c shows, not only calls

into question whether legal stipulation of the subjects could be done, but also inquires of a person's capabilities whether s/he can get to fit well with the most difficult subject: dialectics.[23] Then, does Plato here suggest the necessity of some significant amendments to his former bold proposal at the *Republic* 534d?[24] For Socrates in the *Republic* at the locus was positively proposing to legislate the course of dialectics aiming to nurture the φύλακες of the state Kallipolis. Socrates there said that they 'shall give special heed to the discipline that will enable them to ask and answer questions in the most scientific manner.'[25]

Therefore, the problem at issue was not only the organization of the higher studies in the Council,[26] but also the different possible preparations, which are necessary to cope with persons' innate dissimilarities.[27] These both requirements, thus, run counter to one another, whereas anyone who is to be a member of the Nocturnal Council, without exception, had to 'turn towards the One (πρὸς τὸ ἕν) so as to discern it' (965b) and on discerning 'the identical element which pervades all the four virtues' (965c), to survey and organize all the rest with a single eye to it' (965b).

IV

Therefore, the heart of the problem at issue must be a minimum condition, without which the subsistence of the Council could not be expected : namely a term that each learner themselves should acquire within his soul some knowledge of the subject in question.[28] However, putting the matter more sharply, what is "the subject" in question? It is the Dialectical training, which alone makes one possible to contemplate the One. The Athenian stranger at 965b-d surveys the range of the One and the Many as follows:[29]

(1) The same nature which pervades the many things (διὰ πάντων ······ ταὐτόν 965d); the single unity in the many things (ἐν ······ ἓν ὄν 965d),

(2) The nature in virtue of which the many things may be called by a single name (ἑνὶ ὀνόματι προσαγορεύεσθαι 965d),

(3) The one whereto one ought to pay regard (965b; 965c; 965d) and

(4) The paradigm whereto one turns his eyes from the dissimilar many things to the single form and have an ability to discern all the rest in order to survey and organize them (γνόντα πρὸς ἐκεῖνο συντάξασθαι πάντα συνορῶντα 965b; μὴ μόνον ⋯⋯ πρὸς τὰ πολλὰ βλέπειν πρὸς δὲ τὸ ἕν 965b; πρὸς μίαν ἰδέαν ἐκ τῶν πολλῶν καὶ ἀνομοίων 965c).

It is not immediately obvious whether the nature at issue refers unambiguously to the Form or some other things like a genus, a specie etc. But, I believe, the Athenian's strong requirement for the investigation into its essence, whereto we ought to turn our eyes (εἰς ὃ βλεπτέον), whether it exists by nature as a unity, or as a whole, or as both, or in some other way (965d), primarily has to do with the above mentioned minimum condition: "to turn one's mind from the Many towards the One."

The general pattern of this minimum essential has many similar examples in other dialogues[30]; it covers only a part of the whole range of the One and the Many in question; but without doubt it is the most fundamental position for Plato's whole philosophy.

We should, here, recollect Parmenides' words to the young Socrates in the dialogue *Parmenides*, which may be regarded as a precise presentation of the minimum essential of Plato's philosophy. After his radical criticism to the theory of Forms, Parmenides says to young Socrates: 'nevertheless, if ⋯⋯ one will not allow that there are εἴδη of each single thing, and refuses to distinguish as something an εἶδος of each single thing, he will not even have anything to which to turn his mind.'(135b)[31]

Naturally, the above Parmenides' utterance should make us recollect also Socrates' words at the *Phaedrus* 265d: 'The first method is to take a synoptic view of many scattered particulars and collect them under a single generic term, so as to form a definition in each case and make clear the exact nature of the subject one proposes to expound.'[32]

Thus, the subject of the "turning one's mind from the Many towards the One" is identified with the dialectician's task in the context of "definition." In addition, this context is indeed nothing other than that range,

where we could identify the nature of the above-mentioned isomorphic structure.

V

Now, we should ascertain briefly the typical cases in Plato's dialogues, where the isomorphic structure at issue could be definitely specified. Now, even if we cannot survey the whole range of the passages where the problems concerning the incommensurable make its appearance, the primary candidates for our scrutiny must be, of course, some passages wherein the subject-matter of the incommensurability makes its appearance at the same time with that of definition. Hence, the typical cases, which are to be taken up primarily, must be the *Meno* 82b-85b and the *Theaetetus* 147d-148b respectively.[33]

Meno 82b-85e

Socrates, being urged by Meno to prove his hypothesis 'what we call learning is really the recollection (ἀνάμνησις),' intends to illustrate the case by showing how an uneducated slave-boy can solve a simple geometrical problem, without teaching except for a series of questions.

A crucial point at this passage lies in the following: whether we may suppose that the whole passage covers the whole process of recollection. According to Nehamas, it is not the case, since the slave-boy's coming to have false beliefs (82b9-83e3) should not be regarded as a case of recollection.[34]

Then, exactly when does the slave-boy begin to recollect truths? It is a hard nut to crack. But, I suppose it is immediately after when the boy at 84a replied to Socrates: 'Ἀλλὰ μὰ τὸν Δία, ὦ Σώκρατες, ἔγωγε οὐκ οἶδα.' That is, at that crucial moment when the boy realized his own ignorance. From this moment, however, 'he will understand, recovering the knowledge (ἐπιστήμην) out of himself' (85d), 'if he were repeatedly asked these same questions in a variety of forms.'(85c)[35] Then, he will have in his mind the square itself (τοῦ τετργώνου αὐτοῦ) and the diagonal itself (διαμέτρου αὐτῆς), which can be apprehended only by thought.[36]

Thus being interpreted, the passage is especially instructive in respect

of the following two points:

(1) The cases of slave-boy's false beliefs correspond to ta τὰ πολλά, which are not only *visible* (ὁρατόν as sides or area) and *expressible* (ῥητά) by integers such as 2, 3, 4, but they also exhibit their mutual relationship as that of '*longer than*' or '*less than*' (83c-d). So that they incidentally adumbrate also the appearance of the concept of " ἄπειρον" in the dialogue *Philebus*[37] and

(2) Socrates' pressing words to the boy after his second unsuccessful attempt that 'if you do not want *express* this *as a number* (ἀριθμεῖν), then just show (δεῖξον) me it (one)' (83e) not only correspond to the fact that the length of the sides of the requested square is inexpressible (ἄρρητον) by an integer,[38] but also shows implicitly another epistemological dimension[39] into which the boy will have to step finally, and where he will possibly have knowledge of all and every thing, which are akin one another.[40]

If the above mentioned is anyhow acceptable, I think, the following conclusion is unavoidable:

(1) The incommensurable-commensurable relation and the One-Many relations are so closely isomorphic one another that the former can be used as a capable model dealing with the latter.

(2) Nowhere, however, in the above-elucidated passage, Plato suggests the case that the boy attained an exact understanding of the square itself or the diagonal itself, which can be apprehended only by thought.[41] Hence, the recovering of knowledge itself (85d) is not attained[42] and remains to be a mere dreaming.

(3) Therefore, also, the incommensurables gap between the One and the Many remains to be an ἄρρητος, the inexpressible one. Nowhere Plato discloses explicitly a way by which one can securely shift from the Many to the One.

***Theaetetus* 147d-148b**

In this passage also, we observe substantially the same pattern with the above-mentioned *Meno* passage. Socrates begins by asking Theaetetus what is knowledge (146c). Theaetetus responds to this by *enumerating* different kinds of knowledge (τὰ πολλά). There upon Socrates demands a comprehensive definition of knowledge, illustrating the case of what clay is. Then Theaetetus, realizing swiftly the gist of Socrates' demand, tells his own recent achievement, which he obviously acknowledges as a typical model in order to deal with Socrates' requirement. The episode consists of two scenes:

(1) Theodorus was drawing some squares to demonstrate that the lengths of the sides of squares having an area of three or five feet are not commensurable with the length of the sides of a unit square. He discussed the case of each square individually until he reached the case of the square with area seventeen square feet.

(2) Then, Theaetetus and the young Socrates tried to group infinitely many squares (ἄπειροι τὸ πλῆθος αἱ δυνάμεις) under the one heading (ἕν) by which they could refer to all δυνάμεις. They divided all numbers into two groups: the square numbers which can be obtained as the product of two equilateral sides and the oblong numbers (προμήκη ἀριθμον) which is always enclosed by a *greater* and a *less-* er side (μείζων δὲ καὶ ἐλάττων ἀεί πλευρὰ περιλαμβάνειν). In addition, they called the former as μῆκος and the latter as δυνάμεις.[43]

It is a noteworthy fact that the above passage never discloses the nature of Theaetetus' proof, making an appeal to which he had have to be able to handle all the possible cases of linear incommensurability. Theaetetus does not even suggest his own proofs about it.[44] In addition to this, the passage 148b1-2,[45] which may be counted as a cardinal point of Theaetetus' achievements,[46] can be never regarded as a proof itself.

We cannot detect here even a bit of the proof-process. In order to prove the case definitely; Theaetetus had to appeal to the τετραγωνίζειν, the task of transforming a rectangle into a square of the same area. In addition, the latter should demand appealing to the theory of the mean

proportional, by the medium of which one could find out a mean proportional between two arbitrary line segments.[47] Nevertheless, here, there is no reference to such a proof-process. Theaetetus here is in his perfect silence on such a proof.

However, that is no wonder. The main aim of the passage 147d-148b is to display a paradigmatic instance that may be useful for defining what is knowledge[48]: a vivid demonstration designating the many forms of knowledge by one definition; after all, an example of the "turning one's mind from the Many towards the One."

The upshot is that Plato, leaving the historical Theaetetus' proof of the incommensurable itself (if any) aside, is casting his image into a specimen of the paradigmatic pattern of prospective dialectician, who remains, however, for the present to be still unripe for the task due to his youth.[49] Thus the incommensurable gap between the One and the Many, in the same way as the passage of the *Meno*, remains to be an ἄρρητος.[50]

VI

These are, I think, after all, the facts of the case in Plato's whole dialogues including the *Laws* itself. In spite of the fact that Plato always speaks zealously about the necessity of turning away from the visible world (the Many) to the invisible reality (the One),[51] he never tells explicitly when and how this entire soul's turning from the darkness to the light[52] takes place, except for the metaphorical expressions[53].

In the dialogue *Republic*, this peculiar situation is compared to "the journey's end" (τέλος τῆς πορείας),[54] which can be revealed"[55] only to "a master of dialectics" who demands an account of the essence of each thing."[56]

The phrase "the journey's end" should remind us Diotima's words about the unspeakable in the dialogue *Symposium*: Diotima there says that whosoever may have been thus far guided······'coming now to the end of erotics, will see instantaneously (ἐξαίφνης) something wonderful, beautiful in its nature.'[57] This passage of *Symposium*, as is well known, has its counterparts in the *Phaedrus* 249c6, 250a5; the *Parmenides* 156cff;

the *Seventh Letter* 341c-d.

Diotima's prophetic vision in this locus, according to S. Rosen, 'serves as the bond between discursive and intuitive reason.'[58] It is worthwhile to note that the bond in question exactly corresponds to that "suddenness" (ἐξαφθὲν) in the *Seventh Letter* 341d1. In addition, the locus itself in turn refers to the One, which is called in the Letter itself the fifth, being itself 'that which is the object of knowledge and truly exists' (342b).[59]

Pay attention to a fact carefully that the "suddenness" here functions as a mean proportional. That is, the word "ἐξαφθὲν" acts as a mean proportional between "πυρὸς πηδήσαντος" and "φῶς" by medium of which a philosophically significant communication between master and pupil may narrowly become "expressible" (ῥητά). Such being the case, it should be obvious also that this mean proportional runs parallel to the word "ἐκ ⋯⋯ τοῦ συζῆν" at 341c7 which is in turn another mean proportional between "τὸ πρᾶγμα αὐτὸ" and "ἐν τῇ ψυχῇ γενόμενον[60] and the fact reminds us immediately the Athenian's very similar words at the *Laws* 968c: "διδαχὴ μετὰ συνουσίας πολλῆς."

It is only through this proportional mean that the linear incommensurability between the One and the Many can *be* somehow *squared* with an expressible order, wherein what that remains to be an "ἀπόρρητα" could be transformed into an "ἀπρόρρητα."[61]

These were, I believe, the state of things that we should have ascertained in the *Laws*, exploring into the parallelism between two words "διάκενα" at 820e3 and "ἀτελές" at 960c1.

Thus, Plato's stance concerning the "turning one's mind from the Many towards the One" in the *Laws* fundamentally remains the same as that which we can identify in the other dialogues, among all in the *Republic*.

Therefore, we shall say, going along with the Late professor T. J. Saunders, that 'What is the relationship between the state Magnesia and the state Callipolis? There is no relationship. They are the same Platonic state.'[62] To this, however, I think, we should add the following words: 'It is surely the same Platonic state based on the same metaphysical

principles, which Plato developed in the previous dialogues and with which he could not part even for the moment: the One over the Many.

NOTES

1) They are three branches of secondary grade of learning: arithmetic, geometry, and astronomy (817e), whose detailed study is necessary for only 'a few' (818a).

2) It is a notable fact that the Athenian stranger recognizes that the legislation of the curriculum of "the rest" [fairest] sciences (τάλλα 818d) succeeding the necessary studies is so difficult that its precise enactment should be postponed on a later occasion (818).

3) Glenn R. Morrow, *Plato's Cretan City*, A Historical Interpretation of the *Laws*, Princeton University press, 1993, p.346.

4) *Ibid*

5) Cf. *op. cit.*, p.347.

6) *Op. cit.*, p.506. The words however, it seems to me, could be settled more aptly in the context of the incommensurable and the commensurable, since Plato's concern for the μέτριον in the dialogue *Philebus* has to do with the mathematical theory of proportion, which deals with the linear incommensurability.

7) However see Glenn R. Morrow, *op. cit.* Chapter X 'The Mixed Constitution,' where Morrow investigates Plato's concept of 'μέτριον' which deals with the theory of proportion, but without particular reference to the irrational. Cf. pp. 573ff.

8) *Laws*, 818c.

9) *Laws*, 818a.

10) See *Republic* VII, 535a-c.

11) Concerning the list of words ἄρρητος and ἄλογος in Plato's dialogues, see D. H. Fowler, *The Mathematics of Plato's Academy*, A New Reconstruction, Oxford University Press, 1987, 5.3 'Appendix: The words *alogos* and (*ar) rhe tos* in Plato, Aristotle, and the pre-Socratic philosophers'; The following list is Plato's part of his list except for the Bold letters which show the *Laws'* loci.

ἄλογον (m.): *Timaeus* 42d1. ἀλόγους: *Republic* VII534d5. ἄλογος(f.): *Definitions* 414c7, 416a23. ἄλογον: *Theaetetus* 201d1. *Timaeus* 47d4. ἀλόγου: *Timaeus* 28a3; *Laws* III 696e1. ἀλόγῳ: *Republic* IX591c6; *Timaeus* 69d4. ἄλογον (nt.) : *Gorgias* 465a6, 496b1, 519e3; *Phaedo* 62b2, c6, 68d12; *Symposium* 202a6; *Republic*

X609d9, 11; *Parmenides* 131d2, 144b3; *Theaetetus* 199a3, 203d6, 205c9, e3; *Timaeus* 51e4; *Sophist* 219e4, 238c10, e6, 239a5, 259a1; *Philebus* 55b1; *Axiochus* 365e5. ἀλόγου: *Philebus* 28d6; ἄλογα: *Protagoras* 321c1; *Theaetetus* 202b6, 203 a4, b6; *Sophist* 241a5, 249b1. ἀλογώτερον (nt.): *Charmides* 175c7; *Gorgias* 519d1; ἀλογώτατον (nt.): *Apology* 18c8; ἀλογώτατα: *Philebus* 55c3; ἀλόγως: *Gorgias* 501a6; *Republic* IV439d4; *Phaedrus* 238a1; *Timaeus* 43b2, e3, 53a8; *Laws* **II 669d4, IX 875b8**.

ἄρρητον(f.): *Laws* **VII 788a3**. ἀρρήτων: *Republic* VIII 546c5. ἄρρητον (nt.): *Sophist* 238c10, e6, 239a5; *Laws* **VI754a4**; *Alcibiades* 1122d2. ἄρρητα: *Hippias Major* 303c1; *Symposium* 189b4; *Sophist* 241a5; *Laws* **VII793b3, 822e2**. ἀρρήτων: *Hippias Major* 303b7. ῥηταί: *Theaetetus* 205d9. ῥητάς: *Theaetetus* 202b7. ῥητῶν: *Republic* VIII 546c5. ῥητόν: *Theaetetus* 205e7; *Epistle* VII 341c5. ῥητά: *Hippias Major* 303b8; *Republic* VIII546c1, *Laws* **VII817d3**; *Epistle* VII341d5. ῥητοῖς: *Symposium* 213a2; *Laws* **VIII 850a7**.

12) Concerning Plato's double use of the words ἄρρητος and ἄλογος, see K. von Fritz, *Grundprobleme der Geschichte der antiken Wissenschaften*, de Gruyter Verlag, Berlin/New York, 1971, p.53; p.571. He says: 'Es ist auch interessant zu beobachten, wie allmählich der Begriff ἄλογος wieder auftauchte. Zuerst wird der Begriff ῥητός (rational) im Gegensatz zu ἄρρητος geschaffen. Dann verschwindet der Begriff; und Theaetet, der die Theorie der Irrationalität weiter untersuchte, führte den Begriff ἄλογος wieder ein. '(p. 571) However see also J. Kokkinos' view: 'Platon für den Begriff des Irrationalen zwei gleichbedeutende Namen verwendet, nämlich und das der Pythagoreischen Mystik entstammende neue Wort ἄρρητος Daß er aber die Bezeichnung ἄρρητος für das Irrationale bevorzugt, ist offenkundig. Denn nirgends sonst in seinen Schriften gebraucht er ein zweites Mal den Ausdruck ἄλογος anstelle einer Bezeichnung der Irrationalität oder Inkommensurabilität. Hingegen gibt es zwei Stellen in seinen Schriften, Politeia 546c und Hippias I 303b, wo im gleichen Atemzug das Begriffspaar ⟨rational-irrational⟩ erwähnt und wo rational mit ῥητός und irrational mit dem Beinamen ἄρρητος bezeichnet sind.' (p.126) See J. Kokkinos, *Das mathematische Inkommensurable und Irrationale bei Platon*, Peter Lang, Europäische Verlag der Wissenschaften, 1997, pp. 125 -128. In any way for Plato the words ἄρρητος and ἄλογος were exchangeable concepts both of which signified the incommensurability or irrationality.

13) W. Hamilton's translation. See *Plato, Phaedrus and the Seventh and Eighth Letters*, Penguin Books, 1973, p.136.

14) Among eight cases of Plato's use of ἄρρητος or ἄλογος and their cognate

words except for ἀπόρρητα-ἀπρόρρητα in the *Laws* it seems none of them in straightway entails the mathematical meanings. See the above note 10.

15) Dialectics (963a-966b) and Theology (966c-969c).

16) Cf. *Republic* 531e: 'except a very few (μάλα γέ τινες ὀλίγοι).' By the way, in this occasion, we should pay attention to a fact that at *The Seventh Letter* 341e we can detect an equivalent expression corresponding to this situation; the writer says: 'I do not think that the attempt to put these matters into words would be to men's advantage, except to those few (τισιν ὀλίγοις) who can find out the truth for themselves with a little guidance.'(341e) W. Hamilton's translation. See *ibid.* Cf. also *Republic* 494a.

17) Its solution was Plato's life long concern.

18) Cf. *Politicus* 266a5-b7. In this locus, Eleatic stranger and his interlocutor (the young Socrates) try to distinguish between men and pigs. The Eleatic stranger says that distinction between men and pigs should be defined according to 'the diagonal and diagonal of (the square constructed on) the previous diagonal' (τῇ διαμέτρῳ δήπου καὶ πάλιν τῇ τῆς διαμέτρου διαμέτρῳ). This is a play upon words δύναμις("ability" or "square") and δίπους ("two legged" or "measuring two feet") etc.: cf. Árpád Szabó, *The Beginnings of Greek Mathematics*, D. Reidel Publishing Company, 1978, pp. 68-71. However, if this is a play upon words, the Athenian stranger's utterance about Greek's ignorance about the incommensurability at 819d that 'it seemed to me to be the condition of guzzling pigs rather than of human beings' too may be a play upon words which ought to be interpreted in relation to the above mentioned passage of the *Politicus*.

19) *Seventh letter* 341c-d: 'No treatise by me concerning it exists or ever will exist. It is not something that can be put into words (ῥητὸν γὰρ οὐδαμῶς) like other branches of learning; only after long partnership in a common life (ἐκ πολλῆς συνουσίας γιγνομένης) devoted to this very thing does truth flash upon the soul, like a flame kindled by a leaping spark, and once it is born there it nourishes itself thereafter.' (Translation by Walter Hamilton, *Plato, Phaedrus and The Seventh and Eighth Letters*, Penguin Classics, 1973.

20) See *Laws*, 968c-e. Cf. Walter Burkert, *Lore and Science in Ancient Pythagoreanism*, Harvard University Press, 1972, pp. 461-2. Burkert takes the ἀπρόρρητος as "forbidden" in contradistinction to the ἄρρητον which means "unsayable" i.e. "unspeakable because secret" in Pythagorean tradition.

21) Take for an example Pythagoras' theorem. Then, be it the case that *a* represents the side of the square and *d* represents its diagonal, then the

theorem tells that $d^2 = 2a^2$; So for $a = 5$, $d^2 = 50$; hence $d = \sqrt{50} \fallingdotseq 7$. Then, the number 7 is an approximate value of $\sqrt{50}$. Thus the number 7 can be described as διάμετρος ρητή, the "expressible (rational) diagonal." See Árpád Szabó, *The Beginning of Greek Mathematics*, D. Reidel Publishing Company, 1978, p. 88.

22) 'These it is neither easy to discover for oneself nor is it easy to find another who has made the discovery and learn from him. Moreover, with respect to the limits of time, when and for how long they ought to receive instruction in each subject, it were idle to lay down written regulations; for even the learners themselves could not be sure that they were learning at the opportune time until each of them had acquired within his soul some knowledge of the subject in question.'

23) Cf. *Republic* 532a.

24) In reality, it is not so. See my conclusion below.

25) 'But, surely,' said I, 'if you should ever nurture in fact your children whom you are now nurturing and educating in word, you would not suffer them, I presume, to hold rule in the state, and determine the greatest matters, being themselves as irrational (ἀλόγους) as the lines (γραμμάς) so called in geometry.' 'Why, no,' he said. 'Then you *will provide by law that they shall give special heed to the discipline that will enable them to ask and answer questions in the most scientific manner?*' '*I will so legislate,*' he said, '*in conjunction with you.*' 'Do you agree, then,' said I, 'that we have set dialectics above all other studies to be as it were the coping-stone-and that no other higher kind of study could rightly be placed above it, but that our discussion of studies is now complete?' 'I do,' he said. (Translation by P. Shorey except for the italics and the insertion of Greek)

26) Concerning the matter, Morrow, criticizing commentators' reading, said (see *Plato's Cretan City*, p. 513) that 'τῶν τοιούτων' refers, I am sure, to the organization of the higher studies of the Council, the matter on which the Athenian himself has just offered to give assistance and of which he says he has had much experience (968b). It obviously refers to the same thing as *ta toiauta*, two lines later, where it is said that to arrange such matters properly would involve διδαχή μετά συνουσίας πολλῆς, i.e. just that joint study and inquiry which the Council itself, after it has been established, is to pursue. In other words, the Council must already have been engaged in these studies before any rules can be laid down concerning the order in which they are to pursue them ⋯⋯ a paradoxical condition that is well reflected in the puzzling syntax of the passage.'
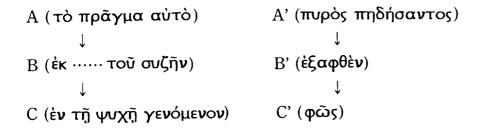
27) Cf. The *Laws* 968e.

28) Cf. 968e: πρὶν ἐντὸς τῆς ψυχῆς ἑκάστῳ ἐπιστήμην γεγονέναι. I follow Bury's reading 968e2 τοῦ μαθήματος.

29) Cf. Michitaro Tanaka, *Platon* II, *Tetsugaku* (1), Iwanami-Shoten, 1981, pp. 73-6.

30) Cf. as some typical cases, *Republic* 596b; *Phaedrus* 265d; *Cratylus* 389d; *Timaeus* 28a.

31) A slightly changed version of R. E. Allen's translation; see R. E. Allen, *Plato's Parmenides*, translation and Analysis, Basil Blackwell, 1983, p.13.

32) W. Hamilton's translation. Cf. *Op. cit.*, p. 81. Cf. also R. Hackforth, *Plato's Phaedrus*, Translated with Introduction and Commentary, Cambridge at the University Press, 1952, p.132, Note 4.

33) In this paper I cannot refer to the other loci where the incommensurable makes its appearance. However, in order to aim at perfection, the followings are at least referred to: *Meno* 86e-87b; *Republic* 534d4-d7; 546b4-c4; *Theaetetus* 148b2-3; *Timaeus* 31b6-32c5.

34) Cf. A. Nehamas, 'Meno's Paradox and Socrates as a Teacher' in Jane M. Day (ed.), *Plato's Meno in focus*, Routledge, 1994, pp. 237-8.

35) W. R. M. Lamb's translation.

36) Cf. *Republic* 510d-e: ʹζητοῦντες δὲ αὐτὰ ἐκεῖνα ἰδεῖν ἃ οὐκ ἂν ἄλλως ἴδοι τις ἢ τῇ διανοίᾳ.ʹ

37) Cf. *Philebus* 16c: 'all beings always said to exist come from the One and the Many and have inherently within themselves the Definiteness and the Indefiniteness' (ὡς ἐξ ἑνὸς μὲν καὶ πολλῶν ὄντων τῶν ἀεὶ λεγομένων εἶναι, πέρας δὲ καὶ ἀπειρίαν ἐν αὑτοῖς σύμφυτον ἐχόντων).' For the reading of this passage, I followed Cynthia Hampton, *Pleasure, Knowledge, and Being*, An Analysis of Plato's *Philebus*, SUNY Series in Ancient Greek Philosophy, State University of New York Press, 1990. p. 23.

38) Árpád Szabó, *The Beginnings of Greek Mathematics*, D. Reidel Publishing Company, 1978, p. 93.

39) Cf. *Meno* 81c-d.

40) Cf. *Phaedo* 73cff.

41) Cf. *Republic* 510c.

42) Cf. K. M. Sayre, *Plato's Analytic Method*, University of Chicago Press, 1969, .58-9, Note 1.

43) The situation strongly suggests that Plato's stand-point at this passage comes close to that of " πέρας-ἄπειρον " in the dialogue *Philebus*.

44) Cf. Árpád Szabó, *Op. cit.*, p. 62.

45) Following Árpád Szabó's suggestion, *Op. cit.*, pp. 42-3 I read the passage as follows: ['Those which yield an oblong number *when squared* we denoted as *dunameis,*] for although the latter are not commensurable with the former in length, nonetheless the areas (ἐπιπέδοις) which they enclose *when* squared (ἃ δύνανται), are commensurable.'

46) If it is historical Theaetetus' achievement. However, it is dubious. Cf. Árpád Szabó, *Op. cit.*

47) Cf. Euclid's proposition VI.13.

48) Cf. *Theaetetus* 148d: Ἴθι δή ⋯⋯ πειρῶ μιμούμενος τὴν περὶ τῶν δυνάμεων ἀπόκρισιν, ὥσπερ ταύτας πολλὰς οὔσας ἑνὶ εἴδει περιέλαβες, οὕτω καὶ τὰς πολλὰς ἐπιστήμας ἑνὶ λόγῳ προσειπεῖν.

49) Cf. *Theaetetus* 142c: 'αὐτῷ μειρικίῳ ὄντι' and Socrates' evaluation words of Theaetetus that 'πᾶσα ἀνάγκη εἴη τοῦτον ἐλλόγιμον γενέσθαι, εἴπερ εἰς ἡλικίαν ἔλθοι.' Socrates' reference to Theaetetus' youth naturally remind us Parmenides' words to the young Socrates in the *Parmenides* 130e: 'Νέος γὰρ εἶ ἔτι, ⋯⋯ ὦ Σώκρατες, καὶ οὔπω σου ἀντείληπται φιλοσοφία ὡς ἔτι ἀντιλήψεται κατ' ἐμὴν δόξαν, ὅτε οὐδὲν αὐτῶν ἀτιμάσεις. νῦν δὲ ἔτι πρὸς ἀνθρώπων ἀποβλέπεις δόξας διὰ τὴν ἡλικίαν.' I take this Parmenides' sayings corresponds to his later words at 135b-c: εἴ γέ τις δή, ὦ Σώκρατες, αὖ μὴ ἐάσει εἴδη τῶν ὄντων εἶναι κτλ.' Cf. also *Theaetetus* 143d; 146b.

50) The dialogue *Theaetetus* ends in failure to define what is knowledge, not because of Plato's real unsuccessful approach to the problem, but because of the absence of the minimal essential for such an investigation; that is, because of losing sight of the One, whereto the young Theaetetus had to pay regard. This interpretation is resonant with Goshen Adalier's recent paper's gist. See Goshen Adalier's view against Burnyeat, M. F., (*The Theaetetus of Plato*, Indianapolis, 1990.) "The Case of Theaetetus," *Phronesis*, Vol. XLVI No. 1, February, 2001.

51) Cf. *Republic* 529b: 'I cannot think of any study as making the mind look upwards, except one which has to do with invisible reality.'

52) Cf. *Republic* 518c: 'we must conclude that education is not what it is said to be by some, who profess to put knowledge into a soul which does not possess it, as if they could put sight into blind eyes. On the contrary, our own account signifies that the soul of every man does possess the power of learning the truth and the organ to see it with; and that, *just as one might have to turn the whole body round in order that the eye should see light instead*

of darkness, so the entire soul must be turned away from this changing world, until its eye can bear to contemplate reality and that supreme splendor which we have called the Good.'(Cornford's translation except for the italics)

53) Cf. L. Wittgenstein, *Philosophische Untersuchungen*, Teil II, X ff. Ludwig Wittgenstein, *Schriften I*, Frankfurt am Mein, 1969.

54) Cf. *Republic* 532e3; the journey here means dialectics and the end of the journey is the truth itself (αὐτὸ τὸ ἀληθές); see *Republic* 532b4, 533a3.

55) *Republic* 533a9.

56) *Republic* 534b3-4.

57) *Symposium* 210e2-6.

58) Stanley Rosen, *Plato's Symposium*, Yale University Press, 1968, p.269.

59) Cf. Walter Burkert, *Lore and Science in Ancient Pythagoreanism*, Harvard University Press, 1972, p. 20: 'Mind (νοῦς) comes closest to this (342d) but even mind does not grasp it completely and unambiguously. ……Because of this inadequacy-even on the part of mind and knowledge-it is easy to contradict and refute where the "fifth" is concerned: 'When we are under the necessity of separating out and revealing the fifth element, anyone who likes to do so has the means of confuting us' (343d). The ring of disappointment at the "contempt" (3410, 345b) to which the most sublime is exposed may echo the contempt ……shown by hearers of the lecture *On the Good*'
However, these circumstances do not always imply that this unchangeable truth could in no way be speakable. The truth, according to the writer of the *Seventh Letter*, once suddenly grasped, occupies the smallest part of soul and never escapes one's memory (344e). The expression remind us Bergson's words about the notion of philosophical intuition which C. H. Kahn aptly quotes to explain the matter: 'there is something simple, infinitely simple, so extraordinarily simple that the philosopher has never succeeded in saying it. And that is why he went on talking all his life …… Thus all the complexity of his doctrine, which could go on ad infinitum, is only the incommensurability between his simple intuition and the means at his disposal for expressing it.'(Bergson, H. H., *La Pensee et le Mouvant*, Paris, 1960, p. 119). Cf. C. H. Kahn, "*Phaedrus* and the limits of writing" in his Plato and the Socratic Dialogue, The philosophical use of a literary form, Cambridge, 1996. pp. 391-2.

60) Pay attention carefully to the fact that two expressions "ἐκ …… τοῦ συζῆν" and "" function respectively as "a proportional means."

A (τὸ πρᾶγμα αὐτὸ)   A' (πυρὸς πηδήσαντος)

↓   ↓

B (ἐκ ⋯⋯ τοῦ συζῆν)   B' (ἐξαφθὲν)

↓   ↓

C (ἐν τῇ ψυχῇ γενόμενον)   C' (φῶς)

In the above schemata B and B' act as "proportional means" respectively, so that A:B = B:C and A':B' = B':C'. Namely, they connect respectively a pair of two extremes, which are ontologically different and incommensurable.

The ancient Greek way of thinking was typically proportional. By means of a third term, or *tertium comparationis*, the Greeks brought two different and seemingly unrelated terms into a new and significant relation. Such a proportional way of thinking penetrated a whole range of Greek culture, and Plato's thinking is no exception to this. The case of the *Seventh Letter* 341c-d is merely one example. Cf. Hideya Yamakawa, *Greek Philosophy and The Modern World*, International Center for Greek Philosophy and Culture, Studies in Greek Philosophy Series, No. 26, pp. 34-5.

61) Cf. *Laws* 968e.

62) Trevor J. Saunders, 'Plato's Later Political Thought', in Richard Kraut (ed.) *The Cambridge Companion to Plato*, Cambridge University.

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One and Many

in

Plato's *Laws*

Hideya YAMAKAWA

In his presentation of the secondary grade curriculum for the young, the Athenian stranger refers to the matter, which it is disgraceful not to know: the problems concerning "the essential nature of the commensurable and the incommensurable."(820c) The reference appears immediately after the Athenian stranger's introduction to three branches of learning: arithmetic, geometry, and astrology (817e), whose detailed study is necessary for only 'a few' (818a).

Referring to the instruction of the essential nature of the commensurable and the incommensurable, what matter did Plato exactly have in his mind? G. R. Morrow said that the introduction of the problem of incommensurable magnitudes to the curriculum was "an innovation to which Plato attaches peculiar importance" and also that although the belief that "all magnitudes are somehow commensurable with one another is natural to us," seeing that this belief is in fact false would be to rise above one's human nature." Although the saying seems to hit the mark, Morrow did not fully explain why Plato's introduction to the learning of incommensurable magnitudes was "an innovation" which had a "peculiar importance" and how the subject-matter of the incommensurable could "emancipate the student from his instinctive sense-bound beliefs and thus, ······raise him above his human nature." (pp. 346-7)

However, it is worth listening to his other words which bear on the significance of mathematical thinking in the context of legislation: "the law, as we learn in the *Philebus*, results from the application of the Limit (πέρας) to an indefinitely varying qualitative continuum (ἄπειρον), and the Limit is conceived of as analogous to mathematical order." The

words could be settled more aptly in the context of the incommensurable.

I insist that Plato here in the dialogue *Laws*, referring to the peculiar importance of the incommensurable, gives an inkling of the fact that the scientific knowledge of the incommensurable has an isomorphic property with that faculty that discerns "the identical element which pervades all the four virtues"(965c). That is to say, Plato identifies the method by which the nature of the incommensurable magnitudes can be explained as a significant model in order to "hold very tight and not to let go"(965 d) until we can adequately explain the essential nature of the object existing as a unity. In other words, the detailed study of the incommensurable is of great advantage to 'a few' (818a) who are expected to be members of the Nocturnal Council and who "must not only be able to pay regard to the many, but must be able also to press towards the one (*pros to hen*) so as to discern it and, on discerning it, to survey and organize all the rest with a single eye to it"(965b), since the fruit of this offers them a convenient paradigm in medium of which they may master an isomorphic way in order to see the *invisible* one.

Plato's reference to the problem of the incommensurable in the *Laws* is at first sight negative. After his labor to explain the necessity of a subject concerning the incommensurable, the Athenian stranger says that it is to be laid down provisionally only, 'like pledges capable of redemption, apart from the rest of our constitution, in case they fail to satisfy either us who enact them or you for whom they are enacted.'(820e). However, the utterance is paradoxical in the same way as the introduction of the Nocturnal Council in the closing books of the *Laws* appears paradoxically to be an appendix, whereas in reality the whole constitution of Magnesia is rooted in the Council. Both facts correspond to one another and put Plato's metaphysical thinking out of our sight.

Focusing the implications of philosophical dimensions of the incommensurable in the *Laws*, I will try to bring them into the relevance to Plato's metaphysical thought in the other dialogues and the *Seventh Letter* VII, where the deepest tenet on the incommensurable or the cognate ideas make their appearance vividly.