Buckling Analysis of Laminated Composite Plates Containing Various Types of Delaminations(種々の層間はく離を有する複合材料積層板の座屈解解析に関する研究)

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号	2429
発行年	1999
URL	http://hdl.handle.net/10097/7702

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学位授与年月日	平成12年3月23日
学位授与の根拠法規	学位規則第4条第1項
研究科，専攻の名称	東北大学大学院工学研究科(博士課程)航空宇宙工学専攻
学　位　論　文　題　目	Buckling Analysis of Laminated Composite Plates Containing Various Types of Delaminations (種々の層間はく離を有する複合材料積層板の座屈解析に関する研究)
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論文内容要旨

1. Introduction

Laminated composite plates have attracted the attention of many users and designers in many fields. While these materials show outstanding advantages, they require new and sophisticated methods of design, fabrication, test and analysis and they have unique modes of failure. Interlaminar crack or *delamination* is one of the most serious failure modes of the laminated composite plates causing substantial reduction in the strength and stiffness of the plate. Particularly, when the plate is subjected to in-plane loads it can reduce the critical buckling load considerably. Delaminations may appear because of several reasons. They may arise in the form of *single embedded delamination*, because of imperfections in the production process; as *multiple delaminations*, because of impact of foreign objects on the plate; or as *edge delamination*, because of the interlaminar stresses.

We are interested to know the effect of delaminations on the buckling load of delaminated composite plates. For this purpose a buckling analysis is required. Performing an unconstrained buckling analysis leads to physically inadmissible buckling modes in some cases because of overlap between sublaminates in the delaminated region. Many researchers tried to solve this problem, however there is not any perfect method for the contact problem. In this study, the buckling analysis of laminated composite plates containing single embedded delamination, multiple embedded delaminations and edge delamination is performed. To eliminate overlap, a stable and reliable contact analysis is proposed using the penalty function method.

2.Buckling and Contact Analysis

2.1. Buckling analysis

To do a buckling analysis we can perform a non-linear analysis by applying a gradually increasing load through iterations, or we can solve a linear eigenproblem. Following the history of the load-displacement in the former method provides us with much information including the buckling load and mode. In the latter case, we obtain just the buckling load and mode in one step. Usually the main concern of the designers is the buckling load. In this case, the eigenproblem solution is the proper method because of its simplicity and speed. The eigenproblem for the buckling analysis is

$$\mathbf{K}_0 \delta = \lambda \mathbf{K}_G \delta \tag{1}$$

where $\mathbf{K}_0$ and $\mathbf{K}_G$ are the elastic and geometric stiffness matrices of the structure, and δ is the generalized global displacement vector. The smallest eigenvalue is the buckling load and its corresponding eigenvector represents the buckling mode. It is known that the transverse shear effects in laminated composite plates are substantial and we need to consider them in analysis. Here, to obtain the stiffness matrix, we use the Mindlin plate theory, which assumes a linear transverse shear deformation through the thickness of the plate.

2.2. Contact analysis

A delaminated plate can buckle in local, global or mixed buckling modes as shown in Figs. 1(a)-1(c). In unconstrained buckling analysis we may obtain physically inadmissible buckling modes as shown in Fig. 1(d). To eliminate the overlap between the sublaminates some constraints should be added in a contact analysis. Here we use the penalty function method to enforce constraints on entire overlapped region. The physical meaning of the constraint comes out as a fictitious spring between the overlapped nodes restricting their relative displacement.

To use this method, we calculate the stiffness of each fictitious spring, then modify the stiffness matrix by adding the effect of all fictitious springs, and finally solve the eigenproblem again. These steps are repeated in iterations until we restrict the maximum overlap to a desired small limit. The modification of the stiffness matrix and solving eigenproblem are clear and we only explain the calculation of the stiffness of each fictitious spring. In this study, to calculate the stiffness of fictitious spring at each node we consider the elastic properties and overlap depth at that node. As an example, to obtain ${}^{ij}k^*$ shown in Fig. 2 we extract two equations related to the lateral displacement of points i and j from Eq. (1), and include the effect of the unknown fictitious spring between them. Then we find the value of fictitious spring, which restricts the overlap at points i and j. After some manipulations we can find

$$ {}^{ij}k^* = \frac{(k_{mm}+k_{mn})R_n - (k_{mn}+k_{nn})R_m + \frac{{}^{ij}\varphi}{1+r}(k_{mn}^2 - k_{mm}k_{nn})}{\frac{{}^{ij}\varphi}{1+r}(k_{mm}+2k_{mn}+k_{nn})} \tag{2} $$

where

$$ R_m = \lambda\sum_{l=1}^{N_{dof}} k_{Gml}\delta_l - \sum_{\substack{l=1 \\ (l\neq m, l\neq n)}}^{N_{dof}} k_{ml}\delta_l \, , R_n = \lambda\sum_{l=1}^{N_{dof}} k_{Gnl}\delta_l - \sum_{\substack{l=1 \\ (l\neq m, l\neq n)}}^{N_{dof}} k_{nl}\delta_l \tag{3} $$

Here, lateral displacements of points i and j are the m-th and n-th elements of the global displacement vector, k and k_G are the elements of elastic and geometric stiffness matrices, N_{dof} is the total degrees of freedom and r is an initially small value which is increased in each iteration.

The eigenvector corresponding to the smallest eigenvalue of Eq. (1) is the buckling mode. Any multiplier of this eigenvector is another valid solution. However, it should be noted that multiplying by a negative value would change the overlapped region. To choose one of the two possible options, we use the Rayleigh quotient to calculate the approximate buckling load of the next iteration and follow the case having lower buckling load.

3.Single Embedded Delamination

We are interested to know the buckling response of laminated composite plates with single embedded delaminations as they may actually appear because of imperfections during production and moreover they can be considered as simplified models of impact induced delaminations. Here, laminated composite plates with single embedded delaminations of elliptical or circular shapes are considered. The buckling and contact analyses developed are used to investigate the effect of many parameters on the buckling load and mode of the plate when it is bearing in-plane compressive or shear loads. In addition, the details of overlap elimination process through iterations are shown.

The results of a case with a circular delamination show that the unconstrained analysis leads to

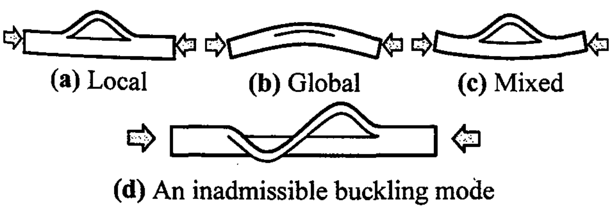

Fig. 1 Buckling Modes in a delaminated plate

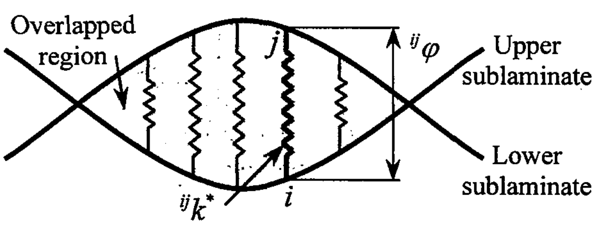

Fig. 2 Inserting fictitious springs to eliminate overlap

inadmissible buckling mode. As we add constraints through iterations, the maximum overlap depth decreases and buckling load increases. The buckling mode can change during iterations. When the buckling mode changes, there is a considerable increase in the buckling load because of the effect of constraints.

Figure 3 shows the effect of the size of delamination on the buckling load and mode in symmetric cross-ply laminated plates of 8, 12 and 20 plies when the delamination is placed between the first and second plies. Three lamination sequences have the same thickness. For small delamination sizes we obtain global buckling modes with high buckling loads. As delamination size increases, mixed and then local buckling modes appear and buckling load decreases considerably. This figure also shows that the buckling load decreases for thinner delaminated layers.

Other results in this chapter show the effect of the delamination shape, fiber angle in the delaminated layer and delamination position through the thickness on the buckling load and buckling mode of delaminated composite plates. The contact area is investigated for different cases. In addition, the buckling load is calculated for different delamination sizes and delamination positions through the thickness when the plate is under in-plane shear load.

4.Multiple Delaminations

Low velocity impact of foreign objects on laminated composite plates may result in multiple embedded delaminations. These delaminations reduce the compressive strength and buckling load of the plate considerably. To study this problem we consider multiple embedded delaminations of elliptical or circular shapes positioned at equal distances through the thickness of the plate. The delaminations are of equal size or of linearly variable size through the thickness.

In this study we simulate each sublaminate as a separate Mindlin plate with a distinct mid-plane. For this purpose a mesh as shown in Fig. 4(a) with several overlaid layers of mesh with the same pattern in the delaminated region is considered. The continuity conditions at the intersection of the sublaminates with the base plate are as shown in Fig. 4(b) with the formulation

$$u_j = u_i - z_j\theta_{xi}, \quad v_j = v_i - z_j\theta_{yi}, \quad w_j = w_i, \quad \theta_{xj} = \theta_{xi}, \quad \theta_{yj} = \theta_{yi} \tag{4}$$

where u, v, w, θ_x, θ_y show translations in the x, y, z directions and rotations in the xz and yz planes, respectively. These relations resemble the displacement conditions in the Mindlin plate theory and ensure continuity at the intersection. To enforce these conditions we consider them as constraints and use the penalty function method.

The numerical results show the effect of the number, size and shape of delaminations on the buckling load. The buckling load of multiply delaminated plate is compared to the buckling load of a plate with a hole

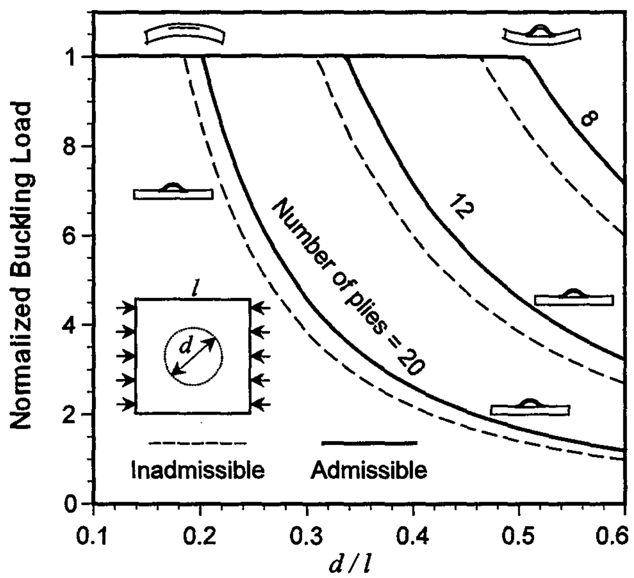

Fig. 3 Effect of delamination size on the buckling load

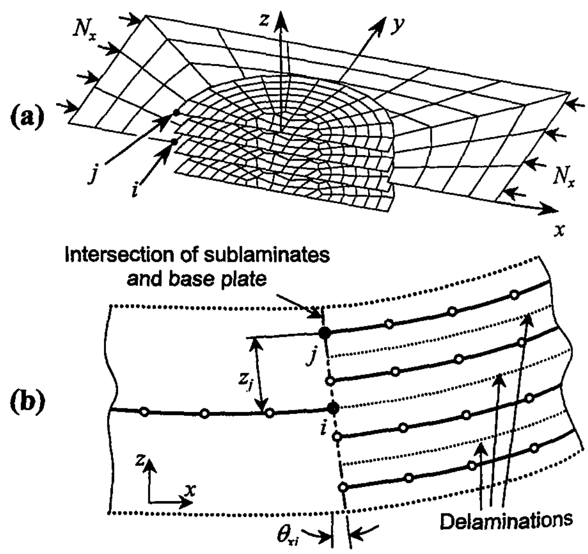

Fig. 4 Finite element mesh and continuity conditions

linearly variable size through the thickness is evaluated. It is very difficult to make a mesh directly for this calculation, however we can make a usable mesh easily by restricting the displacement of some points in the mesh shown in Fig. 4(a). Here we use the relations of Eq. (4) as constraints and employ the penalty function method to enforce them at the restricted points. The results of this case are compared to the results of a plate with a single delamination having the size of the maximum delamination in the multiple delamination case. It is shown that the results of these two cases are almost the same.

5.Edge Delamination

When a laminated composite plate is placed under load the interlaminar stresses near the edge of the plate show rapid changes and these stresses can cause edge delaminations. The buckling load of a plate with edge delamination decreases and we like to know the extent of this decrease. Here a laminated composite plate with a single edge delamination of constant width is considered and several cases are evaluated using the theories explained in the previous chapters.

Generally, a laminated composite plate may be unsymmetric about the mid-plane. Even in symmetric laminated plates, the upper and lower sublaminates may become unsymmetric because of delamination. In these cases, if we apply the load as usual at the mid-plane because of the bending-stretching coupling, displacement in the lateral direction appears from the beginning of the load application and bifurcation buckling does not occur. However, in the real problem, when we apply in-plane load on a plate such that the points on the edge undergo uniform displacements, the resultant of the load is applied at the neutral axis, which is not coincident with the mid-plane for unsymmetric plates. By applying the load at the neutral axis we can make a model more close to the real problem and obtain the buckling load. Instead of changing the position of the application of the load we can apply sufficient moments at the loading edges. Here we use the parallel axis theorem to calculate the lateral position for the application of the load.

The results are compared to the experimental and analytical results available in the literature and it is shown that in the contact analysis we need to add constraints on all overlapped area rather than a single point at the center. Numerical results show the effect of delamination width and depth on the buckling load. In addition, the effect of the boundary conditions on the buckling load is investigated.

6. Conclusion

Buckling analysis of laminated composite plates containing various types of delaminations was performed using finite element method. Mindlin plate theory was employed to account for the transverse shear effects. A stable and reliable contact analysis is introduced to eliminate the overlap between the sublaminates and obtain the admissible result. More accurate continuity conditions at the intersection of sublaminates with the base plate are introduced using penalty function method in multiple delamination case. For edge delaminations, a modified model is used to be able to consider unsymmetric sublaminates. Effect of delaminations on the buckling load and buckling mode of the laminate under in-plane compressive and shear loads was investigated.

It is shown that generally buckling load decreases as delamination size increases. In this process, the variation of the buckling mode can be generally divided into two stages. First, buckling mode varies from global to mixed mode with small reduction in the buckling load, and then buckling mode changes from mixed to local mode with considerable reduction in the buckling load. Unconstrained buckling analysis for delaminated plates leads to overlapped situations in some cases. Ignoring overlap in general results in lower buckling loads. It is necessary to add constraints in all overlapped area to eliminate the overlap. Enforcing constraints to prevent overlap increases buckling load. Particularly, when buckling mode changes after including constraints, this increase is considerable. Delaminated layers with fibers in 0° buckle more easily than those with fibers in 90°. As a general design rule, to increase the buckling load of a delaminated plate, the 0° plies should be situated far from the outer surface of the plate. The shape and orientation of delaminations have substantial effect on the buckling load. Delamination area alone may not provide so much information about the buckling load.

審査結果の要旨

航空宇宙機の構造部材の一つである複合材料積層板は、一般に層間のじん性が低いことから、異物の衝突や疲労等により層間はく離を生じ座屈荷重が低下する。そのため、層間はく離を有する複合材料積層板の座屈荷重の正確な把握は航空宇宙機の構造設計上重要な課題となっている。本論文は、種々の層間はく離を有する複合材料積層板に対して適用できる汎用性の高い有効な座屈解析手法を確立し、航空宇宙機の構造設計上有用な座屈荷重に関する研究成果をまとめたもので全文6章よりなる。

第1章は序論であり、本研究の背景及び目的を述べている。

第2章では、複合材料積層板をミンドリン平板でモデル化し、有限要素法を基礎として固有値問題の定式化を行い、層間はく離の上、下面の接触をペナルティー法を用いて考慮する座屈解析手法を提示している。これは優れた成果である。

第3章では、単一の円形あるいは楕円形の層間はく離を有する矩形複合材料積層板が面内圧縮あるいは面内せん断を受ける場合の座屈荷重について数値解析を行い、層間はく離の上、下面の接触を無視すると座屈荷重を過小に評価することがあることを明らかにしている。さらに、座屈荷重に対する層間はく離の形や大きさ、板厚方向の層間はく離位置、積層構成の影響を明らかにしている。これらは重要な知見である。

第4章では、異物の衝突により発生する層間はく離をモデル化した多重層間はく離を有する矩形複合材料積層板の圧縮座屈荷重を検討している。各層間はく離の大きさが異なる多重層間はく離の場合に、新たな拘束条件を導入し、それをペナルティー法で座屈解析に組入れるという着眼点のよい工夫を行っている。得られた座屈荷重に関する成果は航空宇宙機の構造設計上有用である。

第5章では、自由端での層間はく離を有する複合材料積層板を対象に、面内圧縮荷重下における座屈解析を行っている。数値解析により自由端での層間はく離の幅や板厚方向の位置および境界条件の座屈荷重に対する影響を明らかにしている。これらは重要な成果である。

第6章は結論である。

以上要するに本論文は、種々の層間はく離を有する複合材料積層板に対して適用できる汎用性の高い座屈解析手法を確立し、数値解析を行うことによって航空宇宙機の構造設計上有用な座屈荷重に関する成果を得たものであり、航空宇宙工学の発展に寄与するところが少なくない。

よって、本論文は博士（工学）の学位論文として合格と認める。