A Study on Neural Networks with Applications to Associative Memories(ニューラルネットワークによる連想記憶への応用に関する研究)

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論文内容要旨

1 Introduction

Artificial neural networks have been intensively investigated recently and have become a very popular field of research which is rapidly growing. The application domains of artificial neural networks can be roughly divided into the following categories: association/clustering/classification, pattern completion, regression/generalization and optimization. The present work focuses on the application of the neural networks, and in particular single-layered feedback type networks, to associative memories.

Artificial neural networks used as associative memories present three major advantages over the classical methods for recognition and association. First, the sequential search is replaced by a parallel process. Second, the information is widely distributed in the synaptic weights of the network. Third, an associative memory based on artificial neural networks presents some insensitivity to noise, i.e., error-correction ability. While the first characteristic depends only on the architecture of the network the other two can vary with the synthesis technique used.

The problem of synthesis of a neural network with application to associative memories consists in determination of the interconnection matrix and, if necessary, the thresholds, in order to memorize a given set of prototype vectors. Such a network exhibits information retrieval properties if when the network is initialized with a sufficiently large portion of memory's information (called a "key"), the network is able to recall a complete set of previous stored information. Moreover, given a distorted version of a prototype vector (e.g., due to noise or perturbations an error occurs in each component of the vector) as an initial state of the network, the latter will generally converge to the prototype vector, thus retrieving the complete correct information. In this case the network is performing the functions of noise suppression or error correction.

Various feedback neural network models have been proposed in the literature for the implementation of associative memories. They can in general be classified into discrete/continuous state discrete time networks and continuous state continuous time networks. In particular several design methods for such systems have been developed. However open issues still prevail regarding the design efficiency and implementation. This motivates further research on which this work is based. The representation of the work is organized in five chapters.

Chapter 1 presents some basic notions and features of associative memory in order to provide the required background. Different network architectures as well as different techniques used for learning in such networks are considered. Some insight into the relationship between the efficient information retrieval and the generalization, which is a widely used performance measure employed in the analysis of adaptive learning systems, is also provided. This serves as a basis for the discussion building up in Chapter 4.

Chapter 2 presents systematically some theoretical results pertaining to the dynamic properties of single-layered fully connected feedback type neural networks in a unified framework. The emphasis is on their applicability to associative memories and the analysis of their information retrieval abilities.

Chapter 3 provides a brief overview of the design methods for associative memories which have been proposed for different neural network models. An attempt is made to take a unified view, pointing also the advantages and limitations of these methods. This gives the background and the motivation for developing new design methods, established in Chapter 4 for a particular class of neural networks.

Chapter 4 establishes a synthesis method for a particular class of continuous state discrete time neural networks based on the brain-state-in-a-box (BSB) model. The analysis results concern the location of equilibrium points, qualitative properties of equilibria, the extent of the basin of attraction of asymptotically stable equilibria, and the global dynamical behavior of the network. The problem of storing the desired patterns as asymptotically stable equilibrium points is phrased in term of linear inequalities. Furthermore by adding an objective function and further constraints on interconnection matrix the design problem is formulated as an convex optimization problem. The applicability of the presented method is illustrated by several specific examples emphasizing its advantages over some existing methods.

The thesis ends with *Chapter 5* where conclusions are given and future research suggested.

2 A Synthesis Procedure for a Class of Discrete Time Neural Networks Described on Hypercubes

The neural network model considered belongs to the class of discrete time continuous state networks, with continuous state variables updated in parallel, described by the following difference equation

$$x(k+1) = g(x(k) + \alpha W x(k) + \alpha I) \qquad k = 0, 1, 2 \cdots \tag{S}$$

where x is a real n vector, W is a real $n \times n$ matrix, I is a real n-vector, α is a positive constant, and

$$g(x) = [g(x_1), \cdots, g(x_n)]^T$$

represents the activation function which is chosen as a saturation nonlinearity

$$\mathrm{sat}(\theta) = \begin{cases} 1, & \text{if } x_i > 1 \\ \theta, & \text{if } -1 \leq x_i \leq 1 \\ -1, & \text{if } x_i < -1 \end{cases}$$

The above model is a variant of the brain-state-in-a-box (BSB) model with the component αI added to the linear part. The parallel updating offers two major advantages. First, it allows to exploit the potential of parallel (i.e., synchronous) processing, and second, when network (S) is implemented on a serial processor, as in the case of digital simulations, it offers considerable advantages compared with the digital simulations of the differential equations representing analog networks.

The design problem of associative memory based on the above model is formulated as follows: given a set of binary vectors, $\mathcal{P} = \{p^k\}_{k=1}^m \in \{-1, 1\}^n$, find a matrix W and a vector I such that

(1) The binary vectors $p^1, \cdots, p^m$ are stored as asymptotically stable equilibrium points of system (S).

(2) The basin of attraction of each pattern p^k is as large as possible.

(3) The total number of spurious asymptotically stable equilibria is as small as possible.

(4) The system (S) is globally stable.

The rationale of the proposed synthesis method that addresses the above problems is as follows.

It is well known, [Theorem 2, Chapter 4], that the stability conditions (1) can be written as

$$\sum_{j=1}^{n} w_{ij} p_i^k p_j^k + p_i^k I_i > 0 \qquad i = 1, \cdots, n, \quad k = 1, \cdots, m. \tag{1}$$

As a result any pattern sufficiently close to an $p^k \in \mathcal{P}$ will be recognized as this p^k. The amount of allowed displacement for the correct recognition clearly depends on the extent of the basin of attraction. This is also closely related with the performance of the network in generalization. It is excepted, and in fact confirmed by simulations, that bounding the elements of the interconnection matrix W will improve the generalization properties of the network, hence its recalling abilities. This constraint imposes the conditions $-\bar{w} \leq w_{ij} \leq \bar{w}$, $i, j = 1, \cdots, n$, $\bar{w} > 0$ in addition to the linear inequalities (1). In fact, considering the equivalent forward network embedded in the dynamic recognition process, smoother connection weights should be able to capture more reliable features and this generalizes better.

The problem of reducing the number of spurious equilibria is addressed in the following way. First the condition that $w_{ii} \geq 0$ for $i = 1, \cdots, n$, [Theorem 1, Chapter 4], excludes the existence of stable equilibria inside the hypercube $[-1, 1]^n$, i.e., spurious stable equilibria if they exists, are located at the extreme points of $[-1, 1]^n$. This clearly will improve the attractivity of the desired stored patterns. Second, [Theorem 4, Chapter 4], maximization of the left hand sums of inequalities (1) subject to bounds on elements of matrix W and elements of bias vector I will increase the basin of instability of the given patterns. In particular, if $w_{ii} = 0\ \forall i$, [Corollary 2, Chapter 4], there are no other binary equilibria at unit Hamming distance from the stored patterns. This also will improve the attractivity of the desired stored patterns.

From the above considerations, a synthesis strategy that requires the maximization of the left hand sums of inequalities (1) subject to reasonably small bounds on $w_{ij}, I_i,\ i,j = 1,\cdots,n$ and $w_{ii} = 0\ \forall i$, will partially satisfy the design requirement of having the attractivity of the desired memory vectors as large as possible and a small number of spurious asymptotically stable binary vectors (i.e., asymptotically stable equilibrium binary vectors which are not in the given set $\mathcal{P}$). The synthesis problem is formulated as an optimization problem in W and I

$$
\begin{array}{ll}
\text{maximize} & \tau \\
\text{subject to} & \sum_{j=1}^{n} w_{ij} p_i^k p_j^k + p_i^k I_i \geq \tau, \quad \tau > 0, \quad k = 1,\cdots,m \\
 & -\bar{w} \leq w_{ij} \leq \bar{w} \qquad i \neq j\ i,j = 1,\cdots,n, \quad \bar{w} > 0 \\
 & w_{ii} = 0 \qquad i = 1,\cdots,n \\
 & -\bar{I} \leq I_i \leq \bar{I} \qquad i = 1,\cdots,n\ \ \bar{I} > 0 \\
 & tE + W \geq 0 \qquad 0 < t < 2/\alpha
\end{array}
$$

where E is $n \times n$ identity matrix, and the inequality sign $(tE + W) \geq 0$ means that $(tE + W)$ is positive semidefinite. In a discussion that would allow more details, this last condition is required to guarantee that the network (S) is globally stable, i.e., every solution trajectory of system (S) converges to some equilibrium point.

The synthesis problem as stated above is solved by using interior point methods which combine a very low polynomial worst case complexity with excellent behavior in practice. Without the nonlinear constraint $(tE + W) \geq 0$ the above problem reduces to a linear program (LP). Another advantage of the interior point methods is that their implementations for large scale linear programming, though still involved, are considerable simpler than those of simplex algorithms. Furthermore as the problem size increases (e.g., more than 10000 variables or constraints), so does the frequency of interior point methods outperforming the classic simplex methods.

The applicability of the proposed method is illustrated by several specific examples (Section 4.4) emphasizing its advantages over some existing methods.

3 Conclusions

This work makes contribution to the synthesis of a class of neural networks with piecewise linear saturation functions with application to associative memories. Some unique features of the proposed method are:

- The proposed method leads to significant improvements of the information retrieval capabilities of the considered neural network model compared with other existing methods.
- The design problem is formulated as a convex optimization problem which allows the optimization of

a design parameter that takes into account inaccuracies which arise in the implementation of the network by hardware. This problem is solved efficiently by interior-point methods. It is interesting to note that even some special neural algorithms can be classified into interior point methods and have been presented for problems were variables are constrained in finite space $\{0,1\}$. This suggests that the design of the network can be solved by another network similar to the combined learning/production architecture which is inherent in biological structures; the production phase (referring to present application we consider) corresponds to retrieval of information from the memory and the (learning) design phase corresponds to the stage in which the network (learns) designs its internal architecture.

• The proposed method is also applicable to some other neural network models including sparse interconnected neural networks.

審 査 結 果 の 要 旨

脳の神経回路網の数理モデルであるニューラルネットワークは，連想的に情報を処理するための記憶システム，いわゆる連想記憶のモデルとしても注目され，効率のよい連想記憶を達成するための設計手法の確立を目指す研究が活発に進められてきた。著者は，連想記憶としてのニューラルネットワークの諸性質について論じ，それらの結果をもとに，より効率的な連想記憶を実現する新たな設計手法を見出した。本論文はこれらの成果を取りまとめたもので全文５章よりなる。

第１章は序論であり，本研究の背景と基本となる諸概念について述べている。

第２章では，連想記憶への応用の視点から，リカレント型ニューラルネットワークの種々の数理モデルとそれぞれの安定解析手法とについて述べている。

第３章では，これまでに提案されている主な連想記憶モデルの設計手法の特徴と問題点を明らかにしている。効率のよい連想記憶の実現には，（１）各記憶パターンの安定平衡点への対応付け，（２）安定平衡点の引力圏の大きさ，および（３）記憶パターンに対応しない不要な安定平衡点の削減，を反映する適切な指標を見出す必要がある。本章では，これらの３点から見た従来の諸手法の欠点を指摘し，そのことが引力圏の大きさや不要な安定平衡点の数にどのように反映するかを分析している。

第４章では，離散時間，連続状態のニューラルネットワークに対し，設計のための諸指標を提案して，それらと上記の（１），（２）および（３）との関連を明らかにしている。その結果をもとに，連想記憶の設計問題を定式化して，それが凸計画問題となり，さらに一部の制約を緩和することで，近年注目されている半正定値計画問題に帰着できることを明らかにしている。また，本手法の有用性を種々のシミュレーションにより実証している。これらの成果は，かなりの規模の連想記憶を容易に設計することを可能としたもので，高く評価できる。

第５章は結論である。

以上要するに本論文は，リカレント型ニューラルネットワークの連想記憶能力について論じ，その結果をもとに，効率のよい連想記憶モデルの設計手法を提案し，その有用性を明らかにしたもので，情報処理工学及び制御工学の発展に寄与するところが少なくない。

よって，本論文は博士（工学）の学位論文として合格と認める。