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A study of dynamical syn-eco symbiotic system with bio-economic aspect at interior equilibrium

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Abstract

This paper proposes a multi species ecosystem with optimal harvesting of the prey (S_1), and Super predator (S_3).The predator (S_2) is surviving on the prey and the super predator is surviving on predator. The mathematical model is defined by three non-linear ordinary differential equations .The existence of bionomic equilibrium of the system has been discussed. The Local stability is discussed using Routh-Hurwitz's criteria and global stability by Lyapunov's function. The analytical stability criteria are supported by numerical simulations using Mat lab.

Keywords: Prey, predator, super predator, non-linear differential equations, equilibrium points, local and global stability, Bionomic equilibrium.

INTRODUCTION

Bionomics, which is the management of renewable resources, plays a significant role in the prey-predator models. The ecological literature has emphasized the role of harvesting in promoting the persistence of predator-prey systems. An excellent introduction to optimal management of renewable resources is given by Clark [4]. An optimal control problem for the combined harvesting of two competing species was given by Chaudhuri [1, 2]. The combined harvesting of a two species prey-predator fishery have been discussed by Chaudhuri and SahaRay [3] Mesterton-Gibbons [6], etc.. Pradhan and Chaudhuri [7] proposed a model for the study of selective harvesting in an inshore-offshore fishery. Rui Zhang et al [9] studied about stability of several species models by incorporating the harvesting term. The optimal harvesting policy for a prey – predator system where the prey has no commercial value and the predator is selectively harvested has been given by Ragozin and Brown [8]. Kar and Chaudhuri discussed the harvesting of a two-prey one- predator fishery [5]. In the present paper we have discussed a multi-species model with harvesting of the prey and the super predator and the predator species is not harvested. An analysis for the existence and stability of the equilibria of the system has been made. We have discussed the bionomic equilibrium along with optimal harvesting policy taking simple economic considerations into account. Finally some numerical illustrations are given.

The present investigation is an analytical study of three species system consisting of a prey, predator and super predator. The equilibrium points are identified based on the model equations and these are defined for: (i) fully washed-out state (ii) partially washed-out state and (iii) co-existent state (Interior equilibrium). The local and global stability of the states have been derived. We discuss the possibilities of the existence of a bionomic equilibrium and some numerical examples are given.

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THE MATHEMATICAL MODEL

The ecological model is as follows. We consider a prey, predator and super predator species. The predator is surviving on the prey and the super predator surviving on the predator. There is a harvesting of the prey and the super predator species. The three species multi-system is given by a set of three non-linear ordinary differential equations as:

(i) Equation for the growth rate of prey (S_1)

$$\frac{dN_1}{dt} = a_1N_1 - \alpha_{11}N_1^2 - \alpha_{12}N_1N_2 - q_1E_1N_1 \quad (2.1)$$

(ii) Equation for the growth rate of predator (S_2)

$$\frac{dN_2}{dt} = a_2N_2 - \alpha_{22}N_2^2 + \alpha_{21}N_2N_1 - \alpha_{23}N_2N_3 \quad (2.2)$$

(iii) Equation for the growth rate of Super predator (S_3)

$$\frac{dN_3}{dt} = a_3N_3 - \alpha_{33}N_3^2 + \alpha_{32}N_3N_2 - q_2E_2N_3 \quad (2.3)$$

using the following notation.

$N_i(t)$: Population density of the species S_i at time t, i=1, 2, 3.
a_i : The natural growth rates of S_i, i = 1,2,3
α_{ii} : The rate of decrease of S_i due to its own insufficient resources i = 1,2,3.
α_{12} : The rate of decrease of the prey (S_1) due to inhibition by the predator (S_2).
α_{21} : The rate of increase of the predator (S_2) due to its successful

attacks on the prey (S_1)

α_{23} : The rate of decrease of the predator (S_2) due to inhibition by the super predator (S_3).

α_{32} : The rate of increase of the super predator (S_3) due to its successful attacks on the predator (S_2), $K_i = a_i / \alpha_{ii}$: Carrying capacities of S_i, i = 1, 2, 3.

Further the variables N_1, N_2 and N_3 are non-negative and the model parameters a_i, K_i, α_{ij}, i=1, 2, 3, j=1, 2, 3, are assumed to be non-negative constants. q_1: Catchability coefficient of prey species (S_1),

q_2: catchability coefficient of Super predator (S_3), E_1: Effort applied to harvest the prey (S_1), E_2: Effort applied to harvest the Super predator (S_3), Throughout our analysis, we assume that $a_1 - q_1 E_1 > 0$, $a_3 - q_2 E_2 > 0$

EXISTENCE OF EQUILIBRIUMS

The system under investigation has equilibrium states given by $\frac{dN_i}{dt} = 0$, *i* =1, 2, 3.

The four possible equilibrium points are

(i) $E_1(0,0,0)$ (In the absence of all the species)

(ii) $E_2(\overline{N}_1, \overline{N}_2, 0)$ (In the absence of Super predator)

(iii) $E_3\left(0, N_2^{\phi}, N_3^{\phi}\right)$ (In the absence of predator)

(iv) $E_4(N_1^*, N_2^*, N_3^*)$ (The interior equilibrium)

The three possible equilibrium points are

Case (i): $E_1(0,0,0)$ i.e. the population is extinct and this state always exists.

Case (ii): $E_2(\overline{N}_1, \overline{N}_2, 0)$

$$\overline{N}_1 = \frac{\alpha_{22}(a_1 - q_1 E_1) - \alpha_{12} a_2}{\alpha_{11}\alpha_{22} + \alpha_{21}\alpha_{12}},$$

$$\overline{N}_2 = \frac{a_2 \alpha_{11} + \alpha_{21}(a_1 - q_1 E_1)}{\alpha_{11}\alpha_{22} + \alpha_{21}\alpha_{12}}$$

$\overline{N}_1$ is positive provided the following inequality hold : $\alpha_{22}(a_1 - q_1 E_1) > \alpha_{12} a_2$

Case (iii): $E_3\left(0, N_2^{\phi}, N_3^{\phi}\right)$

$$N_2^{\phi} = \frac{a_2 \alpha_{33} - \alpha_{23}(a_3 - q_2 E_2)}{\alpha_{22}\alpha_{33} + \alpha_{23}\alpha_{32}},$$

$$N_3^{\phi} = \frac{a_2 \alpha_{32} + \alpha_{22}(a_3 - q_2 E_2)}{\alpha_{22}\alpha_{33} + \alpha_{23}\alpha_{32}}$$

N_2^{ϕ} is positive provided the following inequality hold : $a_2 \alpha_{33} > \alpha_{23}(a_3 - q_2 E_2)$

Case (iv): $E_4(N_1^*, N_2^*, N_3^*)$

$$N_1^* = \frac{(a_1 - q_1 E_1)\{\alpha_{22}\alpha_{33} + \alpha_{32}\alpha_{23}\} + \alpha_{12}\alpha_{23}(a_3 - q_2 E_2) - a_2 \alpha_{12}\alpha_{33}}{\alpha_{11}\alpha_{22}\alpha_{33} + \alpha_{11}\alpha_{23}\alpha_{32} + \alpha_{12}\alpha_{21}\alpha_{23}},$$

$$N_2^* = \frac{a_2 \alpha_{11}\alpha_{33} + \alpha_{21}\alpha_{33}(a_1 - q_1 E_1) - \alpha_{11}\alpha_{23}(a_3 - q_2 E_2)}{\alpha_{11}\alpha_{22}\alpha_{33} + \alpha_{11}\alpha_{23}\alpha_{32} + \alpha_{12}\alpha_{21}\alpha_{23}},$$

$$N_3^* = \frac{(a_3 - q_2 E_2)\{\alpha_{22}\alpha_{33} + \alpha_{12}\alpha_{12}\} + a_2 \alpha_{11}\alpha_{32} + \alpha_{32}\alpha_{21}(a_1 - q_1 E_1)}{\alpha_{11}\alpha_{22}\alpha_{33} + \alpha_{13}\alpha_{31}\alpha_{22} + \alpha_{12}\alpha_{21}\alpha_{33}}$$

These points are all positive provided the following inequalities hold

$$(a_1 - q_1 E_1)\{\alpha_{22}\alpha_{33} + \alpha_{32}\alpha_{23}\} + \alpha_{12}\alpha_{23}(a_3 - q_2 E_2) > a_2 \alpha_{12}\alpha_{33}$$

and

$$\{a_2 \alpha_{11}\alpha_{33} + \alpha_{21}\alpha_{33}(a_1 - q_1 E_1)\} > \alpha_{11}\alpha_{23}(a_3 - q_2 E_2)$$

STABILITY ANALYSIS

LOCAL STABILITY

First we consider the local stability of the equilibriums.

The variational matrix of the system in the absence of Prey species at $E_3\left(0, N_2^{\phi}, N_3^{\phi}\right)$ is

$$A = \begin{bmatrix} a_1 - q_1 E_1 - \alpha_{12} N_2^{\phi} & 0 & 0 \\ \alpha_{21} N_2^{\phi} & -\alpha_{22} N_2^{\phi} & -\alpha_{23} N_2^{\phi} \\ 0 & -\alpha_{32} N_3^{\phi} & -\alpha_{33} N_3^{\phi} \end{bmatrix}$$

and the characteristic equation is

$$\left(\lambda - (a_1 - q_1 E_1 - \alpha_{12} N_2^{\phi})\right)\left\{\lambda^2 + \left(\alpha_{22} N_2^{\phi} + \alpha_{33} N_3^{\phi}\right)\lambda + \left(\alpha_{22}\alpha_{33} + \alpha_{23}\alpha_{32}\right) N_2^{\phi} N_3^{\phi}\right\} = 0 \quad (4.a1)$$

From the equation (4.1) we find that the two Eigen values which are the roots of the quadratic equation are real and negative or complex conjugates having negative real parts and the other Eigen value is $(a_1 - q_1 E_1) - \alpha_{12} N_2^{\phi}$. Hence the state is stable only if $(a_1 - q_1 E_1) < \alpha_{12} N_2^{\phi}$. Otherwise the system is unstable.

The variational matrix of the system(2.1)-(2.3) at (N_1^*, N_2^*, N_3^*) is

$$V = \begin{bmatrix} -\alpha_{11} N_1^* & -\alpha_{12} N_1^* & 0 \\ \alpha_{21} N_2^* & -\alpha_{22} N_2^* & -\alpha_{23} N_2^* \\ 0 & \alpha_{32} N_3^* & -\alpha_{33} N_3^* \end{bmatrix}$$

The characteristic equation of V is

$$\mu^3 + b_1 \mu^2 + b_2 \mu + b_3 = 0 \quad (4.a2)$$

where $b_1 = \alpha_{11} N_1^* + \alpha_{22} N_2^* + \alpha_{33} N_3^*$

$$b_2=(\alpha_{11}\alpha_{22}+\alpha_{12}\alpha_{21})N_1^*N_2^*+(\alpha_{22}\alpha_{33}+\alpha_{23}\alpha_{32})N_2^*N_3^*+\alpha_{11}\alpha_{33}N_1^*N_3^*$$
$$b_3=(\alpha_{11}\alpha_{22}\alpha_{33}+\alpha_{11}\alpha_{23}\alpha_{32}+\alpha_{12}\alpha_{21}\alpha_{33})N_1^*N_2^*N_3^*$$

By Routh-Hurwitz's criteria, all Eigen values of the above characteristic equation have negative real parts if and only if $b_1>0, b_3>0$ and $b_3(b_1b_2-b_3)>0$ .Clearly b_1 and b_3 are positive .And after some algebraic deductions, it can be verified that $b_3(b_1b_2-b_3)>0$. Hence the system is locally asymptotically stable.

GLOBAL STABILITY ANALYSIS

In this section, we consider the global stability of the system (2.1)-(2.3) by considering a suitable Lyapunov's function

Theorem1: The Equilibrium point $E_3\left(0, N_2^\phi, N_3^\phi\right)$ is globally asymptotically stable

$$V(0,N_2,N_3)=\left\{N_2-N_2^\phi-N_2^\phi\ln\left[\frac{N_2}{N_2^\phi}\right]\right\}+l\left\{N_3-N_3^\phi-N_3^\phi\ln\left[\frac{N_3}{N_3^\phi}\right]\right\} \tag{4.b1}$$

where l the suitable constant to be determined in the subsequent steps. It can be easily verified that the function V is zero at the equilibrium point $E_3\left(0, N_2^\phi, N_3^\phi\right)$. The time derivative of V along the trajectories of (2.1)-(2.3) is

$$\frac{dV}{dt}=\left(\frac{N_2-N_2^\phi}{N_2}\right)\frac{dN_2}{dt}+l\left(\frac{N_3-N_3^\phi}{N_3}\right)\frac{dN_3}{dt} \tag{4.b2}$$

$$\begin{aligned}\frac{dV}{dt}&=\left(N_2-N_2^\phi\right)\left\{\alpha_{22}N_2^\phi+\alpha_{23}N_3^\phi-\alpha_{22}N_2-\alpha_{23}N_3\right\}\\&+l\left(N_3-N_3^\phi\right)\left\{-\alpha_{32}N_2^\phi+\alpha_{33}N_3^\phi+\alpha_{32}N_2-\alpha_{33}N_3\right\}\end{aligned} \tag{4.b3}$$

Choosing $l=\dfrac{\alpha_{23}}{\alpha_{32}}$, after a small algebraic manipulation, we get,

$$\frac{dV}{dt}=-\alpha_{22}\left(N_2-N_2^\phi\right)^2-\alpha_{33}\frac{\alpha_{23}}{\alpha_{32}}\left(N_3-N_3^\phi\right)^2<0 \tag{4.b4}$$

Therefore the equilibrium point $E_3\left(0, N_2^\phi, N_3^\phi\right)$ is globally asymptotically stable.

Theorem 2: The Equilibrium point $E_4(N_1^*, N_2^*, N_3^*)$ is globally asymptotically stable

$$V(N_1,N_2,N_3)=N_1-N_1^*-N_1^*\ln\left[\frac{N_1}{N_1^*}\right]+l_1\left\{N_2-N_2^*-N_2^*\ln\left[\frac{N_2}{N_2^*}\right]\right\}+l_2\left\{N_3-N_3^*-N_3^*\ln\left[\frac{N_3}{N_3^*}\right]\right\} \tag{4.b5}$$

where l_1 and l_2 are suitable constants to be determined in the subsequent steps. It can be easily verified that the function V is zero at the equilibrium point $E_4(N_1^*, N_2^*, N_3^*)$. The time derivative of V along the trajectories of (2.1)-(2.3) is

$$\frac{dV}{dt}=\left(\frac{N_1-N_1^*}{N_1}\right)\frac{dN_1}{dt}+l_1\left(\frac{N_2-N_2^*}{N_2}\right)\frac{dN_2}{dt}+l_2\left(\frac{N_3-N_3^*}{N_3}\right)\frac{dN_3}{dt} \tag{4.b6}$$

Choosing $l_1=\dfrac{\alpha_{12}}{\alpha_{21}}$, $l_2=\dfrac{\alpha_{23}}{\alpha_{32}}\dfrac{\alpha_{12}}{\alpha_{21}}$ after a small algebraic manipulation, we get,

$$\frac{dV}{dt}=-\alpha_{11}\left(N_1-N_1^*\right)^2-\alpha_{22}\frac{\alpha_{12}}{\alpha_{21}}\left(N_2-N_2^*\right)^2-\alpha_{33}\frac{\alpha_{13}}{\alpha_{31}}\frac{\alpha_{23}}{\alpha_{32}}\left(N_3-N_3^*\right)^2<0 \tag{4.b7}$$

Therefore the interior equilibrium point $E_4(N_1^*, N_2^*, N_3^*)$ is globally asymptotically stable.

BIOECONOMIC ASPECT AT INTERIOR EQUILIBRIUM:

As we have already discussed, a biological equilibrium is given by $\dfrac{dN_i}{dt}=0$, i =1, 2, 3. Bionomics is the study of the dynamics of living resources using economic models. The bionomic equilibrium is said to be achieved when the total revenue obtained by selling the harvested biomass equals the total cost utilized in harvesting it. Let c_1 be the harvesting cost per unit effort for prey species, c_2 be the harvesting cost per unit effort for super predator species, p_1 be the price per unit biomass of the prey, p_2 be the price per unit biomass of the super predator. Therefore net revenue or economic rent at any time given by $R=R_1+R_2$. Where $R_1=(p_1q_1N_1-c_1)E_1$, $R_2=(p_2q_2N_2-c_2)E_2$ here R_1, R_2 represents net revenue for the prey, the super predator respectively, the bionomic equilibrium $\left((N_1)_\infty,(N_2)_\infty,(N_3)_\infty,\left(E_1\right)_\infty,\left(E_2\right)_\infty\right)$ is given by the following equations.

$$a_1N_1-\alpha_{11}N_1^2-\alpha_{12}N_1N_2-\alpha_{13}N_1N_3-q_1E_1N_1=0 \tag{5.1}$$
$$a_2N_2+\alpha_{21}N_2N_1-\alpha_{22}N_2^2-\alpha_{23}N_2N_3=0 \tag{5.2}$$
$$a_3N_3+\alpha_{32}N_2N_3-\alpha_{33}N_3^2-q_2E_2N_3=0 \tag{5.3}$$
$$R=\left(p_1q_1N_1-c_1\right)E_1+\left(p_2q_2N_3-c_2\right)E_2 \tag{5.4}$$

In order to determine the bionomic equilibrium we come across the following cases.

Case (i): if $c_2>p_2q_2N_3$, then the cost is greater than revenue for super predator then the super predator species will not be harvested (E_2=0).Only the prey species is harvested. $\left(c_1<p_1q_1N_1\right)$

$$\left(N_1\right)_\infty=\frac{c_1}{p_1q_1} \tag{5.5}$$

$$\left(N_2\right)_\infty = \frac{\left(a_2\alpha_{33} + \alpha_{21}\alpha_{33}\dfrac{c_1}{p_1q_1} - a_3\alpha_{23}\right)}{\alpha_{22}\alpha_{33} + \alpha_{23}\alpha_{32}} \qquad (5.6)$$

$$\left(N_3\right)_\infty = \frac{\left(a_3\alpha_{22} + a_2\alpha_{32} + \alpha_{32}\alpha_{21}\dfrac{c_1}{p_1q_1}\right)}{\alpha_{22}\alpha_{33} + \alpha_{23}\alpha_{32}} \qquad (5.7)$$

$$\left(E_1\right)_\infty = \frac{1}{q_1}\left\{a_1 - \alpha_{11}\frac{c_1}{p_1q_1} - \alpha_{12}\left(N_2\right)_\infty\right\} \qquad (5.8)$$

Now $\left(E_1\right)_\infty > 0$, if the following inequality holds

$$a_1 > \left(\alpha_{11}\frac{c_1}{p_1q_1} + \alpha_{12}\left(N_2\right)_\infty\right) \qquad (5.9)$$

Case (ii): If $c_1 > p_1q_1N_1$ then the cost is greater than revenue for prey then the prey species is not harvested (E_1=0). Only the super predator species remains operational $c_2 < p_2q_2N_3$

$$\left(N_3\right)_\infty = \frac{c_2}{p_2q_2} \qquad (5.10)$$

$$\left(N_1\right)_\infty = \frac{\left(a_1\alpha_{22} + \alpha_{21}\dfrac{c_2}{p_2q_2} - a_2\alpha_{12}\right)}{\alpha_{11}\alpha_{22} + \alpha_{21}\alpha_{12}} \qquad (5.11)$$

$$\left(N_2\right)_\infty = \frac{\left(a_2\alpha_{11} + a_1\alpha_{21} - \alpha_{11}\dfrac{c_2}{p_2q_2}\right)}{\alpha_{11}\alpha_{22} + \alpha_{21}\alpha_{12}} \qquad (5.12)$$

Now substitute $\left(N_1\right)_\infty, \left(N_2\right)_\infty, \left(N_3\right)_\infty$ in equations (5.1), (5.2) and (5.3) we get

$$\left(E_2\right)_\infty = \frac{1}{q_2}\left\{a_3 + \alpha_{32}\left(N_2\right)_\infty - \alpha_{33}\frac{c_2}{p_2q_2}\right\} \qquad (5.13)$$

Now $\left(E_2\right)_\infty > 0$, if the following inequality holds

$$\left(a_3 + \alpha_{32}\left(N_2\right)_\infty\right) > \alpha_{33}\frac{c_2}{p_2q_2} \qquad (5.14)$$

Case (iii): if $c_1 > p_1q_1N_1$, $c_2 > p_2q_2N_3$, then the cost is greater than the revenue for both the species and they are not harvested.

Case (iv): if $c_1 < p_1q_1N_1$, $c_2 < p_2q_2N_3$, then the revenues for both the species is economical, then both the species are harvested.

$$\left(N_1\right)_\infty = \frac{c_1}{p_1q_1}, \left(N_3\right)_\infty = \frac{c_2}{p_2q_2},$$

$$\left(N_2\right)_\infty = \frac{1}{\alpha_{22}}\left(a_2 + \alpha_{21}\frac{c_1}{p_1q_1} - \alpha_{23}\frac{c_2}{p_2q_2}\right) \qquad (5.15)$$

$$\left(E_1\right)_\infty = \frac{1}{q_1}\left\{a_1 - \alpha_{11}\frac{c_1}{p_1q_1} - \alpha_{12}\left(N_2\right)_\infty\right\} \qquad (5.16)$$

$$\left(E_2\right)_\infty = \frac{1}{q_2}\left\{a_3 + \alpha_{32}\left(N_2\right)_\infty - \alpha_{33}\frac{c_2}{p_2q_2}\right\} \qquad (5.17)$$

$$\left(E_1\right)_\infty > 0 \quad \text{if} \quad a_1 > \left(\alpha_{11}\frac{c_1}{p_1q_1} + \alpha_{12}\left(N_2\right)_\infty\right) \qquad (5.18)$$

$$\left(E_2\right)_\infty > 0 \quad \text{if} \quad \left(a_3 + \alpha_{32}\left(N_2\right)_\infty\right) > \alpha_{33}\frac{c_2}{p_2q_2} \qquad (5.19)$$

Thus the Non-trivial bionomic equilibrium point $\left((N_1)_\infty, (N_2)_\infty, (N_3)_\infty, \left(E_1\right)_\infty, \left(E_2\right)_\infty\right)$ exists if condition (5.18), (5.19) holds.

NUMERICAL SIMULATION:

(1)Leta_1=8,α_{11}=0.01,α_{12}=0.2,q_1=0.5,E_1=10,a_2=2.5,α_{21}=0.3,α_{22}=0.01, α_{23}=0.2,a_3=7, α_{32}=0.01, α_{33}=0.02 q_2 =0.3,E_2 = 15, N_1 =10, N_2 =50 and N_3 =55

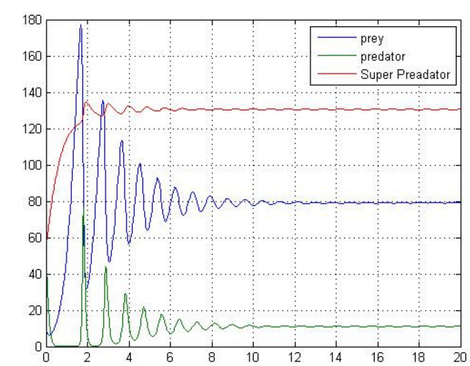

Fig 1.

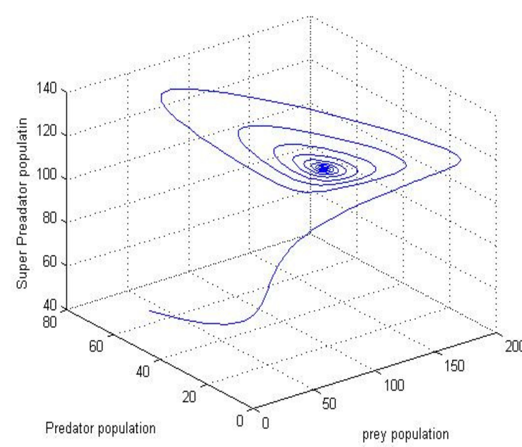

Fig 2.

(2) Leta_1=6,α_{11}=0.01,α_{12}=0.2,q_1=0.5,E_1=10,a_2=2.5,α_{21}=0.3,α_{22}=0.01, α_{23}=0.2, a_3=5.5, α_{32}=0.2, α_{33}=0.02 , q_2 =0.3,E_2 = 17, N_1 =60, N_2 =55 and N_3 =40

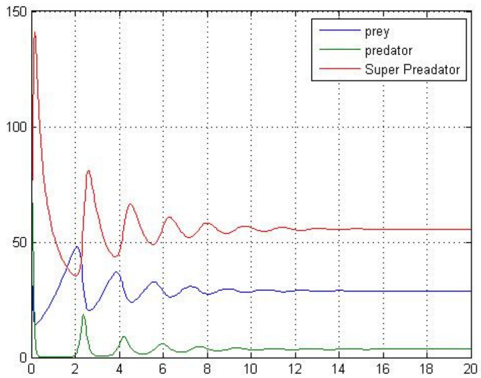

Fig 3.

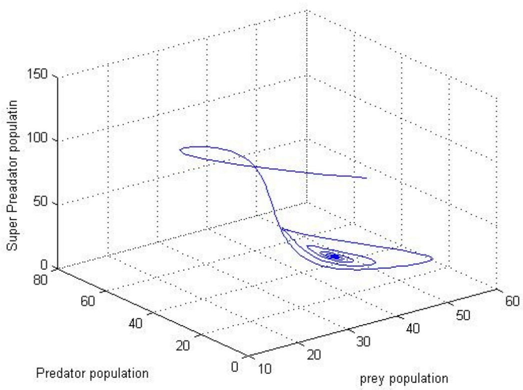

Fig 4.

Figures (1, 3) Represents Variations in the growth rate of the populations.

Figures (2, 4)Represents phase-space trajectories corresponding to the stabilities of the population.

CONCLUDING REMARKS:

A mathematical model with a prey, a predator and super predator species is proposed and analyzed by considering the harvesting of the prey and super predator. The conditions for existence and the stability of the equilibria of the system have been given. It is observed that even under continuous harvesting the populations may be maintained at an appropriate equilibrium level. The numerical simulations support the analytical data given using well defined mathematical models.

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