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Fuzzy multi objective optimization: With reference to multi objective transportation problem

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Abstract
In this paper we present a review of the connection between modern era techniques & fuzzy multi objective optimization (FMOO) to deal with its shortcoming and FMOO used in transportation problem. Multi objective optimization represents an interest area of research since most real life problem have a set of conflict objectives. MOO has its root in late nineteenth century welfare economics, in the works of Edge worth & Pareto. But due to some shortcoming faces, researchers attract to FMOO and they use modern era technique like artificial intelligence. Finally we develop a fuzzy linear programming method for solving the transportation problem with fuzzy goals, available supply & forecast demand and showing a frame for fuzzy multi objective transportation problem (FMOTP) solution.

Keywords: Transportation problem (TP), Multi-objective transportation problem (MOTP), Fuzzy multi objective optimization (FMOO), Artificial Intelligence (AI), Fuzzy linear programming. MSC: 90C08

INTRODUCTION

The word optimization comes from Operation Research(OR), the term OR describe the discipline that focused on the application of information technology for informed decision making in other word OR represent the study of optimal resource allocation. Now a day we sincerely believe that all real life problems have set of conflict objectives so we need FMOO.

A mathematical model of the MOO can be written as follows:

$$\mathrm{Min}_s\, Z = [Z_1(x), Z_2(x), \ldots\ldots\ldots\ldots\ldots\ldots Z_k(x)\,]^T \qquad (1.1)$$

Where

$S = \{\, x \in X : Ax \leq b,\ x \varepsilon R^n, x \geq 0\}$

Where

$Z(x)= Cx$ is the K-dimensional vector of objective function and C is the vector of cost corresponding to each objective function, S is the feasible region which is bounded by the given set of constraints.

Here A is the coefficients matrix, b is available resources, x is n-dimensional vector of decision variables.

When the objective functions and constraints are linear the model is called LMOO and if the objective function and/or constraints are non linear model is called NLMOO. The above model (1.1) is deterministic in nature that is why can be solve by any existing method. But in real life situation input information may be vague. Either A , b or C.

All these cases lead towards FMOO that can be written as follows:

$$\mathrm{Min}_s\, Z \cong [Z_1(x), Z_2(x), \ldots\ldots\ldots\ldots\ldots\ldots Z_k(x)\,]^T \qquad (1.2)$$

Where

$S = \{\, x \in X : \tilde{A}x \tilde{\leq} \tilde{b},\ x \varepsilon R^n, x \geq 0\}$

The above fuzzy model is transformed into crisp by implementing a suitable membership function. Again we classify this model in to two category LFMOO and NLFMOO respectively on the basis of linear and nonlinear function.

There are different methods to handle the solution of problem (1.2), all are using an appropriate membership function to change fuzzy model to crisp model.

Transportation problem [TP] is one of the earliest applications of the linear programming problems. The basic transportation problem was originally developed by Hitchcock [1]. Efficient method of solution derived from the simplex algorithm were developed in 1947, primarily by Dantzig [2] and then by Charnes et al. [3]. The transportation problem can be modeled as a standard linear programming problem, which can then be solved by the simplex method.

The objective of traditional transportation is to determine the optimal transportation pattern of a certain goods from supplier to demand customer so that the transportation cost become minimum and for this purpose we have different method for getting initial and optimal solution. We can get an initial basic feasible solution for the transportation problem by using the North-West corner rule, Row minima, Column minima, Matrix minima, or the Vogel Approximation Method (VAM). To get an optimal solution for the transportation problem, we use the MODI method (Modified Distribution Method). Charnes and Cooper [3] developed the Stepping Stone Method (SSM), which gives an alternative way of determining the optimal solution.

A transportation problem can be represented as a single objective transportation problem

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$$Min Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$ Subject to

$$\sum_{i=1}^{m} x_{ij} = a_i\ ,\quad i=1, 2, \ldots\ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j\ ,\quad j= 1,2, \ldots\ldots, n$$

$$x_{ij} \geq 0,\ i=1,2, \ldots\ldots, m\ ,\ j= 1,2, \ldots\ldots, n \qquad (1.3)$$

But in real world transportation problem, input data or related parameters are often imprecise / fuzzy owing to incomplete or unobtainable information.

A transportation problem can be represented as a multi objective transportation problem

$$Min\, z_k = \sum_{i=1}^{m}\sum_{j=1}^{n} c^{k}_{ij}x_{ij}\ ,\quad k= 1, 2, \ldots\ldots k$$

Subject to

$$\sum_{i=1}^{m} x_{ij} = a_i\ ,\quad i=1, 2, \ldots\ldots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j\ ,\quad j= 1, 2, \ldots\ldots n$$

$$x_{ij} \geq 0,\ i=1, 2, \ldots\ldots, m\ ,\ j= 1,2, \ldots\ldots, n \qquad (1.4)$$

The subscript on Z_k and superscript on c^k_{ij} denote the k-th penalty criterion, $a_i > 0$ for all i, bj > 0 for all j, $c^k_{ij} \geq 0$ for all (i, j), and $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ (balanced condition).

The balanced condition is both a necessary and sufficient condition for the existence of a feasible solution to the transportation problems in both cases.

FUZZY PRELIMINARIES

Definition

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1to each member in X . This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned to the element of the universal set X fall within a specified range [0,1] i.e. $\mu_{\tilde{A}}: X \rightarrow [0,1]$.The assigned values indicate the membership grade of the element in the set A .

The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{ (x, \mu_{\tilde{A}}(x)) x \in X\}$ defined by $\mu_{\tilde{A}}$ for each $x \in X$ is called a fuzzy set. $\mu_{\tilde{A}}(x)$ is the degree of membership of x in $\tilde{A}$. The closer the value of $\mu_{\tilde{A}}(x)$ is to 1, the more x belongs to A.

BACKGROUND OF FMOO

Bellman & Zedah [8] highlighted the main pillar of fuzzy decision making in 1970, that can be summarized as follows:

$$D = G \cap C \qquad (2.1)$$

Where

G is fuzzy goal, C is fuzzy constraints & D is fuzzy decision that characterized by a suitable membership function as follows:

$$\mu_D(x) = \min\,(\mu_G(x),\ \mu_C(x)) \qquad (2.2)$$

The maximization decision is then define as follows

$$\max_{x\in X}\ \mu_D(x) = \max_{x\in X}\ \min\,(\mu_G(x),\ \mu_C(x)) \qquad (2.3)$$

for k fuzzy goals & m fuzzy constraints, the fuzzy decision is defined as follows:

$$D = G_1 \cap G_2 \cap \ldots\ldots \cap G_k \cap C_1 \cap C_2 \cap \ldots\ldots \cap C_m \qquad (2.4)$$

And the corresponding maximizing decision is written as follows:

$$\max_{x\in X}\ \mu_D(x) = \max_{x\in X}\ \min\,(G_1(x), G_2(x), \ldots\ldots G_k, C_1(x), C_2(x), \ldots\ldots, C_m(x)) \qquad (2.5)$$

in [16] we can see details.

From the last 25 years many researchers works have been developed and different argument they put on their papers to overcome the shortcoming in the FMOO.

In FMOO the membership function depends on where the fuzziness is existed. If the fuzziness in the objective function coefficients, the membership function may be represented as follows:

$$\mu_k(Z^k(x)) = \begin{cases} 1 & Z^k(x) \leq L_k \\ \frac{U_k - Z^k(x)}{U_k - L_k} & L_k < Z^k(x) < U_k \\ 0 & Z^k(x) \geq U_k \end{cases} \qquad (2.6)$$

Where

U_k is the worst upper bond and L_k is the best lower bound of the objective function k, respectively. They are calculated as follows:

$$U_k = (Z^k)^{max} = \max_{x\in X} Z^k(x)$$

$$L_k = (Z^k)^{min} = \min_{x\in X} Z^k(x),\quad k= 1,2,3\ldots\ldots K \qquad (2.7)$$

Where the membership function is assumed to be symmetrically triangular functions. The membership function may vary according to the problem or problem solver on his experience. The deterministic model of FMOO will be as follows:

Max β

Subject to

$$\beta \leq \mu_k(Z^k(x)),\ k = 1,2,\ldots K$$

$$\sum_{j=1}^{n} A_{ij}\, x_j = b_i,\quad i= 1,2,3\ldots\ldots m$$

$$x_j \geq 0,\ i= 1,2,3\ldots\ldots m;\ j= 1,2,3\ldots\ldots n;\ k = 1,2,\ldots K$$

$$0 \leq \beta \leq 1 \qquad (2.9)$$

Where β showed the level of satisfaction & treated as auxiliary variable. Model (2.7) can be solved as single objective function by using linear or non linear programming. After the Bellman & Zedah paper several research work has been adopted such as Hannan [18] and Zimmerman [19] handled fuzzy linear programming with multiple objectives by using membership function. Hannan & Zimmerman used discrete & continuous membership function respectively. Boender [29] , Sakawa [16], Baptistella et al [20] impilimented the fuzzy set theory in interactive multi objective decision making. More historical & methedolocal development can be seen in Sakawa[16], Lia & Hwang [17] etc.

Many real life problems have been formulated as FMOO and solved by using appropriate technique. Some of these application involved production, manufacturing, location-allocation problem, environmental management, business, marketing, agriculture, economics, machine controls, engineering application & regression modeling. Well classified details can be found in Lia & Hwang [17] and new literature review [30] assure the same field of applications.

Shortcomings of fmoo solution approaches

FMOO has been characterized by specify criteria to indicate its class. Some of these indicator criteria that consider them ill-structured problems are:

1. There is no available solution technique to solve the model,
2. There is no standard mathematical model to represent the problem,
3. There is no ability to involve the qualitative factors in the model,
4. There is no available solution space to pick up the optional solution, and
5. There is a difficulty to measure the quality of the result solution(s)

If the some of these criteria are existed, then the problem will belong to the second category which is called semi-ill structured problems. But, if all of these criteria and others are not existed, then the problem will belong to the third category which is called well structured problem. Fortunately the first and second categories represent a rich area for investigation, especially in the era of information technology where all the sciences are interchanged in complex manner to a degree that one can find difficulty to separate between sciences. In other words biological sciences, sociology, insects, science… etc attracted the researchers to simulate then by using computer technology which consequently reflects its positive progress on the optimization research work.

Let us now apply these criteria of ill-structured problems on FMOO problems. For FMOO model structure, the following problems are represented an optical stone to more progress in this area. Some of these problems are:

1- Incorporating fuzzy preferences in the model still needs a new methodologies to take it into model account without increasing the model complexity,

2- As mentioned above, the membership function is the corner stone of fuzzy programming and right now, the problem solvers assumed it according the experience As a result, the solution will be different according to the selected membership function. This will lead to another problem which solution is better or qualify more enough to the problem under study. In this case, there is an invitation to implement the progress in information technology to discover an appropriate membership function.

3- Large-scale FMOO models still need more research especially when incorporating large preference information.

Regarding the solution methodologies, there some difficulties to enhance them. Some of them are:

1- Fuzzy integer programming with multi-objectives can be considered a combinatorial optimization problem and as a result it needs an exponential time algorithm to with it.

2- In 0-1 FMOO problems(whatever small scale or large-scale),the testing process of the Pareto-optimal solution is considered NP-hard problem.

3- In fuzzy and non-fuzzy MOO problems, there is a difficulty to construct an initial solution that should be close to the Pareto-optimal solution to reduce the solution time. So, we need a powerful methodology based information technology to deal with this problem.

Because of these shortcomings and others, FMOO attracts the attentions of researchers to enhance the field of FMOO by developing more powerful links between it and other sciences. In this chapter the attention will highlight the links between artificial intelligence and FMOO to overcome all or some of the mentioned problems. This link leads to a new and interesting area of research called “Intelligent Optimization”. In the next subsection, some of intelligent techniques will be introduced briefly.

Some intelligent techniques

Artificial intelligence (AI) is the branch of computer technology that simulates the human being behavior via intelligent machines to perform well and better than him. The researchers of computer science are wondering to extract their idea from the biological systems of human being such as thinking strategies, nervous system and genetics.

Now, we shall classify the intelligent FMOO problems based upon the implemented technique.

Expert system and FMOO

Expert system is an intelligent computer program that consists of three modules:

1) inference engine modules
2) knowledge-base module and user interface module.

This system can produce one of the following functions:

1) conclusion
2) recommendation and
3) advice.

The main feature of the ES is its ability to treat the problems symbolically not algorithmically. So, it can perform a good job regarding both the decision maker's preferences and the qualitative factors that cannot be included in the degree of model complexity.

Generally speaking, ES has been applied to solve different application that can be modeled in MOO. For example, [31] developed an expert system for regional planning system to optimize the industrial structure of an area. In this system, AI paradigms and numeric multiobjective optimization the industrial strrtcture of an area. In this system, AI paradigms and numeric multi objectives optimization techniques are combined to arrive at a hybrid approach to discrete alternative selection. These techniques include: (1) qualitative analysis, (2) various statistical checks and recommendations, (3) robustness and sensitivity analysis, and (4) help for defining acceptable regions for analysis . Abd El-Wahed [32], developed a decision support system with goal programming based ES to solve engineering problems. In this research, the statistical analysis and the decision maker's preferences are combined in an ES to assign the differential weights of the sub-goals in goal programming problems. Also, Rasmey, Abd EL-Wahed and others [33] presented a fuzzy ES to include the qualitative factors that could not be involved in the mathematical model of multi criteria assignment problem in the field of bank processing. The approach depends on evaluating the model solution by using the developed fuzzy ES. If the solution is coincided with the evaluation criteria, the approach is terminated. Otherwise, some modification on the preferences is done in the feedback to resolve the model again and so on until getting a solution coincides with the evaluation criteria.

There is a little research work regarding FMOO have been done. For example: Rasmey et al [34] presented an interactive approach for solving MOO problem with fuzzy preferences in both aspiration level determination and priority structure by using the framework of fuzzy expert system. The main idea of this approach is to convert the MOO problem into its equivalent goal programming model by setting the aspiration levels and priority of each objective function based on fuzzy linguistic variables. This conversion makes the implement of ES in an easy and effective manner.

Shih-Yaug Liu and Jen-Gwo [35] present an Integrated Machine Troubleshooting Expert System (IMTES) that enhances the efficiency of the diagnostic process. The role of fuzzy multi attribute decision-making in ES is determining the most efficient diagnostic process and crates a "meta knowledge base" to control the diagnosis process.

The researchers are invited to investigate the following points that are not covered right now:

1. Applying ES to guide the determination process of the aspiration levels of fuzzy goal programming.
2. Applying ES to handle the DM's preferences in solving interactive FMOO to reduce the solution time and reducing the solution efforts.

ANN and FMOO problem

ANN is a simulation of human being nervous system. The ANN simulator depends on the Third law of Newton "For any action there Is an equal reaction with negative direction" A new branch of computer science is opened for research called "Neural computing" , Neural computing has been viewed as a promising tool to solve problems that involve large data /preference or what is called in optimization large-scale optimization problem . Also, the transformation of FMOO into crisp model needs an appropriate membership function to do this. In other situation, ANN is implemented to solve the FMOO problem without need to defuzzify the mathematical model of FMOO problems. ANN offers an excellent methodology for estimating continuous or discrete membership functions/values. To do that, an enough amount of historical data is needed to train and test the ANN as well as getting the right parameters and topology of it to solve such problem.

There are previous research works which handle ANN in solving some optimization problems as well as FMOO specifically. These works can be classified according to the treating method of FMOO model as follows:

Treating the fuzzy preferences in MOO Problems

Wang [36] presented a feed-forward ANN approach with a dynamic training procedure to solve multi objectives cutting parameter optimization in the presence of fuzzy preferences. In this approach, the DM's preferences are modeled by using fuzzy preference information based ANN. Wang. S and N. P. Archer [37] modeled the uncertainty of multi-objective multi-persons decision making by using fuzzy characteristics. They implemented the back-propagation learning algorithm under monotonic function constraints. Stam. A., M. Sun and M. Haines [38] presented two approaches of ANNs to process the preference ratings which resulted from analytical hierarchy process pair wise comparison matrices. The first Approach implements ANN to determine the eigenvectors of the pair wise comparison matrices. This approach is not capable of generalizing the preference information. So it is not appropriate for approximating the preference ratings if the DM's Judgments are imprecise. The second approach uses the feed- forward ANN to accurately approximate the preference ratings. The results show that this approach is working well with respect to imprecise pair wise judgments. Jian Chen et al [39] developed Decision Neural Network (DNN) to use in capturing and representing the DM's preferences. Then, with DNN, an optimization problem is solved to look for the most desirable solution.

Handling fuzziness in FMOO Models

It is clear that ANN is capable of solving the constrained optimization problems, especially the applications that require on line optimization .Gen. M., K. Ida and R. Kobuchi [40] discussed two – phase approach to solve MOO problems with fuzziness in both objectives and constraints.

Determining the membership functions

Ostermark, Ralf [41] proposed a fuzzy ANN to generate the membership functions to new data . The learning process is reflected in the shape of the membership functions which allow the dynamic adjustment of the functions during the training process. The adopted fuzzy ANN is applied successfully to multi – group classification based multi objectives analysis in the economical field.

From the above analysis, we can deduce that there are many research points still uncovered. It means that the integration area between ANN and FMOO is very rich for more research. These points are summarized as follows:

1. Applying the ANN to solve FMOO Problems in its fuzzy environment without transforming it into crisp model to obtain more accurate, efficient and realistic solutions.
2. Developing more approaches to enhance the process of generating real membership functions.
3. Combine both ES and ANN to develop more powerful approaches to consider the preference information in FMOO problem.

Genetic algorithms and FMOO

Genetic algorithm (GA) is search algorithm that mimics the process of natural evolution. The problem address by GA is searching the feasible space is identifying the best one in the problems that are combinatorial or large scale or ill structured in general. GA encodes the variable of problems in either binary or real valued or vectors. Each code is called chromosome. In binary coding there are two decoding functions to convert from real to binary and vice versa. In addition, mutation, crossover and selection are the three important operators used for generating the new solution within the feasible space.

GA seems desirable for solving MOO because they deal simultaneously with asset of solution which allows the problem solver to find several members of Pareto optimal set in a single run of the algorithm, instead of having to perform a series of separate runs such as the traditional mathematical programming techniques. Additionally, GA are less susceptible to the shape or continuity of the Pareto front, whereas these two issues are a real concern for mathematical programming techniques

FMOO Based GA

Sakawa and others presented a series of papers in this category. The ideas of these works can be summarized in the following:

a) Kato and Sakawa [42] introduces an interactive satisfying method using GA for getting the satisfying solution for the DM from an extended Pareto optimal solution set. In this method, for certain value of α -level cut and reference membership function the solution of large scale multi objective 0-1 programming is obtained by adopting a GA with decomposition procedures.
b) Sakawa and Shibano [43] presents an interactive method for multi objective 0-1 programming problems that involve fuzzy numbers. In this method, the DM specifies the value of α and reference membership value and solving the augmented mini-max problems through GA with double strings to obtain the extended Pareto optimal solution.
c) Sakawa and Yauchi [44] proposed an interactive decision making method for solving multi objective non-convex programming problems with fuzzy numbers through co-evolutionary GAs. In this paper, the authors were trying to overcome the drawbacks of GENCOP III by introducing a method to generate an initial feasible point and a bisection method. This modification leads to a new GENCOP called revised GENCOP III.
d) Sakawa and Kato [45] deals with the general multi-objective 0-1 programming problems that involve positive and negative coefficients. The extended GA with new decoding algorithm for individuals. The double strings map each individual to a feasible solution based on backtracking and individual modification. For more details about the GA and FMOO see [46]
e) Basu [47] applied an interactive fuzzy satisfying method based on evolutionary programming technique for short-term multi-objective hydrothermal scheduling. The multi objective problems is formulated with assuming that the DM has fuzzy goals for each of the objective functions and the evolutionary programming technique based fuzzy satisfying method is applied for generating a corresponding optimal non-inferior solution for the DM's goals.
f) Abd EL_Wahed at al [48] presented a contribution in this area by suggesting an interactive approach to determine the preferred compromise solution for the MOO problems in the presence of fuzzy preferences. Here, the DM evaluates the solution by using a defined set of linguistic variables and consequently the achievement membership function can be constructed for each objective function. The used non-negative differential weights are determined based on the entropy degree of each objective function to support in transforming the MOO into single objective function.

At the end of this section, we are can decide that the implementation of GAs in sloving the FMOO problems are occupied a wide interest of the research more than any other AI search technique. However, there are still some problems in FMOO problems have not been studied yet such as:

1. Large-scale FMOO problems with fuzzy numbers in the objective function and constraints.
2. Combining both ES and GA to handle the fuzzy preferences in MOO problems to get more powerful solution method.
3. Implementing the GA to study both sensitivity and parametric analysis of linear and non-liner FMOO.

Particle Swarm Optimization (PSO)

The basic principles of PSO are represented by asset of moving particles that is initially thrown inside the search space. Each particle characterized by the following features:

a- A position and velocity
b- It knows its position, and the objective function value for this position
c- It knows is neighbors, best previous position and objective function value
d- It remembers its best previous position
e- It is considered that the neighborhood of a particle includes this particle itself.
f- At each time step, the behavior of a given particle is a compromise between three possible choices:
   1. Following its own way
   2. Going towards its best previous position
   3. Going towards the best neighbor's best previous position.

The basic equations of PSO can be formalized as follows:

$$\left.\begin{aligned} V_{t+1} &= C_1 V_t + C_2(P_{i,t} - x_t) + C_3(P_{g,t} - x_i) \\ X_{t+1} &= X_t + V_{t+1} \end{aligned}\right\} \quad (4.1)$$

With
V_t = velocity at time step t
X_t =position at time step t
$P_{i,t}$:=best previous position, at time step t
$P_{g,t}$:=best neighbors previous best, at time step t, (or best neighbor)

$C_1, c_2, {}_3$:= social/cognitive confidence coefficients

PSO has been used in solving some real life applications that involved multi objectives.

Salman et al [49] proposed a PSO to task assignment. PSO follows a collaborative population-based search. PSO system combines local search methods (through self experience) with global search methods (through neighboring experience), attempting to balance exploration and exploitation.

The scan of some database gave an indication that PSO has not applied yet in solving FMOO problems.

Problem formulation FMOO in TP that is FMOTP

Assume that a logistics center seeks to determine the transportation plan of a homogeneous commodity from m sources to n destinations. Each source has an available supply of the commodity to distribute to various destinations, and each destination has a forecast demand of the commodity to be received from various sources. This work focuses on FMOTP.

The following notation is used.

Index sets

i index for source, for all i=1,2,….,m

j index for destination, for all j=1,2,….,n

g index for objectives, for all g=1,2,….,k

Decision variables

Q_{ij} units transported from source i to destination j(units)

Objective functions

Z_1 total production and transportation costs (Rs.)

Z_2 delivery time (hours)

Parameters

p_{ij} production cost per unit delivered from source i to destination j (Rs. / unit)

c_{ij} transportation cost per unit delivered from source i to destination j (Rs. / unit)

t_{ij} transportation time per unit delivered from source i to destination j (Rs./ unit)

$\tilde{s}_i$ total available supply for each source i (units)

$\tilde{D}_j$ total forecast demand of each destination j (units)

a_{ij} hours of machine usage per unit produced by each source i (machine- hour/unit)

M_{imax} maximum machine capacities available for each source i (machine – hour)

B total budget (Rs)

b_{ij} warehouse space per unit delivered from source i to destination j(ft^2 / unit)

W_{imax} maximum warehouse space available for each destination (ft^2)

Fuzzy multi – objective transportation problem (FMOTP) model
Objective functions

Minimize total production and transportation costs

$$\text{Min } Z_1 \cong \sum_{i=1}^{m}\sum_{j=1}^{n}(p_{ij}+c_{ij})\,Q_{ij} \qquad (5.1)$$

Minimize total delivery time

$$\text{Min } Z2 \cong \sum_{i=1}^{n}\sum_{j=1}^{m} t_{ij}\,Q_{ij} \qquad (5.2)$$

The symbol $\cong$ is the fuzzified version of "=" and refers to the fuzzification of the aspiration levels.

Constraints

Constraints on total available supply for each source i

$$\sum_{j=1}^{n} Q_{ij} \le \tilde{s}_i \quad \forall\, i \qquad (5.3)$$

Constraints on total forecast demand for each destination j

$$\sum_{i=1}^{m} Q_{ij} \ge \tilde{D}_j \quad \forall\, j \qquad (5.4)$$

Constraints on total budget

$$\sum_{i=1}^{m}\sum_{j=1}^{n}(p_{ij}+c_{ij})\,Q_{ij} \le B \qquad (5.5)$$

Constraints on machine capacities for each source i

$$\sum_{j=1}^{n} a_{ij}Q_{ij} \le M_{i\,max} \quad \forall\, i \qquad (5.6)$$

Constraints on warehouse space for each destination j

$$\sum_{i=1}^{m} b_{ij}Q_{ij} \le W_{i\,max} \quad \forall\, j \qquad (5.7)$$

Non –negativity constraints on decision variables

$$Q_{ij} \ge 0 \quad \forall\, i \quad \forall\, j \qquad (5.8)$$

Model development
Membership functions

The FLP approach developed herein exhibits greater computational efficiency and flexibility of the fuzzy arithmetic operations by employing the linear membership functions to represent fuzzy numbers for solving the fuzzy multi objective TP. The corresponding non –increasing continuous linear membership functions for all fuzzy objective functions can be formulated as follows.

$$f_g(z_g) = \begin{cases} 1 & z_g \le Z_g{}^{l} \\ \dfrac{Z_g{}^{u} - z_g}{Z_g{}^{u} - Z_g{}^{l}} & Z_g{}^{l} < Z_g < Z_g{}^{u} \\ 0 & z_g \ge Z_g{}^{u} \end{cases} \qquad (5.9)$$

Where $Z_g{}^1$ and $Z_g{}^2$, g= 1,2,…………….k , are the lower and upper bounds , respectively, of the gth objective function z_g .

Moreover, the corresponding non – increasing continuous linear membership functions for the fuzzy constraint ( ) can be defined as follows.

$$f_i(H_i) = \begin{cases} 1 & H_i \le S_i{}^{l} \\ \dfrac{S_i{}^{u} - H_i}{S_i{}^{u} - S_i{}^{l}} & S_i{}^{l} < H_i < S_i{}^{u} \\ 0 & H_i \ge S_i{}^{u} \end{cases} \qquad (5.10)$$

Where

$H_i = \sum_{j=1}^{n} Q_{ij}$, i= 1,2,3…………….m; $S_i^{\,l}$ and $S_i^{\,u}$, i= 1,2,3…………..m,

are lower and upper. The fuzzy resources , respectively and upper bounds of inequality constraint.

Similarly, the corresponding non –decreasing continuous linear membership functions for the fuzzy constraint(4) can be defined as follows.

$$f_j(V_j) = \begin{cases} 1 & V_j \geq D_j^{\,u} \\ \frac{V_j - D_j^l}{D_j^u - D_j^l} & D_j^{\,l} < V_j < D_j^{\,u} \\ 0 & V_j \leq D_j^{\,l} \end{cases} \qquad (5.11)$$

Where

$V_J = \sum_{i=1}^{m} Q_{ij}$, J=1,2,…,n,

are the upper and lower bounds of the fuzzy resources , respectively, of the jth fuzzy inequality constraint. The pattern of the non-increasing continuous linear membership functions $f_j(v_j)$ is similar to $f_g(z_g)$.

Solving the FMOTP

The minimum operator is used to aggregate all fuzzy sets. Introducing the auxiliary variable L enables the original FMOO to be converted into an equivalent ordinary LP form. Consequently, the complete equivalent LP model for solving the FMOTP can be formulated as follows.

Max L (5.12)

Sub. To

$L \leq f_g(z_g) \quad \forall g$

$L \leq f_i(H_i) \quad \forall i$

$L \leq f_j(V_j) \quad \forall j$

$\sum_{i=1}^{m} \sum_{j=1}^{n} (p_{ij} + c_{ij}) Q_{ij} \leq B$

$\sum_{j=1}^{n} a_{ij} Q_{ij} \leq M_{i\,max} \quad \forall i$

$\sum_{i=1}^{m} b_{ij} Q_{ij} \leq W_{i\,max} \quad \forall j$

$Q_{ij} \geq 0 \quad \forall i \quad \forall j$

Solution procedure

Step 1. Formulate the original FMOO model for the TP according to Eqs. (1) to (8).

Step 2. Specify the corresponding linear membership functions for all the fuzzy objective functions and the fuzzy inequality constraints using Eqs. (9) and (11).

Step 3. Introduce the auxiliary variable L, and then transform the original FMOO problem into an equivalent ordinary LP form using the minimum operator to aggregate all fuzzy sets.

Step. 4 Solve the ordinary LP problem and obtain the initial compromise solution. If the decision maker is dissatisfied with the initial solution, the model should be modified until a satisfactory solution is obtained.

CONCLUSIONS

From the above analysis, one can conclude that the implementation of AI techniques to handle FMOO problems has occupied a reasonable attention from the researchers with respect to some AI techniques such as ES, ANN and GAs. But, some of these techniques have not been opened yet such as SA, TS, PSO, DNA and hybrid techniques for handling the problems of FMOO.

The future of research in the area of FMOO is viewed as two directions:

1. Improving the performance of intelligent techniques by developing each technique and by combining two or more of these techniques to get more powerful techniques.
2. implementing the available techniques and the developed ones to handle the FMOO problems

And intelligent FMOO research directions:

This area of research still needs more and more of research such as the following directions:

1. large scale FMOO with mixed integer decision variables needs to be further investigation especially by using the hybrid intelligent system .
2. Measuring the performance of AI techniques in higher dimensional FMODM problems ; where the only test of performance is using benchmark functions. In addition, the theoretical analysis of measuring AI performance needs an extensive attention from the researchers.
3. Theoretical work is required to deal with the FMODM problems in Its fuzzy environment without transforming it into crisp model, where the result solution may be more reasonable than the solution results from the transformation process.
4. Studying the effect of changing the AI techniques parameters on the solution behavior of FMODM problems. In other words, understanding the dynamics of swarm's dynamics (as in PSO) and the pheromones dynamics (as in ACO) on the behavior of optimization process.

Finally in this paper we present MOTP through the membership function its model construction and solution procedure.

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