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Expected Time to Recruitment in an Organization with Two Grades using a Univariate Recruitment Policy Involving Two Thresholds

[1]A. Srinivasan* and [2]V . Vasudevan

[1]P.G. and Research Department of Mathematics, Bishop Heber College, Trichy–17, Tamil Nadu, India

[2]Department of Mathematics, Urumu Dhanalakshmi College, Trichy–19, Tamil Nadu, India

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***Corresponding Author**

Tel : +91-4312770136
Fax : +91-4312770293

Email: mathsrinivas@yahoo.com

Abstract

An organization with two grades subjected to loss of man power due to the policy decisions taken by the organization is considered in this paper. Two mathematical models are constructed and an appropriate univariate recruitment policy, based on shock model approach involving optional and mandatory thresholds for the loss of manhours in each grade is suggested. Performance measures namely mean and variance of the time to recruitment are obtained for both the models when (i) the loss of manhours process forms a sequence of independent and identically distributed exponential random variables (ii) the inter-decision times are independent and identically distributed exponential random variables and (iii) the optional thresholds are exponential random variable and the distributions of the mandatory thresholds have SCBZ property. The analytical results are substantiated by numerical illustrations and the influence of nodal parameters on the performance measures is also analyzed.

Key Words: Manpower planning, Shock models, Univariate recruitment policy, Mean and variance of the time to recruitment.
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Introduction

Exodus of personnel is a common phenomenon in any marketing organization whenever the organization announces revised policies regarding sale target, revision of wages, incentives and perquisites. This in turn produces loss in man-hours, which adversely affects the sales turnover of the organization. Frequent recruitment is not advisable as it will be expensive due to the cost of recruitment. As the loss of manhours is unpredictable, a suitable recruitment policy has to be designed to overcome this loss. One univariate recruitment policy which is based on shock model approach in reliability theory is given as follows : If the total amount of manhours lost crosses a particular level, known as threshold, the organization reaches an uneconomic status, which otherwise be called the breakdown point and the recruitment has to be done at this point. Many models have been discussed using different types of distributions and such models could be seen in [7] and [2]. In [10] the authors have obtained the performance measures namely mean and variance of the time to recruitment for a two graded system when (i) the loss of man hours and the threshold for the loss of man hours in each grade are exponential random variables (ii) the inter decision times are independent and identically distributed exponential random variable forming the same renewal process for both grades and (iii) threshold for the organization is the max (min) of the thresholds for the two grades (max (min) model) using the above cited univariate cumulative policy of recruitment. In [1] the author has studied the maximum model in [10] when both the distributions of the thresholds have SCBZ property. Assuming that the inter decision times are exchangeable and constantly correlated random variables, the performance measures of time to recruitment are derived in [8] according as the loss of manpower and thresholds are discrete or continuous random variables. In [11] the author has extended the results in [8] for geometric thresholds when the inter-decision times for the two grades form two different renewal processes. In [17] the author has studied the results in [8] and [10] using a bivariate policy of recruitment. Recently in [18] these performance measures are obtained when the inter decision times are exchangeable and constantly correlated exponential random variables and the distributions of the thresholds have SCBZ property. In [16] the authors have studied the results in [10] when the thresholds for the organizations are the sum of the thresholds for the grades. This paper has been extended in [11] when threshold distributions have SCBZ property. In [9] the work in [16] is studied when the loss of man power and thresholds are geometric random variables according as the inter decision times for the two grades are correlated random variables or forming two different renewal processes. This author has also obtained the mean time for recruitment for constant combined thresholds using a univariate max policy of recruitment. Recently in [3], [4] and [5] the authors have considered a new univariate recruitment policy involving two thresholds in which one is optional and the other a mandatory and obtained the mean time to recruitment under different conditions on the nature of the thresholds according as the inter decision times are independent and identically distributed random variables or the inter decision times are exchangeable and constantly correlated exponential random variables. In [6] the authors have also obtained the mean time to recruitment when the optional and mandatory thresholds are geometric random variables. In [12], [13], [14] and [15] the authors have extended the results in [3] for a two grade system according as the thresholds are exponential random variables or geometric random variables or SCBZ property possessing random variables or extended exponential random variables. The objective of the present paper is to obtain the variance of time to recruitment for a two graded system using the univariate cumulative recruitment policy considering optional and mandatory thresholds for both the grades. The present paper extends the result in [3] for a two graded system when the distributions of the optional thresholds have the exponential random variable and mandatory thresholds SCBZ property.

Model Description and Analysis for Model - I

Consider an organization having two grades **1** and **2** in which decisions are taken at random epochs in $(0, \infty)$ and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manhours to the organization if a person quits. It is assumed that the loss of manhours are linear and cumulative. Let $\{X_i\}_{i=1}^{\infty}$ be a sequence of independent and a identically distributed exponential random variable with parameter α ($\alpha > 0$) where X_i is the loss of manhours due to i^{th} decision. Let $S_k = \sum_{i=1}^{k} X_i$ be the cumulative loss of man hours in the first k decisions (k = 1,2,…). Let g(.) be the probability density function of X_i, i = 1, 2, 3 … It is assumed that the inter-decision times are independent and identically distributed exponential random variables with probability density function (distribution function) f(.) (F(.)) and parameter θ ($\theta > 0$). Let f_k (.) (F_k (.)) be k fold convolution of f(.) (F(.)). Let f^*(.) (g^*(.)) be the Laplace transform of f(.) (g(.)). It is assumed that loss of manhours process and the process of inter-decision times are statistically independent. Let Y_1 and Y_2 be exponential random variables denoting the optional thresholds for grades **1** and **2** with parameters λ_1 and λ_2 respectively. Let Z_1 and Z_2 be SCBZ random variables denoting mandatory thresholds for grades **1** and **2** with parameters δ_1, η_1, μ_1 and δ_2, η_2, μ_2 respectively. It assumed that $Y_1 < Z_1$ and $Y_2 < Z_2$. The optional and mandatory thresholds Y and Z for the organization are defined as Y = max (Y_1,Y_2) and Z = max (Z_1,Z_2). The recruitment policy employed in this paper is as follows: *If the total loss of manhours crosses the optional threshold level Y, the organization may or may not go for recruitment, but if the total loss of manhours crosses the mandatory threshold Z, recruitment is necessary.* Let β be the probability that the organization is not going for recruitment whenever the total loss of manhours crosses optional level Y. Let W be a continuous random variable denoting the time for recruitment in the organization with probability density function ℓ (.), cumulative distribution function L(.). Let $V_k(t)$ be the probability that there are exactly k–decision epochs in (0, t]. Since the number of decisions made in (0,t] form a renewal process we note that $V_k(t) = F_k(t) - F_{k+1}(t)$ where $F_0(t) = 1$. Let E(W) and V(W) be the mean and variance of time to recruitment respectively.

As in [3] the survival function of W is given by

$$P(W > t) = \sum_{k=0}^{\infty} V_k(t)\, P(S_k < Y) + \sum_{k=0}^{\infty} V_k(t) \times \beta \times P(S_k \geq Y) \times P(S_k < Z) \quad (1)$$

Invoking the law of total probability it can be shown that

$$P(S_k < Y) = (D_1)^k + (D_2)^k - (D_3)^k \quad (2)$$

and

$$P(S_k < Z) = p_2 (D_4)^k + q_2 (D_5)^k + p_1 (D_6)^k + q_1 (D_7)^k - p_1p_2 (D_8)^k - p_1 q_2 (D_9)^k - p_2q_1 (D_{10})^k - q_1q_2 (D_{11})^k \quad (3)$$

where

$D_1 = g^*(\lambda_1)$, $D_2 = g^*(\lambda_2)$, $D_3 = g^*(\lambda_1 + \lambda_2)$, $D_4 = g^*(\delta_2 + \mu_2)$, $D_5 = g^*(\eta_2)$, $D_6 = g^*(\delta_1+\mu_1)$, $D_7 = g^*(\eta_1)$, $D_8 = g^*(\delta_1+\mu_1+\delta_2+\mu_2)$, $D_9 = g^*(\delta_1+\mu_1+\eta_2)$, $D_{10} = g^*(\eta_1+\delta_2+\mu_2)$, $D_{11} = g^*(\eta_1+\eta_2)$, $p_1 = \dfrac{(\delta_1 - \eta_1)}{(\mu_1 + \delta_1 - \eta_1)}$, $q_1 = 1-p_1$, $p_2 = \dfrac{(\delta_2 - \eta_2)}{(\mu_2 + \delta_2 - \eta_2)}$, $q_2 = 1-p_2$.

For m = 1, 2, 3, define

$$E_m(t) = [1 - D_m] \sum_{k=1}^{\infty} F_k(t)\, (D_m)^{k-1} \quad (4)$$

For r = 1, 2, 3, s = 4, 5, 6, …, 11, define

$$E_{r,s}(t) = [1 - D_r D_s] \sum_{k=1}^{\infty} F_k(t)\, (D_r D_s)^{k-1} \quad (5)$$

From (1), (2), (3) (4) and (5) and on simplification we get

$$\begin{aligned} P(W > t) = {} & 1 - E_1(t) - E_2(t) + E_3(t) + \beta \{ -p_2E_4(t) - q_2E_5(t) - p_1E_6(t) - q_1E_7(t) + p_1p_2E_8(t) + p_1q_2E_9(t) \\ & + p_2q_1E_{10}(t) + q_1q_2E_{11}(t) + p_2E_{1,4}(t) + q_2E_{1,5}(t) + p_1E_{1,6}(t) + q_1E_{1,7}(t) - p_1p_2E_{1,8}(t) - p_1q_2E_{1,9}(t) \\ & - p_2q_1E_{1,10}(t) - q_1q_2E_{1,11}(t) + p_2E_{2,4}(t) + q_2E_{2,5}(t) + p_1E_{2,6}(t) + q_1E_{2,7}(t) - p_1p_2\, E_{2,8}(t) - p_1q_2E_{2,9}(t) \\ & - p_2q_1E_{2,10}(t) - q_1q_2E_{2,11}(t) - p_2E_{3,4}(t) - q_2E_{3,5}(t) - p_1E_{3,6}(t) - q_1E_{3,7}(t) + p_1p_2E_{3,8}(t) + p_1q_2E_{3,9}(t) \\ & + p_2q_1E_{3,10}(t) + q_1q_2E_{3,11}(t) \} \end{aligned} \quad (6)$$

Since $\ell(t) = \frac{d}{dt}[1 - P(W > t)]$ and $\ell^*(s) = L\{\ell(t)\}$ from (6) it can be shown that

$$\begin{aligned} \ell^*(s) = {} & e_1^*(s) + e_2^*(s) - e_3^*(s) + \beta\{ p_2 e_4^*(s) + q_2 e_5^*(s) + p_1 e_6^*(s) + q_1 e_7^*(s) - p_1p_2 e_8^*(s) - p_1q_2\, e_9^*(s) \\ & - p_2q_1\, e_{10}^*(s) - q_1q_2\, e_{11}^*(s) - p_2\, e_{1,4}^*(s) - q_2\, e_{1,5}^*(s) - p_1\, e_{1,6}^*(s) - q_1 e_{1,7}^*(s) + p_1p_2 e_{1,8}^*(s) + p_1q_2 e_{1,9}^*(s) \\ & + p_2q_1 e_{1,10}^*(s) + q_1q_2 e_{1,11}^*(s) - p_2 e_{2,4}^*(s) - q_2\, e_{2,5}^*(s) - p_1 e_{2,6}^*(s) - q_1 e_{2,7}^*(s) + p_1p_2 e_{2,8}^*(s) + p_1q_2\, e_{2,9}^*(s) \\ & + p_2q_1 e_{2,10}^*(s) + q_1q_2 e_{2,11}^*(s) + p_2 e_{3,4}^*(s) + q_2\, e_{3,5}^*(s) \end{aligned}$$

$$+ p_1 e^*_{3,6}(s) + q_1 e^*_{3,7}(s) - p_1p_2\, e^*_{3,8}(s) - p_1q_2\, e^*_{3,9}(s) - p_2q_1\, e^*_{3,10}(s) - q_1q_2\, e^*_{3,11}(s) \} \quad (7)$$

where $e^*_m(s) = L[E'_m(t)]$ and $e^*_{r,s}(s) = L[E'_{r,s}(t)]$

$$\left.\begin{aligned} &\text{For } m = 1,2,3, \text{ note that } e^*_m(s) = \frac{[1-D_m]f^*(s)}{[1-f^*(s)D_m]}. \\ &\text{For } r = 1,2,3,\ s = 4,5,\ldots,11, \\ &e^*_{r,s}(s) = \frac{[1-D_rD_s]f^*(s)}{[1-f^*(s)D_rD_s]} \end{aligned}\right\} \quad (8)$$

By hypothesis

$f^*(s) = \theta/(\theta+s)$ and $g^*(s) = \alpha/(\alpha+s)$ (9)

Since $E(W) = -\left\{\frac{d}{ds}\ell^*(s)\right\}_{s=0}$ (10)

and $E(W^2) = \left\{\frac{d^2}{ds^2}\ell^*(s)\right\}_{s=0}$ (11)

using (4),(7),(8), (9) in (10) and (11) and on simplification one can show that

$$\begin{aligned} E(W) = {} & C_1 + C_2 - C_3 + \beta\{ p_2C_4 + q_2C_5 + p_1C_6 + q_1C_7 - p_1p_2C_8 - p_1q_2C_9 - p_2q_1C_{10} - q_1q_2C_{11} - p_2H_{1,4} - q_2H_{1,5} - p_1H_{1,6} \\ & - q_1H_{1,7} + p_1p_2H_{1,8} + p_1q_2H_{1,9} + p_2q_1H_{1,10} + q_1q_2H_{1,11} - p_2H_{2,4} - q_2H_{2,5} - p_1H_{2,6} - q_1H_{2,7} + p_1p_2H_{2,8} + p_1q_2H_{2,9} \\ & + p_2q_1H_{2,10} + q_1q_2H_{2,11} + p_2H_{3,4} + q_2H_{3,5} + p_1H_{3,6} + q_1H_{3,7} - p_1p_2H_{3,8} - p_1q_2H_{3,9} - p_2q_1H_{3,10} - q_1q_2H_{3,11} \} \end{aligned} \quad (12)$$

$$\begin{aligned} E(W^2) = {} & 2[C_1^2 + C_2^2 - C_3^2 + \beta\{ p_2C_4^2 + q_2C_5^2 + p_1C_6^2 + q_1C_7^2 - p_1p_2C_8^2 - p_1q_2C_9^2 - p_2q_1C_{10}^2 - q_1q_2C_{11}^2 \\ & - p_2H_{1,4}^2 - q_2H_{1,5}^2 - p_1H_{1,6}^2 - q_1H_{1,7}^2 + p_1p_2H_{1,8}^2 + p_1q_2H_{1,9}^2 + p_2q_1H_{1,10}^2 + q_1q_2H_{1,11}^2 - p_2H_{2,4}^2 \\ & - q_2H_{2,5}^2 - p_1H_{2,6}^2 - q_1H_{2,7}^2 + p_1p_2H_{2,8}^2 + p_1q_2H_{2,9}^2 + p_2q_1H_{2,10}^2 + q_1q_2H_{2,11}^2 + p_2H_{3,4}^2 + q_2H_{3,5}^2 \\ & + p_1H_{3,6}^2 + q_1H_{3,7}^2 - p_1p_2H_{3,8}^2 - p_1q_2H_{3,9}^2 + p_2q_1H_{3,10}^2 + q_1q_2H_{3,11}^2 \} \end{aligned} \quad (13)$$

where for m = 1,2,3 $C_m = 1/[\theta(1 - D_m)]$ and

for r = 1,2,3, s = 4,5, …, 11, $H_{r,s} = 1/[\theta(1- D_r D_s)]$

We know that $V(W) = E(W^2) - [E(W)]^2$ (14)

While (12) gives the mean time to recruitment, (12) and (13) together with (14) give the variance of the time for recruitment for the present model.

Model Description and Analysis for Model - II

For model II, the optional and mandatory thresholds for the organization are give by $Y= \min(Y_1,Y_2)$ and $Z= \min(Z_1, Z_2)$. All other assumptions and notations are as in model I.

For this model it can be shown that

$P(S_k < Y) = (D_3)^k$ and

$P(S_k < Z) = p_1p_2(D_8)^k + p_1q_2(D_9)^k + p_2q_1(D_{10})^k + q_1q_2(D_{11})^k$

$$L(t) = E_3(t) + \beta\{ p_1p_2E_8(t) + p_1q_2E_9(t) + p_2q_1E_{10}(t) + q_1q_2E_{11}(t) - p_1p_2E_{3,8}(t) - p_1q_2E_{3,9}(t) - p_2q_1E_{3,10}(t) - q_1q_2E_{3,11}(t) \}$$

$$\ell^*(s) = e^*_3(s) + \beta\{ p_1p_2e^*_8(s) + p_1q_2e^*_9(s) + p_2q_1e^*_{10}(s) + q_1q_2e^*_{11}(s) - p_1p_2e^*_{3,8}(s) - p_1q_2e^*_{3,9}(s) - p_2q_1\, e^*_{3,10}(s) - q_1q_2\, e^*_{3,11}(s) \}$$

$$E(W) = C_3 + \beta\{ p_1p_2C_8 + p_1q_2C_9 + p_2q_1C_{10} + q_1q_2C_{11} - p_1p_2H_{3,8} - p_1q_2H_{3,9} - p_2q_1H_{3,10} - q_1q_2H_{3,11}\} \quad (15)$$

and

$$E(W^2) = 2[C_3^2 + \beta\{ p_1p_2C_8^2 + p_1q_2C_9^2 + p_2q_1C_{10}^2 - q_1q_2C_{11}^2 - p_1p_2H_{3,8}^2 - p_1q_2H_{3,9}^2 - p_2q_1H_{3,10}^2 - q_1q_2H_{3,11}^2\}] \quad (16)$$

(15) gives the mean time to recruitment and (15) and (16) together with (14) give variance of the time for recruitment for the present model.

Numerical Illustration and Conclusions

The analytical expression for expectation and variance of time to recruitment are analyzed numerically by varing parameters. The values of the mean and variance of time to recruitment are calculated for both the models and presented in the first part of table by varing the mean $1/\theta$ of the inter decision time, keeping the other parameters fixed. In the second part of the table the corresponding results are tabulated when the mean $1/\alpha$ of the loss of manhours varies, keeping the other parameters fixed.

Table: Effect of θ and α on performance measures.

($\lambda_1 = 2.0$; $\lambda_2 = 4.0$; $\mu_1 = 0.4$; $\mu_2 = 0.5$, $\delta_1 = 0.6$, $\eta_1 = 0.7$, $\delta_2 = 0.5$, $\eta_2 = 0.6$, $\beta = 0.6$, $p_1 = 0.6$, $q_1 = 0.4$, $p_2 = 0.7$, $q_2 = 0.3$)

θ	α	E(W)		V(W)	
		Model I	Model II	Model I	Model II
0.3	0.6	6.18	4.50	37.29	19.18
0.4	0.6	4.64	3.38	20.98	10.79
0.5	0.6	3.71	2.90	13.43	6.90
0.6	0.6	3.09	2.25	9.32	4.79
0.7	0.6	2.65	1.93	6.85	3.52
0.8	0.6	2.32	1.69	5.24	2.70
0.6	0.3	2.38	1.98	5.76	3.81
0.6	0.4	2.62	2.07	6.87	4.13
0.6	0.5	2.86	2.16	8.05	4.46
0.6	0.6	3.09	2.25	9.32	4.79
0.6	0.7	3.33	2.34	10.68	5.13
0.6	0.8	3.56	2.43	12.12	5.48

From the above table we observe the following which agree with reality

(i) the expected time and the variance of time to recruitment decrease, with the mean of the inter-decision time for models I and II.

(ii) mean and variance of time to recruitment increase when the mean loss of manhours decreases for both the models.

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