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An Inflated Probability Model for the Rural Out-Migration

Himanshu Pandey*, Rajendra Tiwari

Department of Mathematics and Statistics, D.D.U.Gorakhpur University, Gorakhpur, 273009 (India)

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***Corresponding Author**

Tel : +919415282207

Email: himanshu_pandey62@yahoo.com

Abstract

The main objective of the paper is to develop an inflated Probability model for the total number of migrants from a household. The suitability of the model is tested through observed data.

Key Words: Inflated Probability Model , Displaced Geometric Distribution, Log- Series Distribution, Method of Moments, Method of Maximum Likelihood

Introduction

Migration is an important source of major socio-economic change of a nation or region. It is expected that migration incoming decades may play a crucial role in achievement of comprehensive policies and programs regarding spatial distribution of population and hence relating to socio economic cultural and demographic situation of a region particularly in developing countries. It is worth mentioning that the problem of human movement bring policy makers and development–planners to a new level of awareness[1], the relationship between migration and other socio-economic factors, resources environment and sustainable development[2-3] on several planning process at the regional as well as national levels.

The rural out–migration [4-7] in developing countries like India may be considered of two types: one is which an adult male aged is 15 years and above migrate alone leaving their wives and children at home; and second male members migrate with their wives, children and other dependent relatives. It is seen that these two types of migration affect the socio-economic and cultural activities of origin and destination places of the migrants differently.

In this paper, some probability models are proposed to study the distribution of households according to number of both types of cut migrants as mentioned above at the household level.

The main objective of the paper is to develop an inflated Probability model for the total number of migrants from a household. The suitability of the model is tested through observed data.

Model

A probability model[8] to describe the distribution of households according to the total number of migrants has been derived on the basis of the following assumption.

(i) Let α be the probability of migration from a household and $(1-\alpha)$ be the probability of not migration from a household.

(ii) Two types of household are observed at the survey point. In first type of households, only males of fifteen years aged and above migrate alone and in second type of households , a male member migrates with his family . Let π_1 and π_2 be the respective proportions of two types of exposure households to the risk of migration such that

$$\pi_1 + \pi_2 = \alpha$$

(iii) The number of migrants occurred in first type of house-hold follows a displaced geometric distribution with parameter p,i.e.,

$$P[\,Y = K\,] = pq^{K-1} \quad , K = 1, 2, 3, \ldots\ldots, p = 1 - q \qquad (2.1)$$

where Y be the no. of migrants from the first type of household.

(iv) In second type of households migrants moves with their family member and follows a log - series distribution with parameter λ, i.e. ,

$$P[\,Z = K\,] = \left[\frac{-\lambda^K}{K\log(1-\lambda)}\right] \quad ,K = 1,2,3, \ldots \qquad (2.2)$$

where Z be the no. of migrants from second type of household.

Let X is the total number of migrants from a household. Therefore from the above assumption (i) to (iv) , with the help of Johnson and Kotz [9], inflated form of the distribution for the total number of migrants from a household is

$$P[\,X = 0\,] = 1 - \alpha \qquad ,K = 0 \quad \text{and}$$

$$P[X=k]=\pi_1 pq^{K-1}+\pi_2\left[\frac{-\lambda^K}{k\log(1-\lambda)}\right], K=1,2,3\ldots \quad (2.3)$$

Estimation:

Probability model consists of four parameter α, p, λ and π. It is difficult to estimate all these four parameters simultaneously. Therefore supposing $\pi_1 = \pi_2$, the remaining parameters are estimated by the method of moment and method of maximum likelihood.

Method of Moments:

The parameters α, p and λ are estimated by equating Zeroth and first cell theoretical frequencies to the observed frequencies of the respective cells and theoretical mean equal to observe mean as follows.

$$(1-\alpha)=\frac{f_0}{f_1} \quad (2.4)$$

$$\pi_1 p+\pi_2\left[\frac{-\lambda}{\log(1-\lambda)}\right]=\frac{f_1}{f} \quad (2.5)$$

$$\frac{\pi_1}{p}+\pi_2\left[\frac{-\lambda}{(1-\lambda)\log(1-\lambda)}\right]=\bar{X} \quad (2.6)$$

where f_0 denotes the no. of observed zeroth cell, f_1 denotes the no. of observed first cell, f denotes the total no. of observations and $\bar{X}$ denotes the observed mean of the distribution.

Method of Maximum Likelihood: After taking the value of λ from the method of moments and the remaining parameters are estimated by this method. Let X be a random variable from a sample of f observation with the probability function (2.3) then the likelihood function for the given sample can be express as:

$$L=(1-\alpha)^{f_0}\left(\pi_1 p+\pi_2\left[\frac{-\lambda}{\log(1-\lambda)}\right]\right)^{f_1}\left(\alpha-\pi_1 p-\pi_2\left[\frac{-\lambda}{\log(1-\lambda)}\right]\right)^{f-f_0-f_1} \quad (2.7)$$

Let $\left[\frac{-\lambda}{\log(1-\lambda)}\right]=A$

, then the likelihood function become the following form.

$$L=(1-\alpha)^{f_0}\left(\pi_1 p+\pi_2 A\right)^{f_1}\left(\alpha-\pi_1 p-\pi_2 A\right)^{f-f_0-f_1} \quad (2.8)$$

Taking log on both the side, we get

$$\log L=f_0\log(1-\alpha)+f_1\log\left(\pi_1 p+\pi_2 A\right)+(f-f_0-f_1)\log\left(\alpha-\pi_1 p-\pi_2 A\right) \quad (2.9)$$

Now partially differentiating (2.7) with respect to α and p respectively and equating to zero we get the following equation.

$$\frac{\partial}{\partial\alpha}\log L=-\frac{f_0}{(1-\alpha)}+\frac{f-f_0}{\alpha}=0 \quad (2.10)$$

and

$$\frac{\partial}{\partial p}\log L=\frac{f_1}{(p+A)}-\frac{(f-f_0-f_1)}{(2-p-A)}=0 \quad (2.11)$$

By solving the equation (2.10) and (2.11), we get the estimated value of α and p in the following form.

$$\alpha = f-\frac{f_0}{f} \quad (2.12) \text{ and}$$

$$p=\frac{2f_1}{f-f_0}+A \quad (2.13)$$

Now the asymptotic variance of (α, p) is obtained by inverting the information matrix whose elements are negative of second order of the likelihood function.

The second order derivations of the equations (2.10) and (2.11) are:

$$\frac{\partial^2}{\partial\alpha^2}\log L=-\left[\frac{f_0}{(1-\alpha)^2}+\frac{f-f_0}{\alpha^2}\right] \quad (2.14)$$

$$\frac{\partial^2}{\partial p^2}\log L=-\left[\frac{f_1}{(p+A)^2}+\frac{(f-f_0-f_1)}{\left(2-(p+A)\right)^2}\right] \quad (2.15)$$

and

$$\frac{\partial^2}{\partial\alpha\partial p}\log L=\frac{\partial^2}{\partial p\partial\alpha}\log L=0 \quad (2.16)$$

Since $E[f_0]=f(1-\alpha)$, $E[f_1]=f(\pi_1 p+\pi_2 A)$, and

$$E[f-f_0-f_1]=f(\alpha-\pi_1 p-\pi_2 A)$$

where E denotes for the Expectation.

The elements of the information matrix follow from (2.14), (2.15) and (2.16) as:

$$\Phi_{11}=E\frac{\left[-\frac{\partial^2}{\partial\alpha^2}\log L\right]}{f}=\frac{1}{(1-\alpha)}+\frac{1}{\alpha}$$

$$\Phi_{22}=E\frac{\left[-\frac{\partial^2}{\partial p^2}\log L\right]}{f}=\left[\frac{(\pi_1 p+\pi_2 A)}{(p+A)^2}+\frac{(\alpha-\pi_1 p-\pi_2 A)}{\left[2-(p+A)\right]^2}\right]$$

and

$$\Phi_{12}=E\left[\frac{-\frac{\partial^2}{\partial\alpha\partial p}\log L}{f}\right]=\Phi_{21}=E\left[\frac{-\frac{\partial^2}{\partial p\partial\alpha}\log L}{f}\right]=0$$

Therefore by inverting the information matrix, the expression for the asymptotic variances of (α, p) can be obtained as:

$$V(\alpha)=\frac{\left[\frac{\Phi_{22}}{(\Phi_{11}\Phi_{22}-\Phi_{12}^2)}\right]}{f}$$

and

$$V(p)=\frac{\left[\frac{\Phi_{11}}{(\Phi_{11}\Phi_{22}-\Phi_{21}^2)}\right]}{f}$$

Table – 1: Observed and Expected Distribution of Households According to the Total Number of Migrants from a Household in Three Types of Villages

Total No. of Migrants from a Household	Types of Villages: Semi-Urban Observed	Semi-Urban Expected Method Of Moment	Semi-Urban Expected Method Of Likelihood	Remote Observed	Remote Expected Method Of Moment	Remote Expected Method Of Likelihood	Growth-Centre Observed	Growth-Centre Expected Method Of Moment	Growth-Centre Expected Method Of Likelihood
0	1032	1032.00	1032.00	871	871.00	871.00	972	972.00	972.00
1	58	57.93	57.89	139	139.00	138.92	124	124.05	120.56
2	23	24.18	24.16	52	48.52	48.49	32	39.97	38.32
3	6	13.81	13.80	15	25.30	25.29	25	20.82	20.26
4	10	8.97	8.97	14	15.59	15.59	12	13.16	13.26
5	8	6.22	6.23	11	10.35	10.36	10	9.05	9.48
6	7	4.48	4.49	10	7.11	7.13	5	6.41	6.99
7	1	13.41	13.46	6	17.13	17.22	5	18.54	21.13
8^{+}	16			16			17		
Total	1161	1161	1161	1134	1134	1134	1202	1202	1202
α		0.1111	0.1111		0.2319	0.2319		0.1913	0.1913
p		0.2347	0.2341		0.2899	0.2893		0.2765	0.2461
λ		0.5885	-		0.4255	-		0.3681	-
π_1		0.0556	0.0556		0.1159	0.1159		0.0957	0.0957
$V(\alpha)$		-	0.0000 85		-	0.000157		-	0.000129
V(p)		-	0.007669		-	0.003794		-	0.004333
χ^2		7.4815	7.4199		7.2052	7.1248		3.5342	3.0096
d.f.		4	5		4	5		4	5

Table-2: Observed and Expected distribution of Households according to the total number of Migrants in Household Cohort (Including International Migrants/Excluding International Migrants) of the Rural Areas of Comilla district of Bangladesh.

Number of Migrants Per Household	Household Cohort: Including International Migrants Observed	Including Expected Method Of Moment	Including Expected Method Of Likelihood	Excluding International Migrants Observed	Excluding Expected Method Of Moment	Excluding Expected Method Of Likelihood
0	1941	1941.12	1941.12	1941	1941.09	1941..09
1	544	544.07	543.91	292	291.91	292.11
2	117	111.87	111.87	67	70.74	70.76
3	50	49.14	49.18	37	32.16	32.12
4	18	24..50	24.54	17	17.27	17.22
5	8			6		
6	6	25.30	25.38	7	21.83	21.70
7	6			3		
8	6			5		
Total	2696	2696	2696	2375	2375	2375
α		0.2800	0.2800		0.1827	0.1827
P		0.4912	0.4912		0.4356	0.4365
λ		0.0978	-		0.1760	-
π_1		0.14	0.14		0.0914	0.0914
V(a)		-	0.000075		-	0.000063
V(P)		-	0.001293		-	0.002032
χ^2		1.9942	1.7738		0.9577	0.9666
d.f.		2	3		2	3

Table – 3: Observed and Expected distribution of the Household according to the total out-migrants of the Rural Area of Rewa.

Total No. of Migrants from a Household	Household Cohort		
	Observed	Expected	
		Method Of Moment	Method Of Likelihood
0	1470	1470.00	1470.00
1	172	171.75	171.92
2	54	56.59	56.67
3	23	29.39	29.57
4	18	18.66	18.67
5	14	12.74	12.67
6	11	8.81	9.01
7	6		
		24.06	23.49
8^+	24		
Total	1792	1792	1792
α		0.1797	0.1797
P		0.2756	0.2761
λ		0.3845	-
π_1		0.0898	0.0898
$V(\alpha)$		-	0.000082
V(P)		-	0.003092
$\chi2$		3.667	3.9928
d.f.		4	5

Conclusion

The proposed model has been applied to data for total out migration at household level taken three different survey wherein one **"RDPG – a sample survey (1978)",** second is Rural Areas of Camilla districts, Bangladesh and third is **"Rewa Survey – 2002".**After testing the suitability of the model, it is found that the observed value of **$\chi2$** are insignificant at 5% level of significant and hence indicating the suitability of the model. From the table, it is found that the estimated value of the risk of migration under the proposed model are 0.1111, 0.2319 and 0.1913 respectively for "Semi-Urban", "Remote", "Growth Centre", type of villages . So we concluded that the risk of migration of Remote (0.2319) is greater than Growth Centre (0.1913) and Semi-Urban (0.1111).

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