

RRST-Physics

Entropy Change of Non-uniform Medium due to Isothermal Propagation of Strong Spherical Shock Waves

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Abstract

The aim of the present paper is to investigate the entropy production of non-uniform gas atmosphere perturbed by strong spherical shock waves. The study has been carried out for case when the motion of the shock is isothermal. Bhowmick conditions for isothermal propagation of shock are used for the purpose of study. The case are explored for freely propagation of shock as well as in the presence of overtaking disturbances. The variation of entropy with propagation distance r and heat capacity γ are obtained.

Key Words: Spherical weak shock, Isothermal flow, Entropy production

Introduction

In this vast space, there is a great variety of organisms ranging from plants, single-cell bacteria and animals to humans. Therefore, we are facing a very complex system in which physical as well as chemical mechanisms are involved in a highly intricate way. Almost all of these mechanisms are linked to the energy input in the form of solar radiation. Most of the energy received by the biosphere is transformed into heat, which means a destruction of available free energy and a production of entropy. As a first approximation, the free energy destruction in the biosphere may be taken as the difference between the energy delivered by the solar radiation and the energy of the waste heat emitted by the earth. Only a small part of this flow of energy is used by the biosphere for the development of living organisms. We evaluated the flow of energy and entropy and obtained the following figures using [1-4]:

Net energy input into the biosphere: 119 000 TW

Entropy production : around 500 TW/K ($\approx 10^{12}$ Watt/ $^{\circ}$ Kelvin)

Energy consumption by living organisms : 95 TW

The bulk of the energy received by the earth is either reflected or degraded into heat and therefore produces entropy. Yet, based on what has been said above, a small part subtracted from the total flow of energy is utilized by the biosphere and used for the development of life. It is clear that life is related to solar energy, which globally generates negentropy if its degradation into heat is avoided. However, this available free energy must be captured and used to carry out transformations with a decrease in entropy.

There are number of other variety of phenomenon which contributes in the entropy production. The perturbation of non-uniform medium by strong shock waves is the one of them. A considerable amount of papers have appeared in this field. Yadav et al. [5] studied the propagation of diverging shock waves in non-uniform medium in presence of overtaking disturbances and computed temperature variation behind shock front. Yadav and Gangwar [6] investigated the change in entropy of non-uniform medium due to propagation of strong spherical converging shock waves.

The present paper is an analytical study of entropy generation in non-uniform medium perturbed by strong spherical shock waves. Propagation of shock waves are considered in two ways (a) when shock wave propagate freely i.e. propagation of shock wave is not influenced by overtaking disturbances and (b) when the propagation influenced by overtaking disturbances. The case (a) is explored by Chester-Chisnell-Whitham method, a very well known theory in shock dynamics. In the case (b), correction in the CCW theory has been imposed by applying Yadav [6] approach. Assuming non-uniformity of the medium obeys

$$\rho_0 = \rho' r^{-\omega}$$

where ρ' and w are constants, the analytical relations for shock velocity and shock strength have been derived for both the cases. The relations for shock strength so obtained are used to calculate the entropy variation of non-uniform medium perturbed by strong spherical shock waves.

Basic Equations

The equations governing the spherically flow enclosed by the shock front are

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{1}{\rho} \frac{\partial p}{\partial r} = 0 \tag{1}$$

$$\frac{\partial}{\partial t} + u \frac{\partial}{\partial r} \rho + \rho \left(\frac{\partial u}{\partial r} + \frac{2u}{r} \right) = 0 \tag{2}$$

$$\left(\frac{\partial}{\partial t} + u \frac{\partial}{\partial r} \right) p \rho^{-\gamma} = 0 \tag{3}$$

where, $u(r, t)$, $p(r, t)$ and $\rho(r, t)$ denote respectively the particle velocity, the pressure and density at a distance 'r' from the origin at time 't' and ' γ ' is the adiabatic index of gas.

Boundary Conditions

Let p_0 and ρ_0 denote the undisturbed values of pressure and density in front of the shock and p , ρ and u be the values of respective quantities at any point immediately after the passage of shock. The Bhowmick[7] conditions for isothermal flow for strong shock ($M \gg 1$) are

$$u = a_0 M \tag{4a}$$

$$\rho = \rho_0 \gamma M^2 \tag{4b}$$

$$p = \rho_0 a_0^2 M^2 = \gamma p_0 M^2 \tag{4c}$$

$$F = - \frac{p_0 a_0^3 M^2}{2} \tag{4d}$$

where, F is the radiative heat flux. And

$$\frac{\Delta s}{R} = \log (\gamma M^2) \tag{5}$$

Theory

Case1: Freely Propagation of Shock

For spherical diverging shocks, the characteristic form of the system of equations (1)- (3) i.e. the form in which each equation contain derivatives in only one direction in (r, t) plane is

$$dp + \rho a du + \frac{\alpha \rho a^2 u}{(u+a)r} dr = 0 \tag{6}$$

Using the values from equations (4, a), (4, b) and (4, c) in equation (6) and simplifying, we get

$$M = Kr^{-\gamma(\alpha+w/2)/(1+\gamma)} \tag{7}$$

The expression for shock velocity can be written as

$$U = Ma_0 = Ka_0 r^{-\gamma(\alpha+w/2)/(1+\gamma)} \tag{8}$$

Case2: Effect of Overtaking Disturbances

To consider the effect of overtaking disturbances, we have used the differential equation valid across characteristic, given by

$$dp - \rho a du + \frac{\alpha \rho a^2 u}{(u-a)r} dr = 0 \tag{9}$$

Now, substituting values from equations (4, a), (4, b) and (4, c) in equation (9) and applying the condition of overtaking, we get

$du_+ + du_- = du$

$$M^* = K \left[r^{-\gamma(\alpha+w/2)/(1+\gamma)} + r^{-\gamma(\alpha+w/2)} \right] \tag{10}$$

Therefore, the expression for shock velocity can be written as

$$U^* = Ka_0 \left[r^{-\gamma(\alpha+w/2)/(1+\gamma)} + r^{-\gamma(\alpha+w/2)} \right] \tag{11}$$

Result and Discussion

The expression (8) representing shock velocity for freely propagation of strong shock in non-uniform atmosphere shows that shock velocity is a function of propagation distance (r), adiabatic index (γ) and constant (w). Due to overtaking disturbances this expression modifies to (11). Initially, taking $U/a_0=10.0000$ at $r=2.0$ for $\gamma=1.41$ and $w=2.0$, variation of shock velocity with propagation distance (r), adiabatic index(γ) and constant (w) have been shown respectively in Figures 1, 2 and 3 for freely propagation of shock as well as with in presence of overtaking disturbances. It is found that for freely propagation (FP), shock velocity decreases with propagation distance (r) as well as it decreases continuously in case of modified shock velocity (EOD) (cf. fig.- 1), it decreases with adiabatic heat index (γ) for both FP and EOD (cf. fig.-2). This agrees with earlier results (Yadav et al.-[8]). The shock velocity also decreases continuously in case of freely propagation (FP) and overtaking disturbances (EOD) with constant (w) (cf. fig.- 3).

The expression (7) and (10) are obtained respectively in absence and in presence of effect of overtaking disturbances (EOD). The variation of shock strength with propagation distance (r), adiabatic heat index (γ) and constant (w) are presented in figures 4, 5 and 6. The shock strength decreases with propagation distance (r) in absence of overtaking

disturbances as well as in presence of overtaking disturbances (cf. fig.-4). It also decreases with adiabatic index (γ) (cf. fig.-5) and with constant (w) (cf. fig.-6).

The expressions for the pressure behind shock front for freely and modified by overtaking disturbances are derived. They are

$$p = \gamma p_0 K^2 r^{-2\gamma(\alpha+w/2)/(4+\gamma)}$$

and

$$p^* = \gamma p_0 K^2 \left[r^{-2\gamma(\alpha+w/2)/(4+\gamma)} + r^{-2(\alpha+w/2)} \right]$$

These expressions are used to numerically compute the results. The variation of pressure with different parameters such as propagation distance (r), adiabatic index (γ) and constant (w) are shown in figures 7, 8 and 9, respectively. The pressure decreases with all the three indices in absence and in presence of overtaking disturbances (cf. fig.-7, 8 and 9).

The expressions for the particle velocity immediately behind the shock for freely propagation and with inclusion of effect of overtaking disturbances are obtained. They are

$$u = K a_0 r^{-\gamma(\alpha+w/2)/(4+\gamma)}$$

and

$$u^* = K a_0 \left[r^{-\gamma(\alpha+w/2)/(4+\gamma)} + r^{-(\alpha+w/2)} \right]$$

These expressions are used to compute the results. The variations of the particle velocity with propagation distance (r), adiabatic index (γ) and constant (w) are given in figures 10, 11 and 12. The particle velocity decreases with propagation distance (r), adiabatic index (γ) and constant (w) in each case of freely propagation and influence of overtaking disturbances (cf. fig.-10, 11 and 12).

The expressions of entropy change due to freely propagation of shock as well as in presence of overtaking disturbances are given by

$$\frac{\Delta s}{R} = \log \gamma K^2 r^{-2\gamma(\alpha+w/2)/(4+\gamma)}$$

and

$$\frac{\Delta s^*}{R} = \log \gamma K^2 \left[r^{-2\gamma(\alpha+w/2)/(4+\gamma)} + r^{-2(\alpha+w/2)} \right]$$

The variation of entropy change in absence of overtaking and in presence of overtaking disturbances with different parameters (r , γ and w) is shown in figures 13, 14 and 15, respectively. The entropy change decreases with different values of propagation distance (r), adiabatic index (γ) and constant (w) (cf. fig.-13, 14 and 15).

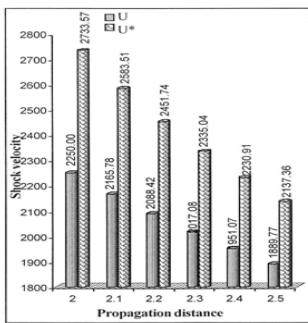


Fig.-1 : Shock velocity (U) Versus Propagation distance (r).

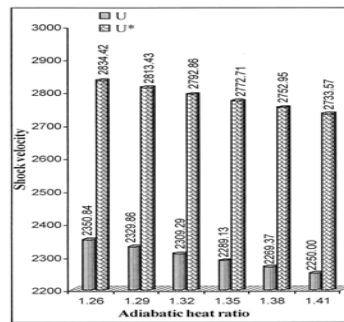


Fig.-2 : Shock velocity (U) Versus Adiabatic heat ratio (gamma).

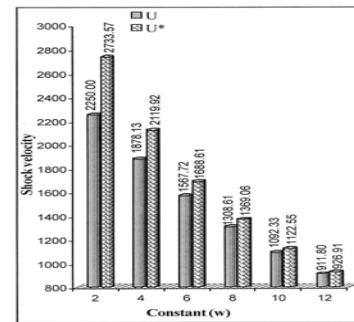


Fig.-3 : Shock velocity (U) Versus Constant (w).

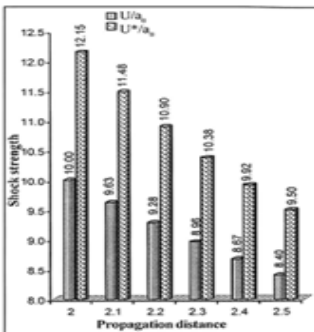


Fig.-4 : Shock strength (U/a_0) Versus Propagation distance (r).

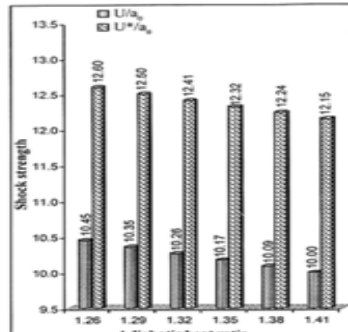


Fig.-5 : Shock strength (U/a_0) Versus Adiabatic heat ratio (gamma).

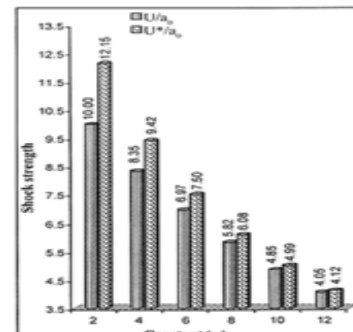


Fig.-6 : Shock strength (U/a_0) Versus Constant (w).

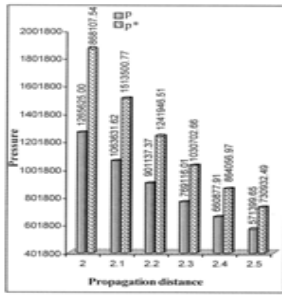


Fig-7 : Pressure (p) Versus Propagation distance (r).

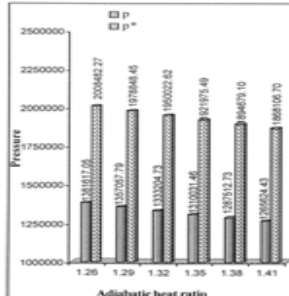


Fig-8 : Pressure (p) Versus Adiabatic heat ratio (gamma).

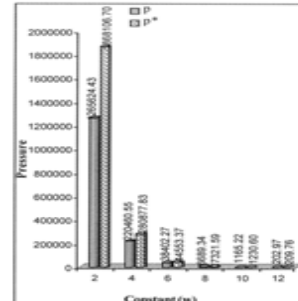


Fig-9 : Pressure (p) Versus Constant (w).

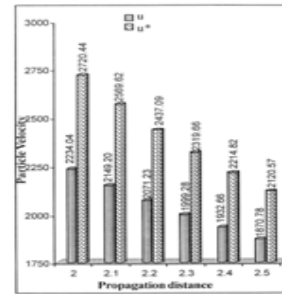


Fig-10 : Particle velocity (u) Versus Propagation distance (r).

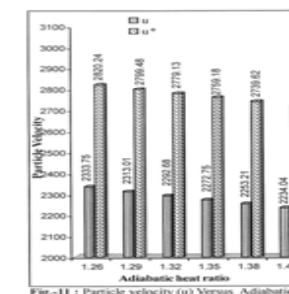


Fig-11 : Particle velocity (u) Versus Adiabatic heat ratio (gamma).

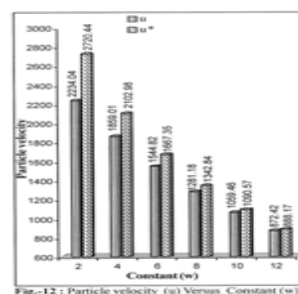


Fig-12 : Particle velocity (u) Versus Constant (w).

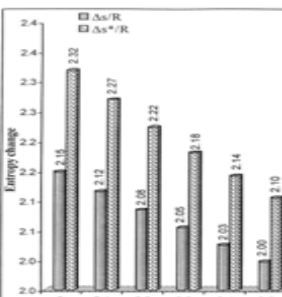


Fig-13 : Entropy change (Delta S/R) Versus Propagation distance (r).

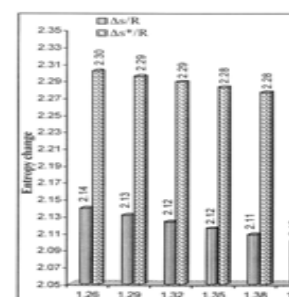


Fig-14 : Entropy change (Delta S/R) Versus Adiabatic heat ratio (gamma).

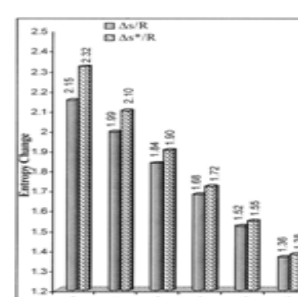


Fig-15 : Entropy change (Delta S/R) Versus Constant (w).

Conclusion

The results of the present study show that shock velocity and shock strength are quite large near the point of explosion. As the shock moves, shock strength and shock velocity decreases sharply with the propagation distance (r) for freely propagation of the shock for isothermal.

In the isothermal case the value of shock strength and shock velocity are large unlike the results reported earlier (Yadav et al. [8]), whereas with the inclusion of overtaking disturbances the results are in agreement with earlier results. The results also show that as the shock moves from the source of explosion, the entropy production is large near the point of explosion and it decreases as shock moves away from the source in isothermal flow.

Therefore, it may be concluded that entropy variation of atmosphere due to isothermal propagation of weak shock is significant in both the cases, when (a) shock moves freely and (b) when it moves under the influence of overtaking disturbances.

References

[1] Strusmia, A., 1986, A detailed Study of entropy jump across shock waves in relativistic fluid dynamics. I.L. Nuova Cimento, 92 B (1), 91-105.

[2] Yadav, R.P. ; Agarwal, P.K. ; Sharma, Atul , 2005, On the entropy propagation due to explosion in sea water. Entropy, 7 (2), 134-147.

[3] Nath, O. , 1988 Self similar flow behind a spherical shock with varying strength in an inhomogeneous medium. Astrophys Sp. Sci, 146, 169-174.

[4] Ojha, S.N., 1987, Self Similar flow behind a spherical shock with varying strength in an inhomogeneous self gravitating medium. Astrophys Sp. Sci., 129, 11-17.

[5] Yadav R.P., Rana D.S., Singh A: 2000, Effect of overtaking disturbances on sound and temperature behind strong shock waves in non-uniform medium., J. Nat. Phy. Sci, 14 (1-2), 49-58.

[6] Yadav, R.P. ; Gangwar, P.K. , 2002, Change in entropy of the universe. Acta Cien. Indica, 28(3), 121-128.

[7] Ray, G.D.; Bhowmick, J.B., 1976, similarity solutions for explosions in radiating Star. Ind. J. Pure App. Maths, 7 (1), 96-103.

[8] Yadav, R.P. ; Agarwal, P.K. ; Sharma, Atul , 2006: On the propagation of blast wave in earth's atmosphere: Adiabatic and isothermal flow, Entropy, 8, 143-168.