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# A Review of Approaches to Model a Risk-averse Newsvendor

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## 1. Introduction

In the SCM literature, the inventory management of one product over one period is often modeled as a newsvendor model. The newsvendor model has been widely studied and used to manage inventory decisions in, for example, the fashion industry (See Khouja 1999; Qin et. al 2011). At the beginning of the selling season, the retailer must choose order quantity  $q$  and receive the  $q$  units with the unit wholesale price  $w$  from a wholesaler. The demand per period of the product  $D$  is uncertain at the time of purchase from the wholesaler. The retailer sells the product at unit retail price  $p$  during the selling season. At the end of the season, the retailer's unsold units can be salvaged for  $v$  per unit. The classical newsvendor model is to find the order quantity that maximizes the retailer's expected profit under the assumption of risk neutrality. Nowadays, the decision makers focus more on risk under an uncertain environment. The assumption of risk neutrality seems not to be applicable to contemporary business environment. Many researchers have studied a risk averseness in newsvendor model especially from the modeling perspective. In this paper, we review the typical four approaches, which are expected utility theory, mean-risk optimization approach, downside risk, and coherent measure of risk, from a modeling perspective and point out that why it is difficult to consider risk attitude in the newsvendor models.

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## 2. Literature review on the risk-averse newsvendor models

We review the risk-averse newsvendor literature in this section. Choi et al. (2011) provide a categorization of the risk-averse newsvendor models. There are four typical approaches to model a risk-averse newsvendor, which are expected utility theory, mean-risk optimization approach, downside risk, and coherent measure of risk.

The expected utility theory of von Neumann and Morgenstern (1944) derives the existence of a non-decreasing utility function from simple axioms about preference relation  $\succsim$  of the decision maker, which are completeness, transitivity, continuity, and independence. In this model, the risk-averse newsvendor maximizes its expected utility function. Lau (1980) is the early paper considering the risk-averse newsvendor maximizing the expected utility. He shows that numerically the optimal order quantity becomes smaller than the risk-neutral one using the utility function approximated by a polynomial. Eeckhoudt et al. (1995) examine the newsvendor model with general utility function. Wang et al. (2009) analyses the classes of utility functions, which are CARA, IARA, and DARA classes, within the expected utility theory framework.

The mean-risk approach is well-known. The mean-variance function categorized in this approach is used in the context of portfolio optimization (Markowitz 1959). A merit of this function is that we can analyze a trade-off between the mean outcome and the variance as its measure of risk. It is also known that the utility function can be approximated by the mean-variance (MV) function if it is quadratic or if it is normally distributed. Chen and Federgruen (2000) model the risk-averse newsvendor in MV model, assuming that the newsvendor's utility function is quadratic. Choi et al (2008) consider the risk-averse players who maximize the expected profit subject to the standard deviation of the profit constraint. Lau and Lau (1999) also study the MV model numerically. Wu et al. (2009) analyze the MV model including stockout cost. Anvari (1987) uses the capital asset pricing model (CAPM) to study a newsvendor facing normal demand distribution. The mean-standard deviation (MS) value function considered in Tsay (2002) and Ohmura and Matsuo (2012a, 2012b) uses the standard deviation as its risk measure, and it is categorized as a mean-risk approach. Although it is not used much in the literature, Tsay (2002) uses this MS function and refers to Bar-Shira and Finkelshtain (1999), which argue that using a value function

that increases in mean and decreases in standard deviation is more robust than the approaches based on expected utility. Chiu and Choi (2013) review the literature focusing on mean-variance analytical models.

Some studies use the downside risk measures. Chance constrained programming is introduced in the field of stochastic programming (Charnes and Cooper, 1959). The Chance constraints are now called as Value-at-Risk (VaR), and are used in newsvendor type formulations as their constraints, limiting the probability of particular events happening. Gan et al. (2005) consider the risk-averse newsvendor with downside risk constraint in the context of supply chain coordination. Tapiero (2005) analyzes VaR inventory management as the “regret-disappointment model” in decision theory.

Recently, some studies use the Conditional Value-at-Risk (CVaR) to model the risk-averse newsvendor. The Conditional Value-at-Risk (CVaR) measures the average value of the profit falling below a certain percentile level. It is also called mean excess loss or tail VaR or expected shortfall. Artzner et al. (1998) suggest four axioms that a risk measure should satisfy, and the risk measure is called a coherent measure of risk if it satisfies the four axioms. CVaR is a coherent measure of risk. And CVaR has better computational characteristics than VaR. Ahmed et al. (2007) solve the CVaR maximization problem for the newsvendor model and shows the existence of an optimal solution. Choi and Ruszczyński (2008) also uses the CVaR and shows that CVaR actually represents a trade-off between the expected profit and a certain risk measure, and thus can be regarded as a special mean-risk criterion. Chen et al. (2009) provides conditions where there exists optimal price and order quantity for both additive and multiplicative demand models.

Krokhmal et al. (2011) surveys the most recent decision making model under uncertainty. Although they do not focus on the newsvendor model, various approaches to decision making and optimization under uncertainty in management science and operations research are presented.

### 3. Risk-neutral newsvendor models

The demand per period of the product is  $D$ , with pdf  $f(D)$  and cdf  $F(D)$ .  $F(D)$  is differentiable, strictly increasing and  $F(D) = 0$ . Let  $D > 0$  and  $\bar{F}(D) = 1 - F(D)$ . At the beginning of the selling season, the retailer must choose order quantity  $q$  and receive the  $q$  units with the unit wholesale price  $w$  from a wholesaler. The retailer sells the product at

unit retail price  $p$  during the selling season. At the end of the season, the retailer's unsold units can be salvaged for  $v$  per unit. To avoid trivial cases, we assume  $v < w < p$ . If the realized demand  $D$  is greater than  $q$ , the retailer incurs unit shortage penalty cost  $s$  for the unsatisfied demand  $x - D$ . The retailer's payoff function,  $\pi_R(q)$ , is represented as follows:

$$\pi_R(q, D) = \begin{cases} \pi_R^-(q) = (p - w)D - (w - v)(q - D) & \text{if } D \leq q \\ \pi_R^+(q) = (p - w)q - s(D - q) & \text{if } D > q \end{cases}$$

The newsvendor model is to find an optimal order quantity  $q^*$  that maximizes the retailer's expected profit. That is, the retailer maximizes the following expected profit.

$$E(\pi_R(q, D)) = \int_0^q \pi_R^-(q) f(D) dD + \int_q^\infty \pi_R^+(q) f(D) dD \quad (1)$$

Since  $F$  is strictly increasing,  $E(\pi_R(q))$  is strictly concave and the optimal order quantity  $q^*$  is uniquely determined. The optimal  $q^*$  satisfies the following first-order condition.

$$(w - v)F(q^*) = (p - w + s)\bar{F}(q^*) \quad (2)$$

Equation (2) shows that the retailer determines the order quantity by balancing the cost of being overstocked and the cost of being understocked. If the retailer overstocks, then the retailer loses  $w - v$  per unit of unsold inventory. This cost is called as overage cost denoted as  $c_o$ . If the retailer understocks, then the retailer incurs the opportunity loss  $p - w$  and the shortage penalty cost  $s$ . This cost is called as underage cost denoted as  $c_u$ . From (2),  $q^*$  satisfies following equation.

$$F(q^*) = \frac{c_u}{c_u + c_o} = \frac{p + s - w}{p + s - v} \quad (3)$$

The right hand side of Equation (3) is known as the critical fractile.

#### 4. Risk-averse newsvendor models

In the risk-averse case, the retailer maximizes its own risk-averse objective function

instead of the expected profit in (1). In the SCM literature, there are four typical approaches to model risk-averse decision making. They are expected utility theory, mean-risk optimization approach, downside risk, and coherent measure of risk. We describe these models in detail.

#### 4.1. Expected utility theory

The expected utility theory of von Neumann and Morgenstern (1944) derives the existence of a non-decreasing utility function from simple axioms about preference relation  $\succsim$  of the decision maker, which are completeness, transitivity, continuity, and independence. In the maximization context, risk-averse decision makers have concave and non-decreasing utility functions and maximize it.

The function  $u(W)$  defines the retailer's utility of the final wealth  $W$  and  $W_0$  is the initial wealth. The retailer's payoff function,  $\pi_R(q)$ , is represented as follows:

$$\pi_R(q) = \begin{cases} \pi_R^-(q) = W_0 + (p - w)D - (w - v)(q - D) & \text{if } D \leq q \\ \pi_R^+(q) = W_0 + (p - w)q - s(D - q) & \text{if } D > q \end{cases}$$

The retailer maximizes the expected utility represented as follows:

$$E(u(\pi_R(q))) = \int_0^q u(\pi_R^-(q))f(D)dD + \int_q^\infty u(\pi_R^+(q))f(D)dD \quad (4)$$

The first and second order derivatives of (4) are as follows:

$$\frac{dE(u(\pi_R(q)))}{dq} = -(w - v) \int_0^q u'(\pi_R^-(q))f(D)dD + (p + s - w) \int_q^\infty u'(\pi_R^+(q))f(D)dD \quad (5)$$

$$\begin{aligned} \frac{d^2E(u(\pi_R(q)))}{dq^2} = & -(w - v)u'(\pi_R^-(q, q))f(q) + (w - v)^2 \int_0^q u''(\pi_R^-(q))f(D)dD \\ & - (p + s - w)u'(\pi_R^+(q, q))f(q) + (p + s - w)^2 \int_q^\infty u''(\pi_R^+(q))f(D)dD \end{aligned} \quad (6)$$

where  $\pi_R^-(q, q) = \pi_R^+(q, q) = W_0 + (p - w)q$ . Since  $w > v$ ,  $p > w$  and  $u''(W) < 0$ ,  $d^2E(u(\pi_R(q)))/dq^2 < 0$ . In addition,  $dE(u(\pi_R(0)))/dq > 0$ . Thus, there exists a unique optimal order quantity  $q^*$  that satisfies the first order condition for  $0 < q^*$ . The optimal order quantity  $q^*$  satisfies the

following.

$$(w - v) \int_0^{q^*} u'(\pi_R^-(q^*)) f(D) dD = (p + s - w) \int_{q^*}^{\infty} u'(\pi_R^+(q^*)) f(D) dD \quad (7)$$

Using the conditional probability, Equation (7) can be rewritten as follows:

$$(w - v) F(q^*) E(u'(\pi_R^-(q^*)) | D \leq q^*) = (p + s - w) \bar{F}(q^*) E(u'(\pi_R^+(q^*)) | D > q^*) \quad (8)$$

Compare (8) with (2). When maximizing the expected utility, the risk-averse retailer determines the order quantity by balancing the expected marginal utility in addition to the cost of underage  $c_u$  and overage  $c_o$ . The risk-averse retailer with the utility function orders less than it the risk-neutral orders, since  $E(u'(\pi_R^-(q^*)) | D \leq q^*) > E(u'(\pi_R^+(q^*)) | D > q^*)$  for  $u''(W) < 0$ . An increase in risk aversion is equivalent to a concave transformation of the utility function. Thus, replacing the  $u(\pi_R)$  with  $k(u(\pi_R))$  in Equation (5), where  $k'(\cdot) > 0$  and  $k''(\cdot) < 0$ , the optimal order quantity  $q^{k*}$  satisfies the following first order condition.

$$\begin{aligned} -(w - v) \int_0^{q^{k*}} k'(u(\pi_R^-(q^{k*}))) u'(\pi_R^-(q^{k*})) f(D) dD \\ + (p + s - w) \int_{q^{k*}}^{\infty} k'(u(\pi_R^+(q^{k*}))) u'(\pi_R^+(q^{k*})) f(D) dD = 0 \end{aligned} \quad (9)$$

Replacing  $q^{k*}$  with  $q^*$  yields

$$\begin{aligned} -(w - v) \int_0^{q^*} k'(u(\pi_R^-(q^*))) u'(\pi_R^-(q^*)) f(D) dD \\ + (p + s - w) \int_{q^*}^{\infty} k'(u(\pi_R^+(q^*))) u'(\pi_R^+(q^*)) f(D) dD \\ < k'(u(\pi_R(q^*, q^*))) \left[ -(w - v) \int_0^{q^*} u'(\pi_R^-(q^*)) f(D) dD \right. \\ \left. + (p + s - w) \int_{q^*}^{\infty} u'(\pi_R^+(q^*)) f(D) dD \right] = 0 \end{aligned} \quad (10)$$

where  $\pi_R(q^*, q^*)$  is the retailer's payoff for  $D = q^*$ . Equation (10) implies  $q^{k*} > q^*$ . Therefore the optimal order quantity will decrease as risk aversion increases.

Eeckhoudt et al. (1995) examine the newsvendor model with general utility function.

They use the newsvendor model including emergency order after demand realization and shows that risk aversion leads to lower order quantities. They also conduct comparative static analysis of retail price and cost (wholesale price in our work). They show that increasing the retail price and cost can affect the order quantity in both directions. In risk-neutral case, order quantity increases as retail price increases and cost decreases, which is clear in Equation (3). However, the effect on the marginal utility makes the optimal decision more complicated as shown in (8).

Wang et al. (2009) analyses this complexity with classes of utility functions within the expected utility theory framework. Utility functions are commonly classified into three categories of the Arrow-Pratt measure of absolute risk aversion represented as  $r(W) = -u''(W)/u'(W)$ . If  $dr(W)/dW < 0$ , then the utility function called as decreasing absolute risk aversion (DARA) utility function. If  $dr(W)/dW = 0$ , then the utility function called as constant absolute risk aversion (CARA) utility function. If  $dr(W)/dW > 0$ , then the utility function called as increasing absolute risk aversion (IARA) utility function. Wang et al. (2008) show that for utility functions except for the some unbounded DARA, the risk-averse newsvendor decreases the order quantity as retail price becomes greater once the price is beyond a threshold value. This result is not consistent with the result based on risk neutrality and is not intuitive. They show that an unbounded DARA utility function such as the logarithmic and power function can avoid this anomalous result. Although such functions are subject to another counterintuitive result known as the St. Petersburg paradox, they claim that the choice of such unbounded DARA utility functions is one alternative for managers facing the risk-averse newsvendor problem.

In the application of utility function to supply chain coordination, the manufacturer's decision on the wholesale price is not a simple problem. Depending on the retailer's form of utility function, the order quantity might increase or decrease in response to the wholesale price increase. It is important to identify the utility function, but the utility function is too conceptual to identify. There is an advantage of the use of risk measurement over the use of utility function.

#### **4. 2. Mean-risk optimization approaches**

Mean-risk analysis is developed in finance especially in portfolio management. It quantifies the problem in a form of mean which is the expected value of the outcome and risk which is variability of the outcome. In the newsvendor problem, the retailer solves the

following minimization problem with risk measure  $\rho(\cdot)$  and certain predefined profit level  $\pi_0$ .

$$\min_q \rho(\pi_R(q)) \quad \text{subject to} \quad E(\pi_R(q)) \geq \pi_0 \quad (11)$$

Alternatively, the following maximization formulation is employed with certain predefined risk level  $\rho_0$ .

$$\max_q E(\pi_R(q)) \quad \text{subject to} \quad \rho(\pi_R(q)) \leq \rho_0 \quad (12)$$

Or maximization of a weighted combination of risk and expected profit is used.

$$\max_q E(\pi_R(q)) - k * \rho(\pi_R(q)) \quad (13)$$

Mean-variance (MV) model formulated by Markowitz (1959) is widely used today. Using this function, we can analyze a trade-off between the mean outcome and the variance as its measure of risk. It is also known that the maximization of utility function is equivalent to the maximization of mean-variance function under the assumptions of the CARA utility function and normally distributed profit. And it can be approximated by maximization of the mean-variance function if it is quadratic or if the profit is normally distributed. Thus, the mean-variance function is often used in finance and economics. However, in newsvendor problem it is difficult to satisfy such assumptions primary because of kinked profit function. Figure 1 shows that the retailer's profit with respect to the realized demand at order

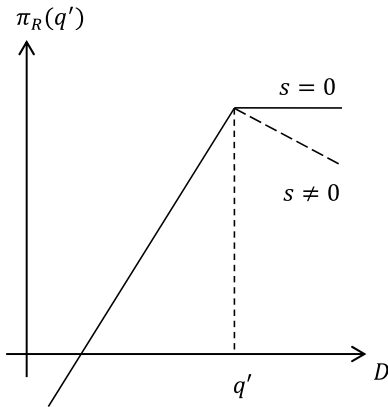


Figure 1: The retailer's profit with respect to the realized demand at order quantity  $q'$

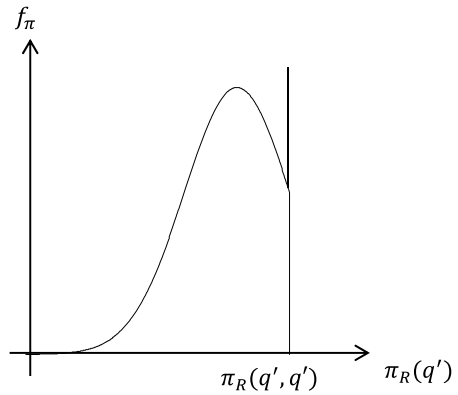


Figure 2: The distribution of the retailer's profit



quantity  $q'$ . Figure 2 is the distribution of the retailer's profit. As shown in Figure 2, even if the demand is normally distributed, the profit is not normally distributed. In addition, the form of the profit distribution changes depending on the retailer's decision variable  $q'$ .

Chen and Federgruen (2000) model the risk-averse newsvendor in the mean-variance criteria, assuming that the newsvendor's utility function is quadratic. They show that the risk-averse newsvendor's decision might be different, depending on whether the objective is profit maximization or cost minimization, which is equivalent in risk-neutral model. The reason is as follows. The distribution of profit is bounded by the profit when the order quantity is equal to the demand, that is,  $\pi_R(q', q')$  in Figure 2. On the other hand, the newsvendor incurs the opportunity cost when the demand exceeds the order quantity. Thus, the distribution of cost is not bounded and the probability of the demand exceeding the order quantity affects the variance of cost. This inconsistency between the profit variance and the cost variance causes the difference of the newsvendor decision. When the mean-variance is applied for the profit maximization, they show that the order quantity is less than it in the risk-neutral case. When it is applied for the cost minimization, they show that the order quantity might be greater than it in the risk-neutral case, depending on the parameter of demand distribution.

Although the consistency between the utility function and the mean-variance formulation exists under the limited assumption mentioned above in newsvendor model, Van Mieghem (2003) claims that the benefits of mean-variance formulation are implementable and useful. It is implementable because only two moments are required and it is useful because it provides "good recommendations," even when the decision maker does not know her utility function. He also quotes the following statement in Markowitz (1991).

*We seek a set of rules which investors can follow in fact—at least investors with sufficient computational resources. Thus we prefer an approximate method which is computationally feasible to a precise one which cannot be computed. I believe that this is the point at which Kenneth Arrow's work on the economics of uncertainty diverges from mine. He sought a precise and general solution. I sought as good an approximation as could be implemented. I believe both lines of inquiry are valuable.*

In that sense, use of the model based on mean-variance formulation is valid for the contract model in the SCM context which seeks to provide guidance in negotiating the terms of the relationship between buyer and seller. The mean-risk approaches using

another risk measure instead of variance have the same benefits and the meaning in the SCM research.

Although the mean-risk approaches have benefits in the SCM research, it is difficult to analyze the effect of the risk aversion on the supply chain because of complexity of the equation. We cannot obtain the closed form of optimal order quantity in general. Thus, the analysis is often conducted numerically. Ohmura (2014) shows the interaction of the risk aversion analytically as much as possible and do numerical analysis using the mean-standard deviation and mean-variance formulation.

#### **4.3. Downside risk approach**

Variance and standard deviation treat over-performance equally as under-performance. It is natural to exclude the upside deviation as risk measurement, because the upside deviation is not undesirable deviation. Downside risk measurement only captures the undesirable deviation. Value-at-Risk (VaR) is a popular downside risk measurement and widely used in practice, since it is considered as a risk measure in the 2001 proposal of the Basel Banking Supervisory Committee. Gan et al. (2005) model the risk-averse newsvendor with VaR constraint. VaR constraint is known as a chance constraint in the Operations Research literature. It was formulated in the well-known paper of Charnes and Cooper (1959). And VaR constraint is often used in finance. Under VaR constraint, the retailer's problem is as follow:

$$\max_q E(\pi_R(q)) \text{ subject to } P\{\pi_R(q) \leq \alpha\} \leq \beta$$

where  $\alpha$  is the retailer's target profit level and the probability that his profit fall below  $\alpha$  is restricted under the probability  $\beta$ . Note that for risk-aversion pair  $(\alpha_1, \beta_1)$  and  $(\alpha_2, \beta_2)$ , if  $\alpha_1 \leq \alpha_2$  and  $\beta_1 \geq \beta_2$ , then  $(\alpha_2, \beta_2)$  means a higher risk aversion than  $(\alpha_1, \beta_1)$ . Although VaR constraint is intuitive and implementable in practice, we need two parameters for analysis of the risk sensitivity. Gan et al. (2005) avoid this problem, setting the target profit  $\alpha$  at the expected profit of the risk-neutral newsvendor. This kind of techniques is required for analysis of the risk sensitivity. And then VaR itself does not seem to be appropriate as a risk measure, since VaR is not a coherent measure of risk.

#### 4.4. Coherent measure of risk

Artzner et al. (1998) suggest four axioms that a risk measure should satisfy, and the risk measure is called a coherent measure of risk if it satisfies the four axioms. The axiomatic approach has become the dominant framework in risk analysis, and has been used in SCM literature. The risk measure  $\rho(\cdot)$  is called coherent if and only if it satisfying the following four axioms.

- *Convexity* :  $\rho(\lambda X + (1 - \lambda)Y) \leq \lambda \rho(X) + (1 - \lambda)\rho(Y)$  for all random variables  $X$  and  $Y$ , and all  $\lambda \in [0,1]$ ;
- *Monotonicity* : If  $X \succcurlyeq_{\text{FSD}} Y$ , then  $\rho(X) \leq \rho(Y)$ ; ( $X \succcurlyeq_{\text{FSD}} Y$  means that  $F_X(z) \leq F_Y(z)$  for all  $z$  where  $F_X(\cdot)$  is cdf of  $X$  and  $F_Y(\cdot)$  is cdf of  $Y$ .)
- *Translation Equivariance* :  $\rho(X + a) = \rho(X) + a$  for  $a \in \mathbb{R}$ ;
- *Positive Homogeneity* :  $\rho(tX) = t\rho(X)$  for  $t \geq 0$ ;

CVaR is a coherent measure of risk. In addition, CVaR has computational advantage in the newsvendor model as shown later. The retailer having CVaR measure criterion maximizes the following function.

$$CVaR_R(\pi_R(q)) = E(\pi_R(q) \mid \pi_R(q) \leq q_{\eta_R}(\pi_R(q)))$$

where  $q_{\eta_R}(\pi_R(q))$  is the  $\eta_R$ -quantile of the retailer's profit. Thus,  $\eta_R \in (0,1]$  and it reflects the degree of risk-aversion for the retailer. When  $\eta_R = 1$ , the retailer is risk neutral. As  $\eta_R$  decreases, the retailer is more risk-averse. Rockafellar and Uryasev (2000, 2002) and Pflug (2006) show the following equivalent definition.

$$CVaR_R(\pi_R(q)) = \max_{\tau \in \mathbb{R}} \left\{ \tau + \frac{1}{\eta_R} E(\min(\pi_R(q) - \tau, 0)) \right\} \quad (14)$$

This also can be rewritten as follows:

$$CVaR_R(\pi_R(q)) = E(\pi_R(q)) - \frac{1}{\eta_R} \rho_{\eta_R}(\pi_R(q)) \quad (15)$$

where  $\rho_{\eta_R}$  is a risk measure which represents the weighted mean deviation from quantile:

$$\rho_{\eta_R}(\pi_R(q)) = \min_{\tau \in \mathbb{R}} E(\max\{(1 - \eta_R)(\tau - \pi_R(q)), \eta_R(\pi_R(q)) - \tau\})$$

The relation in (15) allows us to interpret CVaR as a mean-risk model (Choi and Ruszczyński, 2008). If the shortage penalty  $s = 0$ , then it is known that the optimal order quantity  $q^*$  satisfies following equation.

$$F(q^*) = \eta_R \frac{p - w}{p - v} \quad (16)$$

We can obtain the optimal order quantity in closed form. And it is clear that as the retailer is more risk-averse, the order quantity decreases. This computational advantage is a reason why CVaR model is used recently in the SCM literature.

When the shortage penalty  $s \neq 0$ , the problem is complex because of the distribution of the profit. Considering the shortage penalty, there is no one-to-one mapping between the realization of demand and the resulting profit (See Figure 2). Thus, the optimization requires repeated resorting of profit. Fichtinger (2010) discuss the inventory control with shortage penalty and analyze the newsvendor model under CVaR criterion with shortage penalty.

## 5. Conclusion

There are four typical approaches to model a risk-averse newsvendor. These are expected utility theory, mean-risk optimization approach, downside risk, and coherent measure of risk. In this paper, we have provided a detail review of these approaches within the context of a single player model. The expected utility theory is a major theory in decision making under uncertainty. However, depending on the player's form of utility function, the order quantity might increase or decrease in response to the wholesale price increase. Thus, it is important to identify the utility function, but the utility function is too conceptual to identify. The mean-risk approach is widely used in finance. It is implementable because only two moments are required and it is useful because it provides "good recommendations," even when the decision maker does not know her utility function. However, it is difficult to analyze the effect of risk aversion analytically. Variance and standard deviation treat over-performance equally as under-performance. It is natural to

exclude the upside deviation as risk measurement, because the upside deviation is not undesirable deviation. Downside risk measurement is more desirable risk measurement than variance and standard deviation, since it only captures the undesirable deviation. However, VaR is not a coherent measure of risk. Artzner et al. (1998) suggest four axioms that a risk measure should satisfy, and the risk measure is called a coherent measure of risk if it satisfies the four axioms. The axiomatic approach has become the dominant framework in risk analysis, and has been used in the SCM literature. CVaR is a coherent measure of risk. In addition, CVaR has computational advantage in the newsvendor model. These approaches have both merits and demerits in their concept and application.

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