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POSITIVE SOLUTIONS FOR A CLASS OF INFINITE SEMIPOSITONE PROBLEMS INVOLVING THE p-LAPLACIAN OPERATOR

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We discuss the existence of a positive solution to the infinite semipositone problem

$-\Delta_p u = au^{p-1} - bu^{\gamma} - f(u) - \frac{c}{u^{\alpha}}, \ x \in \Omega, \quad u = 0, \ x \in \partial\Omega,$

where Δ_p is the p-Laplacian operator, $p > 1$, $\gamma > p-1$, $\alpha \in (0,1)$, a,b and c are positive constants, Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, and $f : [0,\infty) \to \mathbb{R}$ is a continuous function such that $f(u) \to \infty$ as $u \to \infty$. Also we assume that there exist $A > 0$ and $\beta > p-1$ such that $f(s) \leq As^{\beta}$, for all $s \geq 0$. We obtain our result via the method of sub- and supersolutions.

1. Introduction

We consider the positive solution to the boundary value problem

$$\begin{cases} -\Delta_p u = au^{p-1} - bu^{\gamma} - f(u) - \dfrac{c}{u^{\alpha}}, & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases} \tag{1}$$

where Δ_p denotes the p-Laplacian operator defined by $\Delta_p z = \operatorname{div}(|\nabla z|^{p-2}\nabla z)$, $p > 1$, $\gamma > p-1$, $\alpha \in (0,1)$, a,b and c are positive constants, Ω is a bounded

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domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, and $f : [0,\infty) \to \mathbb{R}$ is a continuous function. We make the following assumptions:

(H1) $f : [0,+\infty) \to \mathbb{R}$ is a continuous function such that $\lim_{s\to+\infty} f(s) = \infty$.

(H2) There exist $A > 0$ and $\beta > p-1$ such that $f(s) \leq As^\beta$, for all $s \geq 0$.

In [8], the authors have studied the equation $-\Delta u = g(u) - (c/u^\alpha)$ with Dirichlet boundary conditions,where g is nonnegative and nondecreasing and $\lim_{u\to\infty} g(u) = \infty$. The case $g(u) := au - f(u)$ has been study in [7], where $f(u) \geq au - M$ and $f(u) \leq Au^\beta$ on $[0,\infty)$ for some $M, A > 0, \beta > 1$ and this g may have a falling zero. In this paper, we study the equation $-\Delta_p u = au^{p-1} - bu^\gamma - f(u) - (c/u^\alpha)$ with Dirichlet boundary conditions. Our result in this paper include the result of [7] in the case $p = 2$ (Laplacian operator), where say in Remark 2.2. Let $F(u) := au^{p-1} - bu^\gamma - f(u) - (c/u^\alpha)$, then $\lim_{u\to 0^+} F(u) = -\infty$ and hence we refer to (1) as an infinite semipositone problem. In fact, our result in this paper is on infinite semipositone problems involving the p-Laplacian operator.

In recent years, there has been considerable progress on the study of semipositione problems ($F(0) < 0$ but finite) (see [1],[2],[5]). We refer to [6], [7], [8] and [9] for additional results on infinite semipositone problems. We obtain our result via the method of sub- and supersolutions([3]).

2. The main result

We shall establish the following result.

Theorem 2.1. *Let* (H1) *and* (H2) *hold. If* $a > (\frac{p}{p-1+\alpha})^{p-1}\lambda_1$, *then there exists positive constant* $c^* := c^*(a, A, p, \alpha, \beta, \gamma, \Omega)$ *such that for* $c \leq c^*$, *problem* (1) *has a positive solution, where* λ_1 *be the first eigenvalue of the p-Laplacian operator with Dirichlet boundary conditions.*

Remark 2.2. Theorem 2.1 was established in [7] for the case $p = 2$ (the Laplacian operator), $f(u) := g(u) - bu^\gamma$, where the function g satisfy the following assumptions:
• $g(u) \approx bu^\theta$ for some $\theta > \gamma$.
• There exist $A > 0$ and $\beta > 1$ such that $g(u) \leq Au^\beta$, for all $u \geq 0$.
• There exist $M > 0$ such that $g(u) \geq au - M$, for all $u \geq 0$.
In fact, the function f satisfy the hypotheses of Theorem 2.1 in this paper (Since $\lim_{u\to\infty} (g(u)/bu^\theta) = 1$, hence $\lim_{u\to\infty} f(u) = \infty$) and g satisfy the hypotheses of Theorem 2.1 in [7], where (1) changes to equation $-\Delta u = au - g(u) - (c/u^\alpha)$ with Dirichlet boundary conditions.

Proof of Theorem 2.1. We shall establish Theorem 2.1 by constructing positive sub-supersolutions to equation (1). From an anti-maximum principle (see [4, pages 155-156]), there exists $\sigma(\Omega) > 0$ such that the solution z_λ of

$$\begin{cases} -\Delta_p z - \lambda z^{p-1} = -1, & x \in \Omega, \\ z = 0, & x \in \partial\Omega, \end{cases}$$

for $\lambda \in (\lambda_1, \lambda_1 + \sigma)$ is positive in Ω and is such that $\frac{\partial z}{\partial \nu} < 0$ on $\partial\Omega$, where ν is outward normal vector on $\partial\Omega$. Fix $\lambda^* \in (\lambda_1, \min\{\lambda_1 + \sigma, (\frac{p-1+\alpha}{p})^{p-1} a\})$ and let

$$K := \min\Big\{ \Big(\frac{(p/p-1+\alpha)^{p-1}}{2b \|z_{\lambda^*}\|_\infty^{\frac{\gamma p-(p-1)(\alpha-1)}{p-1+\alpha}}} \Big)^{\frac{1}{\gamma-p+1}}, \Big(\frac{a - (\frac{p}{p-1+\alpha})^{p-1}\lambda^*}{3b \|z_{\lambda^*}\|_\infty^{\frac{p(\gamma-p+1)}{p-1+\alpha}}} \Big)^{\frac{1}{\gamma-p+1}},$$
$$\Big(\frac{(p/p-1+\alpha)^{p-1}}{2A \|z_{\lambda^*}\|_\infty^{\frac{\beta p-(p-1)(\alpha-1)}{p-1+\alpha}}} \Big)^{\frac{1}{\beta-p+1}}, \Big(\frac{a - (\frac{p}{p-1+\alpha})^{p-1}\lambda^*}{3A \|z_{\lambda^*}\|_\infty^{\frac{p(\beta-p+1)}{p-1+\alpha}}} \Big)^{\frac{1}{\beta-p+1}} \Big\}$$

Define $\psi = K z_{\lambda^*}^{\frac{p}{p-1+\alpha}}$. Then

$$\nabla\psi = K(\frac{p}{p-1+\alpha}) z_{\lambda^*}^{\frac{1-\alpha}{p-1+\alpha}} \nabla z_{\lambda^*}$$

and

$$\begin{aligned}
-\Delta_p \psi &= -\operatorname{div}(|\nabla\psi|^{p-2}\nabla\psi) \\
&= -K^{p-1}(\frac{p}{p-1+\alpha})^{p-1} \Big\{ (\tfrac{(p-1)(1-\alpha)}{p-1+\alpha}) z_{\lambda^*}^{\frac{-\alpha p}{p-1+\alpha}} |\nabla z_{\lambda^*}|^p + z_{\lambda^*}^{\frac{1-\alpha}{p-1+\alpha}} \Delta_p z_{\lambda^*} \Big\} \\
&= -K^{p-1}(\frac{p}{p-1+\alpha})^{p-1} \Big\{ (\tfrac{(p-1)(1-\alpha)}{p-1+\alpha}) z_{\lambda^*}^{\frac{-\alpha p}{p-1+\alpha}} |\nabla z_{\lambda^*}|^p + z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}} (1 - \lambda^* z_{\lambda^*}^{p-1}) \Big\} \\
&= K^{p-1}(\frac{p}{p-1+\alpha})^{p-1} \left\{ \lambda^* z_{\lambda^*}^{\frac{p(p-1)}{p-1+\alpha}} - z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}} - \Big((\tfrac{(p-1)(1-\alpha)}{p-1+\alpha}) \Big) \frac{|\nabla z_{\lambda^*}|^p}{z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}} \right\}
\end{aligned}$$

Let $\delta > 0$, $\mu > 0$, $m > 0$ be such that $|\nabla z_{\lambda^*}|^p \geq m$ in $\overline{\Omega}_\delta$ and $z_{\lambda^*} \geq \mu$ in $\Omega \setminus \overline{\Omega}_\delta$, where $\overline{\Omega}_\delta := \{x \in \Omega : d(x, \partial\Omega) \leq \delta\}$. Let

$$c^* := K^{p-1+\alpha} \min\Big\{ (\tfrac{p}{p-1+\alpha})^{p-1} \Big((\tfrac{(p-1)(1-\alpha)}{p-1+\alpha}) \Big) m^p, \tfrac{1}{3}\mu^p \Big(a - (\tfrac{p}{p-1+\alpha})^{p-1}\lambda^* \Big) \Big\}.$$

Let $x \in \overline{\Omega}_\delta$ and $c \leq c^*$. Since $(\frac{p}{p-1+\alpha})^{p-1}\lambda^* < a$, we have

$$K^{p-1}(\frac{p}{p-1+\alpha})^{p-1} \lambda^* z_{\lambda^*}^{\frac{p(p-1)}{p-1+\alpha}} < a \Big(K z_{\lambda^*}^{\frac{p}{p-1+\alpha}} \Big)^{p-1}. \tag{2}$$

From the choice of K , we have

$$\frac{1}{2}\left(\frac{p}{p-1+\alpha}\right)^{p-1} \geq bK^{\gamma-p+1}\,\|z_{\lambda^*}\|_\infty^{\frac{\gamma p-(p-1)(\alpha-1)}{p-1+\alpha}} \tag{3}$$

$$\frac{1}{2}\left(\frac{p}{p-1+\alpha}\right)^{p-1} \geq AK^{\beta-p+1}\,\|z_{\lambda^*}\|_\infty^{\frac{\beta p-(p-1)(\alpha-1)}{p-1+\alpha}} \tag{4}$$

and by (3),(4),(H2), we know that

$$-\frac{1}{2}K^{p-1}\left(\frac{p}{p-1+\alpha}\right)^{p-1} z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}} \leq -b\Big(Kz_{\lambda^*}^{\frac{p}{p-1+\alpha}}\Big)^{\gamma} \tag{5}$$

$$-\frac{1}{2}K^{p-1}\left(\frac{p}{p-1+\alpha}\right)^{p-1} z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}} \leq -A\Big(Kz_{\lambda^*}^{\frac{p}{p-1+\alpha}}\Big)^{\beta} \leq -f\Big(Kz_{\lambda^*}^{\frac{p}{p-1+\alpha}}\Big) \tag{6}$$

Since $|\nabla z_{\lambda^*}|^p \geq m$ in $\overline{\Omega}_\delta$, from the choice of c^* we have

$$\begin{aligned}
&-K^{p-1}\left(\frac{p}{p-1+\alpha}\right)^{p-1}\Big(\frac{(p-1)(1-\alpha)}{p-1+\alpha}\Big)\frac{|\nabla z_{\lambda^*}|^p}{z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}}\\
&\leq -K^{p-1}\left(\frac{p}{p-1+\alpha}\right)^{p-1}\Big(\frac{(p-1)(1-\alpha)}{p-1+\alpha}\Big)\frac{m^p}{z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}}\\
&\leq -\frac{c}{\Big(Kz_{\lambda^*}^{\frac{p}{p-1+\alpha}}\Big)^{\alpha}}.
\end{aligned} \tag{7}$$

Hence for $c \leq c^*$, combining (2), (5), (6) and (7) we have

$$\begin{aligned}
-\Delta_p\psi &= K^{p-1}\left(\frac{p}{p-1+\alpha}\right)^{p-1}\\
&\quad\times\left\{\lambda^* z_{\lambda^*}^{\frac{p(p-1)}{p-1+\alpha}} - z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}} - \left(\frac{(p-1)(1-\alpha)}{p-1+\alpha}\right)\frac{|\nabla z_{\lambda^*}|^p}{z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}}\right\}\\
&= K^{p-1}\left(\frac{p}{p-1+\alpha}\right)^{p-1}\lambda^* z_{\lambda^*}^{\frac{p(p-1)}{p-1+\alpha}} - \frac{1}{2}K^{p-1}\left(\frac{p}{p-1+\alpha}\right)^{p-1} z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}}\\
&\quad -\frac{1}{2}K^{p-1}\left(\frac{p}{p-1+\alpha}\right)^{p-1} z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}}\\
&\quad -K^{p-1}\left(\frac{p}{p-1+\alpha}\right)^{p-1}\left(\frac{(p-1)(1-\alpha)}{p-1+\alpha}\right)\frac{|\nabla z_{\lambda^*}|^p}{z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}}\\
&\leq a\Big(Kz_{\lambda^*}^{\frac{p}{p-1+\alpha}}\Big)^{p-1} - b\Big(Kz_{\lambda^*}^{\frac{p}{p-1+\alpha}}\Big)^{\gamma} - f\Big(Kz_{\lambda^*}^{\frac{p}{p-1+\alpha}}\Big) - \frac{c}{\Big(Kz_{\lambda^*}^{\frac{p}{p-1+\alpha}}\Big)^{\alpha}}\\
&= a\psi^{p-1} - b\psi^{\gamma} - f(\psi) - \frac{c}{\psi^{\alpha}}, \quad x \in \overline{\Omega}_\delta.
\end{aligned}$$

Next in $\Omega \setminus \overline{\Omega}_\delta$, for $c \le c^*$ from the choice of c^* and K, we know that

$$\frac{c}{K^\alpha} \le \frac{1}{3} K^{p-1} z_{\lambda^*}^p \left(a - (\frac{p}{p-1+\alpha})^{p-1} \lambda^* \right), \tag{8}$$

and

$$bK^{\gamma-p+1} z_{\lambda^*}^{\frac{p(\gamma-p+1)}{p-1+\alpha}} \le \frac{1}{3} \left(a - (\frac{p}{p-1+\alpha})^{p-1} \lambda^* \right) \tag{9}$$

$$AK^{\beta-p+1} z_{\lambda^*}^{\frac{p(\beta-p+1)}{p-1+\alpha}} \le \frac{1}{3} \left(a - (\frac{p}{p-1+\alpha})^{p-1} \lambda^* \right). \tag{10}$$

By combining (8), (9) and (10) we have

$$\begin{aligned}
-\Delta_p \psi &= K^{p-1} (\frac{p}{p-1+\alpha})^{p-1} \\
&\quad \times \left\{ \lambda^* z_{\lambda^*}^{\frac{p(p-1)}{p-1+\alpha}} - z_{\lambda^*}^{\frac{(p-1)(1-\alpha)}{p-1+\alpha}} - \left(\frac{(p-1)(1-\alpha)}{p-1+\alpha} \right) \frac{|\nabla z_{\lambda^*}|^p}{z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}} \right\} \\
&\le K^{p-1} (\frac{p}{p-1+\alpha})^{p-1} \lambda^* z_{\lambda^*}^{\frac{p(p-1)}{p-1+\alpha}} \\
&= \frac{1}{z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}} \sum_{i=1}^{3} \left(\frac{1}{3} K^{p-1} (\frac{p}{p-1+\alpha})^{p-1} \lambda^* z_{\lambda^*}^p \right) \\
&\le \frac{1}{z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}} \Big\{ \left(\frac{1}{3} K^{p-1} z_{\lambda^*}^p a - \frac{c}{K^\alpha} \right) + K^{p-1} z_{\lambda^*}^p \left(\frac{1}{3} a - bK^{\gamma-p+1} z_{\lambda^*}^{\frac{p(\gamma-p+1)}{p-1+\alpha}} \right) \\
&\qquad + K^{p-1} z_{\lambda^*}^p \left(\frac{1}{3} a - AK^{\beta-p+1} z_{\lambda^*}^{\frac{p(\beta-p+1)}{p-1+\alpha}} \right) \Big\} \\
&\le aK^{p-1} z_{\lambda^*}^{\frac{pp-1}{p-1+\alpha}} - bK^\gamma z_{\lambda^*}^{\frac{\gamma p}{p-1+\alpha}} - AK^\beta z_{\lambda^*}^{\frac{\beta p}{p-1+\alpha}} - \frac{c}{K^\alpha z_{\lambda^*}^{\frac{\alpha p}{p-1+\alpha}}} \\
&\le a \left(K z_{\lambda^*}^{\frac{p}{p-1+\alpha}} \right)^{p-1} - b \left(K z_{\lambda^*}^{\frac{p}{p-1+\alpha}} \right)^\gamma - f \left(K z_{\lambda^*}^{\frac{p}{p-1+\alpha}} \right) - \frac{c}{\left(K z_{\lambda^*}^{\frac{p}{p-1+\alpha}} \right)^\alpha} \\
&= a\psi^{p-1} - b\psi^\gamma - f(\psi) - \frac{c}{\psi^\alpha}, \quad x \in \Omega \setminus \overline{\Omega}_\delta.
\end{aligned}$$

Thus ψ is a positive subsolution of (1). From (H1) and $\gamma > p-1$, it is obvious that $z = M$ where M is sufficiently large constant is a supersolution of (1) with $z \ge \psi$. Thus Theorem 2.1 is proven. □

3. An extension to system (11)

In this section, we consider the extension of (1) to the following system:

$$\begin{cases} -\Delta_p u = a_1 u^{p-1} - b_1 u^{\gamma} - f_1(u) - \dfrac{c_1}{v^{\alpha}}, & x \in \Omega, \\ -\Delta_p v = a_2 v^{p-1} - b_2 v^{\gamma} - f_2(v) - \dfrac{c_2}{u^{\alpha}}, & x \in \Omega, \\ u = 0 = v, & x \in \partial\Omega, \end{cases} \tag{11}$$

where Δ_p denotes the p-Laplacian operator, $p > 1$, $\gamma > p-1$, $\alpha \in (0,1)$, a_1, a_2, b_1, b_2, c_1 and c_2 are positive constants, Ω is a bounded domain in $\mathbb{R}^N$ with smooth boundary $\partial\Omega$, and $f_i : [0,\infty) \to \mathbb{R}$ is a continuous function for $i = 1,2$. We make the following assumptions:

(H3) $f_i : [0,+\infty) \to \mathbb{R}$ is a continuous functions such that $\lim_{s\to+\infty} f_i(s) = \infty$ for $i = 1,2$.

(H4) There exist $A > 0$ and $\beta > p-1$ such that $f_i(s) \leq As^{\beta}$, $i = 1,2$, for all $s \geq 0$.

We prove the following result by finding sub-super solutions to infinite semipositone system (11).

Theorem 3.1. *Let* (H3) *and* (H4) *hold. If* $\min\{a_1,a_2\} > (\frac{p}{p-1+\alpha})^{p-1}\lambda_1$*, then there exists positive constant* $c^* := c^*(a_1,a_2,b_1,b_2,A,p,\Omega)$ *such that for*

$$\max\{c_1,c_2\} \leq c^*,$$

problem (11) *has a positive solution.*

Proof. Let σ be as in section 2, $\tilde{a} = \min\{a_1,a_2\}$ and $\tilde{b} = \max\{b_1,b_2\}$. Choose $\lambda^* \in (\lambda_1, \min\{\lambda_1+\sigma, (\frac{p-1+\alpha}{p})^{p-1}\tilde{a}\})$. Define

$$K := \min\Bigg\{ \Bigg(\frac{(p/p-1+\alpha)^{p-1}}{2\tilde{b}\,\|z_{\lambda^*}\|_\infty^{\frac{\gamma p-(p-1)(\alpha-1)}{p-1+\alpha}}} \Bigg)^{\frac{1}{\gamma-p+1}}, \Bigg(\frac{\tilde{a} - (\frac{p}{p-1+\alpha})^{p-1}\lambda^*}{3\tilde{b}\,\|z_{\lambda^*}\|_\infty^{\frac{p(\gamma-p+1)}{p-1+\alpha}}} \Bigg)^{\frac{1}{\gamma-p+1}},$$
$$\Bigg(\frac{(p/p-1+\alpha)^{p-1}}{2A\,\|z_{\lambda^*}\|_\infty^{\frac{\beta p-(p-1)(\alpha-1)}{p-1+\alpha}}} \Bigg)^{\frac{1}{\beta-p+1}}, \Bigg(\frac{\tilde{a} - (\frac{p}{p-1+\alpha})^{p-1}\lambda^*}{3A\,\|z_{\lambda^*}\|_\infty^{\frac{p(\beta-p+1)}{p-1+\alpha}}} \Bigg)^{\frac{1}{\beta-p+1}} \Bigg\},$$

and
$c^* := K^{p-1+\alpha} \min\Big\{ (\frac{p}{p-1+\alpha})^{p-1} \Big(\frac{(p-1)(1-\alpha)}{p-1+\alpha}\Big) m^p, \frac{1}{3}\mu^p \Big(\tilde{a} - (\frac{p}{p-1+\alpha})^{p-1}\lambda^*\Big) \Big\}$.
By the same argument as in the proof of theorem 2.1, we can show that (ψ_1,ψ_2) $:= (Kz_{\lambda^*}^{\frac{p}{p-1+\alpha}}, Kz_{\lambda^*}^{\frac{p}{p-1+\alpha}})$ is a positive subsolution of (11) for $\max\{c_1,c_2\} \leq c^*$.

Also it is easy to check that constant function $(z_1,z_2) := (M,M)$ is a supersolution of (11) for M large. Further M can be chosen large enough so that $(z_1,z_2) \geq (\psi_1,\psi_2)$ on Ω. Hence for $\max\{c_1,c_2\} \leq c^*$, (11) has a positive solution the proof is complete. □

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