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DIFFERENTIAL INEQUALITIES FOR MEROMORPHIC p-VALENT FUNCTIONS ASSOCIATED WITH GENERALIZED INTEGRAL OPERATOR

RABHA M. EL-ASHWAH

In this paper the author introduced a new generalized integral operator for meromorphic p -valent functions in $U^* = \{z : z \in \mathbb{C} \text{ and } 0 < |z| < 1\}$. The object of this paper is to give an application of this operator to the differential inequalities.

1. Introduction

Let Σ_p be the class of functions of the form:

$$f(z) = z^{-p} + \sum_{k=1}^{\infty} a_k z^{k-p} \quad (p \in \mathbb{N} = \{1, 2, \dots\}), \quad (1)$$

which are analytic and p -valent in the punctured unit disc $U^* = \{z : z \in \mathbb{C} \text{ and } 0 < |z| < 1\} = U \setminus \{0\}$.

For a function $f(z) \in \Sigma_p$, we define the integral operator

$$\mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha,\mu)f(z) \quad (\alpha, \beta, \mu, \lambda \geq 0, \alpha + \beta \neq 0, \mu + \lambda \neq 0)$$

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as follows:

$$\begin{aligned} \mathfrak{S}_{p,\lambda}^{\beta,0}(\alpha, \mu)f(z) &= f(z), \\ \mathfrak{S}_{p,\lambda}^{\beta,1}(\alpha, \mu)f(z) &= \left(\frac{\alpha+\beta}{\mu+\lambda}\right) z^{-p-\left(\frac{\alpha+\beta}{\mu+\lambda}\right)} \int_0^z t^{\left(\frac{\alpha+\beta}{\mu+\lambda}+p-1\right)} f(t)dt \\ &\quad (f \in \Sigma_p; z \in U^*), \\ \mathfrak{S}_{p,\lambda}^{\beta,2}(\alpha, \mu)f(z) &= \left(\frac{\alpha+\beta}{\mu+\lambda}\right) z^{-p-\left(\frac{\alpha+\beta}{\mu+\lambda}\right)} \int_0^z t^{\left(\frac{\alpha+\beta}{\mu+\lambda}+p-1\right)} \mathfrak{S}_{p,\lambda}^{\beta,1}(\alpha, \mu)f(t)dt \\ &\quad (f \in \Sigma_p; z \in U^*), \end{aligned}$$

and, in general,

$$\begin{aligned} &\mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \beta, \mu)f(z) \\ &= \left(\frac{\alpha+\beta}{\mu+\lambda}\right) z^{-p-\left(\frac{\alpha+\beta}{\mu+\lambda}\right)} \int_0^z t^{\left(\frac{\alpha+\beta}{\mu+\lambda}+p-1\right)} \mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha, \mu)f(t)dt \\ &= \mathfrak{S}_{p,\lambda}^{\beta,1}(\alpha, \mu) \left(\frac{1}{z^p(1-z)}\right) * \dots * \mathfrak{S}_{p,\lambda}^{\beta,1}(\alpha, \mu) \left(\frac{1}{z^p(1-z)}\right) * f(z) \quad (2) \\ &\quad [-----m-times-----] \end{aligned}$$

We note that if $f(z) \in \Sigma_p$, then from (1) and (2), we have

$$\begin{aligned} \mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu)f(z) &= \frac{1}{z^p} + \sum_{k=1}^{\infty} \left[\frac{\alpha + \beta}{\alpha + \beta + (\mu + \lambda)k} \right]^m a_k z^{k-p} \\ &\quad (\alpha, \beta, \mu, \lambda \geq 0, \alpha + \beta \neq 0, \mu + \lambda \neq 0; p \in \mathbb{N}; m \in \mathbb{N}_0 = \mathbb{N} \cup \{0\}; z \in U^*). \quad (3) \end{aligned}$$

From (3), it is easy to verify that

$$\begin{aligned} (\mu + \lambda)z(\mathfrak{S}_{p,\lambda}^{\beta,m+1}(\alpha, \mu)f(z))' &= (\alpha + \beta)\mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu)f(z) \\ &\quad - [p(\mu + \lambda) + (\alpha + \beta)]\mathfrak{S}_{p,\lambda}^{\beta,m+1}(\alpha, \mu)f(z). \quad (4) \end{aligned}$$

We note that:

- (i) $\mathfrak{S}_{p,0}^{0,\gamma}(1, 1)f(z) = P_p^\gamma f(z) (\gamma > 0)$ (see Aqlan et al. [1]);
- (ii) $\mathfrak{S}_{1,0}^{0,m}(\alpha, \mu)f(z) = \mathcal{L}^m(\alpha, \mu)f(z) (\alpha > 0, \mu \geq 0, m \in \mathbb{N}_0)$ (see Bulboaca et al. [2]);

(iii) $\mathfrak{S}_{1,0}^{0,\gamma}(1, \mu)f(z) = P_{\mu}^{\gamma}f(z)$ ($\gamma > 0, \mu > 0$) (see Lashin [3]).

Also we note that

- (i) $\mathfrak{S}_{p,1}^{0,m}(\alpha, 1)f(z) = \mathfrak{S}_p^m(\alpha)\overline{f(z)}$ ($\alpha > 0, m \in \mathbb{N}_0$),
 where $\mathfrak{S}_p^m(\alpha)f(z) = \frac{1}{z^p} + \sum_{k=1}^{\infty} \left(\frac{\alpha}{\alpha + 2k}\right)^m a_k z^{k-p}$;
- (ii) $\mathfrak{S}_{p,\lambda}^{0,m}(1, 0)f(z) = \mathfrak{S}_{p,\lambda}^m f(z)$ ($\lambda \geq 0, m \in \mathbb{N}_0$),
 where $\mathfrak{S}_{p,\lambda}^m f(z) = \frac{1}{z^p} + \sum_{k=1}^{\infty} \left(\frac{1}{1 + \lambda k}\right)^m a_k z^{k-p}$.

For our purpose, we introduce.

Definition 1.1. Let H be the set of complex-valued function $h(r, s, t) : \mathbb{C}^3 \rightarrow \mathbb{C}$ (\mathbb{C} is the complex plane) such that:

- (i) $h(r, s, t)$ is continuous in a domain $D \subset \mathbb{C}^3$;
- (ii) $(1, 1, 1) \in D$ and $|h(1, 1, 1)| < 1$;
- (iii) $\left| h\left(e^{i\theta}, e^{i\theta} + \left(\frac{\mu+\lambda}{\alpha+\beta}\right)\zeta e^{i\theta}, \frac{\left(\frac{\alpha+\beta}{\mu+\lambda}\right)^2 e^{i\theta} + \left(2\left(\frac{\alpha+\beta}{\mu+\lambda}\right) + 1\right)\zeta e^{i\theta} + L}{\left(\frac{\alpha+\beta}{\mu+\lambda}\right)^2}\right) \right| \geq 1$

whenever

$$\left(e^{i\theta}, e^{i\theta} + \left(\frac{\mu+\lambda}{\alpha+\beta}\right)\zeta e^{i\theta}, \frac{\left(\frac{\alpha+\beta}{\mu+\lambda}\right)^2 e^{i\theta} + \left(2\left(\frac{\alpha+\beta}{\mu+\lambda}\right) + 1\right)\zeta e^{i\theta} + L}{\left(\frac{\alpha+\beta}{\mu+\lambda}\right)^2} \right) \in D$$

with $\Re(e^{-i\theta}L) > \zeta(\zeta - 1)$ for real θ and for $\zeta \geq 1$.

Definition 1.2. Let H be the set of complex-valued function $h(r, s, t) : \mathbb{C}^3 \rightarrow \mathbb{C}$ (\mathbb{C} is the complex plane) such that:

- (i) $h(r, s, t)$ is continuous in a domain $D \subset \mathbb{C}^3$;
- (ii) $(1, 1, 1) \in D$ and $|h(1, 1, 1)| < 1$;
- (iii) $\left| h\left(e^{i\theta}, e^{i\theta} + \left(\frac{\mu+\lambda}{\alpha+\beta}\right)\zeta, e^{i\theta} + \left(\frac{\mu+\lambda}{\alpha+\beta}\right)\zeta + \left(\frac{\mu+\lambda}{\alpha+\beta}\right)\frac{\left(\frac{\alpha+\beta}{\mu+\lambda}\right)\zeta e^{i\theta} + \zeta - \zeta^2 + L}{\zeta + \left(\frac{\alpha+\beta}{\mu+\lambda}\right)e^{i\theta}}\right) \right| \geq 1$

whenever

$$\left(e^{i\theta}, e^{i\theta} + \left(\frac{\mu+\lambda}{\alpha+\beta}\right) \zeta, e^{i\theta} + \left(\frac{\mu+\lambda}{\alpha+\beta}\right) \zeta + \left(\frac{\mu+\lambda}{\alpha+\beta}\right) \frac{\left(\frac{\alpha+\beta}{\mu+\lambda}\right) \zeta e^{i\theta} + \zeta - \zeta^2 + L}{\zeta + \left(\frac{\alpha+\beta}{\mu+\lambda}\right) e^{i\theta}} \right) \in D$$

with $\Re(L) > \zeta(\zeta - 1)$ for real θ and for $\zeta \geq 1$.

2. Main result

Unless otherwise mentioned, we assume throughout this paper that $\alpha, \beta, \mu, \lambda \geq 0, \alpha + \beta \neq 0, \mu + \lambda \neq 0$ and $m \geq 2$.

In proving our main result, we shall need the following lemma.

Lemma 2.1. [4] *Let $w(z) = a + w_n z^n + \dots$, be analytic in U with $w(z) \neq 0$ and $\zeta \geq 1$. If $z_0 = r_0 e^{i\theta}$ ($0 < r_0 < 1$) and $|w(z_0)| = \max_{|z| \leq r_0} |w(z)|$. Then*

$$z_0 w'(z_0) = \zeta w(z_0) \tag{5}$$

and

$$\Re \left\{ 1 + \frac{z_0 w''(z_0)}{w'(z_0)} \right\} \geq \zeta, \tag{6}$$

where ζ is a real number and

$$\zeta \geq k \frac{|w(z_0) - a|^2}{|w(z_0)|^2 - |a|^2} \geq k \frac{|w(z_0)| - |a|}{|w(z_0)| + |a|}.$$

Theorem 2.2. *Let $h(r, s, t) \in H$ and let $f \in \Sigma_p$ satisfy:*

$$\left(z^p \mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu) f(z), z^p \mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha, \mu) f(z), z^p \mathfrak{S}_{p,\lambda}^{\beta,m-2}(\alpha, \mu) f(z) \right) \in D \subset \mathbb{C}^3 \tag{7}$$

and

$$\left| h \left(z^p \mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu) f(z), z^p \mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha, \mu) f(z), z^p \mathfrak{S}_{p,\lambda}^{\beta,m-2}(\alpha, \mu) f(z) \right) \right| < 1 \tag{8}$$

for all $z \in U$. Then we have

$$\left| z^p \mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu) f(z) \right| < 1 \quad (z \in U)$$

Proof. We define the function $w(z)$ by

$$z^p \mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu) f(z) = w(z),$$

for $f(z) \in \Sigma_p$. With the aid of the identity (4), we obtain

$$z^p \mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha, \mu) f(z) = w(z) + \left(\frac{\mu+\lambda}{\alpha+\beta}\right) zw'(z)$$

and

$$z^p \mathfrak{S}_{p,\lambda}^{\beta,m-2}(\alpha, \mu) f(z) = \frac{\left(\frac{\alpha+\beta}{\mu+\lambda}\right)^2 w(z) + \left(2\left(\frac{\alpha+\beta}{\mu+\lambda}\right) + 1\right) zw'(z) + z^2 w''(z)}{\left(\frac{\alpha+\beta}{\mu+\lambda}\right)^2}$$

we claim that $|w(z)| < 1$ for $z \in U$. Otherwise there exists a point $z_0 \in U$ such that $\max_{|z|<|z_0|} |w(z)| = |w(z_0)| = 1$. Letting $w(z_0) = e^{i\theta}$ and using Lemma 2.1 we see that

$$z^p \mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu) f(z_0) = w(z_0) = e^{i\theta},$$

$$z^p \mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha, \mu) f(z_0) = e^{i\theta} + \left(\frac{\mu+\lambda}{\alpha+\beta}\right) \zeta e^{i\theta},$$

$$z^p \mathfrak{S}_{p,\lambda}^{\beta,m-2}(\alpha, \mu) f(z_0) = \frac{\left(\frac{\alpha+\beta}{\mu+\lambda}\right)^2 e^{i\theta} + \left(2\left(\frac{\alpha+\beta}{\mu+\lambda}\right) + 1\right) \zeta e^{i\theta} + L}{\left(\frac{\alpha+\beta}{\mu+\lambda}\right)^2},$$

where $L = z_0^2 w''(z_0)$ and $\zeta \geq 1$.

Further, an application of (6) in Lemma 2.1 gives

$$\Re \left\{ \frac{z_0 w''(z_0)}{w'(z_0)} \right\} = \Re \left\{ \frac{z_0^2 w''(z_0)}{\zeta e^{i\theta}} \right\} \geq (\zeta - 1),$$

or

$$\Re(e^{-i\theta} L) \geq \zeta(\zeta - 1).$$

Since $h(r, s, t) \in H$, we have

$$\begin{aligned} & \left| h \left(z^p \mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu) f(z_0), z^p \mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha, \mu) f(z_0), z^p \mathfrak{S}_{p,\lambda}^{\beta,m-2}(\alpha, \mu) f(z_0) \right) \right| \\ &= \left| h \left(e^{i\theta}, e^{i\theta} + \left(\frac{\mu+\lambda}{\alpha+\beta}\right) \zeta e^{i\theta}, \right. \right. \end{aligned}$$

$$\left| \frac{\left(\frac{\alpha+\beta}{\mu+\lambda}\right)^2 e^{i\theta} + \left(2\left(\frac{\alpha+\beta}{\mu+\lambda}\right) + 1\right) \zeta e^{i\theta} + L}{\left(\frac{\alpha+\beta}{\mu+\lambda}\right)^2} \right| \geq 1,$$

which contradicts the condition (8) of the theorem. Therefore we conclude that

$$\left| z^p \mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu) f(z_0) \right| < 1 \quad (z \in U).$$

This complete the proof of Theorem 2.2. □

Corollary 2.3. *Let $h(r,s,t) = s$ and $f \in \Sigma_p$ satisfying the conditions (7) and (8) for $m \geq 2$. Then*

$$\left| z^p \mathfrak{S}_{p,\lambda}^{\beta,m+i}(\alpha, \mu) f(z) \right| < 1 \quad (i \geq 0; z \in U).$$

Proof. Since $h(r,s,t) = s \in H$, so with the aid of Theorem 2.2, we have

$$\begin{aligned} & \left| z^p \mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha, \mu) f(z) \right| < 1 \\ \implies & \left| z^p \mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu) f(z) \right| < 1 \quad (m \geq 2) \implies \left| z^p \mathfrak{S}_{p,\lambda}^{\beta,m+i}(\alpha, \mu) f(z) \right| < 1 \quad (i \geq 0). \end{aligned}$$

□

Theorem 2.4. *Let $h(r,s,t) \in H$ and let $f \in \Sigma_p$ satisfy:*

$$\left(\frac{\mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu) f(z)}{\mathfrak{S}_{p,\lambda}^{\beta,m+1}(\alpha, \mu) f(z)}, \frac{\mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha, \mu) f(z)}{\mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu) f(z)}, \frac{\mathfrak{S}_{p,\lambda}^{\beta,m-2}(\alpha, \mu) f(z)}{\mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha, \mu) f(z)} \right) \in D \subset \mathbb{C}^3 \quad (9)$$

and

$$\left| h \left(\frac{\mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu) f(z)}{\mathfrak{S}_{p,\lambda}^{\beta,m+1}(\alpha, \mu) f(z)}, \frac{\mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha, \mu) f(z)}{\mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu) f(z)}, \frac{\mathfrak{S}_{p,\lambda}^{\beta,m-2}(\alpha, \mu) f(z)}{\mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha, \mu) f(z)} \right) \right| < 1 \quad (10)$$

for all $z \in U$. Then we have

$$\left| \frac{\mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu) f(z)}{\mathfrak{S}_{p,\lambda}^{\beta,m+1}(\alpha, \mu) f(z)} \right| < 1 \quad (z \in U).$$

Proof. Let

$$\frac{\mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha,\mu)f(z)}{\mathfrak{S}_{p,\lambda}^{\beta,m+1}(\alpha,\mu)f(z)} = w(z).$$

Then it follows that $w(z)$ is either analytic or meromorphic in U , $w(0) = 1$ and $w(z) \neq 1$. With the aid of the identity (4), we obtain

$$\frac{\mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha,\mu)f(z)}{\mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha,\mu)f(z)} = \left\{ w(z) + \left(\frac{\mu+\lambda}{\alpha+\beta} \right) \frac{zw'(z)}{w(z)} \right\}$$

and

$$\left. \begin{aligned} & \frac{\mathfrak{S}_{p,\lambda}^{\beta,m-2}(\alpha,\mu)f(z)}{\mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha,\mu)f(z)} = \left\{ w(z) + \left(\frac{\mu+\lambda}{\alpha+\beta} \right) \frac{zw'(z)}{w(z)} + \right. \\ & \left. \left(\frac{\mu+\lambda}{\alpha+\beta} \right) \frac{zw'(z) + \left(\frac{\mu+\lambda}{\alpha+\beta} \right) \left(\frac{zw'(z)}{w(z)} + \frac{z^2w''(z)}{w(z)} - \left(\frac{zw'(z)}{w(z)} \right)^2 \right)}{w(z) + \left(\frac{\mu+\lambda}{\alpha+\beta} \right) \frac{zw'(z)}{w(z)}} \right\} \end{aligned}$$

we claim that $|w(z)| < 1$ for $z \in U$. Otherwise there exists a point $z_0 \in U$ such that $\max_{|z|<|z_0|} |w(z)| = |w(z_0)| = 1$. Letting $w(z_0) = e^{i\theta}$ and using Lemma 2.1 with $a = k = 1$, we see that

$$\frac{\mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha,\mu)f(z_0)}{\mathfrak{S}_{p,\lambda}^{\beta,m+1}(\alpha,\mu)f(z_0)} = e^{i\theta},$$

$$\frac{\mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha,\mu)f(z_0)}{\mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha,\mu)f(z_0)} = e^{i\theta} + \left(\frac{\mu+\lambda}{\alpha+\beta} \right) \zeta,$$

and

$$\frac{\mathfrak{S}_{p,\lambda}^{\beta,m-2}(\alpha,\mu)f(z_0)}{\mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha,\mu)f(z_0)} = e^{i\theta} + \left(\frac{\mu+\lambda}{\alpha+\beta} \right) \zeta + \left(\frac{\mu+\lambda}{\alpha+\beta} \right) \frac{\left(\frac{\alpha+\beta}{\mu+\lambda} \right) \zeta e^{i\theta} + \zeta - \zeta^2 + L}{\left(\frac{\alpha+\beta}{\mu+\lambda} \right) e^{i\theta} + \zeta},$$

where $L = \frac{z_0^2 w''(z_0)}{w(z_0)}$ and $\zeta \geq 1$.

Further, an application of (6) in Lemma 2.1 gives

$$\Re(L) \geq \zeta(\zeta - 1).$$

Since $h(r, s, t) \in H$, we have

$$\begin{aligned} & \left| h \left(\frac{\mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu)f(z_0)}{\mathfrak{S}_{p,\lambda}^{\beta,m+1}(\alpha, \mu)f(z_0)}, \frac{\mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha, \mu)f(z_0)}{\mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu)f(z_0)}, \frac{\mathfrak{S}_{p,\lambda}^{\beta,m-2}(\alpha, \mu)f(z_0)}{\mathfrak{S}_{p,\lambda}^{\beta,m-1}(\alpha, \mu)f(z_0)} \right) \right| \\ &= \left| h \left(e^{i\theta}, e^{i\theta} + \left(\frac{\mu+\lambda}{\alpha+\beta} \right) \zeta, \right. \right. \\ & \quad \left. \left. e^{i\theta} + \left(\frac{\mu+\lambda}{\alpha+\beta} \right) \zeta + \left(\frac{\mu+\lambda}{\alpha+\beta} \right) \frac{\left(\frac{\alpha+\beta}{\mu+\lambda} \right) \zeta e^{i\theta} + \zeta - \zeta^2 + L}{\zeta + \left(\frac{\alpha+\beta}{\mu+\lambda} \right) e^{i\theta}} \right) \right| \geq 1, \end{aligned}$$

which contradicts the condition (8) of the theorem. Therefore we conclude that

$$\left| \frac{\mathfrak{S}_{p,\lambda}^{\beta,m}(\alpha, \mu)f(z_0)}{\mathfrak{S}_{p,\lambda}^{\beta,m+1}(\alpha, \mu)f(z_0)} \right| < 1 \quad (z \in U).$$

This complete the proof of Theorem 2.4. \square

Remark 2.5. By specializing the parameters $\alpha, \beta, \mu, \lambda$ and m , we obtain results corresponding to different operators defined in the introduction.

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RABHA M. EL-ASHWAH
Department of Mathematics
Faculty of Science
University of Damietta
e-mail: r_elashwah@yahoo.com