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COMMON FIXED POINT THEOREMS OF COMPATIBLE MAPPING OF TYPE (P) IN INTUITIONISTIC FUZZY METRIC SPACES

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The aim of this paper is to point out results of M. Koireng and Yumnam Rahen [10] on compatible mappings of type (P) in fuzzy metric spaces into intuitionistic fuzzy metric spaces with same terminology and notations.

1. Introduction

Fuzzy set theory was introduced by Zadeh in 1965 [23]. Many authors have introduced and discussed several notions of fuzzy metric space in different ways [11], [5], [6] and also proved fixed point theorems with interesting consequent results in the fuzzy metric spaces [7]. Recently the concept of intuitionistic fuzzy metric space was given by Park [13] and the subsequent fixed point results in the intuitionistic fuzzy metric spaces are investigated by Alaca et al. [1] and Mohamad [12] (see, also [2], [3], [18] and [22]).

2. Preliminaries

The study of fixed points of various classes of mappings have been the focus of vigorous research for many Mathematicians. Among them one of the impor-

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tant result in theory of fixed points of compatible mappings was obtained by G. Jungck [8] in 1986. Since then there have been a flood of research papers involving various types of compatibility such as Compatible mappings of type (A) [9], Semi-compatibility [4], compatible mappings of type (P) [14], compatible mappings of type (B) [16] and compatible mappings of type (C) [17] etc.

The following definitions, lemma and examples are useful for our presentations.

Definition 2.1 (See [19])**.** A binary operation $*\colon [0,1]\times[0,1]\to[0,1]$ is continuous t-norm if the binary operation $*$ satisfying the following conditions:
(i) $*$ is commutative and associative,
(ii) $*$ is continuous,
(iii) $a*1=a$ for all $a\in[0,1]$,
(iv) $a*b\le c*d$ whenever $a\le c$ and $b\le d$ for all $a, b, c, d\in[0,1]$.

Definition 2.2 (See [19])**.** A binary operation $\diamond\colon [0,1]\times[0,1]\to[0,1]$ is continuous t-conorm if the binary operation $\diamond$ satisfying the following conditions:
(i) $\diamond$ is commutative and associative,
(ii) $\diamond$ is continuous,
(iii) $a\diamond 0=a$ for all $a\in[0,1]$,
(iv) $a\diamond b\le c\diamond d$ whenever $a\le c$ and $b\le d$ for all $a, b, c, d\in[0,1]$.

Definition 2.3 (See [1])**.** A 5- tuple $(X,M,N,*,\diamond)$ is called a intuitionistic fuzzy metric space if X is an arbitrary set, $*$ is a continuous t-norm, $\diamond$ is a continuous t-conorm and M,N are fuzzy sets on $X^2\times(0,\infty)$ satisfying the following conditions: for all $x, y, z\in X$ and $s, t>0$
$(IFM-1)$ $M(x,y,t)+N(x,y,t)\le 1$,
$(IFM-2)$ $M(x,y,0)=0$,
$(IFM-3)$ $M(x,y,t)=1$ if and only if $x=y$,
$(IFM-4)$ $M(x,y,t)=M(y,x,t)$,
$(IFM-5)$ $M(x,y,t)*M(y,z,s)\le M(x,z,t+s)$,
$(IFM-6)$ $M(x,y,.)\colon (0,\infty)\to(0,1]$ is left continuous,
$(IFM-7)$ $\lim_{t\to\infty}M(x,y,t)=1$,
$(IFM-8)$ $N(x,y,0)=1$,
$(IFM-9)$ $N(x,y,t)=0$ if and only if $x=y$,
$(IFM-10)$ $N(x,y,t)=N(y,x,t)$,
$(IFM-11)$ $N(x,y,t)\diamond N(y,z,s)\le N(x,z,t+s)$,
$(IFM-12)$ $N(x,y,.)\colon (0,\infty)\to(0,1]$ is right continuous,
$(IFM-13)$ $\lim_{t\to\infty}N(x,y,t)=0$.

Then (M,N) is called an intuitionistic fuzzy metric on X. The functions $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non-nearness between x and y with respect to t, respectively.

Remark 2.4. Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space of the form $(X, M, 1-M, *, \diamond)$ such that t-norm $*$ and t-conorm $\diamond$ are associated, that is, $x \diamond y = 1 - ((1-x)*(1-y))$ for all $x, y \in X$.

Example 2.5. (Induced intuitionistic fuzzy metric space) Let (X, d) be a metric space. Define $a*b = ab$ and $a \diamond b = \min\{1, a+b\}$ for all $a, b \in [0,1]$ and let M_d and N_d be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M_d(x,y,t) = \frac{t}{t+d(x,y)}, \; N_d(x,y,t) = \frac{d(x,y)}{t+d(x,y)}.$$

Then $(X, M_d, N_d, *, \diamond)$ is an intuitionistic fuzzy metric induced by metric d the standard intuitionistic fuzzy metric space.

Definition 2.6 (See [1]). Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. Then

(a) A sequence $\{x_n\}$ in X is said to be convergent to a point x in X if and only if $\lim_{n\to\infty} M(x_n, x, t) = 1$ and $\lim_{n\to\infty} N(x_n, x, t) = 0$ for each $t > 0$.

(b) A sequence $\{x_n\}$ in X is called Cauchy sequence if $\lim_{n\to\infty} M(x_{n+p}, x, t) = 1$ and $\lim_{n\to\infty} N(x_{n+p}, x, t) = 0$ for each $p > 0$ and $t > 0$.

(c) An intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ is said to be complete if and only if every Cauchy sequence in X is convergent in X.

Lemma 2.7 (See [20]). *Let $\{x_n\}$ be a sequence in an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ with $t*t \geq t$ and $(1-t)\diamond(1-t) \leq (1-t)$ for all $t \in [0,1]$. If there exists a number $q \in (0,1)$ such that $M(x_{n+2}, x_{n+1}, qt) \geq M(x_{n+1}, x_n, t)$ and $N(x_{n+2}, x_{n+1}, qt) \leq N(x_{n+1}, x_n, t)$ for all $t > 0$ and $n \in \mathbb{N}$, then $\{x_n\}$ is a Cauchy sequence in X.*

Proof. For $t > 0$ and $q \in (0,1)$ we have,

$$M(x_2, x_3, qt) \geq M(x_1, x_2, t) \geq M(x_0, x_1, \frac{t}{q}),$$

or

$$M(x_2, x_3, t) \geq M(x_0, x_1, \frac{t}{q^2}).$$

By simple induction, we have for all $t > 0$ and $n \in \mathbb{N}$,

$$M(x_{n+1}, x_{n+2}, t) \geq M(x_1, x_2, \frac{t}{q^n}).$$

Thus for any positive number p and real number $t > 0$, we have

$$\begin{aligned} M(x_n, x_{n+p}, t) &\geq M(x_n, x_{n+1}, \frac{t}{p}) * \ldots \cdots * M(x_{n+p-1}, x_{n+p}, \frac{t}{p}), \text{ by } (IFM-5), \\ &\geq M(x_1, x_2, \frac{t}{pq^{n-1}}) * \ldots \cdots * M(x_1, x_2, \frac{t}{pq^{n+p-2}}). \end{aligned}$$

Therefore by $(IFM-7)$, we have

$$M(x_n, x_{n+p}, t) \geq 1 * \ldots \cdots * 1 \geq 1.$$

Similarly, for $t > 0$ and $q \in (0,1)$, we have

$$N(x_2, x_3, qt) \leq N(x_1, x_2, t) \leq N(x_0, x_1, \frac{t}{q}),$$

or

$$N(x_2, x_3, t) \leq N(x_0, x_1, \frac{t}{q^2}).$$

By simple induction, we have for all $t > 0$ and $n \in \mathbb{N}$,

$$N(x_{n+1}, x_{n+2}, t) \leq N(x_1, x_2, \frac{t}{q^n}).$$

Thus for any positive number p and real number $t > 0$, we have

$$\begin{aligned} N(x_n, x_{n+p}, t) &\leq N(x_n, x_{n+1}, \frac{t}{p}) \diamond \ldots \cdots \diamond N(x_{n+p-1}, x_{n+p}, \frac{t}{p}), \text{ by } (IFM-11), \\ &\leq N(x_1, x_2, \frac{t}{pq^{n-1}}) \diamond \ldots \cdots \diamond N(x_1, x_2, \frac{t}{pq^{n+p-2}}). \end{aligned}$$

Therefore by $(IFM-13)$, we have

$$N(x_n, x_{n+p}, t) \leq 0 \diamond \ldots \cdots \diamond 0 \leq 0.$$

This implies that $\{x_n\}$ is a Cauchy sequence in X. This completes the proof. □

Lemma 2.8 (See [20]). *Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space. If $\forall x, y \in X$ and $t > 0$ with positive number $q \in (0,1)$ and $M(x,y,qt) \geq M(x,y,t)$ and $N(x,y,qt) \leq N(x,y,t)$, then $x = y$.*

Proof. If for all $t > 0$ and some constant $q \in (0,1)$, then we have

$$M(x,y,t) \geq M(x,y,\frac{t}{q}) \geq M(x,y,\frac{t}{q^2}) \geq \cdots \geq M(x,y,\frac{t}{q^n}) \geq \ldots,$$

and

$$N(x,y,t) \leq N(x,y,\frac{t}{q}) \leq N(x,y,\frac{t}{q^2}) \leq \cdots \leq N(x,y,\frac{t}{q^n}) \leq \ldots,$$

$n \in \mathbb{N}$ and for all $t > 0$ and $x, y \in X$. When $n \to \infty$, we have $M(x,y,t) = 1$ and $N(x,y,t) = 0$ and thus $x = y$. This completes the proof. □

Definition 2.9 (See [21])**.** Two self mappings A and S of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ are called compatible if

$$\lim_{n\to\infty} M(ASx_n, SAx_n, t) = 1 \text{ and } \lim_{n\to\infty} N(ASx_n, SAx_n, t) = 0$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some $x \in X$.

Definition 2.10. Two self mappings A and S of an intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ are called compatible of type (P) if

$$\lim_{n\to\infty} M(AAx_n, SSx_n, t) = 1 \text{ and } \lim_{n\to\infty} N(AAx_n, SSx_n, t) = 0$$

whenever $\{x_n\}$ is a sequence in X such that $\lim_{n\to\infty} Ax_n = \lim_{n\to\infty} Sx_n = x$ for some $x \in X$.

Theorem 2.11. *Let $(X, M, *)$ be a complete fuzzy metric space and let A, B, S and T be self mappings of X satisfying the following conditions:*
(i) $A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$,
(ii) S and T are continuous,
(iii) the pairs $\{A, S\}$ and $\{B, T\}$ are compatible mappings of type (P) on X,
(iv) there exists $q \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,
$$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Bx, Ty, t) * M(Ax, Ty, t).$$
Then A, B, S and T have a unique common fixed point in X.

The aim of this paper is to extends Theorem 2.11 in the framework of intuitionistic fuzzy metric space.

3. Main Results

Theorem 3.1. *Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space and let A, B, S and T be self mappings of X satisfying the following conditions:*
(i) $A(X) \subseteq T(X)$, $B(X) \subseteq S(X)$,
(ii) S and T are continuous,
(iii) the pairs $\{A, S\}$ and $\{B, T\}$ are compatible mappings of type (P) on X,
(iv) there exists $q \in (0, 1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Bx, Ty, t) * M(Ax, Ty, t)$$

and

$$N(Ax, By, qt) \leq N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(Bx, Ty, t) \diamond N(Ax, Ty, t).$$

Then A, B, S and T have a unique common fixed point in X.

Proof. Since $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$. We define a sequence $\{y_n\}$ such that

$$y_{2n-1} = Tx_{2n-1} = Ax_{2n-2} \quad \text{and} \quad y_{2n} = Sx_{2n} = Bx_{2n-1},\ \forall n \in \mathbb{N}.$$

We shall prove that $\{y_n\}$ is a Cauchy sequence. From (iv), we have

$$\begin{aligned} M(y_{2n+1}, y_{2n+2}, qt) &= M(Ax_{2n}, Bx_{2n+1}, qt) \\ &\geq M(Sx_{2n}, Tx_{2n+1}, t) * M(Ax_{2n}, Sx_{2n}, t) \\ &* M(Bx_{2n+1}, Tx_{2n+1}, t) * M(Ax_{2n}, Tx_{2n+1}, t) \\ &= M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n}, t) \\ &* M(y_{2n+2}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+1}, t) \\ &\geq M(y_{2n}, y_{2n+1}, t) * M(y_{2n+1}, y_{2n+2}, t), \end{aligned}$$

which implies

$$M(y_{2n+1}, y_{2n+2}, qt) \geq M(y_{2n}, y_{2n+1}, t).$$

Similarly, we have

$$M(y_{2n+2}, y_{2n+3}, qt) \geq M(y_{2n+1}, y_{2n+2}, t).$$

Hence, we have

$$M(y_{n+1}, y_{n+2}, qt) \geq M(y_n, y_{n+1}, t). \tag{1}$$

Now

$$\begin{aligned} N(y_{2n+1}, y_{2n+2}, qt) &= N(Ax_{2n}, Bx_{2n+1}, qt) \\ &\leq N(Sx_{2n}, Tx_{2n+1}, t) \diamond N(Ax_{2n}, Sx_{2n}, t) \\ &\diamond N(Bx_{2n+1}, Tx_{2n+1}, t) \diamond N(Ax_{2n}, Tx_{2n+1}, t) \\ &= N(y_{2n}, y_{2n+1}, t) \diamond N(y_{2n+1}, y_{2n}, t) \\ &\diamond N(y_{2n+2}, y_{2n+1}, t) \diamond N(y_{2n+1}, y_{2n+1}, t) \\ &\leq N(y_{2n}, y_{2n+1}, t) \diamond N(y_{2n+1}, y_{2n+2}, t), \end{aligned}$$

which implies

$$N(y_{2n+1}, y_{2n+2}, qt) \leq N(y_{2n}, y_{2n+1}, t).$$

Similarly, we have

$$N(y_{2n+2}, y_{2n+3}, qt) \leq N(y_{2n+1}, y_{2n+2}, t).$$

Hence, we have

$$N(y_{n+1}, y_{n+2}, qt) \leq N(y_n, y_{n+1}, t). \tag{2}$$

Equations (1) and (2) show that $\{y_n\}$ is a Cauchy sequence.

Since X is complete, $\{y_n\}$ converges to some point $z \in X$ and so sequences $\{Ax_{2n-2}\}$, $\{Sx_{2n}\}$, $\{Bx_{2n-1}\}$ and $\{Tx_{2n-1}\}$ also converge to z.

Then, we have

$$AAx_{2n-2} \to Sz \quad \text{and} \quad SSx_{2n} \to Az, \tag{3}$$

and

$$BBx_{2n-1} \to Tz \quad \text{and} \quad TTx_{2n-1} \to Bz. \tag{4}$$

From (iv), we get

$$\begin{aligned} M(AAx_{2n-2}, BBx_{2n-1}, qt) \geq\, & M(SAx_{2n-2}, TBx_{2n-1}, t) * M(AAx_{2n-2}, SAx_{2n-1}, t) \\ & * M(BBx_{2n-1}, TBx_{2n-1}, t) * M(AAx_{2n-2}, TBx_{2n-1}, t). \end{aligned}$$

Using (3) and (4) and taking the limit as $n \to \infty$, we have

$$\begin{aligned} M(Sz, Tz, qt) &\geq M(Sz, Tz, t) * M(Sz, Sz, t) \\ &\quad * M(Tz, Tz, t) * M(Sz, Tz, t) \\ &\geq M(Sz, Tz, t) * 1 * 1 * M(Sz, Tz, t) \\ &\geq M(Sz, Tz, t) \end{aligned}$$

implies

$$M(Sz, Tz, qt) \geq M(Sz, Tz, t).$$

Similarly,

$$\begin{aligned} N(AAx_{2n-2}, BBx_{2n-1}, qt) \leq\, & N(SAx_{2n-2}, TBx_{2n-1}, t) \diamond N(AAx_{2n-2}, SAx_{2n-1}, t) \\ & \diamond N(BBx_{2n-1}, TBx_{2n-1}, t) \diamond N(AAx_{2n-2}, TBx_{2n-1}, t). \end{aligned}$$

Using (3) and (4) and taking the limit as $n \to \infty$, we have

$$\begin{aligned} N(Sz, Tz, qt) &\leq N(Sz, Tz, t) \diamond N(Sz, Sz, t) \\ &\quad \diamond N(Tz, Tz, t) \diamond N(Sz, Tz, t) \\ &\leq N(Sz, Tz, t) \diamond 0 \diamond 0 \diamond N(Sz, Tz, t) \\ &\leq N(Sz, Tz, t) \end{aligned}$$

implies

$$N(Sz, Tz, qt) \leq N(Sz, Tz, t).$$

It follows that

$$Sz = Tz. \tag{5}$$

Now, again from (iv), we have

$$M(Az, BTx_{2n-1}, qt) \geq M(Sz, TTx_{2n-1}, t) * M(Az, Sz, t) * M(BTx_{2n-1}, TTx_{2n-1}, t) * M(Az, TTx_{2n-1}, t).$$

Using (3) and (4) and taking the limit as $n \to \infty$, we have

$$\begin{aligned} M(Az, Tz, qt) &\geq M(Sz, Sz, t) * M(Az, Tz, t) \\ &\quad * M(Tz, Tz, t) * M(Az, Tz, t) \\ &\geq 1 * M(Az, Tz, t) * 1 * M(Az, Tz, t) \\ &\geq M(Az, Tz, t) \end{aligned}$$

implies

$$M(Az, Tz, qt) \geq M(Az, Tz, t).$$

Similarly,

$$N(Az, BTx_{2n-1}, qt) \leq N(Sz, TTx_{2n-1}, t) \diamond N(AAx_{2n-2}, SAx_{2n-1}, t) \diamond N(BTx_{2n-1}, TTx_{2n-1}, t) \diamond N(Az, TTx_{2n-1}, t).$$

Using (3) and (4) and taking the limit as $n \to \infty$, we have

$$\begin{aligned} N(Az, Tz, qt) &\leq N(Sz, Sz, t) \diamond N(Az, Tz, t) \\ &\quad \diamond N(Tz, Tz, t) \diamond N(Az, Tz, t) \\ &\leq 0 \diamond N(Az, Tz, t) \diamond 0 \diamond N(Az, Tz, t) \\ &\leq N(Az, Tz, t) \end{aligned}$$

implies

$$N(Az, Tz, qt) \leq N(Az, Tz, t).$$

It follows that

$$Az = Tz. \tag{6}$$

Now from (iv) and using (5) and (6), we have

$$\begin{aligned} M(Az, Bz, qt) &\geq M(Sz, Tz, t) * M(Az, Sz, t) \\ &\quad * M(Bz, Tz, t) * M(Az, Tz, t) \\ &= M(Az, Az, t) * M(Az, Az, t) \\ &\quad * M(Bz, Az, t) * M(Az, Az, t) \\ &\geq 1 * 1 * M(Bz, Az, t) * 1 \\ &\geq M(Az, Bz, t) \end{aligned}$$

implies

$$M(Az, Bz, qt) \geq M(Az, Bz, t).$$

Similarly,

$$\begin{aligned} N(Az, Bz, qt) &\leq N(Sz, Tz, t) \diamond N(Az, Sz, t) \\ &\diamond N(Bz, Tz, t) \diamond N(Az, Tz, t) \\ &= N(Az, Az, t) \diamond N(Az, Az, t) \\ &\diamond N(Bz, Az, t) \diamond N(Az, Az, t) \\ &\leq 0 \diamond 0 \diamond N(Bz, Az, t) \diamond 0 \\ &\leq N(Az, Bz, t) \end{aligned}$$

implies

$$N(Az, Bz, qt) \leq N(Az, Bz, t).$$

It follows that

$$Az = Bz. \tag{7}$$

From (5), (6) and (7), we have

$$Az = Bz = Tz = Sz.$$

Now, we shall show that $Bz = z$.
Again from (iv), we have

$$\begin{aligned} M(Ax_{2n}, BTz, qt) &\geq M(Sx_{2n}, Tz, t) * M(Ax_{2n}, Sx_{2n}, t) \\ &* M(BTz, Tz, t) * M(z, Tz, t). \end{aligned}$$

Using (5) and (6) and taking the limit as $n \to \infty$, we have

$$\begin{aligned} M(z, Bz, qt) &\geq M(z, Tz, t) * M(z, z, t) \\ &* M(Bz, Tz, t) * M(z, Tz, t) \\ &= M(z, Bz, t) * 1 * M(Az, Az, t) * M(z, Bz, t) \\ &\geq M(z, Bz, t) \end{aligned}$$

implies

$$M(z, Bz, qt) \geq M(z, Bz, t).$$

Similarly,

$$N(Ax_{2n}, Bz, qt) \leq N(Sx_{2n}, Tz, t) \diamond N(Ax_{2n}, Sx_{2n}, t) \diamond N(Bz, Tz, t) \diamond N(z, Tz, t).$$

Using (5) and (6) and taking the limit as $n \to \infty$, we have

$$\begin{aligned} N(z,Bz,qt) &\leq N(z,Tz,t) \diamond N(z,z,t) \diamond N(Bz,Tz,t) \diamond N(z,Tz,t) \\ &= N(z,Bz,t) \diamond 0 \diamond N(Az,Az,t) \diamond N(z,Bz,t) \\ &\leq N(z,Bz,t) \end{aligned}$$

implies

$$N(z,Bz,qt) \leq N(z,Bz,t).$$

It follows that

$$Bz = z. \tag{8}$$

Thus from (8), $z = Az = Bz = Tz = Sz$ and hence z is a common fixed point of the mappings A, B, S and T.

Uniqueness

Let w be another common fixed point of A, B, S and T. Then

$$\begin{aligned} M(z,w,qt) &= M(Az,Bw,qt) \\ &\geq M(Sz,Tw,t) * M(Az,Sz,t) \\ &\quad * M(Bw,Tw,t) * M(Az,Tw,t) \\ &\geq M(z,w,t) \end{aligned}$$

implies

$$M(z,w,qt) \geq M(z,w,t).$$

And

$$\begin{aligned} N(z,w,qt) &= N(Az,Bw,qt) \\ &\leq N(Sz,Tw,t) \diamond N(Az,Sz,t) \\ &\quad \diamond N(Bw,Tw,t) \diamond N(Az,Tw,t) \\ &\leq N(z,w,t) \end{aligned}$$

implies

$$N(z,w,qt) \leq N(z,w,t).$$

Hence $z = w$. This completes the proof. □

Corollary 3.2. *Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space and let A, B, S and T be self mappings of X satisfying the conditions* *(i)-(iii) of Theorem 3.1 and there exists $q \in (0,1)$ such that for all $x, y \in X$ and $t > 0$,*

$$\begin{aligned} &M(Ax, By, qt) \\ &\quad \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(By, Ty, t) * M(By, Sx, 2t) * M(Ax, Ty, t) \end{aligned}$$

and

$$\begin{aligned} &N(Ax, By, qt) \\ &\quad \leq N(Sx, Ty, t) \diamond N(Ax, Sx, t) \diamond N(By, Ty, t) \diamond N(By, Sx, 2t) \diamond N(Ax, Ty, t). \end{aligned}$$

Then A, B, S and T have a unique common fixed point in X.

Corollary 3.3. *Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space and let A, B, S and T be self mappings of X satisfying the conditions (i)-(iii) of Theorem 3.1 and there exists $q \in (0,1)$ such that for all $x, y \in X$ and $t > 0$,*

$$M(Ax, By, qt) \geq M(Sx, Ty, t)$$

and

$$N(Ax, By, qt) \leq N(Sx, Ty, t).$$

Then A, B, S and T have a unique common fixed point in X.

Corollary 3.4. *Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space and let A, B, S and T be self mappings of X satisfying the conditions (i)-(iii) of Theorem 3.1 and there exists $q \in (0,1)$ such that for all $x, y \in X$ and $t > 0$,*

$$M(Ax, By, qt) \geq M(Sx, Ty, t) * M(Sx, Ax, t) * M(Ax, Ty, t)$$

and

$$N(Ax, By, qt) \leq N(Sx, Ty, t) \diamond N(Sx, Ax, t) \diamond N(Ax, Ty, t).$$

Then A, B, S and T have a unique common fixed point in X.

Theorem 3.5. *Let $(X, M, N, *, \diamond)$ be a complete intuitionistic fuzzy metric space. If S and T are continuous self mappings of X, then mappings S and T have a common fixed point in X if and only if there exists a self mapping A of X satisfying the following conditions:*

(i) $A(X) \subseteq T(X) \cap S(X)$,

(ii) the pairs $\{A, S\}$ and $\{A, T\}$ are compatible mappings of type (P) on X,

(iii) there exists $q \in (0,1)$ such that for all $x, y \in X$ and $t > 0$,

$$M(Ax, Ay, qt) \geq M(Sx, Ty, t) * M(Ax, Sx, t) * M(Ay, Ty, t) * M(Ax, Ty, t)$$

and

$$N(Ax,Ay,qt) \leq N(Sx,Ty,t) \diamond N(Ax,Sx,t) \diamond N(Ay,Ty,t) \diamond N(Ax,Ty,t).$$

Then A, S and T have a unique common fixed point in X.

Proof. Necessary part. Let S and T have a common fixed point in X, say z, then $Sz = z = Tz$. Let $Ax = z$ for all $x \in X$, then $A(X) \subseteq T(X) \cap S(X)$ and we know that $\{A,S\}$ and $\{A,T\}$ are compatible mappings of type (P), in fact $A \circ S = S \circ A$ and $A \circ T = T \circ A$ and hence the conditions (i) and (ii) are satisfied. For some $q \in (0,1)$, we have

$$M(Ax,Ay,qt) = 1 \geq M(Sx,Ty,t) * M(Ax,Sx,t) * M(Ay,Ty,t) * M(Ax,Ty,t)$$

and

$$N(Ax,Ay,qt) = 0 \leq N(Sx,Ty,t) \diamond N(Ax,Sx,t) \diamond N(Ay,Ty,t) \diamond N(Ax,Ty,t)$$

for all $x,y \in X$ and $t > 0$. Hence the condition (iii) is satisfied.

Sufficient part. Let $A = B$ in Theorem 3.1. Then A, S and T have a unique common fixed point in X. This completes the proof. □

Corollary 3.6. *Let $(X,M,N,*,\diamond)$ be a complete intuitionistic fuzzy metric space. If S and T are continuous self mappings of X, then mappings S and T have a common fixed point in X if and only if there exists a self mapping A of X satisfying the conditions (i)-(ii) of Theorem 3.5 and there exists $q \in (0,1)$ such that for all $x,y \in X$ and $t > 0$,*

$$\begin{aligned} &M(Ax,Ay,qt) \\ &\quad \geq M(Sx,Ty,t) * M(Ax,Sx,t) * M(Ay,Ty,t) * M(Ax,Sx,2t) * M(Ax,Ty,t) \end{aligned}$$

and

$$\begin{aligned} &N(Ax,Ay,qt) \\ &\quad \leq N(Sx,Ty,t) \diamond N(Ax,Sx,t) \diamond N(Ay,Ty,t) \diamond N(Ax,Sx,2t) \diamond N(Ax,Ty,t). \end{aligned}$$

Then A, S and T have a unique common fixed point in X.

Corollary 3.7. *Let $(X,M,N,*,\diamond)$ be a complete intuitionistic fuzzy metric space. If S and T are continuous self mappings of X, then mappings S and T have a common fixed point in X if and only if there exists a self mapping A of X satisfying the conditions (i)-(ii) of Theorem 3.5 and there exists $q \in (0,1)$ such that for all $x,y \in X$ and $t > 0$,*

$$M(Ax,Ay,qt) \geq M(Sx,Ty,t)$$

and

$$N(Ax,Ay,qt) \le N(Sx,Ty,t).$$

Then A, S and T have a unique common fixed point in X.

Corollary 3.8. *Let $(X,M,N,*,\diamond)$ be a complete intuitionistic fuzzy metric space. If S and T are continuous self mappings of X, then mappings S and T have a common fixed point in X if and only if there exists a self mapping A of X satisfying the conditions (i)-(ii) of Theorem 3.5 and there exists $q \in (0,1)$ such that for all $x, y \in X$ and $t > 0$,*

$$M(Ax,Ay,qt) \ge M(Sx,Ty,t) * M(Sx,Ax,t) * M(Ax,Ty,t)$$

and

$$N(Ax,Ay,qt) \le N(Sx,Ty,t) \diamond N(Ax,Sx,t) \diamond N(Ax,Ty,t).$$

Then A, S and T have a unique common fixed point in X.

Example 3.9. Let $X = \{\frac{1}{n} : n \in \mathbb{N}\} \cup \{0\}$ with $*$ continuous t-norm and $\diamond$ continuous t-conorm defined by $a * b = ab$ and $a \diamond b = \min\{1, a+b\}$ respectively, for $a, b \in [0,1]$. For each $t \in [0,\infty)$ and $x, y \in X$, define (M,N) by

$$M(x,y,t) = \begin{cases} \frac{t}{t+|x-y|}, & \text{if } t > 0, \\ 0, & \text{if } t = 0, \end{cases}$$

and

$$N(x,y,t) = \begin{cases} \frac{|x-y|}{t+|x-y|}, & \text{if } t > 0, \\ 1, & \text{if } t = 0. \end{cases}$$

Clearly $(X,M,N,*,\diamond)$ is an intuitionistic fuzzy metric space.

Define $A(x) = B(x) = \frac{x}{6}$ and $S(x) = T(x) = \frac{x}{2}$ on X. It is clear that $A(X) \subseteq T(X)$ and $B(X) \subseteq S(X)$.

Now

$$\begin{aligned} M(Ax,By,\frac{t}{3}) &= \frac{\frac{t}{3}}{\frac{t}{3} + \frac{|x-y|}{6}} = \frac{2t}{2t+|x-y|} \\ &\ge \frac{t}{t + \frac{|x-y|}{2}} = \frac{2t}{2t+|x-y|} \\ &= M(Tx,Sy,t), \end{aligned}$$

and

$$N(Ax,By,\frac{t}{3}) = \frac{\frac{|x-y|}{6}}{\frac{t}{3}+\frac{|x-y|}{6}} = \frac{|x-y|}{2t+|x-y|}$$
$$\leq \frac{\frac{|x-y|}{2}}{t+\frac{|x-y|}{2}} = \frac{|x-y|}{2t+|x-y|}$$
$$= N(Tx,Sy,t).$$

Thus all the conditions of Theorem 3.1 are satisfied and so A, B, S and T have a unique common fixed point 0.

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