LE MATEMATICHE
Vol. LXVII (2012) – Fasc. II, pp. 27–37
doi: 10.4418/2012.67.2.3

(α,β,T)-CONVEX VAGUE SETS

ARSHAM BORUMAND SAEID

By using vague sets we generalize the notion of convex sets and introduce the notion of (α,β,T)-convex vague sets and study their properties, where T is a triangular norm on $[0,1]$.

1. Introduction

Let C be a set in a real or complex vector space. C is said to be convex if, for all x and y in C and all t in the interval [0,1], the point $(1-t)x+ty$ is in C. In other words, every point on the line segment connecting x and y is in C. Convex sets play a key role in quantum logics and quantum information science. For instance, in quantum mechanics and classical theory, the state of a quantum mechanical system forms a convex set.

Zadeh proposed the theory of fuzzy sets [6]. Since then it has been applied in wide varieties of fields like Computer Science, Management Science, Medical Sciences, Engineering problems etc. to list a few only.

Also, a range of fuzzy values for an event can be expressed as a convex set. A fuzzy interpretation of convexity is that any mixture of two distributions in a set is also in the set. In [6], Zadeh introduced the concept of convex fuzzy sets, which is an important kind of extension of classical convex sets from the

Entrato in redazione: 13 novembre 2011

AMS 2010 Subject Classification: 03E72, 52A99
Keywords: Convex sets, Vague sets, (α,β,T)-convex ((α,β,T)-concave) vague sets, (α,β)-cut, α-cut

viewpoint of cut set. After that the theory and applications about convex fuzzy sets have been studied intensively.

The notion of vague set theory introduced by W. L. Gau and D. J. Buehrer [3], as a generalizations of Zadeh's fuzzy set theory [6] and vague sets are studied by many researchers [1, 2, 4, 5].

In [3], the concept of convex vague sets are introduced and studied, A. Borumand Saeid et. al [1] introduce the notion of t-convex vague sets.

In this paper, we generalize the notion of t-convex vague sets and introduce the notion of (α, β, T)-convex vague sets and study their properties in details.

Here we review some concepts of vague set theory.

Let $U = \{u_1, u_2, ..., u_n\}$ be the universe of discourse. The membership function for fuzzy sets can take any value from the closed interval $[0;1]$. Fuzzy set A is defined as the set of ordered pairs $A = \{(u;\mu_A(u)) \mid u \in U\}$ where $\mu_A(u)$ is the grade of membership of element u in set A. The greater $\mu_A(u)$, the greater is the truth of the statement that 'the element u belongs to the set A'. But Gau and Buehrer [3] pointed out that this single value combines the 'evidence for u' and the 'evidence against u'. It does not indicate the 'evidence for u' and the 'evidence against u', and it does not also indicate how much there is of each. Consequently, there is a genuine necessity of a different kind of fuzzy sets which could be treated as a generalization of Zadeh's fuzzy sets [6].

Definition 1.1. [3] A vague set A in the universe of discourse U is characterized by two membership functions given by:

1- A truth membership function

$$t_A : U \to [0,1]$$

and

2- A false membership function

$$f_A : U \to [0,1]$$

where $t_A(u)$ is a lower bound of the grade of membership of u derived from the evidence for u, and $f_A(u)$ is a lower bound of the negation of u derived from the evidence against u and

$$t_A(u) + f_A(u) \leq 1.$$

Thus the grade of membership of u in the vague set A is bounded by a sub interval $[t_A(u), 1 - f_A(u)]$ of $[0,1]$. This indicates that if the actual grade of membership is $\mu(u)$, then

$$t_A(u) \leq \mu(u) \leq 1 - f_A(u).$$

The vague set A is written as

$$A = \{(u, [t_A(u), 1 - f_A(u)]) \mid u \in U\},$$

where the interval $[t_A(u), 1 - f_A(u)]$ is called the vague value of u in A and is denoted by $V_A(u)$.

Definition 1.2. [3] A vague set A of a set U is called

1- the zero vague set of U if $t_A(u) = 0$ and $f_A(u) = 1$ for all $u \in U$,

2- the unit vague set of U if $t_A(u) = 1$ and $f_A(u) = 0$ for all $u \in U$,

3- the α-vague set of U if $t_A(u) = \alpha$ and $f_A(u) = 1 - \alpha$ for all $u \in U$, where $\alpha \in (0,1)$.

Let $D[0,1]$ denotes the family of all closed sub-intervals of $[0,1]$. Now we define refined minimum (briefly, *rmin*) and order " $\leq$ " on elements $D_1 = [a_1, b_1]$ and $D_2 = [a_2, b_2]$ of $D[0,1]$ as:

$$rmin(D_1, D_2) = [min\{a_1, a_2\}, min\{b_1, b_2\}]$$

$$D_1 \leq D_2 \Longleftrightarrow a_1 \leq a_2 \ \wedge \ b_1 \leq b_2$$

Similarly we can define $\geq$, $=$ and *rmax*. Then concept of *rmin* and *rmax* could be extended to define *rinf* and *rsup* of infinite number of elements of $D[0,1]$.

It is that $L = \{D[0,1], rinf, rsup, \leq\}$ is a lattice with universal bounds $[0,0]$ and $[1,1]$.

For $\alpha, \beta \in [0,1]$ we now define (α,β)-cut and α-cut of a vague set.

Definition 1.3. [3] Let A be a vague set of a universe X with the true-membership function t_A and false-membership function f_A. The (α,β)-cut of the vague set A is a crisp subset $A_{(\alpha,\beta)}$ of the set X given by

$$A_{(\alpha,\beta)} = \{x \in X \mid V_A(x) \geq [\alpha,\beta]\},$$

where $\alpha \leq \beta$.

Clearly $A_{(0,0)} = X$. The (α,β)-cuts are also called vague-cuts of the vague set A.

Definition 1.4. [3] The α-cut of the vague set A is a crisp subset A_α of the set X given by $A_\alpha = A_{(\alpha,\alpha)}$.

Note that $A_0 = X$ and if $\alpha \geq \beta$ then $A_\beta \subseteq A_\alpha$ and $A_{(\beta,\alpha)} = A_\alpha$. Equivalently, we can define the α-cut as

$$A_\alpha = \{x \in X \mid t_A(x) \geq \alpha\}.$$

Definition 1.5. Let f be a mapping from the set X to the set Y and let B be a vague set of Y. The inverse image of B, denoted by $f^{-1}(B)$, is a vague set of X which is defined by $V_{f^{-1}(B)}(x) = V_B(f(x))$ for all $x \in X$.

Conversely, let A be a vague set of X. Then the image of A, denoted by $f(A)$, is a vague set of Y such that:

$$V_{f(A)}(y) = \begin{cases} rsup_{z\in f^{-1}(y)} V_A(z) & \text{if } f^{-1}(y) = \{x : f(x) = y\} \neq \emptyset, \\ [0,0] & \text{otherwise.} \end{cases}$$

We assume for concreteness that X is a real Euclidean space E^n.

Definition 1.6. [3] A vague set A is convex if and only if the set Γ_α defined by

$$\Gamma_\alpha = \{x \mid t_A(x) \geq \alpha_t, 1 - f_A(x) \geq \alpha_f\}$$

are convex for all $\alpha_t, \alpha_f \in (0,1]$.

An alternative and more direct definition of convexity is the following:
A is convex if and only if

$$t_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(t_A(x_1), t_A(x_2))$$

$$1 - f_A(\lambda x_1 + (1-\lambda)x_2) \geq \min(1 - f_A(x_1), 1 - f_A(x_2))$$

for all $x_1, x_2 \in X$ and all $\lambda \in [0,1]$.

Definition 1.7. [3] A vague set A is strongly convex if for any two distinct point x_1 and x_2 in X and $\lambda \in (0,1)$

$$t_A(\lambda x_1 + (1-\lambda)x_2) > \min(t_A(x_1), t_A(x_2))$$

$$1 - f_A(\lambda x_1 + (1-\lambda)x_2) > \min(1 - f_A(x_1), 1 - f_A(x_2))$$

Theorem 1.8. *[3] If A and B are (strongly) convex, so is their intersections.*

Definition 1.9. [1] A vague set $A = \{(u; [t_A(u), 1 - f_A(u)]) \mid u \in U\}$, is said to be t-convex vague set if

$$t_A(\lambda x_1 + (1-\lambda)x_2) \geq T(t_A(x_1), t_A(x_2))$$

$$1 - f_A(\lambda x_1 + (1-\lambda)x_2) \geq T(1 - f_A(x_1), 1 - f_A(x_2))$$

for all $x, y \in E$ and $\lambda \in [0,1]$ and t-norm T. In some sense, this is consistent with the definition of vague sets.

In the same way, we can also define the notion of t-concave vague set.

Definition 1.10. [1] A vague set $A = \{(u;[t_A(u), 1-f_A(u)]) \mid u \in U\}$, is said to be t-concave vague set if

$$t_A(\lambda x_1 + (1-\lambda)x_2) \leq S(t_A(x_1), t_A(x_2))$$

$$1 - f_A(\lambda x_1 + (1-\lambda)x_2) \leq S(1-f_A(x_1), 1-f_A(x_2))$$

for all $x, y \in E$ and $\lambda \in [0,1]$ and t-conorm S.

2. t-norm based graded convex vague sets

In a natural way, in this section, we generalized the notion of t-convex vague set from the viewpoint of t-norm based fuzzy logic, and proposed the notions of (α,β,T)-convex (concave) vague sets. In what follows, T always denote a left continuous t-norm.

Definition 2.1. A vague set $A = \{(u;\mu_A(u)) \mid u \in E\}$, is said to be (α,β,T)-convex vague set if

$$t_A(\lambda x_1 + (1-\lambda)x_2) \geq T(T(t_A(x_1), t_A(x_2)), \alpha)$$

$$1 - f_A(\lambda x_1 + (1-\lambda)x_2) \geq T(T(1-f_A(x_1), 1-f_A(x_2)), \beta)$$

for all $x, y \in E$ and $\alpha, \beta, \lambda \in [0,1]$. The number α may be considered as the degree to which A is convex, the number β may be considered as the degree to which the complementary set of A is concave. In some sense, this is consistent with the definition of vague sets.

In the same way, we can also define the notion of (α,β,T)- concave vague set.

Definition 2.2. An vague set $A = \{(u;[t_A(u), 1-f_A(u)]) \mid u \in E\}$, is said to be (α,β,T)-concave vague set if A^c is (α,β,T)-convex, i.e.,

$$t_A(\lambda x_1 + (1-\lambda)x_2) \leq S(S(t_A(x_1), t_A(x_2)), \alpha)$$

$$1 - f_A(\lambda x_1 + (1-\lambda)x_2) \leq S(S(1-f_A(x_1), 1-f_A(x_2)), \beta)$$

for all $x, y \in E$ and $\alpha, \beta, \lambda \in [0,1]$. The number α may be considered as the degree to which A is concave, the number β may be considered as the degree to which the complementary set of A is convex.

In crisp case, the intersection of any two convex sets is also convex set, the convex fuzzy sets defined by Zadeh preserve this property as well. In the following, we generalize the intersection, union and complement of vague sets using triangular norms, it's dual conorms and inverse order and involutive operators, define $\cap_T$, $\cup_S$ and A^c as follows:

$$A^c = \{(u;[f_A(u), 1-t_A(u)]) \mid u \in E\}$$

$$A \cap_T B = \{(u;[T(t_A(u), t_B(u)), 1-S(f_A(u), f_B(u))]) \mid u \in E\}$$

$$A \cup_S B = \{(u;[S(t_A(u), t_B(u)), 1-T(f_A(u), f_B(u))]) \mid u \in E\}$$

Theorem 2.3. *If A and B are (α, β, T)-convex vague sets, then $A \cap_T B$ is a $(T(\alpha,\alpha), T(\beta,\beta), T)$-convex vague set.*

Proof. Let $C = A \cap_T B$. Then

$$t_C(\lambda x_1 + (1-\lambda)x_2) = T(t_A(\lambda x_1 + (1-\lambda)x_2), t_B(\lambda x_1 + (1-\lambda)x_2))$$

and

$$1 - f_C(\lambda x_1 + (1-\lambda)x_2) = T(1 - f_A(\lambda x_1 + (1-\lambda)x_2), 1 - f_B(\lambda x_1 + (1-\lambda)x_2)).$$

By hypothesis we have

$$t_A(\lambda x_1 + (1-\lambda)x_2) \geq T(T(t_A(x_1), t_A(x_2)), \alpha)$$
$$t_B(\lambda x_1 + (1-\lambda)x_2) \geq T(T(t_B(x_1), t_B(x_2)), \alpha)$$
$$1 - f_A(\lambda x_1 + (1-\lambda)x_2) \geq T(T(1 - f_A(x_1), 1 - f_A(x_2)), \beta)$$
$$1 - f_B(\lambda x_1 + (1-\lambda)x_2) \geq T(T(1 - f_B(x_1), 1 - f_B(x_2)), \beta)$$

therefore

$$\begin{aligned} t_C(\lambda x_1 + (1-\lambda)x_2) &= T(t_A(\lambda x_1 + (1-\lambda)x_2), t_B(\lambda x_1 + (1-\lambda)x_2)) \\ &\geq T(T(T(t_A(x_1), t_A(x_2)), \alpha), T(T(t_B(x_1), t_B(x_2)), \alpha)) \\ &\geq T(T(T(t_A(x_1), t_A(x_2)), T(t_B(x_1), t_B(x_2), T(\alpha, \alpha)) \\ &= T(T(t_C(x_1), t_C(x_2)), T(\alpha, \alpha)) \end{aligned}$$

also

$$\begin{aligned} &1 - f_C(\lambda x_1 + (1-\lambda)x_2) \\ &= T(1 - f_A(\lambda x_1 + (1-\lambda)x_2), 1 - f_B(\lambda x_1 + (1-\lambda)x_2)) \\ &\geq T(T(T(1 - f_A(x_1), 1 - f_A(x_2)), \beta), T(T(1 - f_B(x_1), 1 - f_B(x_2)), \beta)) \\ &\geq T(T(T(1 - f_A(x_1), 1 - f_A(x_2)), T(1 - f_B(x_1), 1 - f_B(x_2), T(\beta, \beta)) \\ &= T(T(1 - f_C(x_1), 1 - f_C(x_2)), T(\beta, \beta)). \end{aligned}$$

□

In the same way, we can prove the following conclusion.

Theorem 2.4. *If A and B are (α,β,T)-concave vague sets, then $A\cup_S B$ is (α,β,T)-concave vague set.*

The following conclusion are obvious.

Theorem 2.5. *If A is a (α,β,T)-convex vague set, then A^c is (α,β,T)-concave vague set, and vice versa.*

By definition, the following two conclusions are obvious.

Theorem 2.6. *Let T_1 and T_2 be two left continuous t-norms and T_1 is weaker then T_2, i.e. $T_1(x,y)\le T_2(x,y)$ for any $(x,y)\in[0,1]^2$, then every (α,β,T_2)-convex (or concave) vague set is also (α,β,T_1)-convex (or concave) vague set.*

Remark 2.7. Since T_G is the strongest t-norm on $[0,1]$ and the Drastic product T_Δ is the weakest t-norm on $[0,1]$, so for any $\alpha,\beta\in(0,1]$ and any t-norm T, every (α,β,T_G)-convex (or concave) vague set is also an (α,β,T)-convex (or concave) vague set, every (α,β,T)-convex (or concave) vague set is also an (α,β,T_Δ)-convex (or concave) vague set, where the drastic product T_Δ is defined as follows:

$$T_\Delta(a,b)=\begin{cases} a & \text{if } b=1,\\ b & \text{if } a=1,\\ 0 & \text{otherwise.}\end{cases}$$

Corollary 2.8. *If A and B are (α,β,T_G)-convex vague sets, then $A\cap_T B$ is $(T(\alpha,\alpha),T(\beta,\beta),T)$-convex vague set for any t-norm T.*

Corollary 2.9. *If A and B are (α,β,T_G)-concave vague set, then $A\cup_S B$ is (α,β,T)-concave vague set for any t-norm T, where S is the dual conorm of T.*

Theorem 2.10. *Let $\alpha_i,\beta_i\in(0,1]$, $i=1,2$ and $\alpha_1\le\alpha_2$, $\beta_1\le\beta_2$, then every $(\alpha_1,\beta_1,\ T)$-convex (or concave) vague set is also (α_2,β_2,T)-convex (or concave) vague set.*

If t-norm T is continuous, then we have

Theorem 2.11. *Let $\alpha_i,\beta_i\in(0,1]$, $i=1,2,...,n,...$, $\lim_{i\to\infty}\alpha_i=\alpha$, $\lim_{i\to\infty}\beta_i=\beta$ and T is continuous. if A is (α_i,β_i,T)-convex (or concave) vague set for any $i\in N$, then A is also (α,β,T)-convex (or concave) vague set.*

It is well known that every level set of Zadeh's convex fuzzy set is a convex subset. In what follows, we will discuss the properties of various level sets of (α,β,T)-convex (or concave) vague set.

Theorem 2.12. *Let A be a vague set on E. If for any $a,b \in [0,1]$, both the sets of $A_a^1 = \{x \in E \mid t_A(x) \geq a\}$ and $A_b^2 = \{x \in E \mid 1 - f_A(x) \geq b\}$ are convex subsets in E, then A is an (α,β,T)-convex vague set.*

Proof. Assume that for any $a \in [0,1]$, both A_a^1 and A_a^2 are convex subsets in E. Let $a = T(T(t_A(x),t_A(y)),\alpha)$. For any $x,y \in E$, then $t_A(x) \geq a$, $t_A(y) \geq a$, thus $x,y \in A_a^1$. Since A_a^1 is a convex set, thus for any $\lambda \in [0,1]$ we have, $\lambda x + (1-\lambda)y \in A_a^1$. Therefore, $t_A(\lambda x + (1-\lambda)y) \geq a = T(T(t_A(x),t_A(y)),\alpha)$. On the other hand, let $b = T(T(1-f_A(x),1-f_A(y)),\beta)$, then $1-f_A(x) \geq b$, $1-f_A(y) \geq b$, thus $x,y \in A_b^2$. Since A_b^2 is a convex set, thus for any $\lambda \in [0,1]$ we have, $\lambda x + (1-\lambda)y \in A_b^2$. Hence $1 - f_A(\lambda x + (1-\lambda)y) \geq b = T(T(1-f_A(x),1-f_A(y)),\beta)$. Sum up, A is an (α,β,T)-convex vague set. □

The converse of above theorem is not true.

Example 2.13. Let $A = \{(x;[t_A(x),1-f_A(x)]) \mid x \in E\}$ be a vague set defined by:

$$t_A(x) = \begin{cases} 0.4x - 0.4 & \text{if } 1 \leq x < 2, \\ 0.4x - 0.2 & \text{if } 2 \leq x < 3 \end{cases}$$

and

$$f_A(x) = \begin{cases} 1.4 - 0.4x & \text{if } 1 \leq x < 2, \\ 1.2 - 0.4x & \text{if } 2 \leq x < 3 \end{cases}$$

It is clear that A is a $(1,1,T_\Delta)$-convex vague set, but the sets $A_a^1 = \{x \in E \mid t_A(x) \geq a\}$ and $A_b^2 = \{x \in E \mid 1 - f_A(x) \geq b\}$ are not convex subsets in E, for any $a,b \in [0,1]$.

Theorem 2.14. *If A is an (α,β,T_G)-convex vague set on E, then for any $a \leq \alpha$, $b \leq \beta$, both A_a^1 and A_b^2 are convex subsets in E.*

Proof. Assume that A is an (α,β,T_G)-convex vague set. For any $a \leq \alpha$, let $x,y \in A_a^1$ and $\lambda \in [0,1]$, then $t_A(x) \geq a$, $t_A(y) \geq a$. Since $t_A(\lambda x + (1-\lambda)y) \geq T_G(T_G(t_A(x),t_A(y)),\alpha)$, thus $T_G(T_G(t_A(x),t_A(y)),\alpha) \geq T_G(T_G(a,a),\alpha) = T_G(a,\alpha) = a$. Hence $t_A(\lambda x + (1-\lambda)y) \geq a$, this shows that $\lambda x + (1-\lambda)y \in A_a^1$. To sum up, A_a^1 is a convex subset in E.

Similarly, we can prove that A_b^2 is also a convex subset in E. □

Corollary 2.15. *If A is an (α,α,T_G)-convex vague set on E, then for any $a \leq \alpha$, A_a is convex subset in E.*

For graded concave vague sets, some analogous conclusions can be obtained.

Theorem 2.16. *Let A be a vague set on E. If for any $a,b \in [0,1]$, both the sets of $B_a^1 = \{x \in E \mid t_A(x) \leq 1-a\}$ and $B_b^2 = \{x \in E \mid f_A(x) \geq b\}$ are convex subsets in E, then A is an (α,β,T_G)-concave vague set*

Theorem 2.17. *If A is an (α,β,T_G)-concave vague set on E, then for any $a \leq \alpha$, $b \leq \beta$, both B_a^1 and B_b^2 are convex subsets in E.*

Corollary 2.18. *If A is an (α,β,T_G)-concave vague set on E, then for any aleqα, $B_a = \{x \in E \mid t_A(x) \leq 1-a \text{ and } f_A(x) \geq a\}$ is convex subset in E.*

Theorem 2.19. *If A is an (α,β,T_G)-convex vague set on E, then for any $\lambda \leq \alpha$ and $\lambda \leq \beta$, the following lower cut set*

$$A_\lambda = \begin{cases} 1 & t_A(x) \geq \lambda, \\ \frac{1}{2} & t_A(x) < \lambda \leq 1 - f_A(x), \\ 0 & \lambda > 1 - f_A(x) \end{cases}$$

is a convex vague set in E.

Proof. Assume that A is an (α,β,T_G)-convex vague set. We only need to prove that for any $x,y \in E$, $\lambda \in [0,1]$, $A_\lambda(\gamma x + (1-\gamma)y) \geq \min(A_\lambda(x), A_\lambda(y))$. If $A_\lambda(x) = 0$ or $A_\lambda(y) = 0$, the conclusion is obvious.

If $A_\lambda(x) = 1$ and $A_\lambda(y) = 1$, then $t_A(x) \geq \lambda$ and $t_A(y) \geq \lambda$, it follows that

$$\begin{aligned} t_A(\gamma x + (1-\gamma)y) &\geq T_G(T_G(t_A(x), t_A(y)), \alpha) \\ &\geq T_G(T_G(\lambda,\lambda),\alpha) \\ &= \lambda. \end{aligned}$$

thus $A_\lambda(\gamma x + (1-\gamma)y) = 1 \geq \min(A_\lambda(x), A_\lambda(y))$.

If $A_\lambda(x) = 1$ and $A_\lambda(y) = \frac{1}{2}$, then $t_A(x) \geq \lambda$ and $1 - f_A(y) \geq \lambda$. Since

$$1 - f_A(\gamma x + (1-\gamma)y) \geq T_G(T_G(1 - f_A(x), 1 - f_A(y)), \beta)$$

by hypothesis we have $t_A(x) \leq 1 - f_A(x)$, then

$$\begin{aligned} 1 - f_A(\gamma x + (1-\gamma)y) &\geq T_G(T_G(1 - f_A(x), 1 - f_A(y)), \beta) \\ &\geq T_G(T_G(\lambda,\lambda),\beta) \\ &= \lambda. \end{aligned}$$

Therefore $A_\lambda(\gamma x + (1-\gamma)y) = \frac{1}{2} \geq \min(A_\lambda(x), A_\lambda(y))$.

If $A_\lambda(x) = \frac{1}{2}$ and $A_\lambda(y) = \frac{1}{2}$, then $1 - f_A(x) \geq \lambda$ and $1 - f_A(y) \geq \lambda$. Similarly, we can see $A_\lambda(\gamma x + (1-\gamma)y) = \frac{1}{2} \geq \min(A_\lambda(x), A_\lambda(y))$. Sum up, A_λ is a convex vague set in E. □

Similarly, we can prove the following conclusions.

Theorem 2.20. *If A is an (α,β,T_G)-concave vague set on E, then for any $\lambda \geq 1-\alpha$ and $\lambda \geq 1-\beta$, the lower cut set A_λ is a concave vague set in E.*

Theorem 2.21. *If A is an (α,β,T_G)-concave vague set on E, then for any $\lambda \geq \alpha$ and $\lambda \geq \beta$, the following upper cut set*

$$A^\lambda = \begin{cases} 1 & f_A(x) \geq \lambda, \\ \frac{1}{2} & f_A(x) < \lambda \leq 1-t_A(x), \\ 0 & \lambda > 1-t_A(x) \end{cases}$$

is a convex vague set in E.

Theorem 2.22. *If A is an (α,β,T_G)-convex vague set on E, then for any $\lambda \geq 1-\alpha$ and $\lambda \geq 1-\beta$, the upper cut set A^λ is a concave vague set in E.*

3. CONCLUSION

The notion of vague set theory introduced by W. L. Gau and D. J. Buehrer [3], as a generalizations of Zadeh's fuzzy set theory [6] and convex vague sets are generalization of convex vague sets. In this paper, a kind of graded convex (concave) vague sets have been established based on triangular norms, some properties of their various cut sets have also been presented. As we have seen, the main conclusions obtained are suitable for all t-norms.

It is our hope that this work would other foundations for further study of the theory of vague sets and convex sets.

In our future study of structure of vague, may be the following topics should be considered:

(1) To get more results in convex sets and application;

(2) If we take other t-norms as the underlying t-norm, what properties can be obtained?

(3) To define another types of convex vague sets.

Acknowledgments

The author is extremely grateful to Prof. Salvatore Giuffrida and to the anonymous referees for giving them many valuable comments and helpful suggestions which helps to improve the presentation of this paper.

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ARSHAM BORUMAND SAEID
Depatment of Mathematics
Shahid Bahonar University of Kerman
Kerman, Iran
e-mail: arsham@uk.ac.ir