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Coasean Fictions: Law and Economics Revisited

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After the 1960s, a strong academic movement developed in the United States around the idea that the study of the law, as well as legal practice, could be strengthened through economic analysis.[2] This trend was started through the works of R.H. Coase and Guido Calabresi, and grew with later developments introduced by Richard Posner and Robert D. Cooter, as well as Gary Becker.[3] Law and Economics (L&E) is the name given to the application of modern economics to legal analysis and practice.

Proponents of L&E have portrayed the movement as improving clarity and logic in legal analysis, and even as a tool to modify the conceptual categories used by lawyers and courts to think about legal problems.[4] In the most extreme interpretation of L&E, the tools of microeconomic analysis are seen as allowing legal analysis to identify which laws have greater merits for society.[5] The supporters of this academic movement assume it allows for a clearer understanding of the process through which inefficient laws are discarded while efficient legislation is enacted and implemented.[6]

As an academic movement, L&E has had significant influence on contract law, torts, criminal law, environmental legislation, privatization, deregulation, and natural resource management, as well as many other dimensions of relevance for the application of law and adjudication in legal disputes. Posner, for example, has promoted the application of the main tenets of L&E to the law of contracts through the concept of efficient breach of contracts.[7] In fact, Posner thinks that the law should be designed to ensure that assets and opportunities are in the hands of those who can and would pay most to have them.

The underlying premise of L&E is that economic theory has succeeded in providing a solid foundation for the idea that competitive markets allocate

resources efficiently.[8]   The purported rigor of economic analysis is perceived to be linked to the basic theoretical contributions of economic theory. It is clear that supporters of L&E firmly believe that economic theory is a solid scientific edifice producing results that can be readily used, and in many cases, translated to the realm of the legal profession.

For some analysts, applying economic analysis to the study of law poses a significant challenge to more conventional approaches. For example, Kornhauser thinks that economic analysis of law is disquieting because explaining normativity is a central pre-occupation of the philosophy of law, but the model of self-interested maximization of preferences does not admit a concept of normativity.[9]   According to Kornhauser, the logic of this commitment to self-interested maximization of preferences appears to imply a denial of the need for a distinct concept of law in the explanation and evaluation of social institutions.[10]

The juxtaposition of justice—or ethics—versus efficiency is also a problematic question. For example, Egger thinks that efficiency cannot serve as a substitute for ethics.[11] Likewise, Arnold embraces the idea that the "criterion of the marketplace (as efficiency has been called)" and the ethics-based criterion are really very different in their essential characteristics, and in their consequences, when used to determine adjudication.[12] The bottom line for him is that the law must *at least* be perceived as fair—not just efficient—in order to maintain social stability.[13]

Ultimately, the central issue is that economic analysis conceives human conduct as being part of a system of relations in which there is no room for ethical considerations. In economic theory, markets make the plans of selfish individuals mutually compatible, but economic laws are imposed on agents without their knowing about them. For example, in the theoretical constructs of modern economics, agents may ignore the contents of the law of supply and demand, but they will eventually suffer its effects and will have to act accordingly. They have no inter personal choices to make— they only have to select the bundles of commodities that maximize their

satisfaction. This is why there is no room for deliberation on justice or ethics in economic theory. Agents do not ask questions about the ethical nature of their acts. There are simply different allocation schemes—some are considered "efficient," while others are not. L&E is just a way of subordinating legal institutions to the dictates of modern economic theory, a discourse that claims to be scientific in nature.

Although these considerations may imply a critique of L&E, the underlying premise concerning the scientific soundness of economic theory is not questioned. The critical analysis presented in this article takes a different tack. This article examines the theoretical shortcomings of modern economic analysis and illustrates the implications for L&E. This paper is concerned with the poverty of results attained by modern economic theory in its quest for a scientific explanation of markets and their dynamics, and how this undermines the basic tenets of L&E. The objective is not to criticize the lack of realism of some of the assumptions and restrictions found in the theoretical discourse of economics, but rather to expose the logical incoherence of modern economic theory. In doing so, we[14] will unravel some of the critical limitations of L&E and show that faith in the scientific robustness of economic theory is misplaced.[15] Contrary to what advocates of L&E assert, modern economic theory is in poor shape, and the mainstream research program, centered on general equilibrium theory, is a dead-end street.

In the first section of this article we examine the underlying assumptions of Coase's Theorem, establishing its links with the theory of a bilateral monopoly. This section analyzes the implications of the indeterminacy of contract first identified by Edgeworth in the case of a bilateral monopoly. The second section focuses on the relationship between the Coase Theorem and n-commodity models. More precisely, the analysis centers on the scope of the Coase Theorem and the limitations of general equilibrium theory. These limitations stem from the inability of the theory to deliver robust and scientifically sound results to the critical question of how equilibrium prices

are formed. In the final section we summarize our main conclusions. Throughout, the role of mathematics is touched upon.

I. COASE'S THEOREM

Ronald Coase, an English economist working at the University of Chicago, was awarded the 1991 Nobel Prize in economics for his contributions on transactions costs and their relevance to institutions. Interestingly, it was not Coase, but another economist named George Stigler, who coined the phrase "the Coase Theorem," stating that "under perfect competition, private and social costs will be equal."[16] This statement essentially means that with zero transaction costs, rights for the use of factors of production will be exchanged until the value of production is maximized.

According to Coase,

> [i]f rights to perform certain actions can be bought and sold, they will tend to be acquired by those for whom they are most valuable either for production or enjoyment. In this process, rights will be acquired, subdivided, and combined, so as to allow those actions to be carried out which bring about that outcome which has the greatest value on the market.[17]

A typical example illustrating this process is the case of a factory polluting neighboring fields. In that example, the conventional economist's remedy is to impose restrictions on the factory's owner and to force payment to redress damages, but Coase argues against this solution because it may lead to socially inefficient outcomes.[18] By this, Coase means that in the absence of transaction costs, people can always negotiate without cost to trade rights whenever this leads to increases in the value of production.

A. Zero-Cost Assumption

Foregoing or negotiating these trade rights to others must match the costs of carrying out the transactions required—the new allocation of rights will

be attained only if the cost of the transactions needed to achieve that outcome is less than the increase in value that such rearrangement makes possible.[19] The use of the term "greatest value" in this context is a direct reference to the maximization of welfare, or utility, in general equilibrium analysis. In general, the term "efficiency" refers to Pareto-optimality or its variant, Hick-Kaldor efficiency, and pertains to economic circumstances in which no one can be made better off without someone else being made worse off.[20]

From the point of view of efficiency, and when transaction costs are zero, "the ultimate result (which maximizes the value of production) is independent of the legal system" or the initial allocation of rights.[21] This statement epitomizes the Coase Theorem, and Stigler first formulated it as such.[22] Coase has expressed concern over the fact that the transaction costs assumption plays such a critical role: "the world of zero transaction costs… is the world of modern economic analysis, and economists therefore feel quite comfortable handling the intellectual problem it poses, remote from the real world though they may be."[23] Coase states that he is more concerned with the problems of the real world, where transaction costs are positive.[24]

This appears to put the question of realism—or lack thereof—firmly in the center of the debate: although the real world with positive transaction costs is more interesting, under the restrictive assumption of zero transaction costs, the Coase Theorem holds true. However, as will be seen later, upholding this statement is a risky proposition. In fact, there is no rational foundation for this conjecture.

First, consider a situation that consists of two agents and two commodities.[25] The most important examples used by Coase correspond to what is known in economic theory as the case of a bilateral monopoly: trade that takes place between two agents in a two-commodity economy. Between the two agents, they own the entire stock of the two commodities; thus, there is no competition from other owners of these commodities. In

these models, the entire available stock of the two commodities is allocated between the two agents. Hovenkamp states that in these cases, economic efficiency cannot be proven on ordinarily neoclassical premises.[26]

The case of a bilateral monopoly, then, covers a very unique and perhaps less interesting example that perfectly illustrates the flaws in Coase's theorem. For one thing, because there are only two commodities involved, a bilateral monopoly excludes the possibility of arbitraging between several commodities, a point we will return to below. It is important to bear in mind that any meaningful discussion regarding economic efficiency and its theoretical foundations needs to be considered in the framework of richer and more complex cases. That point is indeed covered by n-commodity models and is examined by general equilibrium theory. At this initial stage, however, it is important to examine the nature of the problem posed by Coase in terms of the theory of a bilateral monopoly.

Figure 1 is a graphic description of the situation through an Edgeworth Box. Here, W is the initial allocation of commodities M and N between agents I and II.[27] The indifference curves, meeting at point W, mark the boundaries of the transactions: for Agent I (respective to Agent II), any point to the left (respective to the right-hand side of the figure) of the indifference curve crossing W will be rejected because it worsens his or her position.[28]

Figure 1
Coase's Theorem, Multiple Equilibria, and a Bilateral Monopoly
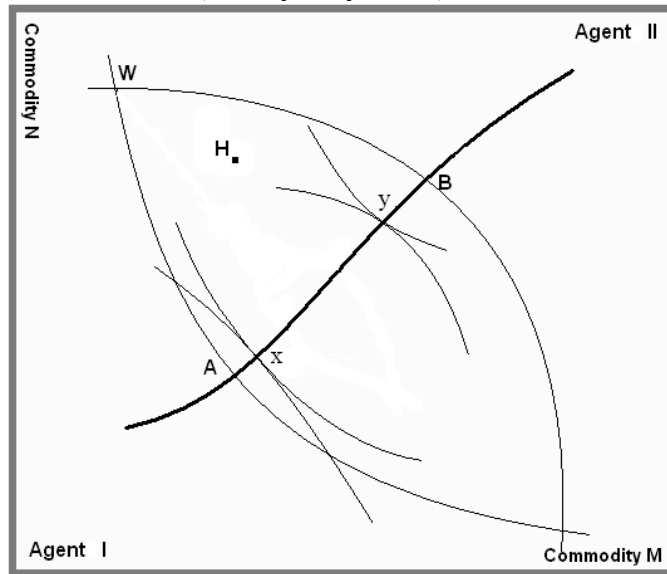
Prices are rates of substitution between the two commodities and are dictated by the slope of straight lines going through point W. For example, if the new allocation is x, the straight line connecting Wx corresponds to the price at which the deal was closed. Evidently, in that case the price of commodity M was rather high, explaining why Agent I would give up a large share of his initial stock of commodity N and obtain a relatively low amount of commodity M in return. The price of M, in terms of N, was higher than if the deal had been closed at point Y.

The set of points in the curve AB corresponds to points where the indifference curves of the agents are tangent—that is, where the indifference curves of both agents have the same slope. The locus of such points is called the contract curve because once agents reach one of those points, there are no new trading options that are mutually beneficial.

Efficiency is attained on the contract curve AB because utility maximization conditions are simultaneously met for both agents. All points in the contract curve are equilibrium points: on the contract curve both agents will refuse entering new negotiations because the ratios of the marginal utilities of each agent will be equal. On the contract curve there are no advantages to be gained by entering into new contracts, and thus, there are no incentives for further trade: the two-agent economy is in equilibrium.

B. Multiple Equilibria, Efficiency, and Collapse

Coase and his followers have stated that where agents can trade freely the outcome will be an efficient one. Edgeworth's analysis reveals that there are multiple efficient solutions to the problem of bilateral monopolies: every point in the contract curve is an efficient solution (i.e., Pareto-efficient allocations). Thus, in the simplest case of a bilateral monopoly, the final settlement agreed upon lies somewhere on the contract curve, but where exactly it will lie cannot be ascertained beforehand.[29]

Edgeworth asks the following question: why should an isolated pair of traders exchange every portion of their commodities at the same price?[30] If mutually advantageous exchanges are allowed, there is a trajectory towards a point of the contract curve, but the final equilibrium is path-dependent. At each stage, trade occurs over a portion of the stock of each commodity. Allocations change and, at every new exchange, the relevant segment of the contract curve is determined by the new set of indifference curves that cross through the point representing the new allocation.

Ultimately, the point of arrival on the contract curve depends on the negotiating abilities of both agents. For example, if Agent I offers to change the allocation and move to point Y, Agent II may or may not want to do this. Agent II may see that if he or she accepts the gains made by Agent I, then the resulting position would be disproportionately higher in comparison to his or her own gains (as shown by the movement to a much

higher indifference curve for Agent I). The reciprocal applies for Agent II with respect to point X. It is important to emphasize the fact that the negotiating process is based solely on maximization functions; nothing in this model allows us to expect the two agents will continue bargaining once they have reached a point on the contract curve.

An interesting aspect of this was first noted by Samuelson: at any point off the contract curve, there is a movement back toward the contract curve—a movement that improves the position of both agents—but in many cases of bilateral monopolies, a final equilibrium may be reached off the contract curve.[31] The explanation is that one or both agents involved in Edgeworth's box may be unwilling to discuss the possibility of making a mutually favorable movement for fear that the discussion may imperil the existing tolerable *status quo*.[32]

Coase dismisses the problem, claiming that the contention is "difficult to understand."[33] In a sense, this is true. Under the conditions and assumptions of the Edgeworth box, given an initial *status quo*, the indifference curve going through the point marking the initial allocation defines the boundaries of acceptable negotiations between the agents (and the relevant segment of the contract curve). No deal will be accepted outside these bounds. Also, if the model involves the usual assumptions, agents will continue their calculations and offers of possible contracts until a point on the contract curve is attained.

What happens when we allow for exchanges *outside* the contract curve but inside the lentil defined by the agents' indifference curves going through point W? This is an open possibility for the Coase Theorem and can be accommodated within the framework of the Edgeworth box. In a Coasean world, it appears that agents carry out negotiations without actual trading until they strike a deal. Presumably, the deal is closed when both parties are satisfied and their indifference curves are tangent. What happens, however, if agents find a trading opportunity that mutually improves their positions but is outside of the contract curve? Consider, for

example, the change from the initial allocation W to point H: this trade improves the agents' positions (both move to higher indifference curves), and it therefore makes sense to strike a deal that is mutually beneficial. However, the new allocation is not socially efficient. The interesting point here is that once trade has taken place, the new allocation may be irreversible. For example, when consumption and/or production actually occur, commodities and inputs are transformed. In a Coasean example, suppose two parties accept a transaction of rights and as a consequence a stream is polluted and fish die. If the resulting allocation of rights is mutually beneficial but is not on the contract curve, the Coase Theorem does not hold.[34]

The ultimate question is whether negotiations will collapse. This is a question of theoretical importance and empirical relevance, especially in the context of L&E. Cooter seems to believe that the bargaining process to reach a suitable deal is inherently unstable and may collapse, even with zero transaction costs.[35] The truth is that agents in the model operate through maximization functions, and nothing prepares them for the case where the initial offer is rejected. The model is basically static and does not reproduce the dynamics of negotiations. This is why the model does not tell us where on the contract curve the process ends. It is clear that agents can refuse bids, potentially bringing the process to a halt. In actual practice, this may happen as agents try to rearrange the allocation of their rights through trade. If the process collapses, the agents will not reach points on the contract curve. All of this is independent of the transaction costs. The point raised by Samuelson—that equilibrium may be reached off the contract curve— has been neglected by the L&E debate.

Although efficiency can be attained if we assume a deal will be settled at a point on the contract curve, there still exists a difficult ethical issue. A stronger agent will certainly take advantage of the weaknesses of the other agent and reap a much higher benefit. For example, in Coase's classic example, the railroad company may have more information or may outlast

the farmer, and the deal will be lopsided in favor of the railway's owners.[36] The unevenness is more serious when the asymmetries in negotiating capabilities are more significant. For example, the railroad company may be able to outwait the other party as it tries to determine a solution. In the end, the railroad and the farmer might reach a "mutually beneficial arrangement," but the benefits will most certainly be concentrated on one side of the table.

C. Invariance

The existence of multiple equilibria raises yet another problem. In Coase's own words, the "ultimate result (which maximizes the value of production) is independent of the legal position if the pricing system is assumed to work without cost."[37] In the same breath, Coase asserts that the "long-run equilibrium position" is the same whether the business is held responsible or not.[38] Thus, Coase affirms the "reciprocal nature of the problem," meaning that both sides will negotiate and reach an agreement that will be efficient and invariant to the underlying legal rule.[39] But multiple equilibrium positions mean there is no unique long-run equilibrium. It also means, as is illustrated by the Edgeworth box, that agents will have incentives to engage in strategic behavior and this will influence the end result. Thus, there is no reason to expect that the result will be invariant.

It should be noted that the definition of the invariance component not only eschews the distributional question that we have examined here, but also does not make any economic sense. Take the example presented by Coase, in which ranchers own cattle that stray into farmland and destroy crops.[40] In one case the ranchers are held liable, while in the other they are not.[41] According to Coase, the end result is the same: in one instance, the ranchers must pay for the damage done, while in the other, the farmers would be "willing to pay (up to) the value of the damage to induce the cattle-raiser to stop it, so that for the cattle-raiser to continue his operations

and bring about this crop damage would mean foregoing this sum."[42] Here it should be noted that for the farmer, this is a typical lose-lose situation: whether he absorbs the damages or buys the right of ranchers to do damage, he suffers a loss. On the other hand, this is a typical win-win situation from the perspective of the rancher: his cattle can forage on other lands and damage crops, or he can receive income (up to) the value of the meat he could produce with the cattle feeding on land that is not his. For farmers there is a real cost, while ranchers merely forego the sum in question.

The critical issue here is that there is no reason to expect that negotiations will lead to the same result regardless of the initial distribution of rights. In fact, there is no reason to expect that the end result will always be the same. The Edgeworth box framework helps clarify this: every point in the relevant interval of the contract curve is a candidate for the conclusion of the deal. The end point depends on how negotiations define a new trajectory every time the process starts. Thus, there is no reason to expect that invariance follows when things such as liabilities are determined in different forms.

Multiplicity of equilibria raises serious distributional questions that cannot be shunted aside. From the perspective of equity and fairness, the Coase Theorem is called into question because if there are gains from trade, the distribution may be severely lopsided. Upholding the theorem comes at a very high cost because from the perspective of ethics, the law is more concerned with fairness than with efficiency. Market theory, on the other hand, is interested in efficiency and is not disturbed by questions of fairness.[43] Sustaining the Coase Theorem transforms fairness into just another commodity. From an ethics perspective, the theorem becomes meaningless.

In economic theory, competitive markets are those in which agents are passive with respect to prices (i.e., they are price-takers).[44] In the case of a bilateral monopoly, on the other hand, agents may make offers and modify them, thereby actively changing the prices at which the deal can be closed. In this sense, a bilateral monopoly is not a competitive market. However,

as we have seen, efficiency is attained when points on the contract curve are reached. The problem is that it is precisely *because* agents actively change prices—and adopt a strategic behavior—that distributional questions arise. Cooter is misguided in stating that whenever non-competitive markets arise there is no room for allocations with Pareto-optimality.[45] In the case of a bilateral monopoly, Pareto-optimality is trivial, but distribution becomes a major issue due to the presence of multiple equilibria.[46]

A disproportionate part of the debate over the Coase Theorem revolves around the assumption of zero transaction costs. This is unfortunate because the central assumption is not whether there are zero transaction costs but, rather, whether we are looking at a two-commodity world or an *n-commodity* economy. In the next section we examine what happens when this two-commodity assumption is relaxed.

II. THE GENERAL CASE: N-COMMODITY MODELS

In a two-commodity world, the only prices involved pertain to those two commodities.[47] This is why, in examining Coase's examples, the only parameters taken into consideration by both agents are their indifference curves and the relative price of the two commodities involved in the transaction.[48] This is a heavy-handed assumption.

In a typical Coasean example, when a locomotive emits sparks and burns farmers' fields adjacent to the railway, the railroad owner may be compelled or required to compensate the farmers for their losses. But, how much is the railroad owner willing to pay? Or, should the farmer pay the railroad to help prevent the burning fields? Answering these questions, however, not only depends on the price of the wheat in the fields and the cost of installing a shield to prevent sparks from burning the fields, but also on the prices of a myriad of other commodities and services relevant to the cost structures of railways and farming, as well as on the prices of all the other commodities that railway owners and farmers are willing to buy and sell. In fact, the

answers depend on the cost structures of alternative investments that railway owners and farmers may want to consider.

In economic theory, the parameters of the demand and supply functions are not only the prices of the commodities directly involved in each market—the parameters also include the prices of all the other commodities. The reason for this is that agents need to take into account their total income in order to have an accurate measure of their budget constraints. Total income depends on the prices of the commodities in the original endowments of each agent. This is why general equilibrium theory is so appealing: it is supposed to be capable of dealing with the general case of systems of prices in interdependent markets. Likewise, this is why the explanatory power of partial equilibrium and a bilateral monopoly is so limited.

We have noted that the critical assumption of the Coase Theorem is not that transaction costs are zero, but rather that we are in a market in which all other prices are irrelevant. This assumption restricts the theorem to the least interesting of cases (bilateral monopolies) and, in accepting the assumption, we disregard a series of difficult problems that economic theory has failed to solve. This is also a very problematic assumption because it is a theory that rests on the premise that markets allocate resources efficiently. The reason for this is that the notion of efficiency attains its full stature in a general model where n-commodities coexist in interdependent markets. And this is where modern economic theory has failed to provide a model that shows how market forces lead an economy to an equilibrium—and an efficient—allocation. We now turn to examine two of the most serious problems of general equilibrium theory: arbitraging and the dynamic formation of equilibrium prices.

A. Arbitrages and Price Systems

Examining the complications that arise when dealing with a three-commodity economy will shed light on how the Coasean world of economic

theory is restricted to the simplistic and misleading two-commodity economy. To examine this issue we start by considering what happens when the simple two-commodity assumption is relaxed. Here we discover the first problem: arbitraging is now a possibility and agents can opt for indirect exchanges.

When we discussed the context of a bilateral monopoly, we excluded the possibility that agents could engage in arbitrages—a possibility that was excluded by the fact that in a two-commodity world there is only one relative price. That fact is eliminated when considering the more complex three-commodity economy. In general, when there is a move to an n-commodity economy there are $n(n-1)/2$ relative prices.[49]

In a three-commodity model, there is a new phenomenon where commodities can be traded through direct or indirect exchanges.[50] In indirect trading, commodities are obtained in order to use them as a means of exchange. For example, consider three commodities, each one owned by one agent: commodity (A) can be traded for (B), or the owner of (A) can first trade it for (C) and then proceed to exchange (C) against (B). Benetti has carried out the complete analysis of this problem and distills its deep theoretical implications.[51]

According to Benetti, the price of one commodity exchanged directly with another commodity may or may not coincide with the price when indirect trades occur. If the price of these two paths does not coincide, gains are likely to take place. This opens the door to arbitraging, a very important phenomenon that was absent from the two-commodity model.

The problems arising from the transition from a two- to a three-commodity world were first analyzed by Walras in the following terms.[52] There are now three "special" markets as commodities are traded pairwise: [A:B], [A:C], and [B:C]. In each of these markets, let $p_{a,b}$ ($p_{b,a}$), $p_{a,c}$ ($p_{c,a}$), and $p_{b,c}$ ($p_{c,b}$) be the prices obtained in the direct exchanges between commodities (A), (B), and (C). The prices in parentheses are the reciprocals, such that $p_{a,b} = 1/p_{b,a}$.[53]

In this context, there are two prices for each commodity; the first price corresponds to direct trading and the second price corresponds to indirect trading.[54]   For example, $p_{a,b}$ is the price when trading takes place directly between commodities (A) and (B).   But agents can resort to indirect exchanges. Suppose one agent trades (A) for (C) and then proceeds to trade (C) against (B). In that case, the relation between prices obtained directly and indirectly can be expressed in the following manner:

$$p_{a,c} = \alpha \, p_{a,b} \, / \, p_{c,b} \qquad \text{(Equation 1)}$$

When $\alpha = 1$, the prices of direct and indirect exchanges coincide.[55] Walras shows that when the price of any two commodities is proportional to their prices expressed in a third commodity, there is a price system in a very precise sense—there is only one price for each commodity, and, regardless of the trajectory of transactions that is followed, the end result is always the same.[56] This result occurs when markets are connected through a set of consistent prices. Agents are indifferent to the path of transactions—they can go to either direct or indirect exchanges, and the result will be identical.

When markets are connected through this price system, there are no opportunities for arbitraging. But what happens when we have a situation in which $\alpha \neq 1$? Walras provides an example with the following set of prices (reciprocals in parentheses):[57]

$$p_{c,b} = 4 \; (p_{b,c} = .25)$$
$$p_{c,a} = 6 \; (p_{a,c} = 0.166)$$
$$p_{b,a} = 2 \; (p_{a,b} = 0.5)$$

In direct trading, one unit of (C) obtains six units of (A). However, the result is different if we proceed through indirect exchanges: one unit of (C) buys four units of (B), and these four units of (B) can purchase eight units of (A). Evidently, the owner of commodity (C) has an interest in using the

indirect route to obtain (A) and will go to make an offer to owners of commodity (B). But in direct trading, one unit of (B) purchases 0.25 units of commodity (C). Going through indirect exchange, one unit of (B) buys two units of (A) and these can be traded for 0.332 units of commodity (C). The owner of commodity (B) can obtain a gain through indirect exchanges and will, therefore, proceed to meet agents with commodity (A).

Finally, in direct trading, one unit of (A) buys 0.5 units of commodity (B). The result is different in indirect exchanges: one unit of (A) provides 0.166 units of (C), and that quantity can be traded for 0.66 units of (B). Agents with commodity (A) will prefer the indirect route to obtain (B).

Figure 2
Arbitrages: Direct and Indirect Exchanges in a Three Commodity Economy
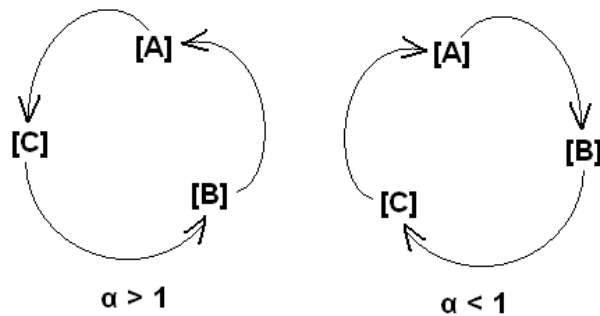
The end result is that an agent with commodity (A) will refuse to trade with an agent with commodity (B), while an agent with commodity (C) will refuse offers from agents with commodity (A), and agents with commodity (B) will reject trades with agents making an offer with commodity (C). In this example, when α > 1, the movement in the diagram follows a

counterclockwise sense (if instead $\alpha < 1$, the movement would follow a clockwise direction).

The direction of the arrows in Figure 2 indicates the direction of offers necessary to engage in a trade. As can be seen, a new problem arises in the absence of a price system ($\alpha = 1$) because there will be no trading at all. An agent with commodity (B), for example, will go to the market where direct trades between commodities (A) and (B) take place and will find that market empty. This means that there will be a supply of commodity (B) against commodity (A), but there will be no reciprocal demand. In general terms, in all markets there will be supplies but no demands (and reciprocal demands but no supplies). Ultimately, there is no reason to expect that $\alpha = 1$ and that a price system—a consistent set of prices—will prevail. And when $\alpha \neq 1$, there are incentives for speculation. In strict terms, speculation is defined as behavior that tends to take advantage of inconsistencies in the price system (or between price systems). This is why the idea of "buy cheap, sell dearly" is a key principle of speculation.

When a market process gets started, why should prices be consistent? Walras tackles this problem by assuming the existence of a price system, which is equivalent to assuming the following relationship:[58]

$$P_{a,b} \cdot P_{b,c} \cdot P_{c,a} = 1 \qquad \text{(Equation 2)}$$

This assumption (which implies $\alpha = 1$) allows Walras to eliminate arbitrages and to set the stage for the market process to start operating.[59] Working with a price system means there is only one price for each commodity: regardless of the path of transactions, the end result is always the same. This eliminates all incentives for speculative behavior, but it also constitutes an assumption that is very difficult to justify.

This is the tradition in price-formation analysis—a price system is assumed even *before* the market process begins. Benetti argues that the reason for this indefensible assumption is that if arbitrages are not

eliminated, the dynamic market process collapses because the price-adjustment process cannot function. In every market there will always be a positive demand of one good and a negative supply of that same good (or vice-versa).[60] This means that there will be no encounters between prospective traders. Of course, it could be advanced that with only three traders, the traders could rapidly decipher what is occurring and independently establish a price system. That, however, is not an adequate answer for a general situation where many agents and n-commodity settings exist, as it will not be possible for the agents to decipher such a condition.

It is now possible to see why the simplifying assumptions of the Coasean world rely on bilateral monopolies. This fiction allows Coase to hide all prices from the agents in his examples—*except* the prices of the two commodities that are involved in his examples—and thus prevent the agents from engaging in arbitrages. It is evident, though, that when agents engage in arbitraging the simple world of bilateral monopolies falls apart. If the Coase Theorem relies upon the notion that equilibrium is attained through a dynamic process of negotiations, then it would make sense—and it would be realistic—to permit arbitraging by agents. However, as we have seen, the adjustment process that could conceivably lead to equilibrium breaks down once we introduce the possibility of arbitraging. In light of this reality, Coase prefers the simplistic and less interesting world of a bilateral monopoly. It is ironic that the L&E movement rests on the premise that economic theory is robust, but the examples that are used rely exclusively on cases of a bilateral monopoly—cases that exclude arbitraging. In so doing, the followers of L&E turn their back on the general case that is the main objective of modern economic theory.

B. The Auctioneer, Stability, Gross Substitutes, and Money

Law and economics is based on the notion that markets allocate resources in an efficient manner. For example, Cooter states that "a central insight of microeconomics is that free exchange tends to move resources to their

highest valued use, in which case the allocation of resources is said to be Pareto-efficient."[61]   The rest of this article examines the validity of this "insight."

Just how do market forces lead economies to equilibrium allocations? This is a critical question for those who believe in the efficiency of the market. In equilibrium, a vector of prices, **p***, ensures that supply equals demand in every market.[62] How are the prices in **p*** formed? To examine this question we must refer to general equilibrium theory, a powerful construct first proposed by Leon Walras (1952) and developed through the works of John Hicks, Paul Samuelson, Kenneth Arrow, Gerard Debreu, and many others. General equilibrium theory (GET) is concerned with the role of the price system in attaining equilibrium allocations that make individual consumption and production plans compatible. The central question for GET is how equilibrium prices are formed.

Pareto-efficiency is attained when the economy reaches a position of general equilibrium. The demonstration of this was carried out by Samuelson through the two fundamental theorems of welfare economics.[63] The first theorem states that any competitive equilibrium is a Pareto-efficient allocation of resources whose proof requires rather weak assumptions.[64]   The second theorem states that any Pareto-efficient allocation can be sustained by a competitive equilibrium.[65] A corollary of this is that efficiency is a property of equilibrium allocations only. Outside of equilibrium allocations, it is not possible to claim that markets allocate resources efficiently.

An equilibrium allocation for an n-commodity economy is represented by an n-dimensional vector of quantities of commodities supplied **y***, an n-dimensional vector of quantities demanded **x***, and an n-dimensional vector of prices **p***.[66] At prices **p***, supply equals demand for every commodity at the same time: $y_i^* = x_i^*$ for all i.[67] Because the quantities supplied and demanded are the result of maximizing behavior of agents, the allocation is a Pareto-optimum.

Although economic theory pays more attention to equilibrium allocations, the critical question is just how these positions are reached through the interplay of market forces. For example, Walras thought that this was the central question for economic theory.[68] There are two classes of models that have been developed to answer this question, and to this date the results are quite unsatisfactory. Thus, the belief of Coase and his followers on the virtues of market systems, in terms of attaining efficiency allocations, is not justified.

The two classes of models differ in how trading is treated outside of equilibrium positions.[69] In the first class, trading is not authorized outside of equilibrium. Price formation proceeds through a "groping" process. Prices are adjusted until the difference between supply and demand (the excess demand) is annulled. Once equilibrium is attained, transactions can proceed at the equilibrium prices, and the new allocation is Pareto-efficient. With respect to the second class, models are called "trading process models" because they allow for trading to take place outside of equilibrium.[70] In these models, the economy-wide allocation changes through the price formation process.[71] The final equilibrium point is path-dependent: there are as many equilibria as there are exchange trajectories.[72]

The two classes of models share one common trait—in each class the agents are assumed to play a passive role vis-à-vis prices. In other words, the agents are price-takers, which is logical because the models were developed to try to prove that competitive market forces do lead an economy to an equilibrium (in which Pareto-optimality exists). However, this left an unresolved problem, first identified by Koopmans: if in this model all agents are price-takers, then who adjusts prices?[73]

Mathematically, the problem was represented through a system of differential equations. For each price, the adjustment process is a function of the excess demand:[74]

$$\dot{p}_i = \frac{dp}{dt} = H_i[Z_i(P)] \qquad \text{(Equation 3)}$$

In this equation Z_i is the excess demand for commodity i (the difference between D^i and S^i, the demand and supply of commodity i, respectively). This equation states that price adjustment proceeds through a function that respects the sign of excess demands: because of the law of supply and demand, when demand is greater than supply, the price must rise. Similarly, if it is less than supply, it must fall. When supply equals demand, there is no change because that price is considered to be an equilibrium price: $dp/dt = 0$.[75]

Although it is reasonable to assume that when demand exceeds supply then prices will increase, when one also assumes perfect competition and price-taking behavior, how exactly are prices changed? This is a well-known problem in economic theory: exactly whose behavior is the equation intended to represent? As will be seen, the answer to that question is unanimously considered unsatisfactory.[76]

The general equilibrium theory equation (3) represents the behavior of a special agent called the auctioneer—this is the most important point of the equilibrium theory.[77] This agent is not a consumer, nor a producer, but an entity charged with the role of adjusting prices in accordance with the law of supply and demand.[78] The auctioneer collects information from all agents, calculates aggregate supply and demand for each commodity, and considers the sign of the excess demand: if the result is positive, the price increases, and if the result is negative, the price diminishes or remains constant.[79] The auctioneer then announces a new set of prices, and all agents recalculate their supplies and demands while a new iteration takes place.[80]

It is important to note that the auctioneer announces a system of prices. As mentioned above, using a system of prices ($p_{a,b} = p_{a,c}/p_{b,c}$ with $\alpha = 1$) is equivalent to working in a setting with only one price for each commodity even before the market mechanism starts operating. This is a restrictive assumption, but it is quite necessary for the functioning of the general equilibrium model. Because all agents calculate their supplies and demands

with the same set of prices, the auctioneer can aggregate these quantities and calculate the excess demand, and from there the auctioneer can proceed to adjust prices. If no price system exists, multiple prices would arise, the quantities of individual agents' supply and demand would not be consistent, and the whole process would break down. Working with price systems is an abuse from the standpoint of the theory's objective, but is essential for the model to operate.

Of course, the presence of the auctioneer is highly problematic. The problematic presence of the auctioneer has been recognized by all the theoreticians that have developed general equilibrium theory. Because the auctioneer aggregates information that is supposed to exist on a decentralized basis, this fictitious agent contradicts the objective of general equilibrium theory.[81] The model is supposed to represent the dynamics of a private, decentralized, market economy. Instead, the presence of the auctioneer, which behaves like a central authority, takes the model closer to the realm of a centrally planned economy. This fictitious agent is required in both classes of models, with or without trading out of equilibrium positions.[82]

The problems do not stop here. Even if we accept this strange fiction in the model of a market economy, we still have to adjust the form of the supply and demand functions in such a way that prices converge to their equilibrium position (and excess demands become zero as all markets clear). In the first class of models, where trading is not allowed outside of equilibrium positions, the mathematical tools that are used translate themselves into arbitrary and highly restrictive assumptions. These assumptions are needed to modify the form of the supply and demand functions in such a way that convergence to equilibrium becomes a possibility. One of these assumptions is that all commodities are gross substitutes (GS).[83] Two commodities are said to be GS when a rise in the price of one leads to an increase in the demand for the other (coffee and tea are a good example). If all commodities are GS, it is possible to use a

mathematical theorem that allows us to state that market forces (as represented by the auctioneer) lead the economy to equilibrium and Pareto-optimality. The assumption is not only an ad-hoc condition, but is also quite extreme and has been unanimously declared unsatisfactory.[84]

Another assumption is that at the aggregate level the weak axiom of revealed preferences (WARP) holds. This axiom states that agents who have chosen a bundle of commodities, Q, when they could afford (at the going prices) bundle R, will only choose R when prices have changed and they can no longer afford Q.[85] The axiom makes perfect sense at the individual level, but it does not hold when using aggregate demand functions. The assumption of WARP at the market level is also used to prove global stability, but the price is too high.

At the end of the 1950s, the analyses of Kenneth J. Arrow, Leonid Hurwicz, and H.D. Block were able to prove global stability using either GS or WARP.[86] They ventured a conjecture that in general, a *tâtonnement* process—where no trading takes place before equilibrium is reached—is globally stable.[87] The only restrictions that the process had to respect were acceptable because they were part and parcel of the general equilibrium model (such as Walras' Law and homogeneity of degree zero for the excess demand functions).[88] However, a paper published by Herbert Scarf showed through the use of counterexamples that the general equilibrium was unstable once those restrictive assumptions were removed.[89] Later, papers by Rolf R. Mantel, Hugo Sonnenschein, and Gerard Debreu confirmed the fact that additional assumptions on excess demand functions will always be needed in order to prove global stability.[90] Their work is known as the Sonnenschein-Mantel-Debreu (SMD) theorem and shows that the properties of individual excess demand functions (Walras' Law, continuity and homogeneity of degree zero) that are transferred to aggregate excess demand functions are not enough to ensure compliance with the weak axiom of revealed preferences.[91] The implication is devastating for stability (the formation of equilibrium prices) in the context of general equilibrium

models. The SMD theorem states that the dynamics of price changes in a general equilibrium model can behave in very strange manners—the theorem is one of "anything goes." The bottom line here is that it is not possible to prove the stability of price dynamics, and convergence to equilibrium, without ad-hoc and restrictive conditions on the excess demand functions. This conclusion is most unfortunate for those who, like Coase, believe there is a theory somewhere showing that the competitive forces of the market lead to equilibrium and efficiency.

Non-tâtonnement models abandon the assumption that trade does not take place out of equilibrium (that is, it occurs during the price formation process).[92] These models were developed by Takashi Negishi, Hirofumi Uzawa, F.H. Hahn, and Franklin Fisher, and do not require the restrictive assumptions needed to ensure stability in tâtonnement processes (i.e., gross substitutability or WARP).[93] They do, however, share with those models the presence of the uncomfortable auctioneer. Also, as with tâtonnement processes, trading models do not allow for actual consumption and production to take place until equilibrium has been reached.[94]

These models are path-dependent because at each stage in the price-adjustment process, the stock of commodities possessed by individual agents changes, and the new demand plans are affected by these changes. Although it could be argued that this is more realistic, it entails a new problem: uniqueness of equilibrium is lost. This is an unwelcome result from the standpoint of comparative statistics, which is a key dimension in economic analysis.[95] This is why these models have not received the acceptance one would expect given the fact that they do not need the restrictive assumptions of GS or WARP.

An additional problem in general equilibrium models makes itself highly visible in this class of processes: the absence of money. Everything that has been stated up until this moment refers to theoretical models in which there is no money. Prices are relative prices, described by rates of substitution between commodities.[96] They are not prices expressed in terms of fiat

money units.[97] It is surprising to realize that these models—which may be very ingenious intellectual constructs with a large dose of mathematical complexity—are, in reality, models of barter economies. The reason for this involves deep theoretical problems.[98] The absence of money poses a serious difficulty in non-tâtonnement models because they are based on the so-called "orderly markets assumption": for each commodity there exists a market such that after trading, there may be unsatisfied sellers or unsatisfied buyers of that commodity, but not both.[99] This assumption may sound reasonable because we know that buyers and sellers look for each other in a market. The problem, however, is that for these transactions to take place, a medium of exchange is required. That fact is not easily resolved because the introduction of money poses its own unresolved problems.[100] The main problem is that the purchasing power of money has to remain positive at all times in order to satisfy the needs of the model. Yet, when introducing fiat money into the picture, it is difficult to guarantee that it will always have a strictly positive price (i.e., positive purchasing power).

That money cannot be introduced into general equilibrium models is a disturbing fact. Coase and his followers do not seem to be aware of this and have always ignored this fundamental difficulty in pure market theory. This ignorance explains their loyalty to the belief that, somewhere, there is strong and consistent scientific proof of the efficiency properties of markets.

We have shown that there is no rational proof—and thus no scientific validity—for the notion that the dynamics of market systems lead to efficient (equilibrium) allocations. Many of the critical assumptions used in general equilibrium models are so outrageous that it is difficult to understand how some would think that the rigor and soundness of economic theory can enrich legal analysis and practice—part of the explanation rests in downright ignorance of economic theory.

III. CONCLUSION: AN APPRAISAL OF LAW AND ECONOMICS

Examining the Coase Theorem relies on a critical analysis of economic theory. The fundamental shortcomings of the most developed theory of the market, general equilibrium theory, as well as the restrictions imposed by the use of partial equilibrium and cases of a bilateral monopoly, undermine the assertions of the Coase Theorem. In the case of a bilateral monopoly, this construct involves serious distributional problems, and the invariance component of the theorem is seriously called into question. In addition, it is possible that the negotiations process may stop when mutually beneficial transactions take place outside of the contract curve. In those cases, social efficiency in the restricted Pareto-optimum sense will not be the outcome.

The simplicity of the bilateral monopoly cases used to illustrate the Coase Theorem is indeed misleading. In reality, a bilateral monopoly is used as an artificial device to camouflage two essential points. First, references to dynamic price-formation processes require working in a multi-commodity setting. When agents make decisions in terms of supply and demand of one commodity, they take into account many prices in the economy, and not just the price of that single commodity. This is due to the fact that they have to consider their budget constraints, which may involve many commodities; this is also why supply and demand functions in the general theory of markets have as parameters an n-dimensional vector of the prices of the n-commodities.

The second point is that using a bilateral monopoly framework allows L&E to abstract from all other prices, therefore preventing agents to engage in arbitraging operations. This is important, as arbitraging would be disastrous for the Coase Theorem because the negotiating process would not necessarily lead to any kind of efficiency allocation.

Faith in the idea that markets allocate resources efficiently is severely shaken by the set of difficulties in general equilibrium theory discussed in this article. The shortcomings of general equilibrium theory in stability theory should alert anyone tempted by the L&E movement and its

applicability to fields of legal practice. The bottom line is that we do not have a theory showing how, if at all, markets reach equilibrium allocations. Because efficiency, in terms of Pareto-optimality, is an attribute *only* of equilibrium allocations, very serious negative implications exist for anyone claiming that markets allocate resources efficiently.

We have concentrated our critique of L&E based on the fact that economic theory is in a very sad state. Proponents of L&E seem to ignore this, appearing instead to believe that there exists somewhere a robust theoretical construct that satisfactorily explains how markets allocate resources efficiently—this article has shown such faith to be groundless. This should be enough to dismiss L&E as another example of the triumph of ideology over science.   In addition, the extreme version of L&E transforms justice into a commodity and represents a disturbing backward movement in social thought. The critiques raised in this article should also suffice to call into question the idea that the main objective of legal systems is efficiency, and that efficiency is attained through the market system. There are no grounds to believe in the efficiency of the market system.

One final thought on the role of mathematics is important.   In its development, economics as a discipline has been obsessed with the use of mathematical models to build a theory of competitive markets. The only function for the very awkward assumptions mentioned above—for example, GS or WARP—was to allow the theoretician to have access to certain mathematical theorems. Functioning in this manner, economic theory has sacrificed the construction of relevant economic concepts for the sake of using mathematical tools.   This is not how scientific discourse should advance, and the followers of L&E are probably not aware of this. In fact, they may have fallen victim to the illusion of scientific rigor conferred by the use, and abuse, of mathematics.

[2] *See, e.g.,* Robert D. Cooter, *The Best Right Laws: Value Foundations of the Economic Analysis of Law*, 64 NOTRE DAME L. REV. 817 (1989).

[3] *Id.*; R. H. COASE, THE FIRM, THE MARKET AND THE LAW 95 (1988); Guido Calabresi, *Some Thoughts on Risk Distribution and the Law of Torts*, 70 YALE L.J. 499 (1961); RICHARD A. POSNER, ECONOMIC ANALYSIS OF THE LAW (1992); Gary S. Becker, *Crime and Punishment: An Economic Approach*, 76 J. POL. ECON., Mar. – Apr. 1968, at 169; Gary S. Becker, *A Theory of Competition Among Pressure Groups for Political Influence*, 98 Q.J. ECON. 371 (1983).

[4] COASE, *supra* note 3, at 119.

[5] RICHARD A. POSNER, ECONOMIC ANALYSIS OF THE LAW 254-255, 535-536 (4th ed. 1992).

[6] *Id.*

[7] *Id.* at 89-90.

[8] Robert D. Cooter, *The Coase Theorem*, *in* THE NEW PALGRAVE: ALLOCATION, INFORMATION, AND MARKETS 64-70 (John Eatwell, Murray Milgate & Peter Newman eds., 1989).

[9] Lewis Kornhauser, *Legal Philosophy: The Economic Analysis of Law,* STANFORD ENCYCLOPEDIA OF PHILOSOPHY (May 16, 2006).

[10] *Id.*

[11] John B. Egger, Comment, *Efficiency is Not a Substitute for Ethics, in* TIME, UNCERTAINTY, AND DISEQUILIBRIUM 117 (Mario J. Rizzo ed., 1979).

[12] Roger A. Arnold, *Efficiency vs. Ethics: Which Is the Proper Decision Criterion in Law Cases?*, J. LIBERTARIAN STUD., Winter 1982, at 55-56 (1982).

[13] *Id.*

[14] By employing "we" instead of "I," I invite readers to consider whether they find what I say to be consistent with their own experience, rather than to presume I can speak for any of us or "all" of us in some definitive sense.

[15] This is not the only part of Coase's work that has been criticized from the perspective of economic theory. Clower and Howitt reject Coase's theory of the firm, among other things, because Coase takes for granted that "firms, instead of being an essential component of market transactions, are *alternatives* to markets . . . as if somehow 'the price mechanism' existed independently of the business firms that make markets." Robert Clower & Peter Howitt, *Money, Markets, and Coase*, *in* IS ECONOMICS BECOMING A HARD SCIENCE? 189, 199 (Antoine d' Autume & Jean Cartelier eds., 1997) (emphasis in original).

[16] GEORGE J. STIGLER, THE THEORY OF PRICE 113 (3d ed. 1966).

[17] COASE, *supra* note 3, at 12.

[18] *Id.* at 96.

[19] *Id.* at 12.

[20] Pareto-optimality implies that at least one person is made better off while nobody's position is made worse off. Kaldor-Hicks efficiency corresponds to outcomes in which those that are made better off could *in theory* compensate those that are made worse off, thus leading to Pareto-optimality.

[21] COASE, *supra* note 3, at 14.

[22] STIGLER, *supra* note 16.

[23] COASE, *supra* note 3, at 15.

[24] *Id.*

[25] In this context, a commodity is just another name for rights. Coase reminds us that "[l]awyers . . . habitually think of what is bought and sold as consisting of a bundle of rights." COASE, *supra* note 3, at 11.

[26] Herbert Hovenkamp, *Law and Economics in the United States: A Brief Historical Survey*, 19 CAMBRIDGE J. ECON. 331 (1995).

[27] The quantity of commodity N is measured in the vertical axes. The amount of commodity M is measured in the horizontal axes. Indifference curves are the loci of points of uniform utility for given combinations of both commodities in the consumption basket of an agent. A key assumption here is diminishing marginal utility. This is represented by the fact that the curves have a negative slope and are convex to the origin: as one agent has less of one good he or she needs more of the other to compensate for the loss of utility. The slope of an indifference curve is the marginal rate of substitution between goods (the additional amount of goods M required to compensate for the reduction in the amount of goods N required in order to maintain utility constant). In our example, agent I has most of commodity N and very little of commodity M. Agent II has most of commodity M and very little of commodity N. For Agent I (respecting Agent II) any shift in a northeast (respecting southwest) direction will improve his or her situation as it will move the agent to a higher indifference curve.

[28] JAMES QUIRK & RUBIN SAPOSNIK, INTRODUCTION TO GENERAL EQUILIBRIUM THEORY AND WELFARE ECONOMICS 103-147 (1968).

[29] Edgeworth's results are known as the "indeterminate contract." F.Y. EDGEWORTH, MATHEMATICAL PSYCHICS: AN ESSAY ON THE APPLICATION OF MATHEMATICS TO THE MORAL SCIENCES 29-33 (1881) (explicitly and correctly criticizing Walras and Jevons for thinking that bilateral trade results in a single point of equilibrium).

[30] Prices are determined by the slope of straight lines going through point W.

[31] PAUL A. SAMUELSON, FOUNDATIONS OF ECONOMIC ANALYSIS 238 (2d ed. 1948).

[32] *Id.* at 251.

[33] COASE, *supra* note 3, at 161.

[34] The theorem could specify that recontracting is not only authorized, but a central piece of the negotiating process so that trading outside of the contract curve would be forbidden.

[35] Cooter, *supra* note 8, at 68.

[36] COASE, *supra* note 3, at 136.

[37] COASE, *supra* note 3, at 104.

[38] *Id.*

[39] Hovenkamp, *supra* note 26, at 336.

[40] COASE, *supra* note 3, at 13.

[41] *Id.*

[42] *Id.*

[43] This is a good time to reflect on the definition of efficiency. The only criterion offered by economic theory is Pareto-efficiency and its relative, Kaldor-Hicks efficiency. But this criterion is very limited: "A state can be Pareto-optimal with some people in extreme misery and others rolling in luxury, so long as the miserable cannot be made better off without cutting into the luxury of the rich." AMARTYA SEN, ON ETHICS AND ECONOMICS 32 (1987).

[44] Franklin M. Fisher, *Adjustment Processes and Stability*, *in* 1 THE NEW PALGRAVE: A DICTIONARY OF ECONOMICS 26 (John Eatwell, Murray Milgate & Peter Newman eds., 1987).

[45] Cooter, *supra* note 8, at 67.

[46] *Id.*

[47] The price is the ratio at which the commodities are traded. Thus, when two commodities are involved, as in a bilateral monopoly, there is only one relative price (units of commodity x needed to obtain one unit of commodity y) and its reciprocal.

[48] From the perspective of the number of prices involved, a bilateral monopoly is analogous to a partial equilibrium model. However, although in partial equilibrium there is also one relative price involved, this is not the result of assuming a two-commodity world. Partial equilibrium is compatible with an n-commodity economy but the assumption is that distribution and wealth effects associated with price variations in the market examined under partial equilibrium are negligible.

[49] LÉON WALRAS, ELEMENTS OF PURE ECONOMICS OR THE THEORY OF SOCIAL WEALTH 161 (William Jaffé trans., 4th ed. 1954).

[50] *Id.* at 153.

[51] Benetti's analysis reveals that this problem is linked to the issue of a unit of account and how essentially the same difficulty presents itself in classical, neo-classical, and Marxian economic theories. Carlo Benetti, *Economie monétaire el économie de troc: la question de l'unité de compte commune*, 38 ECONOMIE APPLIQUEE 85, 88 (1985).

[52] WALRAS, *supra* note 49, at 158.

[53] *Id.* at 159.

[54] *Id.* at 160.

[55] *Id.* at 158.

[56] *Id.* at 159.

[57] *Id.*

[58] *Id.* at 161.

[59] *Id.*

[60] Benetti, *supra* note 51.

[61] Cooter, *supra* note 8, at 64.

[62] A vector of prices is a set of prices, one for each commodity: $p = (p_1, p_2, p_3, \ldots, p_n)$.

[63] SAMUELSON, *supra* note 31.

[64] Lionel W. McKenzie, *General Equilibrium*, *in* 2 The New Palgrave: A Dictionary of Economics 498, 510 (John Eatwell, Murray Milgate & Peter Newman eds., 1987).

[65] *Id.*

[66] Hukuane Nikaido, Convex Structures and Economic Theory 263 (1968).

[67] *Id.*

[68] Walras, *supra* note 49, at 121.

[69] Franklin Fisher, Disequilibrium Foundations of Equilibrium Economics 37, 20-21, 27-29 (1983).

[70] *Id.* at 27.

[71] *Id.*

[72] These processes are also known as non-tâtonnement models. Takashi Negishi, *The Stability of a Competitive Economy: A Survey*, 30 Econometrica 635, 635-639 (1962).

[73] Tjalling C. Koopmans, Three Essays on the State of Economic Science (1957).

[74] Fisher, *supra* note 44.

[75] *Id.*

[76] F. H. Hahn, *Auctioneer*, *in* 1 The New Palgrave: A Dictionary of Economics (John Eatwell, Murray Milgate & Peter Newman eds., 1987) (summarizing these issues in the entry and listed references).

[77] Fisher, *supra* note 44.

[78] *Id.*

[79] *Id.*

[80] *Id.*

[81] *Id.*

[82] Fisher, *supra* note 69, at 21, 33.

[83] *Id.* at 23.

[84] *Id.*

[85] *Id.* at 26.

[86] Kenneth J. Arrow & Leonid Hurwicz, *On the Stability of the Competitive Equilibrium I*, 26 Econometrica 522, 522-552 (1958), *available at* http://links.jstor.org/browse (click to view alphabetical list of journals); Kenneth J. Arrow et al., *On the Stability of the Competitive Equilibrium II*, 27 Econometrica 82, 82-109 (1959), *available at* http://links.jstor.org/browse (click to view alphabetical list of journals).

[87] Arrow et al., *supra* note 86, at 106.

[88] Walras's Law "is an expression of the interdependence among the excess demand equations of a general equilibrium system that stems from the budget constraint." Don Patinkin, *Walras's Law*, *in* 4 The New Palgrave: A Dictionary of Economics 863 (John Eatwell, Murray Milgate & Peter Newman eds., 1987). It means that the sum of excess demands over all the markets in the economy must equal zero and this applies whether or not all markets are in (general) equilibrium. If there is excess supply in one market then it must correspond to this positive excess demand in at least one other market. This is a direct result of the architecture of the model and can be readily observed in the summation of the budget constraints of all agents. On the other hand,

functions possessing the property of homogeneity of degree zero are such that f(tx) = f(x). This means that multiplying all prices by a scalar does not alter the contents of agents' decisions.

[89] Herbert Scarf, *Some Examples of Global Instability of the Competitive Equilibrium*, INT'L ECON. REV., Sept. 1960, at 157.

[90] Rolf R. Mantel, *On the Characterization of Aggregate Excess Demand*, 7 J. ECON. THEORY 348 (1974); Hugo Sonnenschein, *Do Walras' Identity and Continuity Characterize the Class of Community Excess Demand Functions?,* 6 J. ECON. THEORY 345 (1973); Gerard Debreu, *Excess Demand Functions*, J. MATH. ECON., Mar. 1974, at 15.

[91] Mantel, *supra* note 90; Sonnenschein, *supra* note 90; Debreu, *supra* note 90.

[92] FISHER, *supra* note 69, at 31.

[93] *Id*. at 31-33.

[94] *Id.*

[95] In comparative statics, changes in equilibrium values as a result of changes in parameters are crucial. If we ignore which equilibrium values will be attained in a path-dependent process, then comparing equilibrium values may be meaningless.

[96] WALRAS, *supra* note 49, at 158-159 (providing a good and elegant example).

[97] *Id.*

[98] Carlo Benetti, *Money and Prices: The Limits of the General Equilibrium Theory*, *in* THE FLAWED FOUNDATIONS OF GENERAL EQUILIBRIUM: CRITICAL ESSAYS ON ECONOMIC THEORY 48 (Frank Ackerman et al. eds., 2004) (analyzing the source of these problems and their implications).

[99] FISHER, *supra* note 69, at 33-34.

[100] F. H. Hahn, *On Some Problems of Proving the Existence of an Equilibrium in a Monetary Economy*, *reprinted in* THE THEORY OF INTEREST RATES 126 (F. H. Hahn & F. P. R. Brechling eds., 1965). *But see* Benetti, *supra* note 98 (critiquing Hahn's approach).