Distributed and dynamic traffic congestion controls without requiring demand forecasting: Tradable network permits and its implementation mechanisms

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Abstract

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Kentaro Wada

Cities worldwide still face heavily traffic congestion due to urbanization and increasing travel demand. To address the problem, there has been a great deal of research into transportation management (TDM) schemes. A common characteristic of almost all the TDM schemes is based on demand forecasting and requires detailed demand information. However, it is almost impossible for a road manager to obtain such private information due to an asymmetric information between road managers and road users.

To resolve the asymmetric information problem, Akamatsu et al. (2006) and Akamatsu (2007a,b) proposed a new TDM scheme—*tradable network permits (TNP) scheme*—and proved its efficiency properties. In this thesis, we extend the theory of the TNP in two important directions: (i) to develop a supply side control based on the TNP scheme; (ii) to design implementation mechanisms for the TNP scheme in three different situations. The common objective of the control and mechanisms is to achieve an efficient allocation of network capacity without requiring demand forecasting. To accomplish this objective, we employed an evolutionary approach to achieving an optimal supply level while acquiring demand information sequentially. In the following, the results of this thesis are summarized.

Chapter 3 proposes a distributed signal control policy based on the TNP scheme. The proposed signal control policy can determine the green time proportion of each intersection by using only local information. An equilibrium traffic assignment under the proposed policy coincides with a system optimal traffic flow pattern that minimizes the total transportation cost in a network. Furthermore, we construct an evolutionary implementation method for the proposed policy and prove that the day-to-day traffic flow dynamics under the scheme converge to the system optimal traffic pattern.

Chapter 4 proposes an implementation mechanism for trading markets of network permits on general networks. Specifically, we make use of a hybrid mechanism that consistently combines an auction mechanism with a path capacity control; these are repeated on a day-to-day basis. The former phase involves selling bundles of permits, and the latter phase involves adjustment of the number of bundles of permits, which corresponds to the path capacities. We prove that the proposed mechanism has the following desirable properties: truthful bidding is a dominant strategy for each user on each day, and the permit allocation pattern under the mechanism converges to an approximate dynamic system optimal allocation pattern in the sense that the achieved social surplus reaches its maximum value when the number of users is large. Furthermore, we show that the proposed mechanism can be extended to obviate path enumeration by introducing a column generation procedure.

Chapter 5 also proposes an implementation mechanism for trading markets, considering a more general situation where network permits for a specific day are sold in multiple period markets. Under such circumstances, the road manager needs to allocate a bottleneck capacity to these markets, as well as allocate permits to users. As a first step in implementing these markets, we design a dynamic auction mechanism in which the number of permits for each market is fixed. This mechanism can determine optimal permit allocation, along with the actual sequence of time under a certain condition. It is proved that the truthful bidding is a dominant strategy for each user, and that it guarantees that the market choice of the user is optimal. We then derive an adjustment rule of the number of permits sold for each market and demonstrated that combining the dynamic auction and the adjustment rule maximizes the social surplus in a finite number of iterations.

Chapter 6 develops an evolutionary mechanism for a hybrid scheme of the TNP and congestion pricing, considering multiple negative externalities (i.e., queuing congestion and flow congestion). Specifically, we first describe a mechanism consisting of trading rules of the permit markets and users' behaviors expressed by a stochastic learning model. We then derive a stochastic dynamics of the learning process from the mechanism. Finally, we show that that the stochastic dynamics converges to an equilibrium state, and traffic flow pattern at equilibrium is efficient in the sense that the social surplus is maximized.

Overall, this thesis contributes to the development of distributed traffic congestion controls without requiring demand forecasting, and in particular provides further insights into the market-based schemes of managing traffic congestion.

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Chapter 1

Introduction

1.1 Background

Traffic congestion remains a significant problem faced by cities worldwide. The Japanese Ministry of Land, Infrastructure, Transport and Tourism estimated that the time loss due to traffic congestion in Japan in 2006 was 3.5 billion person-hours[1], with the Tokyo metropolitan area accounting for about 25% of the total loss (Road Bureau, MLIT, 2007). Traffic congestion also has negative side effects including waste of fuel, air pollution, greenhouse gas emissions, and increasing traffic accidents (e.g., Parry, Walls, and Harrington, 2007). Furthermore, traffic congestion could be more severe with continuously increasing vehicle ownership and worldwide urbanization. Indeed, there are more than one billion vehicles worldwide, and the number will increase primarily in South and East Asia, Eastern Europe, and South America. Within the next 20 years, it is predicted that the number of vehicles will reach two billion (Sperling and Gordon, 2009).

The traditional remedy to traffic congestion is to build new roads or to expand road capacities. However, these are very costly. In addition, improvements to a congested traffic network do not necessarily lead to a reduction in traffic congestion, which relates to a set of known paradoxes in the transportation field. For example, the "*Pigou-Knight-Downs paradox*" (Downs, 1962) states that expanding road capacity can induce new demand without reducing traffic congestion; the "*Braess paradox*" (Braess, 1968) states that creating a new road can raise total travel costs. One reason for these paradoxes is that new road capacities elicit their own demand (induced demand); another reason is that each road user does not

[1] The monetary equivalent of the time loss is about 11 trillion yen.

recognize the real cost of a trip including the additional cost he or her impose on others, which leads to socially inefficient choices of travel modes, paths, and departure times (Arnott and Small, 1994). Therefore, we have to pay attention to establish control methods to manage travel demand and encourage the efficient use of existing infrastructure.

Based on this motivation, there has been a great deal of research into transportation demand management (TDM) schemes. TDM schemes can be roughly divided into two types: price-based regulation and quantity-based regulation. The next section reviews past efforts to develop both price-based and quantity-based TDM schemes.

1.2 Literature review

1.2.1 Price-based regulation

Congestion pricing is a representative scheme of the price-based approach, and was first advocated by Pigou (1920) and Knight (1924). Standard congestion pricing (i.e., marginal cost pricing) is theoretically desirable for reducing traffic congestion in a distributed manner: if a road manager imposes the marginal cost of road use on road users, an optimal traffic flow pattern for the system is achieved at Wardrop equilibrium (Wardrop, 1952). For decades, various types of pricing schemes for both static and dynamic situations have been proposed; see textbooks written by Button and Verhoef (1998), Yang and Huang (2005), and Small and Verhoef (2007), and surveys written by Maruyama (2009), Tsekeris and Voß (2009), and de Palma and Lindsey (2011), for references and comprehensive reviews. However, there exist two major limitations to implement congestion pricing schemes[2].

The first and most serious limitation is that the congestion pricing scheme requires detailed demand forecasts/estimations to calculate optimal toll levels[3]; to obtain reliable forecasts, the road manager requires accurate demand information (e.g., the willingness to pay, value of time, and desired arrival time). However, it is almost impossible for the road manager to obtain such private information due to an *asymmetric information* between road managers and road users. If the scheme is implemented with imperfect information, this may inevitably result in an economic loss. For instance, suppose that the road manager estimates a

[2] Other limitations/complications of congestion pricing schemes have been described by de Palma and Lindsey (2011).

[3] Some weakness of forecast-based schemes were deeply discussed in Daganzo (2007).

demand function higher than the actual demand function; i.e., optimal congestion tolls based on the incorrect demand function are higher than tolls based on the actual demand function. As a result, although traffic congestion is reduced, there is an excessive occurrence of road users choosing not to make trips (i.e., there are losses of consumer surplus arise) and the social surplus may decrease to a level lower than that before the scheme was implemented.

The second limitation results from the necessity of imposing a time-varying (or dynamic) congestion toll. Negative externalities of traffic congestion are created due to the temporal concentration of travel demands as well as the spatial concentration, and thus dynamic pricing schemes are important in mitigating these externalities effectively. As an example of such schemes, dynamic congestion pricing models that explicitly consider queuing congestion have been studied by several researchers (e.g., Vickrey, 1969; Arnott, de Palma, and Lindsey, 1990, 1993; Mun, 1999; Kuwahara, 2007; Doan, Ukkusuri, and Han, 2011). However, these studies were limited to simple networks (e.g., a single bottleneck or parallel link) even though they derived an optimal dynamic congestion toll. For more general networks with a many-to-one (or one-to-many) origin-destination (OD), Carey and Srinivasan (1993) and Nie (2011) showed a marginal cost analysis for a convex reformulation of the system optimal dynamic traffic assignment (SO-DTA) problem of the seminal works of Merchant and Nemhauser (1978a,b). Ziliaskopoulos (2000) conducted the similar analysis of a SO-DTA problem that is expressed by the cell transmission model (Daganzo, 1994). These models, however, face the so-called *holding-back* problem, which is that vehicles are arbitrarily held back on links although downstream capacity is available (Doan and Ukkusuri, 2012). Friesz, Kwon, and Mookherjee (2007), Ban and Liu (2009), and Lin, Unnikrishnan, and Waller (2011) formulated dynamic second-best toll pricing models for general networks as mathematical programs with equilibrium constraints (MPEC) and developed solution algorithms for the problems. However, convergence of these algorithms have not been well addressed: there is no guarantee that an optimal solution is obtained by these methods. In conclusion, no study established a theory of dynamic congestion pricing for general networks[4].

The primary factors preventing extension of the theory are the intractabilities of analyzing the dynamic traffic equilibrium assignment for general networks (see, for example,

[4] Simulation-based approaches that evaluate path marginal costs using traffic simulation or dynamic loading models can be founded in Ghali and Smith (1995), Peeta and Mahmassani (1995), and Shen, Nie, and Zhang (2007).

Kuwahara and Akamatsu, 1993; Heydecker and Addison, 1996; Akamatsu, 2001; Peeta and Ziliaskopoulos, 2001; Szeto and Wong, 2011). Alternatively, there is an approach that attempts to model traffic dynamics in cities at an aggregate level (Daganzo, 2007; Geroliminis and Daganzo, 2008). These papers proposed and tested a "macroscopic fundamental diagram" that relates the number of vehicles in the area to the area's average density. Following this approach, Geroliminis and Levinson (2009) proposed a dynamic cordon pricing scheme. Gonzales and Daganzo (2012) applied the scheme to the morning commute problem with multiple transport modes. While the approach offers a remarkable method for managing complex urban traffic systems, it does not resolve the first limitation described above: it has to estimate demand information on heterogeneous commuters.

To overcome the problem of asymmetric information, Sandholm (2002, 2005, 2007) proposed an evolutionary method to implement a scheme without a demand function. The method leads traffic flow patterns to a system optimal state by exploiting a trial-and-error toll adjustment procedure, which relies on the description of a static traffic assignment as a potential game (Monderer and Shapley, 1996; Sandholm, 2001). More specifically, Sandholm (2002, 2005, 2007) demonstrated that evolutionary dynamics of traffic flows (i.e., day-to-day dynamics determined by aggregating users' route choice behavior) converges to an equilibrium in a way that minimizes the total transportation cost in a network; i.e., the dynamics converges to the minimum point of a Beckmann-type potential function (Beckmann, McGuire, and Winsten, 1955). In the field of transpiration science, Yang, Meng, and Lee (2004) and Han and Yang (2009) also proposed a similar trial-and-error method[5]. However, these methods cannot be applied to dynamic cases directly since dynamic traffic assignment problems generally do not have potential functions. Moreover, even if the methods can be extended to such cases, there remains a serious problem of economic losses due to queuing congestion in disequilibrium states.

[5] Yang, Xu, He, and Meng (2010) developed an iterative toll adjustment method for the case that both demand and cost functions are unknown, which corresponds to the solution algorithm for a traffic equilibrium problem with asymmetric link flow interactions formulated as a variational inequality.

1.2.2 Quantity-based regulation

The second approach, quantity-based regulation, directly restricts the use of road usage by assigning priority-service permits to road users using particular rules; e.g., license numbers based rationing[6] and advance highway bookings (e.g., Akahane and Kuwahara, 1996; Wong, 1997; Teodorović and Edara, 2005; Edara and Teodorović, 2008). Unlike the price-based regulation, these schemes can achieve a quantitative policy target (e.g., an appropriate level of congestion) without requiring detailed user information by issuing the number of permits less than the target level. However, there may be cases in which road users cannot select their desired choice (e.g., their desired route and arrival time) if the permits are assigned according to unrefined rules (e.g., a simple "quota" scheme). Such an infringement on freedom of choice necessarily causes economic losses.

To circumvent this problem, we need to add an appropriate mechanism in which each user can choose his or her desired permit. A market-based quantitative scheme, which is called the *tradable permit* scheme[7], includes such a mechanism and has recently received much attention in the transportation field as an alternative to traditional congestion pricing (e.g., Goddard, 1997; Verhoef, Nijkamp, and Rietveld, 1997; Viegas, 2001). In this scheme, each road user is free to choose permits through a trading market, which will lead to increase the efficiency of quantity-based regulation[8]. Verhoef et al. (1997) discussed the possibilities of using tradable permits in the various type of regulations of road transport externalities. Viegas (2001) argued a tradable mobility rights scheme that can be used both for private car driving in the tolled area and for riding public transport from the perspective of quality and equity in urban mobility. Teodorović, Triantis, Edara, Zhao, and Mladenović (2008) proposed an auction-based congestion pricing, in which drivers who want to enter downtown have to participate a downtown time slot auction. While formulating an allocation problem of the time slots, this study did not address how to set their prices, which is the core problem of

[6] This is the most simplest scheme and has been applied worldwide (e.g., Athenes, Mexico City and Beijing). For the scheme, Han, Yang, and Wang (2010) analyzed the price of anarchy (Roughgarden and Tardos, 2004) for a general network.

[7] For environmental protection, various tradable permit schemes were proposed and analyzed (e.g., Dales, 1968; Montgomery, 1972; Tietenberg, 1980).

[8] Another line of researches on improving quantity-based regulations, Daganzo (1995) and later Daganzo and Garcia (2000) demonstrated that a hybrid scheme of combining pricing and rationing has possibilities of achieving Pareto-improvement.

auction mechanisms. More recently, Yang and Wang (2011) proposed a tradable travel credit scheme, and several extensions have been made by Wang and Yang (2012), Wang, Yang, Zhu, and Li (2012), Nie (2012), Chen and Yang (2012), and Wu, Yin, Lawphongpanich, and Yang (2012). Basically, under the scheme, the road manager initially distributes credits to all eligible travelers and predetermined a link-specific charge. Credits are freely tradable among credits holders. They then showed that, if the manager can appropriately set total number of credits and link charges, a desirable traffic flow pattern is achieved. However, when to do this, it is apparent that the manager requires detailed user information. As Nie (2012) puts it,

> *Suffice it to say here that the information that the government would need to run a mobility credit market is as much as the information required to operate a conventional pricing scheme. Therefore, the mobility credit market does not reduce the administrative burden of the government, unlike in the case of emission control.*

Thus, the main advantage of the scheme over the ordinary congestion pricing is to improve equity and socially acceptability, rather than to resolve the two limitations described above. Furthermore, although the above studies give on the possibilities and some useful insights into tradable permit schemes for managing traffic congestion, no study exists to provide *time-dependent* tradable permits for eliminating *bottleneck congestion*.

As one possible way to both to eliminate bottleneck congestion and to resolve the asymmetric information problem, Akamatsu, Sato, and Nguyen (2006), and Akamatsu (2007a,b) proposed a novel system of "tradable bottleneck permits" (we call a system of tradable bottleneck permits for general networks "tradable network permits" system). Their proposed scheme comprises two parts:

a) the road manager issues a right (bottleneck permit or network permits) that allows the permit holder to pass through a bottleneck during a pre-specified time period,

b) a trading market is established for network permits that are differentiated on the basis of a pre-specified time.

Under this scheme, the arrival flow rate at a bottleneck in any time period is, from definition of the scheme, equal to the number of permits issued for that time period. This implies that we can completely eliminate the occurrence of queuing congestion by setting the number of

permits issued per unit time to be less than or equal to the bottleneck capacity. Since permit prices are determined through the trading market, the asymmetric information problem is also resolved.

As we have seen, it is expected that the tradable network permits scheme will be the most efficient TDM scheme for using the limited resource of road capacity. Indeed, for a single bottleneck, Akamatsu et al. (2006) showed that the proposed scheme has the following desirable properties: (1) Pareto improvement for both the road manager and all users can be achieved; (2) the equilibrium under the scheme achieves a dynamic system optimal traffic assignment, i.e., the scheme can achieve the desirable state in a distributed manner; (3) the "self-fininacing principle" holds for the equilibrium. Akamatsu (2007a,b) extended these properties (2) and (3) to general networks.

However, there is still a potential for improvement of efficiency of the tradable network permits scheme by incorporating not just *demand side* conditions but also *supply side* conditions. This is analogous to the *welfare theorem* of microeconomic theory, i.e., an efficient resource allocation can be obtained at a demand-supply equilibrium in competitive markets.

Another point to consider is the implementation issues of tradable network permits. In particular, despite the main results of the scheme were built on the assumption that a competitive equilibrium can be achieved in the trading markets, micro mechanisms that attains the equilibrium have not been studied in depth. In other words, trading processes were treated as a black-box[9]. Therefore, we have to address a question of what trading rules (of buying or selling tradable permits) encourage competition and achieve the efficient equilibrium. For a single bottleneck case, Wada and Akamatsu (2010) designed an auction mechanism for implementing the tradable permit market. They then showed that (1) the network permit allocation achieved by the mechanism is efficient and (2) the mechanisms is strategy-proof, which means that a dominant strategy employed by each user is the truthful revelation of the value of the permits. However, whether or not the desirable properties hold for more *spatially* and *temporally* general situations is a problem yet to be studies.

[9] This is true for other tradable permit schemes mentioned above.

1.3 Purpose of the thesis

The purpose of this thesis is twofold: (i) to extend a theory of tradable network permits to include supply side conditions and (ii) to design implementation mechanisms of tradable network permits in three different situations. More specifically, it first explores properties of a signal control policy based on tradable network permits. Next we design auction mechanisms for implementing a hybrid scheme of tradable network permit markets according to auction theory (Milgrom, 2004; Cramton, Shoham, and Steinberg, 2006). Finally, we develops an evolutionary mechanism for implementing a hybrid scheme of tradable network permits and congestion pricing.

Overall, this thesis contributes to the development of distributed traffic congestion controls without requiring demand forecasting, and in particular provides further insights into the market-based schemes of managing traffic congestion. The following subsection briefly describes the results in this thesis.

1.3.1 Overview of the results in this thesis

A distributed signal control policy based on tradable network permits

Chapter 3 proposes a distributed signal control policy based on the tradable network permits. Main idea of underlying the proposed policy is to exploit useful information on the equilibrium permit prices provided by the trading markets. This enables us to appropriately incorporate demand conditions into the signal control policy. Specifically, the proposed policy has two desirable characteristics. First, it can determine a green time proportion (i.e., capacity allocation) of each intersection by using only local information. Second, an equilibrium traffic assignment under the proposed policy coincides with a system optimal traffic flow pattern that minimizes the total transportation cost in a network. Moreover, we construct an evolutionary implementation method for the proposed policy and prove that the day-to-day traffic flow dynamics under the scheme converges to the system optimal traffic pattern.

Auction mechanisms for implementing tradable network permit markets

In the following two chapters, Chapter 4 and Chapter 5, we design auction mechanisms for implementing tradable network permit markets in general networks and in a dynamic setting, respectively. In order to account for individual strategic behavior, we here conduct a game-theoretic analysis of a dynamic traffic assignment with *atomic* users. Under this setting, we need to establish mechanisms in which no user has incentive to manipulate the markets. Furthermore, unlike the single bottleneck case treated in Wada and Akamatsu (2010), we have to deal with more complex markets, including network structures and multiple period markets. Nevertheless, we can resolve these complexities by relying on suitable adaptation of the supply side control developed in Chapter 3.

Chapter 4 focuses on trading markets on general networks. Under the situation, a naive formulation of a problem of finding a dynamic system optimal allocation of network permits leads to a NP-hard problem due to the complex relationship between link and path. To avoid such computational infeasibility, we develop a hybrid mechanism that consistently combines an auction mechanism with a path capacity control, which are repeated on a day-to-day basis. The former phase involves selling bundles of permits, and the latter phase involves adjustment of the number of bundles of permits, which corresponds to the path capacities. We prove that the proposed mechanism has two desirable properties: (1) truthful bidding is a dominant strategy for each user on each day, and (2) the permit allocation pattern under the mechanism converges to an approximate dynamic system optimal allocation pattern in the sense that the achieved social surplus reaches its maximum value when the number of users is large. Furthermore, we show that the proposed mechanism can be extended to obviate path enumeration by introducing a column generation procedure.

Chapter 5 considers a more general situation where network permits for a specific day are sold in multiple period markets. Under this situation, the road manager faces a problem of how to allocate a bottleneck capacity to individual market as well as allocating permits to users in each market. As a first step for implementing the markets, we design a dynamic auction mechanism in which the number of permits for each market is fixed. This mechanism can determine an optimal permits allocation with the actual sequence of time under a certain condition. It is proved that the mechanism is strategy-proof, which also guarantees that the market choice of the user is optimal. Then we derive an adjustment rule of the number of permits sold for each market and demonstrate that combining the dynamic auction and the

adjustment rule maximizes the social surplus in a finite number of iterations.

An evolutionary mechanism for a hybrid scheme of tradable network permits and congestion pricing

Chapter 6 develops an evolutionary mechanism for a hybrid scheme of tradable network permits and congestion pricing, considering multiple negative externalities (i.e., queuing congestion and flow congestion). Specifically, we first describe a mechanism consisting of trading rules of the permit markets and users' behaviors expressed by a stochastic learning model. We then derive a stochastic dynamics of the learning process from the mechanism. Finally, we show that that the stochastic dynamics converges to an equilibrium state, and traffic flow pattern at equilibrium is efficient in the sense that the social surplus is maximized.

1.4 Organization

This thesis is organized into a series of self-contained chapters. Before presenting results in the thesis, Chapter 2 reviews a basic framework and properties of tradable network permits. Chapter 3 extends the above theory to include supply side controls. In then investigates implementation mechanisms of tradable network permits. Chapter 4 designs an auction mechanism for implementing trading markets for general networks. Chapter 5 considers a more general situation where network permits are sold in multiple period markets and constructs an implementation mechanism for these markets. Chapter 6 develops an evolutionary mechanism for a hybrid scheme of tradable network permits and congestion pricing, considering multiple negative externalities (i.e., queuing congestion and flow congestion). Finally, Chapter 7 presents the conclusions of this thesis and discusses some topics for future work.

Chapter 2

Tradable network permits: Basic framework and its properties

This chapter reviews a basic framework and properties of the tradable network permits system from Akamatsu (2007a,b). Section 2.1 outlines the framework of the tradable network permits system. Section 2.2 provides a mathematical model that describes the equilibrium under the tradable network permit system. Section 2.3 shows that the equilibrium coincides with a dynamic system optimal assignment. Section 2.4 further shows other desirable properties of tradable network permits: advantages over congestion pricing, self-financing principle and Pareto improvement. Finally, Section 2.5 presents the recent development of micro mechanisms for implementing tradable network permits.

2.1 A system of tradable network permits in transportation networks

2.1.1 Networks

We consider dynamic traffic flows on a general network with multiple origin-destination (OD) pairs (i.e., a transportation network with general topology). The network consists of a set N of nodes, a set A of directed links, and a set W of OD pairs. The node set N includes a subset O of origin nodes from which users start their trip, and a subset D of destination nodes at which users terminate their trips. Each element of N (i.e., each node) is identified by k, and each element of A (i.e., each link) is denoted by a pair (k,l) of the upstream node k

and the downstream node l. The time interval $[0, T]$ for which we assign the dynamic traffic flow is fixed. We assume that the travel demand Q_{od} that makes trips for the time interval $[0, T]$ is a given constant.

We also assume, without any loss of generality, that each link in a network consists of a free flow segment and a single bottleneck segment. The travel time to pass through the free flow segment of link (k, l) is a constant t_{kl} (i.e., t_{kl} is independent of time and flow). The bottleneck of each link is represented by a point queue model with constant capacity μ_{kl}.

2.1.2 Network permits and trading markets

A road manager aims to restrain traffic congestion on the network and minimize the "social transportation cost." To achieve this, the manager regulates the traffic flow rates entering into each bottleneck in the network by using "*time-dependent network permits.*" The network permit is a right that allows the permit holder to pass through a pre-specified bottleneck at a pre-specified time. In this study, we assume that the road manager can issue time-dependent network permits for all bottlenecks (i.e., links) in the network. This implies that the traffic flow entering into link (k, l) at time t consists of only users who have a "time t permit for link (k, l)," and users without this permit cannot pass through this link at this time.

Throughout this study, we assume that the number of permits issued for each link for each unit time is equal to or less than the traffic capacity of each link in the network. This means that queuing congestion never occurs in the network under this permits-issue scheme. This may be easily seen from this explanation of permits: the inflow rate of each link is equal to (or less than) the number of permits issued, and hence the inflow rate cannot exceed the traffic capacity of each link, which implies that queuing congestion at each link can never occur.

For assigning the network permits to users, we can consider two representative schemes: "*market selling scheme*" and "*free distribution scheme*" (Akamatsu et al., 2006). In the market selling scheme, the road manager sells all the network permits to users in network permits markets. All sales from selling the permits result in revenue for the road manager in this scheme. In the free distribution scheme, the road manager distributes all the permits to users for free according to methods that consider the equity among users, and permits can then be traded freely among users in the network permits markets. Thus, all income transfers

take place only among the users in this scheme.

We here explain only the former scheme ("market selling scheme") that is exploited in the following chapters. The permits issued for each link (bottleneck) are put on sale by the road manager. There are as many markets for trading permits as there are links, and each market is dedicated for trading the permits for each link. The permits for each link are further distinguished by a specified time allowable to use the link. Under the network permits system, each user who would like to use a path must have a set of permits corresponding to a set of links included in the path before making a trip. To fulfill this requirement, each user is assumed to purchase the needed set of permits in the trading markets. The price of each permit is determined by an auction system, which implies that the price is adjusted so as to clear the excess demand for each type of permit. We also assume that the markets are perfectly competitive; that is, neither a monopoly nor oligopoly occurs (this assumption will be relaxed in later chapters).

2.1.3 Dynamic travel costs in general networks

Each user makes a single trip (for the time interval $[0, T]$) from an origin $o \in O$ (e.g., residential zone) to a destination $d \in D$ (e.g., central business district (CBD)) in the network. The user chooses a destination arrival time and a path between the origin and the destination so as to minimize his or her disutility (or "generalized transportation cost"). The transportation cost for a single trip of a network user consists of the following three types of costs: (a) "schedule cost," (b) "travel cost," (c) "permit purchase cost."

(a) The "schedule cost" for a user is the cost due to the difference between the user's desired arrival time and the actual arrival time t. The desired arrival time is assumed to be the same for all users and is equal to $\hat{t}$. The schedule cost is represented by the function $s_d(t)$ of destination arrival time t, which is common to all users that have same destination.

(b) The "travel cost" is the monetary equivalent of the travel time needed for a trip from the origin to the destination. The travel time of a path between the origin-destination pair $r \in R_{od}$ is defined as the sum of travel times of the links included in the path. Note that the travel time of each link (k, l) is a constant t_{kl} at equilibrium under the permits system, in which no queuing occurs.

(c) The "permit purchase cost" is the total payment for purchasing a set of link permits

required for going through a path from the origin to the destination. To put it another way, the permit purchase cost of a user is defined as the sum of permit prices of the links included in the path used. Each link permit price $p_{kl}(t)$ varies depending on what path is taken and at what time because the permits for each link are further differentiated by the specified time and each permit is priced depending on the time and the link.

2.2 Equilibrium under the tradable network permits

For the settings above, it is assumed that an equilibrium traffic flow pattern is achieved. At equilibrium, the following five conditions should hold. Here we describes the equilibrium by using the arc-node formulation[1].

1a) Flow conservation at each node:

Conservation of the dynamic traffic flow in a network is represented as the equality of inflow and outflow at each node at each time point. To formalize this, let $y^o_{kl}(t)$ be the flow rate arriving at link (k, l) at time t with respect to origin o, and $z^o_{kl}(t)$ be the flow rate departing from link (k, l) at time t with respect to origin o. Then the flow conservation is represented as

$$\sum_{l \in NO(k)} y^o_{kl}(t) - \sum_{l \in NI(k)} z^o_{lk}(t) = -q_{od}(t)\delta_{kd}, \qquad \forall t \in [0, T],\ \forall o \in O,\ \forall k \in N \tag{2.1}$$

where $q_{od}(t)$ is a OD flow rate for a OD pair (o, d) arriving at the destination d at time t, and δ_{kd} is Kronecker's delta (i.e., 1 if $k = d$, zero otherwise); $NO(k)$ is a set of downstream nodes of the links incident from node k; $NI(k)$ is a set of upstream nodes of the links incident to node k.

1b) First-In-First-Out conditions on each link:

We assume that the dynamic traffic flow in our model should satisfy the First-In-First-Out (FIFO) property on each link (i.e., we assume that passing can be neglected). As shown in the literature (see, for example, Kuwahara and Akamatsu, 1993; Akamatsu and Kuwahara,

[1] The path-based formulation can be found in Akamatsu (2007a,b)

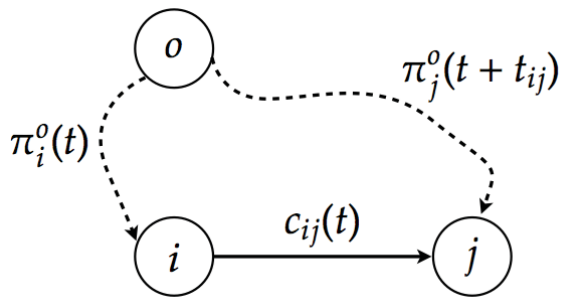


Figure 2.1 Equilibrium conditions for path choice

1994), the FIFO condition for each link can be written as

$$y_{kl}^o(t) = z_{kl}^o(t + t_{kl}(t)) \cdot (1 + dt_{kl}(t)/dt), \tag{2.2}$$

where $t_{kl}(t)$ is the travel time of link (k, l) for a user entering into the link at time t. Note here that $t_{kl}(t)$ is a constant regardless of the arrival time when there is no queue. Hence, at equilibrium under the permits system (i.e., when there is no queue in the network), the FIFO condition (2.2) reduces to the following simpler representation:

$$y_{kl}^o(t) = z_{kl}^o(t + t_{kl}) \qquad \forall t \in [0, T],\ \forall o \in O,\ \forall kl \in A \tag{2.3}$$

2) Equilibrium conditions for path choice:

Consider a user arriving at node k at time t. If the user chooses link (k, l), the arrival time at node l is $t + t_{kl}$. Hence, at equilibrium, link (k, l) should be on the minimum path for a user arriving at node l at time $t + t_{kl}$ if there exists a user entering into link (k, l) at time t. Denoting $\pi_k^o(t)$ as the minimum path cost from the origin o to node k for a user arriving at the node at time t, we can represent this condition as

$$\begin{cases} \pi_l^o(t + t_{kl}) = c_{kl}(t) + \pi_k^o(t) & if \quad y_{kl}^o(t) > 0 \\ \pi_l^o(t + t_{kl}) \leq c_{kl}(t) + \pi_k^o(t) & if \quad y_{kl}^o(t) = 0 \end{cases} \qquad \forall t \in [0, T],\ \forall o \in O,\ \forall kl \in A \tag{2.4}$$

where $c_{kl}(t)$ is the transportation cost for a user who enters into link (k, l) at time t:

$$c_{kl}(t) \equiv p_{kl}(t) + \alpha t_{kl} \tag{2.5}$$

3) Flow conservation for OD flow rates and OD travel demand:

Each OD travel demand Q_{od} have to be assigned to each time point in the interval $[0, T]$; that is, the time-dependent OD flow rates should satisfy

$$\int_0^T q_{od}(u)du = Q_{od} \qquad od \in W \tag{2.6}$$

4) Equilibrium conditions for destination arrival time choice:

At equilibrium, no one can improve his or her own generalized transportation cost by changing the destination arrival time unilaterally. It follows from the path choice equilibrium condition in **2)** that the generalized transportation cost for a user arriving at the estination d at time t is $s_d(t) + \pi_d^o$. Therefore, the equilibrium condition for the user's arrival time choice can be expressed as

$$\begin{cases} \rho_{od} = \pi_d^o(t) + s_d(t) & if \quad q_{od}(t) > 0 \\ \rho_{od} \leq \pi_d^o(t) + s_d(t) & if \quad q_{od}(t) = 0 \end{cases} \qquad \forall t \in [0, T],\ \forall od \in W. \tag{2.7}$$

5) Demand-supply equilibrium (market clearing) conditions in each link permit market:

Since the trading markets are assumed to be perfectively competitive, the price $p_{kl}(t)$ of each permit type adjusted to clear the excess demand for each type of permit. More precisely, at equilibrium, if the price of a certain type of permit is positive, the quantities supplied and the quantities demanded for the permit are equal; for the permit whose supply quantity exceeds the quantity demanded, the price is zero. Note here that, for each link (k, l) and each allowance time t, the demand of the time t permit of the link is equal to the inflow rate $y_{kl}(t)$. On the other hand, the maximum supply (upper bound) of the time t permit of link (k, l) is given by the link capacity μ_{kl}. Therefore, the demand-supply equilibrium condition for the permits market is represented as

$$\begin{cases} y_{kl}(t) = \mu_{kl} & if \quad p_{kl}(t) > 0 \\ y_{kl}(t) \leq \mu_{kl} & if \quad p_{kl}(t) = 0 \end{cases} \qquad \forall t \in [0, T],\ \forall kl \in A \tag{2.8}$$

2.3 Efficiency of the equilibrium under tradable network permits

In order to examine the efficiency of the equilibrium allocation patterns defined in (2.1)-(2.8), consider the following optimization problem [P-M]:

$$\min_{(\mathbf{q},\mathbf{y})\geq\mathbf{0}} \cdot \sum_{o\in O}\sum_{d\in D}\int_0^T q_{od}(t)s_d(t) + \alpha \sum_{(k,l)\in A}\int_0^T y_{kl}(t)t_{kl}dt \tag{2.9}$$

subject to

$$\int_0^T q_{od}(u)du = Q_{od} \qquad \forall od \in W \tag{2.10}$$

$$y_{kl}(t) = \sum_{o\in O} y^o_{kl}(t) \qquad \forall t \in [0,T],\ \forall kl \in A \tag{2.11}$$

$$y_{kl}(t) \leq \mu_{kl} \qquad \forall t \in [0,T], \forall kl \in A \tag{2.12}$$

$$\sum_{l\in NO(k)} y^o_{kl}(t) - \sum_{l\in NI(k)} y^o_{lk}(t-t_{kl}) = -q_{od}(t)\delta_{kd}, \qquad \forall t \in [0,T],\ \forall k \in N,\ \forall o \in O \tag{2.13}$$

This is the problem of finding a dynamic traffic flow pattern that minimizes the total generalized transportation cost in the network, subject to the physical constraints of flows representing the network performance[2]. Specifically, the objective function is the generalized transportation cost (the sum of schedule cost and travel time) expensed by all users in the network. The first constraint (2.10) is conservation of the OD demand, the third constraint (2.12) is the traffic capacity constraints on each link. The final constraint (2.13) is the conservation of dynamic link flows at each node (2.1) combined with the FIFO condition on each link (2.3).

Now the most important property that characterizes the equilibrium assignment can be derived by the fact that [P-M] is an equivalent optimization problem to the equilibrium conditions (2.1)–(2.8). That is,

Proposition 2.1 (Akamatsu, 2007a,b) *For any networks with many-to many OD pairs in which [P-M] has feasible solutions, the equilibrium assignment under the system of time-dependent tradable link permits minimizes the "social transportation cost" defined by* (2.9).

[2] Note that "permit purchase costs" should not be counted as the "social cost" because they are just income transfers between the users and the road manager.

Proof This proposition can be proven by showing that a necessary and sufficient condition for the optimality of the optimization problem [P-M] coincides with the equilibrium conditions (2.1)–(2.8). See Akamatsu (2007a,b) for a complete proof.

The equilibrium generalized transportation cost $\boldsymbol{\rho}$, the permit prices $\mathbf{p}$, and the minimum path cost $\boldsymbol{\pi}$ are given as the optimal Lagrange multipliers for constraints (2.10), (2.12), and (2.13). Thus, the equilibrium prices/costs can be obtained as the solution of the dual problem [D-M], of the problem [P-M]:

$$\max_{(\boldsymbol{\rho},\boldsymbol{\pi},\mathbf{p})\geq\mathbf{0}} \cdot \sum_{od\in W} \rho_{od} Q_{od} - \sum_{(k,l)\in A} \int_0^T p_{kl}(t)\mu_{kl} dt \tag{2.14}$$

subject to

$$\rho_{od} \leq s_d(t) + \pi_d^o(t) \qquad \forall t \in [0,T],\ od \in W \tag{2.15}$$

$$\pi_l^o(t + t_{kl}) \leq \pi_k^o(t) + (\alpha t_{kl} + p_{kl}(t)) \qquad \forall t \in [0,T],\ \forall kl \in A,\ o \in O \tag{2.16}$$

It it obvious that the object function of this problem is the social transportation cost represented as [total generalized transportation cots]−[total permits payments].

2.4 Other desirable properties

This section briefly summarizes other desirable properties of tradable network permits: advantages over congestion pricing, self-financing principle, and Pareto improvement.

2.4.1 Advantages over congestion pricing

The equilibrium permit prices shown in Section 2.2 can be interpreted as the optimal dynamic toll levels for a congestion pricing scheme in which the road manager imposes a time-dependent toll for each link in the network so as to eliminate queuing congestion (e.g., Yang and Meng, 1998). If the road manager can predict users' behaviors (i.e., inflow rate of each link $y_{kl}(t)$) accurately and set an appropriate toll levels based on the condition (2.8) (i.e., queuing congestion never occurs), the equilibrium traffic flow pattern that arises under the congestion pricing scheme is coincides with one under the tradable network permits; it can achieve the system optimal traffic flow pattern that minimizes the social transportation

cost. In other words, the tradable network permits and the congestion pricing scheme are equivalent under the case that the road manager has perfect information.

In contrast, in imperfect information cases, large differences can arise from the discrepancy in the amount and accuracy of the information needed for the road manager to implement these schemes. Specifically, in the tradable network permits, the road manager needs to know only the traffic capacity of each link. In the congestion pricing scheme, on the other hand, the road manager is required to know accurate information on users' behaviors (i.e., precise demands) in addition to the traffic capacity; it is almost impossible for the road manager to obtain such private information. If the congestion pricing scheme is implemented with imperfect information, toll levels based on the condition (2.8) may not be appropriate (i.e., queuing congestion may occur), which does not minimize the social transportation cost and cause an additional economic loss due to queuing congestion.

Considering the differences between the two schemes, the desirable transportation demand management scheme must be found. In generalized terms, this becomes the problem of comparing between "quantity-based regulation" and "price-based regulation." According to the standard theory in the field of economics (see, for example, Weitzman, 1974; Laffont, 1977), quantity-based regulation produces more efficient outcomes than price-based regulation if a regulation authority has only imperfect information on the demand side conditions (i.e., demand functions) while having perfect information on the supply side conditions (i.e., supply functions). For the our problem, we obtain a similar conclusion, although the underlying assumptions of our problem are different from those in conventional economic theory.

2.4.2 Self-financing principle

In order to improve social acceptability of the market selling scheme of the tradable network permits, it may be necessary to use the revenue (or redistribute it to road users) in a socially desirable way. As a representative scheme of the redistribution, Akamatsu et al. (2006) considered the case in which the revenue is used for financing the capacity expansion for a single bottleneck. Under a certain condition[3], they proved the total equilibrium revenue of tradable network permits is equal to the cost required for increasing the bottleneck capacity to a socially optimal level, which has been well known as the "*self-financing principle*"

[3] This condition is that the investment cost function is homogeneous of degree 1 with respect to link capacities.

(Mohring and Harwitz, 1962). Akamatsu (2007a,b) also extended this property to general networks.

2.4.3 Pareto improvement

In a signal bottleneck, by introducing the tradable network permits, queuing congestion can be eliminated completely, and a Pareto improvement can be achieved (Akamatsu et al., 2006). More specifically, each user's (equilibrium) generalized transportation cost does not change with and without system because there is one-to-one correspondence between the queuing delay (at equilibrium without the system) and the permit price (at equilibrium with the system). As Newell (1987) puts it: "*one could convert the worthless expense of queuing into money*." From this property, in market selling scheme, it is easy to see that users' utility levels do not change because users pay a monetary that equals to queuing delay. On the other hand, the total payments make the revenue of the road manager increasing. Therefore, we conclude that the Pareto improvement can be achieved[4]. For a tandem bottleneck network, Yodoshi and Akamatsu (2008) demonstrated that Pareto improvement can be achieved, if revenue from the permits is redistributed to users. However, for networks with many bottlenecks, the correspondences between equilibrium queuing delays and the permit prices are not straightforward because the equilibrium model without the system has complex structure.

2.5 Implementation issues

Although the tradable network permits scheme has the theoretically-desirable properties mentioned above, we should address two closely related problems to implement the scheme:

- the procedures for trading/purchasing network permits are cumbersome;
- no micro mechanisms that realize traffic/market equilibrium have been reported in previous studies.

The former problem must be resolved for the scheme to be socially acceptable. As a way to meet this requirement, Akamatsu (2007a) suggested (but not analyzed) a "multi-agent

[4] From the similar discussion, we obtain the Pareto improving result in free distribution scheme of tradable network permits (Akamatsu et al., 2006).

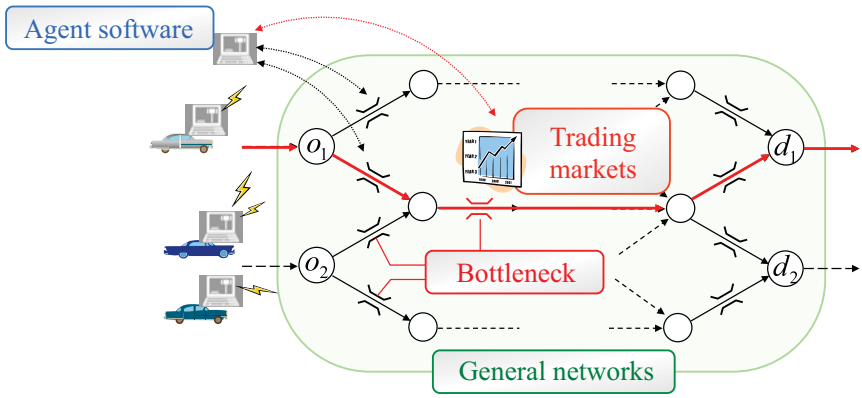


Figure 2.2 Concept of a multi-agent system

system" (see Figure 2.2). In this system, vehicle-installed agent software chooses, on behalf of the user, an optimal path and arrival/departure times based on the user's preferences and deals with procedures for purchasing network permits on an e-market. The system is required to have three properties:

1. each agent can choose a path and an arrival time using local information (autonomy of behavior);

2. the algorithm for the agent's behavior is simple (briefness of agent behavior rules);

3. the whole system can converge to equilibrium (stability of aggregate dynamics).

The essential components that determine (theoretical) properties of such a system are (i) a trading rule for the e-market and (ii) a path/arrival time choice rule for each agent; if these rules are appropriately designed, the multi-agent system can achieve the dynamic system optimal traffic assignment. In other words, these design problems provide micro mechanisms with which to attaining traffic/market equilibrium, thus addressing the latter problem.

Kikuchi and Akamatsu (2008), and Wada, Akamatsu, and Kikuchi (2008) addressed the latter issue (ii) for general networks, assuming that trading markets for network permits are described by a tâtonnement process. Specifically, they first defined the micro behavior of the agent (i.e., a kind of perturbed best response model) based on evolutionary game theory (e.g., Vega-Redondo, 2003; Sandholm, 2010). They then derived day-to-day dynamics of

aggregated traffic flows and permit prices, and proved that mean dynamics of the aggregate variables (flows and prices) converge to a socially optimal state.

For a single bottleneck, Wada and Akamatsu (2010) constructed micro mechanisms consists of both trading rules (auction protocol) and agents' behavior model. More specifically, they designed an auction mechanism based on the Vickrey-Clarke-Groves (VCG) mechanism, which is a benchmark mechanism in auction theory (e.g., Milgrom, 2004; Cramton et al., 2006). By analyzing a deterministic day-to-day traffic flow dynamics obtained from the micro mechanisms, they proved that the dynamics converges to a socially optimal state.

Another line of research focuses on implementation issues under demand uncertainties. Akamatsu et al. (2006) studied the case that users randomly arrive at a bottleneck (i.e., demand uncertainty). In this case, queuing congestion may occur even if the number of permits issued for the bottleneck is equal to its capacity. As a result, the road manager faces a trade-off between total queuing delays and total schedule costs when issuing permits. For a single bottleneck, Akamatsu et al. (2006) experimentally showed that if the number of permits issued for the bottleneck is less than the capacity (about 80%), both mean and variance of economic losses (i.e., queuing delays and schedule costs) are small. Nagae and Gai (2009) considered a stochastic situation where users randomly cancel their trips and proposed a *refundable-tradable bottleneck permits* (R-TBP) scheme. They first revealed that social optimal allocation can be realized if the road manager can determine the proper issue amount of the R-TBP. They then developed an algorithm trial-and-error process by which optimal allocation is achieved only by observable information.

2.6 Conclusion

This chapter reviewed a basic framework and several properties of the tradable network permits scheme. The remarkable feature of this scheme is to achieve the most efficient resource allocation without requiring detailed user information. Under the scheme, the road manager only needs to issue network permits according to the bottleneck capacity; this is contrast to the conventional TDM schemes mentioned in Chapter 1, where the manager bears a great burden (e.g., demand forecasts, price settings). This feature is brought about through the market institution, i.e., permits prices are created by users' autonomous decision-making in the trading markets. Thus, to strengthen the properties of the scheme, Chapter 4 and

Chapter 5 construct micro mechanisms for attaining market equilibrium in more spatially and temporally general situations than a single bottleneck case (Wada and Akamatsu, 2010). This is not the only benefit from the market institution. Trading markets further provides useful information on the equilibrium permit prices, which reflect demand conditions. This suggests that there is every possibility of developing a non-forecast-based supply side control using the information. On the basis, the next chapter explores a supply side control based on the tradable network permits.

Chapter 3

Distributed signal control based on tradable network permits

This chapter[1] proposes a distributed signal control policy based on a tradable network permits system taking into account a semi-dynamic traffic flow. Main idea of underlying the proposed policy is to exploit useful information on the equilibrium permit prices provided by the trading markets. This enables us to appropriately incorporate demand conditions into the signal control policy. Specifically, this signal control policy can achieve a global optimal traffic flow pattern by exploiting only local information: it can determine the green time proportion of each intersection by using intersection information only. An equilibrium traffic assignment under the proposed policy achieves a system optimal traffic flow pattern that minimizes the total transportation cost in a network. Moreover, we construct an evolutionary implementation method for the proposed policy and prove that the day-to-day traffic flow dynamics under the scheme converges to the system optimal traffic assignment.

This chapter is organized as follows. Section 3.1 reviews existing studies on a framework of combining signal control and traffic assignment. In Section 3.2, we describe the preconditions used throughout the chapter and define a system optimal traffic flow pattern. In Section 3.3, we show a framework that combines the tradable network permits system and a novel signal control policy and its properties. In Section 3.4, we show an evolutionary implementation method of the proposed control policy. Section 3.5 concludes the chapter.

[1] This chapter is based on joint research with Takashi Akamatsu, presented in *the 4th International Symposium on Dynamic Traffic Assignment* (Wada and Akamatsu, 2012).

3.1 Related works

A framework of combining signal control and traffic assignment was first introduced by Allsop (1974) and a considerable amount of research into developing this framework (see Cascetta, Gallo, and Montella, 2006; Ghatee and Hshemi, 2007, for references and a detailed review) has been conducted. These studies are divided into two major categories, i.e., global and local optimization approaches (Marcotte, 1983; Cantarella and Improta, 1991).

The former approaches aim to determine an optimal signal settings by solving a *global optimization of the signal setting problem* (Gossp) that optimizes a global network performance function. In general, the Gossp is formulated as a bi-level problem (or a mathematical problems with equilibrium constraints), i.e., the upper level is a signal setting problem, and the lower level is a traffic equilibrium assignment problem. Various solution methods for the bi-level problem have been proposed by numerous authors, e.g., Marcotte (1983); Sheffi and Powell (1983); Heydecker and Khoo (1990); Yang and Yangr (1995); Chiou (1999); Cipriani and Fusco (2004) and Ghatee and Hshemi (2007). However, this global approach faces a difficulty caused by the non-convexity of the Gossp. Since the Gossp may have many local minima, there is no guarantee that the global optimal solution is obtained by those various solution methods. For instance, Cascetta, Gallo, and Montella (1998) showed experimentally that different methods might produce different solutions. In addition, this approach requires knowledge of the entire network information (e.g., origin-destination information) to solve the Gossp. Thus, the optimal signal setting is difficult to obtain using global solution methods.

The latter approach consists of iteratively adjusting signal parameters based on a local criterion. Representative classical signal control policies are the *equisaturation* policy (Webster, 1958) and the *delay-minimization* policy [2] (Allsop, 1974; Gartner, 1974). The equisaturation policy sets a green time proportion so as to equalize the saturation level of all directions at an intersection, and the delay-minimization policy updates the signal parameters by solving a current total delay minimization problem. Since these policies determine the signal parameters in terms of a *short run* objective for a current traffic flow pattern, *long run* interactions between user behaviors and the signal control policy are not properly considered. As a result, equilibrium states consistent with the policies may be unstable, which

[2] This policy is also called *iterative optimization assignment* (IOA).

may lead to poor performance (Smith, 1979b; Dickson, 1981). Smith also pointed out that policies induce inefficient user of network capacity.

To solve this problem, Smith (1979a, 1980, 1981) proposed a *capacity-maximization* policy P_0, which takes into consideration the aforementioned long run iterations. In policy P_0, a stable equilibrium consistent with the policy is achieved by using only local information. Smith, van Vuren, Heydecker, and van Vliet (1987) showed experimentally that this policy is better than the above two policies in terms of total delay at high congestion levels. Furthermore, in Smith and van Vuren (1993) and Smith and Mounce (2011), a day-to-day traffic flow dynamics globally converges to a stable equilibrium when a responsive version of the policy P_0 is employed. However, the policy generally cannot achieve the global optimal traffic flow pattern because it is one of the local optimization approaches.

As we have seen, a global optimization approach may not be promising way to establish a robust signal control policy, which does not require detailed user information. Instead, it will be more effective to develop local optimization approaches that can obtain a stable equilibrium in a distributed manner. In that sense, our signal control policy is designed in the spirit of policy P_0. Note that the equilibrium states with these policies are different in that the equilibrium traffic assignment under the proposed policy coincides with the optimal one, although the equilibrium under the policy P_0 does not[3]. Thus, the proposed policy is the first one that can achieve a global optimal signal setting in a distributed manner, which is guaranteed to be applicable to situations with asymmetric information.

3.2 System optimal traffic assignment in signal-controlled networks

The objective of a signal control policy, which we propose in this chapter, is to eliminate queuing congestion and to achieve a system optimal traffic flow pattern that minimizes the total transportation cost. This section defines this objective. After describing a discrete-time semi-dynamic traffic flow on a signal-controlled network, we then formulate a *global optimization of the signal setting problem* (Gossp) that provides the system optimal traffic flow pattern.

[3] Ghali and Smith (1993) showed that combining the policy P_0 and a marginal cost pricing scheme can achieve the global optimal traffic flow pattern. However, when to implement this, the entire network information is needed.

3.2.1 Signal-controlled networks

We consider a discrete-time semi-dynamic traffic flow (e.g., Kikuchi and Akamatsu, 2007; Nakayama, 2008) on general networks with multiple origin-destination (OD) pairs. The semi-dynamic traffic assignment model assumes that steady state traffic equilibrium is realized in each time period, and traffic states only change between time periods. More specifically, in this model, a queue on a link that cannot exit the link is propagated to the next time period, which affects the next traffic state. This model approximately represents a continuous-time queue evolution.

The network consists of a set N of nodes and a set A of directed links. Each node represents each road section between signalized intersections; each link represents each direction in a signalized intersection. The node set includes a subset O of origin nodes from which users start their trip, and a subset D of destination nodes at which users terminate their trips. A set of OD pairs is denoted by W. Each element of A (i.e., each link) is denoted by a pair (k, l) of the upstream node k and the downstream node l. The time interval for which we assign the semi-dynamic traffic flow is fixed, and each time period is denoted by $t \in T$. We assume that each OD pair's travel demand Q_{od} in the time interval is a given constant.

We assume, without any loss of generality, that each link in a network consists of a free-flow segment and single bottleneck segment (i.e., signalized intersection). The travel time to pass through the free-flow segment of link (k, l) is a constant t_{kl} (i.e., t_{kl} is independent of time and flow)[4]. The bottleneck of each link has a constant saturation flow rate μ_{kl}, and a signal delay may occur.

Signal setting constraints

We also assume that all links are signal-controlled. A set of signalized intersections is denoted by J, and a set of green phases at an intersection j in a time period t is denoted by $E_j(t)$. A cycle time and a loss time of each intersection are given exogenously. Let $g_{e,j}(t)$ be the green time proportion of phase e at intersection j in time period t. For each intersection j in each time period t, the sum of the green time proportions and a loss time proportion l_j equals to 1:

[4] The free-flow travel time includes the travel time to pass through a node (i.e., road section).

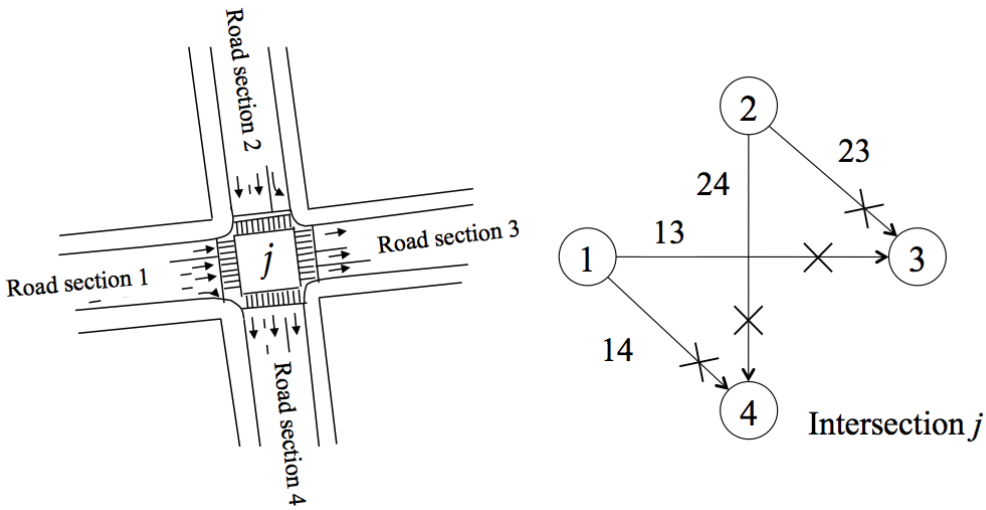


Figure 3.1 Network representation of a signalized intersection

$$\sum_{e \in E_j(t)} g_{e,j}(t) + l_j = 1 \qquad \forall j \in J,\ \forall t \in T. \tag{3.1}$$

Each signalized intersection consists of multiple links (Figure 3.1), and each link belongs to multiple green phases. A set of links of phase e at intersection j in time period t is denoted by $A_{e,j}(t)$. Then, the green time proportion $G_{kl}(t)$ of link (k, l) is the sum of the green times of the phases to which the link belongs, i.e.,

$$G_{kl}(t) = \sum_{e: kl \in A_{e,j}(t)} g_{e,j}(t) \qquad \forall kl \in A,\ \forall t \in T. \tag{3.2}$$

As mentioned before, our objective is to eliminate queuing congestion and to minimize the total transportation cost. Thus, in addition to above signal setting constraints, we here consider capacity constraints, i.e., the inflow rate $y_{kl}(t)$ of each link cannot exceed the traffic capacity $G_{kl}(t)\mu_{kl}$ of each link:

$$y_{kl}(t) \leq G_{kl}(t)\mu_{kl} \qquad \forall kl \in A,\ \forall t \in T, \tag{3.3}$$

Then over-saturated delay (i.e., queuing delay) at each link never occurs.

Transportation costs in signal-controlled networks

Each user makes a single trip from an origin $o \in O$ to a destination $d \in D$ in the network. The transportation cost for a single trip of a network consists of two types of cost: (a) "schedule cost," (b) "travel cost." (a) The schedule cost for a user is the cost due to the difference between the user's desired arrival time period and the actual arrival time period t. The desired arrival time is assumed to be the same for all users and is equal to $\hat{t}$. The schedule cost is represented by the function $s_d(t)$ of the arrival time t at destination d, which is common to all users.

(b) The travel cost is the monetary equivalent of the travel time required for a trip from the origin and the destination. The travel times are divided in two parts, the free-flow travel time and the signal delay. The free-flow travel time of link (k, l) is a constant t_{kl}. The signal delay that arises at each intersection under the capacity constraints (3.3) is not a *queuing delay* (i.e., over-saturated delay) but the *waiting time* caused by the red phase of the intersection. Since this under-saturated delay is often smaller than the time in queue, the change (or externality) of the delay due to the flow and the green time is assumed to be negligible, i.e., the signal delay of the link (k, l) is assumed to be a constant d_{kl}.

3.2.2 Global optimization of signal setting problem

Under the setting, the total transportation cost is defined as the sum of all user costs. Thus, we formulate a global optimization of signal setting problem [Gossp] to determine both the system optimal traffic flow pattern and the optimal green time proportions:

$$TTC^* \equiv \min_{(\mathbf{q},\mathbf{y},\mathbf{g})\geq \mathbf{0}} . \sum_{t\in T}\sum_{o\in O}\sum_{d\in D} q_{od}(t)s_d(t) + \alpha \sum_{t\in T}\sum_{kl\in A} y_{kl}(t)[t_{kl} + d_{kl}] \tag{3.4}$$

subject to

$$\sum_{t\in T} q_{od}(t) = Q_{od} \qquad \forall od \in W \tag{3.5}$$

$$\sum_{l\in NO(k)} y^o_{kl}(t) - \sum_{l\in NI(k)} y^o_{lk}(t) = -q_{od}(t)\delta_{kd} \qquad \forall k \in N,\ \forall o \in O, t \in T \tag{3.6}$$

$$y_{kl}(t) \leq G_{kl}(t)\mu_{kl} \qquad \forall kl \in A,\ \forall t \in T \tag{3.7}$$

$$\sum_{e\in E_j(t)} g_{e,j}(t) + l_j = 1 \qquad \forall j \in J,\ \forall t \in T \tag{3.8}$$

where

$$y_{kl}(t) = \sum_{o \in O} y^o_{kl}(t) \qquad \forall kl \in A,\ \forall t \in T \tag{3.9}$$

$$G_{kl}(t) = \sum_{e:kl \in A_{e,j}(t)} g_{e,j}(t) \qquad \forall kl \in A,\ \forall t \in T, \tag{3.10}$$

where $q_{od}(t)$ is the OD flow rate for a user arriving at the destination at time period t, and α is a coefficient that converts travel time into the monetary equivalent.

This is the problem of finding a semi-dynamic traffic flow pattern $(\mathbf{q}^*, \mathbf{y}^*)$ and green time proportions $\mathbf{g}^*$ that minimize the social transportation cost in the network[5]. Specifically, the first term of the objective function is the total schedule cost expensed by all users, and the second term is the monetary equivalent of the total travel time paid by all users. The first constraint (3.5) is the conservation of the OD demand; the second constraint (3.6) is the conservation of the semi-dynamic traffic flows at each node[6]. The next constraint (3.7) is the traffic capacity constraints on each bottleneck. The final constraint (3.8) is the condition that should be satisfied by the green time proportions.

We should note here that this problem does not incorporate users' behaviors (i.e., destination arrival time choice, path choice). In addition, when to solve this problem, the entire network information is needed. As shown later in Section 3.3, however, a framework that combines the tradable network permits system and a distributed signal control policy can achieve an equilibrium traffic assignment that coincides with the system optimal traffic flow pattern.

[5] This problem does not necessarily have a feasible solution due to the capacity constraint (3.7) on each link. However, if the assignment time interval is large enough that we can make the OD flow rates smaller than the maximum capacity of the underlying network, the problem [Gossp] always has feasible solutions. Thus, throughout this chapter, we only deal with the case where the problem [Gossp] has feasible solutions.

[6] In the semi-dynamic traffic assignment model, an inflow rate and an outflow rate of each link are different when a queue exists. However, in the present case there is no need to distinguish between the in- and outflow rates because a queue never occurs.

3.3 Distributed signal control policy based on tradable network permits

This section presents a framework that combines the tradable network permits system and a signal control policy. In Subsection 3.3.1, we first describe behaviors of agents in our framework. In Subsection 3.3.2, we show the equilibrium that takes place after introducing the tradable network permits, and provide an optimization problem that is equivalent to the equilibrium. From the problem, we can see that the problem [Gossp] includes users' behaviors. In Subsection 3.3.3, we propose a signal control policy based on the tradable network permits. In Subsection 3.3.4, we introduce the most important property that characterizes the equilibrium assignment under the proposed signal control policy.

3.3.1 Behavoirs of agents

A road manager aims to restrain traffic congestion on the network and minimize the total transportation cost. To achieve this, the manager regulates the traffic flow rates entering each bottleneck (i.e., signalized intersection) in the network using time-dependent network permits. We assume that the number of permits issued for each link for each time period is equal to or less than the traffic capacity, which results in eliminating over-saturated delay at each link. We also assume that trading markets are perfectly competitive. In addition, the manager employs a signal control policy based on tradable network permits for controlling green time proportions of each intersection. The precise definition of the signal control policy is introduced in Subsection 3.3.3.

Each road user, on the other hand, chooses a destination arrival time and a path so as to minimize own generalized transportation cost. Under the system of network permits each user must purchase a set of permits corresponding to a set of links included in the user's chosen path. Thus, the generalized transpiration cost is the sum of the transportation cost described above (i.e., schedule cost and travel cost) and the permit purchase cost that is defined as the sum of permit prices of the links included in the path used.

3.3.2 Equilibrium under the tradable network permits system

Let us now assume that a signal setting is fixed. At equilibrium, the following three conditions should hold in addition to physical conditions of the traffic flows (3.5), (3.6), and (3.7) (also see Chapter 2 for a detailed discussion).

1) Equilibrium conditions for path choice:

Consider a user arriving at node k at time period t. At equilibrium, no user can improve his or her own cost by changing the path choice unilaterally (i.e., no user has the incentive to deviate from his or her strategy). Hence, if there exists a user entering into link (k, l) at time period t, link (k, l) should be on the minimum path for a user arriving at node l. Denoting $\pi_k^o(t)$ as the minimum path cost from the origin o to node k for a user arriving at time period t, we can represent this condition as

$$\begin{cases} \pi_l^o(t) = \alpha(t_{kl} + d_{kl}) + p_{kl}(t) + \pi_k^o(t) & \text{if} \quad y_{kl}^o(t) > 0 \\ \pi_l^o(t) \leq \alpha(t_{kl} + d_{kl}) + p_{kl}(t) + \pi_k^o(t) & \text{if} \quad y_{kl}^o(t) = 0 \end{cases} \qquad \forall kl \in A,\ \forall o \in O,\ \forall t \in T, \quad (3.11)$$

where $p_{kl}(t)$ is the permit price of link (k, l) at time period t.

2) Equilibrium conditions for destination arrival time choice:

At equilibrium, no one can improve his or her own generalized transportation cost by changing the destination arrival time unilaterally. It follows from the path choice equilibrium condition above that the generalized transportation cost for a user arriving at the destination d at time period t is $s_d(t) + \pi_d^o(t)$, where $s_d(t)$ is the schedule cost for a user arriving at the destination d at time period t. Therefore, the equilibrium condition for the user's arrival time choice can be expressed as

$$\begin{cases} \rho_{od} = \pi_d^o(t) + s_d(t) & \text{if} \quad q_{od}(t) > 0 \\ \rho_{od} \leq \pi_d^o(t) + s_d(t) & \text{if} \quad q_{od}(t) = 0 \end{cases} \qquad \forall od \in W,\ \forall t \in T, \quad (3.12)$$

where ρ_{od} represents the minimum generalized transportation cost between the origin and the destination.

3) Demand-supply equilibrium conditions in each link permits market:

Since the trading markets are assumed to be perfectly competitive, the price $p_{kl}(t)$ of each permit type is adjusted to clear the excess demand for each type of permit. As we shown in Chapter 2, the demand-supply equilibrium condition for the permits market is represented as

$$\begin{cases} y_{kl}(t) = G_{kl}(t)\mu_{kl} & \text{if} \quad p_{kl}(t) > 0 \\ y_{kl}(t) \leq G_{kl}(t)\mu_{kl} & \text{if} \quad p_{kl}(t) = 0 \end{cases} \qquad \forall kl \in A,\ \forall t \in T. \tag{3.13}$$

For a given a signal setting $\mathbf{g}$, the equilibrium conditions under the tradable network permits mentioned above, which determine the OD flow rates $\mathbf{q}$, link inflow rates $\mathbf{y}$, and permit prices $\mathbf{p}$, are equivalent to the following linear programming program [TAP-P] (Chapter 2):

$$\min_{(\mathbf{q},\mathbf{y})\geq \mathbf{0}} . \sum_{t\in T}\sum_{o\in O}\sum_{d\in D} q_{od}(t)s_d(t) + \alpha \sum_{t\in T}\sum_{kl\in A} y_{kl}(t)[t_{kl} + d_{kl}] \tag{3.14}$$

subject to

$$\sum_{t\in T} q_{od}(t) = Q_{od} \qquad \forall od \in W \tag{3.15}$$

$$\sum_{l\in NO(k)} y^o_{kl}(t) - \sum_{l\in NI(k)} y^o_{lk}(t) = -q_{od}(t)\delta_{kd} \qquad \forall k \in N,\ \forall o \in O, t \in T \tag{3.16}$$

$$y_{kl}(t) \leq G_{kl}(t)\mu_{kl} \qquad \forall kl \in A,\ \forall t \in T \tag{3.17}$$

$$y_{kl}(t) = \sum_{o\in O} y^o_{kl}(t) \qquad \forall kl \in A,\ \forall t \in T. \tag{3.18}$$

This is the problem of finding the semi-dynamic traffic flow pattern that minimizes the total generalized transportation cost in the network *under the condition that the signal setting is fixed.* In other words, the problem [TAP-P] is a sub-problem of the problem [Gossp], which does not involve the determination of the signal settings. Hence, we see that the problem [Gossp] contains user behaviors under the tradable network permits system.

This problem further provides information on the equilibrium prices/costs as well as the equilibrium flow patterns. Specifically, the optimal Lagrange multipliers for constraints (3.15), (3.16), and (3.17) give the generalized transportation cost $\boldsymbol{\rho}$, the minimum path cost from the origin to each node $\boldsymbol{\pi}$, and the link permit prices $\mathbf{p}$, respectively. These Lagrange multipliers $(\boldsymbol{\rho}, \boldsymbol{\pi}, \mathbf{p})$ represent the equilibrium prices under the system of tradable permits and are given as a solution of the dual problem, [TAP-D], of the problem [TAP-P]:

$$\max_{(\boldsymbol{\rho},\boldsymbol{\pi},\mathbf{p})\geq\mathbf{0}} . \sum_{od\in W} \rho_{od} Q_{od} - \sum_{t\in T}\sum_{kl\in A} p_{kl}(t) G_{kl}(t) \mu_{kl} \tag{3.19}$$

subject to

$$\rho_{od} \leq s_d(t) + \pi_d^o(t) \qquad \forall od \in W,\ \forall t \in T \tag{3.20}$$

$$\pi_l^o(t) \leq \pi_k^o + \alpha(t_{kl} + d_{kl}) + p_{kl}(t) \qquad \forall kl \in A,\ \forall o \in O,\ \forall t \in T. \tag{3.21}$$

3.3.3 A distributed signal control policy

In the previous subsection, we described behaviors of agents (i.e., users) that use road capacities. Our signal control policy, on the other hand, is described as behaviors of agents (i.e., suppliers) that supply road capacities. Each supplier is an owner of each intersection in the network and chooses the green time proportion (i.e., capacity allocation) of the intersection so as to maximize his or her profit for given constant permit prices $\mathbf{p}$. This profit maximization problem at the intersection k is formulated as

$$\max_{\mathbf{g}_j\geq\mathbf{0}} . \sum_{t\in T}\sum_{e\in E_j(t)}\sum_{kl\in A_{e,j}(t)} G_{kl}(t) p_{kl}(t) \mu_{kl} \tag{3.22}$$

subject to

$$\sum_{e\in E_j(t)} g_{e,j}(t) + l_k = 1. \qquad \forall t \in T \tag{3.23}$$

$$G_{kl}(t) = \sum_{e:kl\in A_{e,j}(t)} g_{e,j}(t) \qquad \forall kl \in A_{e,j}(t)\ \forall e \in E_j(t),\ \forall t \in T. \tag{3.24}$$

Then the signal control policy is derived as a necessary and sufficient condition for the optimality of the profit maximization problem:

$$\begin{cases} \phi_j(t) = \sum_{kl\in A_{e,j}(t)} p_{kl}(t)\mu_{kl} & \text{if} \quad g_{e,j}(t) > 0 \\ \phi_j(t) \geq \sum_{kl\in A_{e,j}(t)} p_{kl}(t)\mu_{kl} & \text{if} \quad g_{e,j}(t) = 0 \end{cases} \qquad \forall e \in E_j(t),\ j \in J,\ \forall t \in T, \tag{3.25}$$

where $\phi_j(t)$ is the Lagrange multiplier corresponding to the constraint (3.23).

The proposed signal control policy can determine the green time proportion of the intersection by using information on permit prices and saturation flows of the intersection; it does

not require knowledge of entire network information (e.g., origin-destination information). More specifically, the policy (3.25) chooses the green time proportion of each intersection such that

Less profitable phases receive no green time.

3.3.4 Efficiency of the equilibrium under the proposed signal control policy

The most important result in this chapter can be derived by combining user behaviors and the proposed signal control policy in the previous subsections. In particular, we obtain the following proposition on the relationship between the global optimization of signal setting problem [Gossp] and the equilibrium conditions (3.11), (3.12), and (3.13) consistent with the propose policy (3.25).

Proposition 3.1 *For any networks with many-to-many OD pairs in which the problem [Gossp] has feasible solutions, the traffic equilibrium assignment under the tradable network permits consistent with the proposed signal control policy minimizes the total travel cost defined by* (3.4).

Proof This proposition can be proved by showing that a necessary and sufficient condition for the optimality of the optimization problem [Gossp] coincides with the equilibrium conditions (3.11), (3.12), and (3.13) and the proposed signal control policy (3.25). See Appendix 3.A for a complete proof.

To understand the Proposition 3.1 intuitively, we present another interpretation of the proposed control policy, i.e., the proposed policy preferentially allocates capacities to the phases that reduce the social transportation cost more. This comes from the fact that the Lagrange multiplier $\mathbf{p}$ (i.e., the permit prices) corresponding to the capacity constraint (3.7) equals to the decrease in the value of the objective function (3.4) by augmenting 1 unit of link capacity (Ahuja, Magnanti, and Orlin, 1993).

In addition, our signal control policy is similar to the simple type of policy P_0 by Smith (1987) in the sense that these policies can achieve a stable equilibrium in a distributed manner. However, his setting allows intersections to over-saturate, and his policy P_0 cannot achieve the system optimal assignment even if the under-saturated delay is constant.

3.4 Evolutionary implementation method for the proposed signal control policy

To obtain the system optimal traffic flow pattern $(\mathbf{q}^*, \mathbf{y}^*)$, the proposed signal control policy requires the optimal permit price $\mathbf{p}^*$, which may not be instantly available through tradable permits markets. To resolve this difficulty, we construct an evolutionary (i.e., day-to-day) implementation method for the proposed policy.

The scheme consists of two phases, a *traffic equilibrium assignment phase* and a *signal setting adjustment phase*, which are repeated on a day-to-day basis. In the former phase, an equilibrium traffic flow pattern under the tradable network permit arises under the condition that green time proportions are fixed. In addition, the manager obtains the permit price information. In the latter phase, green time proportions are adjusted by a modified version of the proposed signal control policy.

This natural procedure corresponds to Benders decomposition algorithm (Benders, 1962; Lasdon, 1970; Geoffrion and Graves, 1974) for the problem [Gossp] (see Appendix A for a basic framework of the Benders decomposition). Hence, we first decomposes the problem [Gossp] into two problems, a master problem and a sub-problem. We then explain the procedure of the implementation method. Finally, we prove that a day-to-day traffic flow dynamics under the scheme converges to the system optimal traffic flow pattern.

3.4.1 Applying Benders decomposition principle to the Gossp

Let us decompose the problem [Gossp] into two problems based on Benders decomposition principle:

$$TTC^* = \min_{\mathbf{g}\geq\mathbf{0}} . \sum_{t\in T}\sum_{o\in O}\sum_{d\in D} q_{od}(\mathbf{g}(t))s_d(t) + \alpha \sum_{t\in T}\sum_{kl\in A} y_{kl}(\mathbf{g}(t))[t_{kl} + d_{kl}] \tag{3.26}$$

subject to

$$\sum_{e\in E_j(t)} g_{e,j}(t) + l_k = 1 \qquad \forall j \in J,\ \forall t \in T \tag{3.27}$$

$$G_{kl}(t) = \sum_{e:kl\in A_{e,j}(t)} g_{e,j}(t) \qquad \forall kl \in A,\ \forall t \in T \tag{3.28}$$

where $\mathbf{q}(\mathbf{g})$ and $\mathbf{y}(\mathbf{g})$ are obtained by

$$\min_{(\mathbf{q},\mathbf{y})\geq\mathbf{0}} \cdot \sum_{t\in T}\sum_{o\in O}\sum_{d\in D} q_{od}(t)s_d(t) + \alpha \sum_{t\in T}\sum_{kl\in A} y_{kl}(t)[t_{kl}+d_{kl}] \tag{3.29}$$

subject to

$$\sum_{t\in T} q_{od}(t) = Q_{od} \qquad \forall od \in W \tag{3.30}$$

$$\sum_{l\in NO(k)} y^o_{kl}(t) - \sum_{l\in NI(k)} y^o_{lk}(t) = -q_{od}(t)\delta_{kd} \qquad \forall k \in N,\ \forall o \in O, t \in T \tag{3.31}$$

$$y_{kl}(t) \leq G_{kl}(t)\mu_{kl} \qquad \forall kl \in A,\ \forall t \in T \tag{3.32}$$

$$y_{kl}(t) = \sum_{o\in O} y^o_{kl}(t) \qquad \forall kl \in A,\ \forall t \in T, \tag{3.33}$$

where $(\mathbf{q}(\mathbf{g}),\ \mathbf{y}(\mathbf{g}))$ is an optimal solution of the lower level problem for a parameter $\mathbf{g}$. The upper level problem (master problem) determines the optimal signal setting so as to minimize the total transportation cost; the lower level problem (sub-problem) is the same as the problem [TAP-P] shown in Subsection 3.3.2, which represents the traffic equilibrium assignment under the tradable network permits.

To understand that the master problem corresponds to our signal control policy, we need to show the relationship between the master problem and the permit prices. Then we recall the problem [TAP-D] which is the dual problem of the problem [TAP-P]:

$$TTC(\mathbf{g}) \equiv \max_{(\boldsymbol{\rho},\boldsymbol{\pi},\mathbf{p})\geq\mathbf{0}} \cdot \sum_{od\in W} \rho_{od}Q_{od} - \sum_{t\in T}\sum_{kl\in A} p_{kl}(t)G_{kl}(t)\mu_{kl} \tag{3.34}$$

subject to

$$\rho_{od} \leq s_d(t) + \pi^o_d(t) \qquad \forall od \in W,\ \forall t \in T \tag{3.35}$$

$$\pi^o_l(t) \leq \pi^o_k + \alpha(t_{kl}+d_{kl}) + p_{kl}(t) \qquad \forall kl \in A,\ \forall o \in O,\ \forall t \in T. \tag{3.36}$$

From the duality theorem, the optimal value of the objective function of [TAP-D] coincides with the optimal value of the objective function of [TAP-P], i.e.,

$$\begin{aligned} TTC(\mathbf{g}) &= \sum_{od\in W} \rho_{od}(\mathbf{g})Q_{od} - \sum_{t\in T}\sum_{kl\in A} p_{kl}(\mathbf{g}(t))G_{kl}(t)\mu_{kl} \\ &= \sum_{t\in T}\sum_{o\in O}\sum_{d\in D} q_{od}(\mathbf{g}(t))s_d(t) + \alpha \sum_{t\in T}\sum_{kl\in A} y_{kl}(\mathbf{g}(t))[t_{kl}+d_{kl}], \end{aligned} \tag{3.37}$$

where $(\boldsymbol{\rho}(\mathbf{g}), \mathbf{p}(\mathbf{g}))$ is an optimal solution of the [TAP-D] for a parameter $\mathbf{g}$, i.e., it is an extreme point of the convex feasible region Ω that consists of the constraints (3.35) and (3.36). By using the optimal value function (5.40), we can transform the master problem into the following problem:

$$\min_{\mathbf{g}\geq\mathbf{0}} .TTC(\mathbf{g}) = \sum_{od\in W} \rho_{od}(\mathbf{g})Q_{od} - \sum_{t\in T}\sum_{kl\in A} p_{kl}(\mathbf{g}(t))G_{kl}(t)\mu_{kl} \tag{3.38}$$

$$= \min_{\mathbf{g}\geq\mathbf{0}} .\left[\max_{(\boldsymbol{\rho}^{(s)},\mathbf{p}^{(s)})\in V(\Omega)} .\left(\sum_{od\in W} \rho_{od}^{(s)}Q_{od} - \sum_{t\in T}\sum_{kl\in A} p_{kl}^{(s)}(t)G_{kl}(t)\mu_{kl}\right)\right] \tag{3.39}$$

subject to Eq. (3.27) and Eq. (3.28).

where $V(\Omega)$ is all (a finite set of) extreme points of the convex feasible region Ω. In the problem (3.39), the coefficients of the green time proportions are permit prices, i.e., it indicates that green time proportions are determined based on permit prices.

Problem (3.39) is equivalent to the original problem [Gossp] if all extreme points are known. However, it is difficult to obtain the extreme points in advance because a number of extreme points is generally too large. Hence, we consider a relaxation problem of (3.39) that has a subset of extreme points in $V(\Omega)$ and produce an lower bound on the optimal objective value of the problem (3.39) (or the problem [Gossp]). This relaxed problem is called the *restricted master problem*. We then employ an iterative approach by adding an extreme point to the restricted master problem to improve the lower bound. Note that an extreme point is generated by solving the problem [TAP-D] for a fixed signal setting $\mathbf{g}$.

3.4.2 Procedure of the evolutionary implementation method

The evolutionary implementation method for the proposed signal control policy corresponds to solving the above two problems iteratively. Hence, the day-to-day procedure of the scheme can be summarized in the following steps:

Step 0 : *Initial setting.* Set $s = 1$. Determine the initial green time proportions $\mathbf{g}^{(1)}$. Start with a set of extreme points $V^{(1)}(\Omega) = \{\emptyset\}$ and a lower bound $\theta^{(1)} = 0$.

Step 1 : *Traffic equilibrium assignment phase.* For a fixed signal setting $\mathbf{g}^{(s)}$, the consistent equilibrium $(\mathbf{q}^{(s)}, \mathbf{y}^{(s)}, \mathbf{p}^{(s)})$ arises under the tradable network permits system, which is shown in Subsection 3.3.2. The equilibrium generalized transportation cost $\boldsymbol{\rho}^{(s)}$ is also

determined. If the total transportation cost $TTC(\mathbf{g}^{(s)})$ equals the lower bound $\theta^{(s)}$, then stop. Otherwise go to **Step 2**.

Step 2 : *Signal setting adjustment phase.* Add an extreme point to the set: $V^{(s+1)}(\Omega) \equiv \{V^{(s)}(\Omega) \cup (\boldsymbol{\rho}^{(s)}, \mathbf{p}^{(s)})\}$, and produce the signal setting $\mathbf{g}^{(s+1)}$ by a modified version of the proposed signal control policy and update the lower bound $\theta^{(s+1)}$. Let $s = s + 1$. Go to **Step 1**.

At Step 1, through the trading markets, the equilibrium permit allocation pattern and permit prices are determined, which leads to the equilibrium traffic flow pattern shown in Subsection 3.3.2. As an example of concrete mechanisms for the trading markets, we may utilize the auction mechanism constructed in Chapter 4. In this mechanism, the net utility, which corresponds to the generalized transpiration cost $\boldsymbol{\rho}^{(s)}$, can be obtained through a proxy-ascending auction.

At Step 2, the road manager considers all extreme points information $(\boldsymbol{\rho}^{(s)}, \mathbf{p}^{(s)})$ for the current day and past days and adjusts the green time proportion of each intersection. This corresponds to solving the following linear programming problem that is equivalent to the restricted master problem:

$$\min_{\mathbf{g},\theta \geq \mathbf{0}} .\theta \tag{3.40}$$

subject to

$$\theta \geq \sum_{od \in W} \rho_{od}^{(s)} Q_{od} - \sum_{t \in T} \sum_{kl \in A} p_{kl}^{(s)}(t) G_{kl}(t) \mu_{kl} \qquad \forall (\boldsymbol{\rho}^{(s)}, \mathbf{p}^{(s)}) \in V^{(s+1)}(\Omega), \tag{3.41}$$

Eq. (3.27) and Eq. (3.28).

The optimal value of the objective function is the lower bound $\theta^{(s+1)}$ on the optimal value of the problem (3.39). A modified signal control policy is derived as the necessary and sufficient optimality condition of the problem:

$$\begin{cases} \theta = \sum_{od \in W} \rho_{od}^{(s)} Q_{od} - \sum_{t \in T} \sum_{kl \in A} p_{kl}^{(s)}(t) G_{kl}(t) \mu_{kl} & \text{if} \quad \eta^{(s)} > 0 \\ \theta \geq \sum_{od \in W} \rho_{od}^{(s)} Q_{od} - \sum_{t \in T} \sum_{kl \in A} p_{kl}^{(s)}(t) G_{kl}(t) \mu_{kl} & \text{if} \quad \eta^{(s)} = 0 \end{cases}$$

$$\{(\boldsymbol{\rho}^{(1)}, \mathbf{p}^{(1)}), \ldots, (\boldsymbol{\rho}^{(s)}, \mathbf{p}^{(s)})\} \in V^{(s+1)}(\Omega) \tag{3.42}$$

$$\begin{cases} 1 = \sum_s \eta^{(s)} & \text{if} \quad \theta > 0 \\ 1 \geq \sum_s \eta^{(s)} & \text{if} \quad \theta = 0 \end{cases} \tag{3.43}$$

$$\begin{cases} \phi_j(t) = \sum_s \eta^{(s)} \sum_{kl \in A_{e,j}(t)} p_{kl}^{(s)}(t) \mu_{kl} & \text{if} \quad g_{e,j}(t) > 0 \\ \phi_j(t) \geq \sum_s \eta^{(s)} \sum_{kl \in A_{e,j}(t)} p_{kl}^{(s)}(t) \mu_{kl} & \text{if} \quad g_{e,j}(t) = 0 \end{cases} \quad \forall e \in E_j(t), j \in J, \forall t \in T, \tag{3.44}$$

where $\eta^{(s)}$ is the Lagrange multiplier for the constraint (3.41). The modified version of the proposed signal control policy (i.e., (3.42), (3.43), and (3.44)) is not a complete distributed policy but is described as behaviors of a central agent and the local agents.

The central agent coordinates the local agents to improve the lower bound. Specifically, the central agent determines the weight parameter $\eta^{(s)}$ for each day so as to maximize the lower bound based on the conditions (3.42) and (3.43). If the condition (3.42) of day s is bounded, all weight 1 is allocated to the parameter of day s, i.e., $\eta^{(s)} = 1$. Note that we assume that the lower bound is $\theta > 0$ (i.e., $\sum_s \eta^{(s)} = 1$) because the lower bound becomes positive within a few iterations from the starting point $\theta^{(1)} = 0$.

After allocating the weight to the parameter of day s by the central agent, the local agents determine the green time proportion of each intersection based on the permit prices on day s. Thus, the behavior of each local agent described as the condition (3.44) coincides with the proposed signal control policy described in Subsection 3.3.3: *Less profitable phases receive no green time*.

3.4.3 Convergence of the evolutionary implementation method

In the implementation method, the optimal solution of the original problem [Gossp] is achieved when the lower bound $\theta^{(s)}$ coincides with the transportation cost $TTC(\mathbf{g}^{(s)})$. This is because the traffic flow pattern $(\mathbf{q}^{(s)}, \mathbf{y}^{(s)})$ is a feasible solution of the original problem and is not equal to the lower bound, except for the optimal solution. On the other hand, as we showed in Subsection 3.4.1, the restricted master problem coincides with the problem [Gossp] when

all extreme points are known, which implies that the restricted master problem is sure to produce the optimal signal setting by the time all extreme points are generated. Furthermore, a new extreme point is always generated in each Step 1 before the procedure terminates. These statements suggest the following convergence result of the day-to-day traffic flow dynamics under the implementation method.

Proposition 3.2 *For any networks with many-to-many OD pairs in which the problem [TAP-P] has feasible solutions, the day-to-day traffic flow dynamics under the implementation method globally converges to the optimal traffic flow pattern in a finite number of steps.*

Proof See Appendix 3.B for the proof.

3.5 Conclusions

In this chapter, we proposed a distributed signal control policy based on a tradable network permits system. The main feature of the proposed policy is the determination of a green time proportion for each intersection by exploiting only local information. We proved that the equilibrium traffic assignment under the tradable network permits consistent with the proposed signal control policy coincides with the system optimal traffic flow pattern that minimizes the total transportation cost. Finally, we constructed an evolutionary implementation method for the proposed policy and proved that the day-to-day traffic flow dynamics under the scheme converges to the system optimal traffic assignment.

We showed the implementation method of the proposed signal control policy (supply side conditions). However, we should note here that (demand side) traffic equilibrium under the tradable network permits system is assumed to be achieved. Therefore, the next chapter constructs an implementation mechanism that can attain the equilibrium.

While this chapter only dealt with the green time proportion as a signal control parameter, an interesting direction for future research is the problem that includes other control parameters (e.g., number of phases, cycle length, and offsets) for the determination of a detailed signal setting within each time period. Since queuing congestion is eliminated under our framework, we can easily deal with detailed signal settings.

Appendix 3.A Proof of Proposition 1

We will show that a necessary and sufficient condition for the optimality of the optimization problem [Gossp] coincides with the equilibrium conditions (3.11), (3.12), and (3.13), and the proposed signal control policy (3.25). To derive the optimality conditions, we first define the Lagrangean function $\mathcal{L}$ for the problem [Gossp]:

$$\begin{aligned}\mathcal{L} \equiv TTC &+ \sum_{od\in W} \rho_{od}\left\{Q_{od} - \sum_{t\in T} q_{od}(t)\right\} \\ &+ \sum_{t\in T}\sum_{o\in O}\sum_{k\in N} \pi_k^o(t)\left\{q_{od}(t)\delta_{kd} + \sum_{l\in NO(k)} y_{kl}^o(t) - \sum_{l\in NI(k)} y_{lk}^o(t)\right\} \\ &+ \sum_{t\in T}\sum_{kl\in A} p_{kl}(t)\left\{\sum_{o\in O} y_{kl}^o(t) - \sum_{e:kl\in A_{e,j}(t)} g_{e,j}(t)\mu_{kl}\right\} + \sum_{t\in T}\sum_{j\in J}\phi_j(t)\left\{\sum_{e\in E_j(t)} g_{e,j}(t) + l_k - 1\right\}\end{aligned} \tag{3.45}$$

where the TTC is the objective function of [Gossp] defined in (3.4); the variables $\{y_{kl}(t)\}$ are eliminated by substituting the constraint (3.9) into the objective function and the constraint (3.7); the variables $\{G_{kl}(t)\}$ are also eliminated by substituting the constraint (3.10) into the constraint (3.8); and $\boldsymbol{\rho}$, $\boldsymbol{\pi}$, $\mathbf{p}$, and $\boldsymbol{\phi}$ are Lagrange multipliers corresponding to the constraint (3.5), (3.6), (3.7), and (3.8), respectively. Then, the necessary and sufficient conditions for the optimality of [Gossp] are given by the following Kuhn-Tucker conditions:

$$\begin{cases}\partial\mathcal{L}/\partial q_{od}^*(t) = 0 & \text{if} \quad q_{od}^*(t) > 0 \\ \partial\mathcal{L}/\partial q_{od}^*(t) \geq 0 & \text{if} \quad q_{od}^*(t) = 0\end{cases} \qquad \forall od \in W,\ \forall t \in T \tag{3.46}$$

$$\begin{cases}\partial\mathcal{L}/\partial y_{kl}^{o*}(t) = 0 & \text{if} \quad y_{kl}^{o*}(t) > 0 \\ \partial\mathcal{L}/\partial y_{kl}^{o*}(t) \geq 0 & \text{if} \quad y_{kl}^{o*}(t) = 0\end{cases} \qquad \forall kl \in A,\ \forall o \in O,\ \forall t \in T \tag{3.47}$$

$$\begin{cases}\partial\mathcal{L}/\partial g_{e,j}^*(t) = 0 & \text{if} \quad g_{e,j}^*(t) > 0 \\ \partial\mathcal{L}/\partial g_{e,j}^*(t) \geq 0 & \text{if} \quad g_{e,j}^*(t) = 0\end{cases} \qquad \forall e \in E_j(t),\ \forall j \in J,\ \forall t \in T \tag{3.48}$$

$$\partial\mathcal{L}/\partial \rho_{od}^* = 0 \qquad \forall od \in W \tag{3.49}$$

$$\partial\mathcal{L}/\partial \pi_k^{o*}(t) = 0 \qquad \forall k \in N,\ \forall o \in O,\ \forall t \in T \tag{3.50}$$

$$\begin{cases}\partial\mathcal{L}/\partial p_{kl}^*(t) = 0 & \text{if} \quad p_{kl}^*(t) > 0 \\ \partial\mathcal{L}/\partial p_{kl}^*(t) \leq 0 & \text{if} \quad p_{kl}^*(t) = 0\end{cases} \qquad \forall kl \in A,\ \forall t \in T. \tag{3.51}$$

It can be easily seen that conditions (3.49), (3.50), and (3.51) reduce to the physical conditions (3.15), (3.16), and the demand-supply equilibrium condition (3.13), respectively. To examine conditions (3.46), (3.47), and (3.48), we calculate the partial derivatives of the Lagrangean function:

$$\partial\mathcal{L}/\partial q^*_{od}(t) = s_d(t) + \pi^o_d(t) - \rho_{od} \tag{3.52}$$

$$\partial\mathcal{L}/\partial y^{o*}_{kl}(t) = \alpha(t_{kl} + d_{kl}) + p_{kl}(t) + \pi^o_k(t) - \pi^o_l(t) \tag{3.53}$$

$$\partial\mathcal{L}/\partial g^*_{e,j}(t) = \sum_{kl\in A_{e,j}(t)} p_{kl}(t)\mu_{kl} - \phi_j(t) \tag{3.54}$$

Substituting (3.52) into (3.46), we have the same form of conditions as in equilibrium condition (3.12); similarly, we see that (3.47) and (3.48) reduce to the equilibrium condition (3.11) and the proposed signal control policy (3.25), respectively. Thus, the Lagrange multipliers $\boldsymbol{\rho}^*$, $\boldsymbol{\pi}^*$, $\mathbf{p}^*$, and $\boldsymbol{\phi}^*$ in the optimality conditions (3.46)–(3.51) coincide with the equilibrium permit prices, the equilibrium minimum path costs, equilibrium generalized transportation costs in equilibrium conditions (3.11)–(3.13), and the Lagrange multiplier corresponding to the constraint (3.23); the optimal flow patterns $(\mathbf{q}^*, \mathbf{y}^*)$ and the optimal signal setting $\mathbf{g}^*$ also coincide with the equilibrium consistent with the proposed signal control policy.

Appendix 3.B Proof of Proposition 2

We will show that a new extreme point is always generated in every **Step 1** until the convergence criterion is satisfied. We denote the green time proportions on day s by $\mathbf{g}^{(s)}$, the lower bound by $\theta^{(s)}$, and the extreme point of the day by $(\boldsymbol{\rho}, \mathbf{p}) \in V^{(s)}(\Omega)$ bounded by condition (3.42). Then

$$\theta^{(s)} = \sum_{od\in W} \rho_{od}Q_{od} - \sum_{t\in T}\sum_{kl\in A} p_{kl}(t)G^{(s)}_{kl}(t)\mu_{kl} \tag{3.55}$$

holds. In contrast, for another point $(\boldsymbol{\rho}', \mathbf{p}')$ in the set $V^{(s)}(\Omega)$,

$$\sum_{od\in W} \rho_{od}Q_{od} - \sum_{t\in T}\sum_{kl\in A} p_{kl}(t)G^{(s)}_{kl}(t)\mu_{kl} \geq \sum_{od\in W} \rho'_{od}Q_{od} - \sum_{t\in T}\sum_{kl\in A} p'_{kl}(t)G^{(s)}_{kl}(t)\mu_{kl} \tag{3.56}$$

is satisfied by condition (3.42). From the duality theorem, the optimal value of the objective function of [TAP-P] coincides with the optimal value of the objective function of [TAP-D]

for the parameter $\mathbf{g}^{(s)}$, that is,

$$TTC(\mathbf{g}^{(s)}) = \sum_{od\in W} \rho_{od}^{(s)} Q_{od} - \sum_{t\in T}\sum_{kl\in A} p_{kl}^{(s)}(t) G_{kl}^{(s)}(t)\mu_{kl} \tag{3.57}$$
$$= \sum_{t\in T}\sum_{o\in O}\sum_{d\in D} q_{od}^{(s)} s_d(t) + \alpha \sum_{t\in T}\sum_{kl\in A} y_{kl}^{(s)}(t)[t_{kl} + d_{kl}]$$

where $(\boldsymbol{\rho}^{(s)}, \mathbf{p}^{(s)}, \mathbf{q}^{(s)}, \mathbf{y}^{(s)})$ is the optimal solution of [TAP-P] and [TAP-D], which represents the equilibrium state under the signal setting $\mathbf{g}^{(s)}$. In addition, since $(\mathbf{q}^{(s)}, \mathbf{y}^{(s)})$ is a feasible solution of the original problem [Gossp],

$$\sum_{t\in T}\sum_{o\in O}\sum_{d\in D} q_{od}^{(s)} s_d(t) + \alpha \sum_{t\in T}\sum_{kl\in A} y_{kl}^{(s)}(t)[t_{kl} + d_{kl}] \geq TTC^* \tag{3.58}$$

holds. TTC^* represents the optimal value of the objective function of the problem [Gossp]. From the above discussion, the following relationship is satisfied:

$$\sum_{od\in W} \rho_{od}^{(s)} Q_{od} - \sum_{t\in T}\sum_{kl\in A} p_{kl}^{(s)}(t) G_{kl}^{(s)}(t)\mu_{kl} \tag{3.59}$$
$$= \sum_{t\in T}\sum_{o\in O}\sum_{d\in D} q_{od}^{(s)} s_d(t) + \alpha \sum_{t\in T}\sum_{kl\in A} y_{kl}^{(s)}(t)[t_{kl} + d_{kl}]$$
$$\geq TTC^*$$
$$\geq \theta^{(s)}$$
$$= \sum_{od\in W} \rho_{od} Q_{od} - \sum_{t\in T}\sum_{kl\in A} p_{kl}(t) G_{kl}^{(s)}(t)\mu_{kl}.$$

Hence, $(\boldsymbol{\rho}^{(s)}, \mathbf{p}^{(s)}) \neq (\boldsymbol{\rho}, \mathbf{p})$ is achieved when $\theta^{(s)} < TTC(\mathbf{g}^{(s)})$, i.e., a new extreme point is generated until the convergence criterion is satisfied. Since the number of extreme points is finite, we can conclude that the day-to-day traffic flow dynamics converges to the optimal traffic flow pattern in a finite number of steps.

Chapter 4

A hybrid implementation mechanism of tradable network permits system which obviates path enumeration

This chapter[1] designs an auction mechanism for implementing the tradable network permit markets on general networks. An important factor that affects the success of such a market mechanism is incentive of an individual market participant. More specifically, it is well known that a well-designed market mechanism will encourage a competition and increase efficiency, and otherwise the poor efficiency may arise due to participants' strategic manipulations (e.g., McMillan, 2002). Hence, we have to design trading rules in which no participant has incentive to manipulate the markets with careful consideration of the possibilities of the manipulations.

To accomplish this, the present chapter conducts a game-theoretic analysis of a dynamic traffic assignment with *atomic* users, while the previous chapters consider continuous flows. A difficulty of treating atomic users on general networks is that a naive formulation of a dynamic system optimal allocation of network permits leads to a NP-hard problem owing to the complex relationship between link and path. As a result, it is almost impossible to apply the Vickrey-Clarke-Groves (VCG) mechanism, which is a benchmark in auction theory (Milgrom, 2004; Cramton et al., 2006).

[1] This chapter is based on joint research with Takashi Akamatsu, is accepted in *the 20th International Symposium on Transportation and Traffic Theory* (Wada and Akamatsu, 2013). A preliminary version published in *Journal of Japan Society of Civil Engineers, Ser. D3 (Infrastructure Planning and Management)* (Wada and Akamatsu, 2011).

To avoid such computational infeasibility, we develop a hybrid implementation mechanism that consistently combines an auction mechanism with a path capacity control, which are repeated on a day-to-day basis. The former phase involves selling of bundles of permits corresponding to the paths, and the latter phase involves adjustment of the number of bundles, which corresponds to the path capacity. We prove that the proposed mechanism has two desirable properties: (1) truthful bidding is a dominant strategy for each user on each day, and (2) the permit allocation pattern under the mechanism converges to an approximate dynamic system optimal allocation pattern in the sense that the achieved social surplus reaches its maximum value when the number of users is large. Furthermore, we show that the proposed mechanism can be extended to obviate path enumeration by introducing a column generation procedure.

This chapter is organized as follows. Section 4.1 discusses related works. Section 4.2 describes pre-conditions used through the chapter. Section 4.3 defines a dynamic system optimal allocation of network permits and discusses the impossibility of employing the VCG mechanism. Section 4.4 presents ideas of a novel auction mechanism that is readily implementable for general networks. Section 4.5 shows details of the proposed mechanism and clarify its properties. Section 4.6 constructs an extended mechanism which obviates path enumeration. Section 4.7 demonstrates convergence properties of the proposed mechanism by a numerical example. Section 4.8 concludes the chapter.

4.1 Related works

this chapter is mainly concerned with dynamic traffic assignments (DTA), some types of transportation demand management (TDM) schemes (i.e., dynamic congestion pricing schemes and tradable permits schemes) and combinatorial auctions. The first two areas provide an analytical framework for modeling and managing traffic congestion in transportation networks, whereas the third area provides a foundation for constructing an auction mechanism to implement trading markets. In particular, auctions for bundled items with network structure are relevant to our study.

Dynamic traffic assignment models

Due to the successful incorporation of queuing phenomena into transportation network analysis, there has been much research into DTA models (e.g., Vickrey, 1969; Kuwahara and Akamatsu, 1993; Cascetta, 2001). For instance, departure time choice models have been developed by Smith (1984b), Daganzo (1985), Newell (1987), and Iryo and Yoshii (2007), while dynamic user equilibrium (DUE) models have been developed by Kuwahara and Akamatsu (1993), Smith (1993), Heydecker and Addison (1996), Akamatsu (2001), and Iryo (2011) and many others (see Peeta and Ziliaskopoulos, 2001; Szeto and Wong, 2011, for comprehensive reviews). These studies analyzed the properties of user equilibrium and discussed the effectiveness of dynamic congestion pricing as shown in the next subsection. However, few studies have discussed the asymmetric information problem and the effectiveness of quantity-based regulation for eliminating queues.

Dynamic congestion pricing schemes

Dynamic congestion pricing is a natural extension of the static congestion pricing and is a benchmark TDM scheme to eliminate queuing congestion. Despite its importance, most studies have been limited to simple networks (e.g., a single bottleneck) because analyzing DTA models for more general networks is usually intractable (e.g., Arnott et al., 1990, 1993; Kuwahara, 2007; Doan et al., 2011). However, there have been some attempts to overcome this difficulty. For example, Ziliaskopoulos (2000) and Nie (2011) studied dynamic marginal cost analyses for system optimal DTA problems with many-to-one (or one-two many) OD pairs; Yang and Meng (1998) derived an optimal toll based on a time-space network for general networks; Friesz et al. (2007) formulated a dynamic second-best toll pricing problem for general networks as mathematical programs with equilibrium constraints and developed a solution algorithm, but they did not address theoretical questions (e.g., algorithm convergence). In effect, no study has established a theory of dynamic congestion pricing for general networks in which queues arise. Furthermore, implementations of the abovementioned schemes unsurprisingly face the difficulty associated with asymmetric information.

To address the asymmetric information problem, some studies have developed evolutionary (trial-and-error) implementation methods for congestion pricing in static settings (Sandholm, 2002, 2007; Yang et al., 2004; Han and Yang, 2009). These methods set toll

levels based on *realized* traffic flow patterns. The studies then demonstrated that an appropriate adjustment process of route choice (e.g., Smith, 1984a) converges to an equilibrium that minimizes the total transportation cost in the network[2]. This result relies on the fact that there is an equivalent optimization problem (or a Beckmann-type potential function) for a static user equilibrium. However, the properties of static and dynamic congestion pricing are different since the mechanisms of flow and queuing congestion are totally different. The DUE model cannot also be reduced to an optimization problem in general. Thus, it is not easy to generalize the methods to dynamic settings. Further, the methods need to set a discriminatory toll to achieve an optimal state when users have heterogeneous costs (e.g., value of time), but information on such heterogeneities cannot be gathered by these methods, which means that this approach is not a panacea for the problem even in static settings.

Tradable permits schemes for managing traffic congestion

A tradable permits scheme that combines a quantity-based regulation and a market institution has been studied for environmental protection (Montgomery, 1972; Tietenberg, 1980). The capabilities and applicability of this scheme have been increasing, because the emergence of the Internet enables a new market to be established inexpensively. For managing traffic congestion, a few researchers have studied such a scheme as an alternative to congestion pricing. Verhoef et al. (1997) discussed the possibilities of using tradable permits in the various types of regulations for road transport externalities; e.g., vehicle ownership permits, tradable parking permits, and tradable permits in the regulation of road usage. Teodorović et al. (2008) proposed an auction-based congestion pricing, for which drivers who want to enter a downtown area have to participate a downtown time slot auction. Although it formulated the allocation problem for the time slots, their study did not address how to set their prices, which is the core problem of auction mechanisms. Moreover, the existing studies provide some useful insights into tradable permit schemes for managing traffic congestion, but none describes *time-dependent* tradable permits for eliminating *bottleneck congestion*.

In addition, it is worth mentioning the *tradable travel credit scheme* proposed by Yang

[2] Yang et al. (2004) and Han and Yang (2009) did not explicitly consider an adjustment process unlike Sandholm (2002, 2007). Instead, they assumed that user equilibrium traffic flow patterns are realized for any given temporal link toll patterns, which may imply that it takes a time to obtain each equilibrium by the adjustment process.

and Wang (2011), which is superficially similar to but fundamentally different from the tradable network permits scheme[3]. Basically, under the tradable travel credit scheme, the road manager initially distributes credits to all eligible travelers and predetermines a link-specific credit charge. Credits are freely tradable among the credit holders in a market. Yang and Wang (2011) showed that, if the manager can appropriately set the total number of credits and the link-specific credit charges, a desirable traffic flow pattern is achieved. However, it is apparent that this scheme requires detailed demand information unlike the tradable network permits[4]. Further, it is fair to say that this scheme is not be a quantity-based regulation for managing congestion but rather a redistribution scheme for income. Indeed, the main advantage of this scheme over the standard congestion pricing is the improvement in equity and social acceptability, not a direct reduction in traffic congestion.

Auction mechanisms for networked items

Since the pioneering work of Rassenti et al. (1982), who proposed airport time slot auctions, there has been a considerable amount of work on combinatorial auctions (e.g., de Vries and Vohra, 2003; Cramton et al., 2006), which allow bids on combinations of items and thus enhance the economic efficiency when bidders have preferences for sets of items (e.g., spectrum rights, airport time slots, railroad segments, and paths in networks). The most celebrated such auction is the VCG mechanism (Vickrey, 1961; Clarke, 1971; Groves, 1973). This mechanism is strategy-proof and can achieve allocative efficiency. However, to maintain these properties, it requires the auctioneer to solve complex combinatorial optimization problems to determine the allocation and prices (Vickrey payments). Therefore, the VCG mechanism is computationally intractable in many circumstances, including ours (see Section 4.3).

In this regard, several authors have showed that such a intractability can be avoided under some restricted circumstances in which combinations of items have network structures.

[3] Similar schemes of the tradable travel credit were also discussed in Viegas (2001) and Verhoef et al. (1997).

[4] Nie (2012) pointed out this fact in the context of comparison with tradable permits for emission control: "*Suffice it to say here that the information that the government would need to run a mobility credit market is as much as the information required to operate a conventional pricing scheme. Therefore, the mobility credit market does not reduce the administrative burden of the government, unlike in the case of emission control.*"

Bikhchandani et al. (2002) demonstrated that the VCG outcome can be computed by solving two linear programs in the case that a winner determination problem reduces to a spanning tree problem or a shortest path problem[5]. Nisan and Ronen (2001) derived the Vickrey payments for a shortest path problem, and Hershberger and Suri (2001) developed an efficient algorithm to compute those payments. However, the auctions cannot be implemented for trading markets because these are reverse auctions that cannot handle multiple buyers (i.e., users).

The studies on bandwidth auctions for communication networks are also related to our study in the sense that they also focus on an allocation problem for a network capacity that is a limited resource (e.g., Koutsopoulos and Iosifidis, 2010). The studies consider the case in which each bidder (e.g., provider) purchases a quantity of bandwidth over a path in a network. Lazar and Semret (1999) proposed the "progressive second price auction" for allocating a divisible quantity of bandwidth over a certain path. Dramitinos et al. (2007) proposed a multi-unit Dutch auction, which allocates an indivisible quantity of bandwidth over a certain path. Both of these auction mechanisms can induce truth-telling. However, in contrast to the mechanism that is proposed in this chapter, neither takes into account the route choice problem of the bidders (i.e., each bidder is interested in a single fixed path). From the above discussion, we conclude that there is no network auction mechanism that enables us to assign network capacities (i.e., network permits) to multiple users who choose a route in a network, and thus, the proposed mechanism is a major contribution of this chapter.

4.2 Model

4.2.1 Networks

In this chapter, we consider discrete-time dynamic traffic flows on a general network. The network consists of a set N of nodes and a set A of directed links. The node set N includes a subset O of origin nodes from which users start their trips, and a subset D of destination nodes at which users terminate their trips. A set of origin-destination (OD) pairs is denoted by W. Each element of A (i.e., each link) is identified by a sequential natural number a.

The time interval $[0, I]$ for which we assign the dynamic traffic flow is fixed. We assume

[5] Bikhchandani et al. (2002) also dealt with more general cases.

that each OD pair's potential travel demand Q_{od} in the time interval $[0, I]$ is a given constant. The time interval $[0, I]$ is discretized into small intervals of length Δt: each time point is represented by $t = m\Delta t$, where $m = 0, 1, 2, \ldots, M$. Each time interval $[t, t + \Delta t]$ is denoted by $t \in T$ and we call this interval *time period* t.

We also assume, without any loss of generality, that each link in a network consists of a free-flow segment and a single bottleneck segment. The travel time to pass through the free-flow segment of link a is a constant t_a. We then assume that travel time t_a is represented by a natural multiplier of Δt (i.e., an integer n_a satisfies $t_a = n_a \Delta t$). The bottleneck of each link is represented by a *point queue model* with constant capacity μ_a = vehicles/time interval Δt.

4.2.2 Road network manager and users

A road manager aims to restrain traffic congestion in the network and maximize the social surplus. To achieve this, the manager regulates the traffic flow rates entering each bottleneck in the network using time-dependent network permits. The precise definition and setup of the network permit system are described in Subsection 4.2.3.

Within the time interval $[0, I]$, each atomic user $i \in \mathcal{N}_{od}$ (i.e., $|\mathcal{N}_{od}| = Q_{od}$) makes *at most* a single trip in the network from an origin (e.g., residential zone) to a destination (e.g., the central business district). This means that all users do not necessarily make trips, which corresponds to the conventional traffic assignments with elastic demand (see also Subsection 4.5.1). The user chooses a destination arrival time period and a path between the origin and destination so as to maximize his or her utility. Under the system of network permits, each user must purchase a bundle of permits corresponding to a set of links included in the user's chosen path. This implies that choosing a destination arrival time period and a path directly corresponds to purchasing time-dependent network permits in the trading markets.

4.2.3 Network permits and trading markets

In this chapter, we assume that the manager can issue time-dependent network permits for all bottlenecks (i.e., links) in the network. We also assume that the number of permits issued for each link in each time period is equal to or less than the traffic capacity of each link in the network. This means that queuing congestion never occurs in the network under this

permit-issue scheme.

The permits issued for each link (bottleneck) are put on sale by the road manager. Each user who would like to use a path must purchase a bundle of permits corresponding to a set of links included in the user's preferred path. In the trading markets, prices and the allocation of time-dependent permits are determined through an auction mechanism. The detailed trading rules are given in Section 4.5.

It must be admitted that the procedures for trading network permits seem unrealistic at first glance, but implementation of these would become feasible with futuristic vehicles in which an agent software is installed to manage driving, navigation and safety. From this perspective, the mechanism proposed in this chapter can be viewed as the protocol of a multi-agent system in which the agent software executes the procedures for trading network permits on behalf of users.

4.2.4 Dynamic travel costs and user utility in general networks

The transportation cost for a single trip made by a network user consists of "schedule cost" and "travel cost." The schedule cost for user i is the cost due to the difference between the user's desired arrival time period t_i and the actual arrival time period t. The schedule cost is represented by a function $s_i(t, t_i)$ of both destination arrival time and desired arrival time. The travel cost is the monetary equivalent of the travel time for a trip from the origin to the destination. The travel times differ among the paths. The travel time of a path between the OD pair is defined as the sum of travel times of the links included in the path. Note that the travel time of each link a is a constant t_a under the permit system since there is no queuing. Hence, the travel time T_r for path $r \in R_{od}$ between the OD pair is also constant:

$$T_r = \sum_{a \in A} t_a \delta_{a,r(o,d)}, \tag{4.1}$$

where $\delta_{a,r(o,d)}$ is a typical element of the path-link incidence matrix for the node pair (o, d); it is 1 if link a is on path r connecting the OD pair (o, d) and zero otherwise.

We suppose that each user has a private valuation $v_{i,r}(t)$ for each path r and each destination arrival time period t. This valuation $v_{i,r}(t)$ represents a nonnegative value of trip between OD pair along path r in time period t. For example, to show a correspondence with

conventional traffic assignments, we can specify the valuation as

$$v_{i,r}(t) \equiv w_i - (s_i(t, t_i) + \alpha_i T_r)\,, \tag{4.2}$$

where w_i is a parameter, which is interpreted as the trip utility (or willingness-to-pay) between the OD pair, and α_i is a coefficient that converts travel time into a monetary equivalent.

Each user is assumed to have a quasi-linear utility function (we use the term "payoff" interchangeably with "utility"). Specifically, each user's utility $u_{i,r}(t)$ for path r in time period t is represented as the difference between private valuation and the "permit purchase cost $P_r(t)$" determined in an auction:

$$u_{i,r}(t) \equiv v_{i,r}(t) - P_r(t). \tag{4.3}$$

The permit purchase cost is the total payment for purchasing the bundle of link permits required for traveling along a path and arriving at the destination in a certain time period.

4.3 Dynamic system optimal allocation of network permits

The objective of an auction mechanism, such as that designed in this chapter, is to achieve a network permit allocation pattern that maximizes a social surplus (i.e., dynamic system optimal allocation). The social surplus is defined as the sum of user's valuations. This excludes user payments to the road manager to purchase permits because these payments are simply income transfers between the users and the road manager. Thus, we formulate an optimization problem [DSO] of providing the dynamic system optimal allocation of network permits:

$$\max_{(\mathbf{f},\mathbf{y})} .\ SS(\mathbf{f}) \equiv \sum_{od\in W}\sum_{i\in\mathcal{N}_{od}}\sum_{t\in T}\sum_{r\in R_{od}} v_{i,r}(t) f_{i,r}(t) \tag{4.4}$$

subject to

$$\sum_{t\in T}\sum_{r\in R_{od}} f_{i,r}(t) \le 1 \qquad \forall i \in \mathcal{N}_{od},\ \forall od \in W \tag{4.5}$$

$$\sum_{od\in W}\sum_{i\in\mathcal{N}_{od}} y_{i,a}(t) \le \mu_a \qquad \forall a \in A,\ \forall t \in T \tag{4.6}$$

$$y_{i,a}(t) = \sum_{r\in R_{od}} f_{i,r}(t + T_{a,r})\delta_{a,r(o,d)} \qquad \forall a \in A,\ \forall t \in T,\ \forall i \in \mathcal{N}_{od}\ \forall od \in W \tag{4.7}$$

$$f_{i,r}(t), y_{i,a}(t) \in \{0, 1\} \qquad \forall a \in A,\ \forall r \in R_{od},\ \forall t \in T,\ \forall i \in \mathcal{N}_{od},\ \forall od \in W, \tag{4.8}$$

where $f_{i,r}(t)$ denotes the allocation of a bundle of permits to user i and $y_{i,a}(t)$ denotes the allocation of a network permit to user i. Specifically, $f_{i,r}(t)$ is 1 if user i is allocated a bundle of permits for a set of links required to travel along path r and to arrive in time period t and is zero otherwise. Hence, $y_{i,a}(t)$ is 1 if user i is allocated a network permit for link a in time period t and is zero otherwise.

The combinatorial optimization problem of finding an efficient network permit allocation pattern $(\mathbf{f}^*, \mathbf{y}^*)$, subject to the physical constraints on flows representing the network performance. The first constraint (4.5) is the condition that each user makes at most one trip in the interval $[0, I]$. The second constraint (4.6) is the capacity constraint on each link. The third constraint (4.7) expresses the flow conservation between link flows and path flows for each user; that is, the link flow $y_{i,a}(t)$ entering into link a in time period t is the sum of the flows on all paths going through that link and arriving at the destination at time $t + T_{a,r}$. The travel time required for arriving at the destination from the upstream node k (of the link a) through path r (containing link a) is given by:

$$T_{a,r} = \sum_{a' \in A} t_{a'} \delta_{a',r(k,d)}, \tag{4.9}$$

where $\delta_{a',r(k,d)}$ is a typical element of the path-link incidence matrix for node pair (k, d).

Although the road manager seeks to solve the problem [DSO] to achieve the system optimal permit allocation pattern, solving the problem directly poses two major difficulties: (i) the objective function of the problem includes users' private valuations, and (ii) the problem is NP-hard (i.e., no polynomial-time algorithm exists for it). The first difficulty comes from the obvious fact that the manager cannot accurately obtain such private information. The second difficulty comes from the fact that the problem [DSO] is an integer multicommodity flow problem.

One possible way to address these difficulties might be to apply conventional combinatorial auctions to this problem. For example, the VCG mechanism can overcome the first difficulty, at least in principle, because it gives users an incentive (Vickrey payment) to report their valuations truthfully (i.e., strategy-proofness). However, the VCG mechanism cannot overcome the second difficulty because the above mentioned problem [DSO] must be solved *exactly* to determine the optimal permit allocation and to compute the Vickrey payments (i.e., it is computationally infeasible). One natural approach to handling the problem is to seek a sub-optimal solution instead of the optimal solution. However, the VCG mechanism allow-

ing nonoptimal allocations is not strategy-proof, as each user has an incentive to bid false valuations to increase one's own utility (Nisan and Ronen, 2007). Therefore, it is difficult to apply the VCG mechanism directly to the trading markets.

4.4 Day-to-day auction mechanism: an auction mechanism with day-to-day capacity control

In this section, we propose a novel auction mechanism including a day-to-day capacity control, which is readily implementable for general networks. We call this mechanism the *day-to-day auction mechanism*. To avoid computational infeasibility such as that in the case of the VCG mechanism, the proposed mechanism employs an evolutionary approach. Although the evolutionary approach cannot be employed for the one-shot auctions that are typically treated in auction theory, it can be utilized for a tradable network permits scheme in which the auction is opened to morning commuters each day.

Before describing the proposed mechanism, we introduce some modifications of the model. In the proposed mechanism, we consider time-dependent permit allocation patterns and their day-to-day dynamics. We then denote the day by $s \in S$. Suppose that each user behaves myopically and makes one's own choice so as to maximize the following utility defined for each day s:

$$u_{i,r}^{(s)}(t) \equiv v_{i,r}(t) - P_r^{(s)}(t). \tag{4.10}$$

This implies that the user considers only his or her allocation of the bundles and payment on each day, so the user's true valuations are constant for all days.

4.4.1 Reformulation of the DSO problem with path capacities and the Benders decomposition principle

The day-to-day auction mechanism is based on the idea of reformulating the problem [DSO] by introducing *non-individual* variables and then applying the Benders decomposition principle (see Appendix A for a basic framework) to obtain two problems, a master problem and a sub-problem. We then solve these problems on day-to-day basis. Further, in order to

obtain an efficient permit allocation with imperfect information about users, the mechanism also exploits an auction mechanism to solve the sub-problem.

We let $F_r(t), Y_a(t) \in \mathbb{Z}_+$ denote a non-individual path variable and a non-individual link variable, respectively. By using these variables, the problem [DSO] with non-individual variables is formulated as

$$\max_{(\mathbf{f},\mathbf{F},\mathbf{Y})} . \; SS(\mathbf{f},\mathbf{F}) \equiv \sum_{od\in W}\sum_{i\in\mathcal{N}_{od}}\sum_{t\in T}\sum_{r\in R_{od}} v_{i,r}(t) f_{i,r}(t) \tag{4.11}$$

subject to

$$\sum_{t\in T}\sum_{r\in R_{od}} f_{i,r}(t) \le 1 \qquad \forall i \in \mathcal{N}_{od},\ \forall od \in W \tag{4.12}$$

$$\sum_{i\in\mathcal{N}_{od}} f_{i,r}(t) \le F_r(t) \qquad \forall r \in R_{od},\ \forall t \in T,\ \forall od \in W \tag{4.13}$$

$$Y_a(t) \le \mu_a \qquad \forall a \in A,\ \forall t \in T \tag{4.14}$$

$$Y_a(t) = \sum_{od\in W}\sum_{r\in R_{od}} F_r(t+T_{a,r})\delta_{a,r(o,d)} \qquad \forall a \in A,\ \forall t \in T \tag{4.15}$$

$$f_{i,r}(t) \in \{0,1\},\ F_r(t), Y_a(t) \in \mathbb{Z}_+ \qquad \forall a \in A,\ \forall r \in R_{od},\ \forall t \in T,\ \forall i \in \mathcal{N}_{od},\ \forall od \in W. \tag{4.16}$$

Each non-individual path variable $F_r(t)$ in Eq.(4.13) is interpreted as a *path capacity* that is the number of bundles of permits sold for the path. Constraint (4.12) is the condition that each user makes at most one trip. Constraint (4.13) is the path capacity constraint on each path. Constraints (4.14) and (4.15) are the conditions that the path capacity satisfies constraints stemming from link capacities.

This problem includes two types of variables, individual variables $\mathbf{f}$ and non-individual variables $(\mathbf{F},\mathbf{Y})$, and is naturally becomes a bi-level problem based on Benders decomposition principle:

$$\max_{(\mathbf{F},\mathbf{Y})} . \sum_{od\in W}\sum_{i\in\mathcal{N}_{od}}\sum_{t\in T}\sum_{r\in R_{od}} v_{i,r}(t) f_{i,r}(\mathbf{F}(t)) \tag{4.17}$$

subject to Eq. (4.14), Eq. (4.15), and $F_r(t), Y_a(t) \in \mathbb{Z}_+$,

where $\mathbf{f}(\mathbf{F})$ is an optimal solution of the following problem for a parameter $\mathbf{F}$:

$$\max_{\mathbf{f}\ge\mathbf{0}} . \sum_{od\in W}\sum_{i\in\mathcal{N}_{od}}\sum_{t\in T}\sum_{r\in R_{od}} v_{i,r}(t) f_{i,r}(t) \tag{4.18}$$

subject to Eq. (4.12) and Eq. (4.13),

The upper level problem (master problem) determines the optimal path capacity that maximizes the social surplus. The lower level problem (sub-problem) determines the efficient allocation of bundles of permits under the condition that each path capacity is fixed. Note that the sub-problem reduces to independent sub-problems in terms of OD pairs because path capacities differ among OD pairs. Furthermore, the sub-problem (4.18) is the *Hitchcock transportation problem* and so a linear relaxation of the sub-problem satisfies *total unimodularity* (e.g., Papadimitriou and Steiglitz (1982)). Thus, we can obtain an integer solution by solving a linear relaxation of the sub-problem because the path capacities are integer valued.

To demonstrate a clear relationship between the master problem and the sub-problem, we consider the following dual problem of the sub-problem:

$$Z(\mathbf{F}) \equiv \min_{(\boldsymbol{\pi},\mathbf{P})\geq \mathbf{0}} . \sum_{od\in W}\sum_{i\in \mathcal{N}_{od}} \pi_i + \sum_{od\in W}\sum_{t\in T}\sum_{r\in R_{od}} F_r(t)P_r(t) \tag{4.19}$$

subject to

$$\pi_i \geq v_{i,r}(t) - P_r(t) \qquad \forall r \in R_{od},\ \forall t \in T,\ \forall i \in \mathcal{N}_{od},\ \forall od \in W \tag{4.20}$$

where $(\boldsymbol{\pi}, \mathbf{P})$ are Lagrange multipliers for constraints (4.12) and (4.13). As shown in 4.5.1, these Lagrange multipliers equal to the user payoffs and competitive equilibrium bundle prices that are realized in an auction as shown in Subsection (we call these variables *demand information*). From the duality theorem, the optimal value of the objective function (4.19) coincides with the optimal value of the objective function (4.18); that is,

$$Z(\mathbf{F}) = \sum_{od\in W}\sum_{i\in \mathcal{N}_{od}} \pi_i(\mathbf{F}) + \sum_{od\in W}\sum_{t\in T}\sum_{r\in R_{od}} F_r(t)P_r(\mathbf{F}(t)) = \sum_{od\in W}\sum_{i\in \mathcal{N}_{od}}\sum_{t\in T}\sum_{r\in R_{od}} v_{i,r}(t)f_{i,r}(\mathbf{F}(t)), \tag{4.21}$$

where $(\boldsymbol{\pi}(\mathbf{F}), \mathbf{P}(\mathbf{F}))$ is an optimal solution of the dual problem (4.19) for a parameter $\mathbf{F}$. Hence, $(\boldsymbol{\pi}(\mathbf{F}), \mathbf{P}(\mathbf{F}))$ is an extreme point of the convex feasible region Ω_{SD} that consists of the constraints (4.20) and non-negative constraints. By using the function (4.21), we can transform the master problem into the following problem:

$$\max_{(\mathbf{F},\mathbf{Y})} . Z(\mathbf{F}) = \sum_{od\in W}\sum_{i\in \mathcal{N}_{od}} \pi_i(\mathbf{F}) + \sum_{od\in W}\sum_{t\in T}\sum_{r\in R_{od}} F_r(t)P_r(\mathbf{F}(t)) \tag{4.22}$$

$$= \max_{(\mathbf{F},\mathbf{Y})} . \left[\min_{(\boldsymbol{\pi}^{(s)},\mathbf{P}^{(s)})\in V} . \sum_{od\in W}\sum_{i\in \mathcal{N}_{od}} \pi_i^{(s)} + \sum_{od\in W}\sum_{t\in T}\sum_{r\in R_{od}} F_r(t)P_r^{(s)}(t) \right] \tag{4.23}$$

subject to Eq. (4.14), Eq. (4.15), and $F_r(t), Y_a(t) \in \mathbb{Z}_+$,

where V is the finite set of all extreme points of the convex feasible region Ω_{SD}. From this formulation, we see that path capacities are adjusted on the basis of the demand information. Moreover, this problem is equivalent to the following problem:

$$\max_{\theta \geq 0,\ (\mathbf{F},\mathbf{Y})} . \theta \tag{4.24}$$

$$\text{subject to} \quad \text{Eq. (4.14), Eq. (4.15)}, \quad F_r(t), Y_a(t) \in \mathbb{Z}_+,$$

$$\theta \leq \sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} \pi_i^{(s)} + \sum_{od \in W} \sum_{t \in T} \sum_{r \in R_{od}} F_r(t) P_r^{(s)}(t) \qquad \forall (\boldsymbol{\pi}^{(s)}, \mathbf{P}^{(s)}) \in V \tag{4.25}$$

Problem (4.24) is equivalent to the problem [DSO] (with non-individual variables) if all extreme points are known. However, it is difficult to obtain the extreme points in advance because the number of extreme points is generally too large. Hence, we consider a relaxation problem (4.24) that has a subset of the extreme points in V and produces an upper bound on the optimal objective value of the problem [DSO]. This relaxed problem is called the *restricted master problem* [RMP]. We then employ an iterative approach by adding an extreme point to the problem [RMP] to improve the upper bound. Note that an extreme point is generated by solving the problem (4.19) for fixed path capacities $\mathbf{F}$.

The procedure of the proposed mechanism corresponds to solving the above two problems, iteratively. One of the greatest differences between the Benders decomposition and the proposed mechanism is whether or not coefficient parameters $\mathbf{v}_i$ (i.e., truthful valuations of each user) are initially given. As mentioned in Section 4.3, the manager cannot observe such private information. Nevertheless, the proposed mechanism can obtain the demand information by exploiting an auction mechanism for solving the sub-problem.

4.4.2 Interpretation as an auction mechanism with day-to-day capacity control

The day-to-day auction mechanism comprises an *auction phase* and a *path capacity adjustment phase*; the two phases are repeated on a day-to-day basis (Fig.4.1). In the *auction phase* corresponding to the sub-problem, the manager sells bundles of permits to the users through an ascending auction under the condition that each capacity is fixed. In addition, the bundle prices are determined during the ascending auction so as to maximize each user's payoff. In the *path capacity adjustment phase* corresponding to the restricted master problem, the manager adjust each path capacity to an appropriate level by considering the demand information

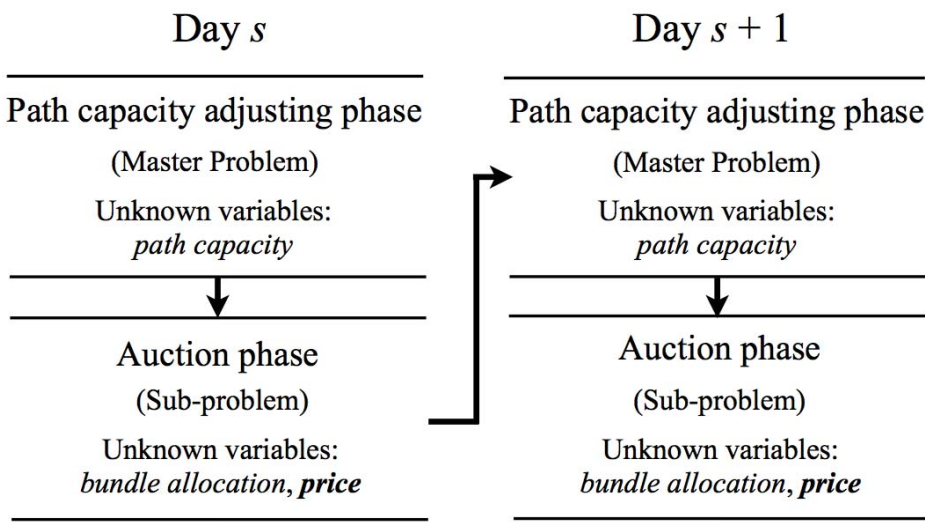


Figure 4.1 Procedure of the proposed mechanism

that was determined in the previous auction phases. Hence, the procedure of the day-to-day auction mechanism can be summarized as follows (more details of the mechanism and its properties can be found in Section 4.5):

Step 0: Set $s = 1$. Determine the initial path capacities $\mathbf{F}^{(1)}$. Start with a set of extreme points $V^{(1)} = \{\emptyset\}$ and a convergence criterion $\underline{\theta}^{(1)} = \infty$.

Step 1: *Auction phase* (Subsection 4.5.1). For fixed path capacities $\mathbf{F}^{(s)}$, the manager sells bundles of permits through an ascending auction. The user payoffs and the bundle prices $(\boldsymbol{\pi}^{(s)}, \mathbf{P}^{(s)})$ are also determined. If the social surplus $SS^{(s)}$ achieved in the ascending auction is equal to or greater than the convergence criterion $\underline{\theta}^{(s)}$ that is defined in Subsection 4.5.2, then stop. Otherwise go to **Step 2**.

Step 2: *Path capacity adjustment phase* (Subsection 4.5.2). Add an extreme point to the set; i.e., $V^{(s+1)} \equiv \{V^{(s)} \cup (\boldsymbol{\pi}^{(s)}, \mathbf{P}^{(s)})\}$. Produce the path capacities $\mathbf{F}^{(s+1)}$ by solving the problem [RMP] and update the convergence criterion $\underline{\theta}^{(s+1)}$. Let $s = s + 1$. Go to **Step 1**.

Note that *stop* in **Step 1** means that optimal path capacities are obtained. Therefore, once the above procedure stops, the manager no longer adjusts the path capacities and sells bundles through the auction with the same optimal path capacities each day.

4.4.3 Comparisons of the proposed mechanism and iterative combinatorial auctions

Our approach presented in this chapter is related to iterative combinatorial auctions (e.g., Parkes and Ungar, 2000a; de Vries, Schummer, and Vohra, 2007) that implement the VCG outcome (i.e., allocation and prices) in the sense that the original social surplus maximization problems (winner determination problem) are reformulated by adding new variables and constraints. Indeed, the problem [DSO] (with aggregate flows) corresponds to the formulation **(P2)** presented in Bikhchandani et al. (2002). However, the objectives of the reformulations are different. Our objective is to decompose the original problem to more simple and small problems. On the other hand, their main objective is to characterize the competitive equilibrium based on LP and to develop iterative auctions (e.g., ascending auctions) using LP algorithms. More specifically, they first focus on a strong (or extended) formulation for the original problem that has integral property, i.e., the linear relaxation of the strong formulation characterizes competitive equilibrium (see Bikhchandani and Ostroy, 2002). Then, the linear relaxation problem is solved by using a dual-based algorithm (i.e., primal-dual algorithm and subgradient algorithm), which is interpreted as (within-day) auction mechanisms (e.g., Parkes and Ungar, 2000a; de Vries, Schummer, and Vohra, 2007).

The iterative combinatorial auctions may overcome the problem of the VCG mechanism discussed in Section 4.3 (and Appendix B). Specifically, these auctions can obtain the VCG outcome by solving only one linear program. Also, these auctions do not require bidders to report bids for possible combination of all items, which enables bidders to participate in auctions without revealing private information more than necessary.

However, it may be difficult or undesirable to apply the iterative combinatorial auctions to our situation directly, while we employ a similar (non-combinatorial) auction to implement the sub-problem. First, a strong formulation of the winner determination problem has an exponential of number of variables if the number of bidders or items is large like our situation. Second, in general, these auctions need to set a bidder-specific price to achieve the efficient allocation. Employing such a price necessarily causes unfairness among road users.

In contrast, in the proposed mechanism, the sub-problem (winner determination problem) is just the transportation problem not a combinatorial optimization problem. The size of the winner determination problem is also small because the sub-problem reduces to the

independent sub-problems in terms of OD pairs. Furthermore, the proposed mechanism achieves an efficient allocation for a fixed path capacity by setting a bundle-specific price. From the above discussion, we conclude that our proposed mechanism will be more suitable for controlling transportation networks.

4.5 Details of proposed mechanism and its properties

This section presents the details and properties of each phase of the day-to-day auction mechanism. Subsection 4.5.1 gives a detailed explanation of the auction phase. Subsection 4.5.2 gives the detailed path capacity adjustment rule. Subsection 4.5.3 analyzes the proposed overall mechanism combining two phases and proves that the day-to-day dynamics of the network allocation pattern converges to the dynamic system optimal allocation when the number of users is large.

4.5.1 Auction phase

Let $\mathbf{F}^{(s)}$ be the path capacities as determined in the path capacity adjustment phase on day $s-1$. Then, the sub-problem for each OD pair is given by the following linear program:

$$SS_{od}^{(s)} \equiv \max_{\mathbf{f}^{(s)} \geq \mathbf{0}} . \sum_{i \in \mathcal{N}_{od}} \sum_{t \in T} \sum_{r \in R_{od}} v_{i,r}(t) f_{i,r}^{(s)}(t) \tag{4.26}$$

$$\text{subject to} \quad \text{Eq. (4.12) and Eq. (4.13)}$$

The necessary and sufficient optimality conditions of the problem are given by the following Kuhn-Tucker conditions:

$$\begin{cases} \pi_i^{(s)} = v_{i,r}(t) - P_r^{(s)}(t) & \text{if} \quad f_{i,r}^{(s)}(t) = 1 \\ \pi_i^{(s)} \geq v_{i,r}(t) - P_r^{(s)}(t) & \text{if} \quad f_{i,r}^{(s)}(t) = 0 \end{cases} \qquad \forall r \in R_{od},\ \forall t \in T, \forall i \in \mathcal{N}_{od} \tag{4.27}$$

$$\begin{cases} \sum_{i \in \mathcal{N}_{od}} f_{i,r}^{(s)}(t) = F_r^{(s)}(t) & \text{if} \quad P_r^{(s)}(t) > 0 \\ \sum_{i \in \mathcal{N}_{od}} f_{i,r}^{(s)}(t) \leq F_r^{(s)}(t) & \text{if} \quad P_r^{(s)}(t) = 0 \end{cases} \qquad \forall r \in R_{od},\ \forall t \in T \tag{4.28}$$

$$\pi_i^{(s)}, P_r^{(s)}(t) \geq 0 \qquad \forall r \in R_{od},\ \forall t \in T,\ \forall i \in \mathcal{N}_{od},\ \forall od \in W. \tag{4.29}$$

Note that the allocation variables $\mathbf{f}^{(s)}$ are integer because each sub-problem (4.26) satisfies total unimodularlity. The solution $(\mathbf{f}^{(s)}, \boldsymbol{\pi}^{(s)}, \mathbf{P}^{(s)})$ consists of a competitive equilibrium allocation, the payoffs and the prices, respectively. In the competitive equilibrium, each user

acquires the bundle of permits that maximizes his or her utility (i.e., (4.27)) for the given set of competitive equilibrium prices that satisfy the market clearing condition (4.28). Further, all users who acquire bundles have nonnegative payoffs (i.e., the user's willingness-to-pay is greater than the price), which is consistent with conventional traffic assignments with elastic demand.

The concept of the competitive equilibrium for indivisible items is a natural extension of the classical economic concept but for divisible items. Here the necessary and sufficient condition for the existence of this competitive equilibrium is that the optimal solution to the linear relaxation problem of the sub-problem is integer (Bikhchandani and Mamer, 1997). In addition, it has been shown that the competitive equilibrium, if it exists, is efficient (Bikhchandani and Ostroy, 2002). This can be summarized as follows:

Lenmma 4.1 *In the tradable network permit markets on day s, there always exists a competitive equilibrium that provides an efficient network permit allocation pattern for a fixed path capacity.*

Proof See Bikhchandani and Mamer (1997) and Bikhchandani and Ostroy (2002).

Note that the set of competitive equilibrium prices discussed above is not necessarily strategy-proof. However, Leonard (1983) showed that *minimal* competitive equilibrium prices such that the payment for each user is equal to the decrease in the value of the social surplus by adding the user to the auction are equivalent to Vickrey payments that produce the strategy-proofness. In addition, Leonard (1983) formulated the problem of finding the minimal competitive equilibrium prices:

$$\min_{(\boldsymbol{\pi}^{(s)},\mathbf{P}^{(s)})\geq \mathbf{0}} \cdot \sum_{t\in T}\sum_{r\in R_{od}} F_r^{(s)}(t)P_r^{(s)}(t) \tag{4.30}$$

subject to Eq. (4.20),

$$\sum_{i\in \mathcal{N}_{od}} \pi_i^{(s)} + \sum_{t\in T}\sum_{r\in R_{od}} F_r^{(s)}(t)P_r^{(s)}(t) = SS_{od}^{(s)}. \tag{4.31}$$

The problem minimizes equilibrium competitive prices (or maximizes user payoffs) subject to the condition that the solution of this problem also solves the dual of the sub-problem.

From the above discussions, we find that the sub-problem can be solved through the VCG mechanism in a computationally efficient manner: the allocation problem (4.26) is

merely the transportation problem and Vickrey payments are computed by solving only one linear program (4.30). However, there remains the problem of complication of the bidding rule: each user has to submit sealed bids reporting the value of all bundles of permits. This bidding rule is also undesirable in terms of the privacy, as users are required to reveal more of their private information than is necessary.

Ascending proxy auction

The proposed mechanism employs an ascending auction to resolve the problems of sealed bid auctions and to produce outcomes (i.e., allocation and prices of bundles of permits) in an informationally efficient manner. More specifically, we employ the (exact) ascending auction proposed by Demange, Gale, and Sotomayor (1986) (we call this the *DGS auction*). In this auction, users report only the "names" of bundles of permits in which they are interested. The procedure of the DGS auction corresponds to solving the sub-problem using a primal-dual algorithm, which is described as follows (see also Bikhchandani et al., 2002, and Appendix C):

Step 0: *Initialization*. Set $\mathbf{P}^{(s)} = \mathbf{0}$ for all bundles.

Step 1: *Bidding phase.* Each user reports "names" of the bundles that maximize one's own payoff under the current prices $\mathbf{P}^{(s)}$, i.e., a demand set $D_i(\mathbf{P}^{(s)}) \equiv \arg\max_{r,t}[v_{i,r}(t) - P_r^{(s)}(t)]$. If each user can be allocated a bundle from his or her demand set, then stop because $\mathbf{P}^{(s)}$ are equilibrium prices. Otherwise go to **Step 2**.

Step 2: *Price adjustment phase.* The manager chooses a *minimal overdemanded set* $M(\mathbf{P}^{(s)})$ and raises the prices of the bundles in that set (i.e., $P_r^{(s)}(t) = P_r^{(s)}(t) + 1,\ \forall(r,t) \in M(\mathbf{P}^{(s)})$). Go to **Step 1**.

Here, an overdemanded set is a set of bundles for which the number of users demanding only bundles in that set exceeds the number of bundles sold in the auction, and the minimal overdemanded set is an overdemanded set of bundles with no proper overdemanded subset.

In the DGS auction, the prices of the bundles converge to the minimal competitive equilibrium prices if each user reports the demand set truthfully (i.e., a myopic best response strategy) because the *minimal* overdemanded set is chosen in **Step 2**. Hence, the outcome of

the DGS auction is equal to the VCG outcome. Further, the truthful reporting of the demand set constitutes a Nash equilibrium for each user in each **Step 3**.

In a practical implementation of the DGS algorithm, it is hard for each user to report the demand set in each bidding phase, i.e., the transaction cost is too large. We therefore introduce a proxy agent system to support the bidding of users. Proxy systems are popular and have been installed in many Internet auctions (e.g., eBay and Yahoo). Under such a system, each user reports valuations to a proxy agent for some bundles that interest the user. Then, the proxy agent bids in the auction on the basis of the information received from the user. This system not only reduces the transaction cost of the bidding phase but also prevents strategic behaviors (e.g., a non-myopic best response strategy) in each bidding phase.

Let us now introduce the proxy agent system proposed by Parkes and Ungar (2000b) into the DGS auction. **Step 0** and **Step 1** are then modified as follows:

Step 0': Before starting the auction, each user reports information of valuations for some bundles to one's own proxy agent. Set $\mathbf{P}^{(s)} = \mathbf{0}$ for all bundles.

Step 1': Based on the information received and the current prices, each proxy agent submits each user demand set $D_i(\mathbf{P}^{(s)})$. If each user can be allocated a bundle from one's own demand set, then stop because the $\mathbf{P}^{(s)}$ are equilibrium prices. Otherwise go to **Step 2**.

In **Step 1'**, the user needs to update information if the proxy agent does not have enough information to submit the demand set. Since the proxy DGS auction restricts user strategies (in each bidding phase) to a myopic best response strategy, the dominant strategy is truthful reporting of the valuations to the proxy agent. From what has been discussed above and **Lemma 4.1**, we obtain the following proposition:

Proposition 4.1 *The network permit allocation pattern achieved under the proxy DGS auction for implementing the tradable network permit markets on day s is efficient, and the prices of bundles of permits converge to the minimal competitive equilibrium prices. The dominant strategy for each user is truth reporting of the valuations of bundles to the proxy agent.*

Proof See Demange et al. (1986) and Parkes and Ungar (2000b).

4.5.2 Path capacity adjustment phase

In the path capacity adjustment phase, the road manager first generates the demand information (i.e., payoffs and prices). The prices $\mathbf{P}^{(s)}$ can be obtained directly in the auction phase for all OD pairs. The payoffs $\boldsymbol{\pi}^{(s)}$, however, are computed indirectly. In the proxy DGS auction, since each user reports his or her true valuations for interesting bundles to the proxy agent, the manager can obtain his or her winning valuation $v^*_{i,r}(t)$. Then, the manager calculates a total payoff $\Pi^{(s)}$ from the duality theorem:

$$\Pi^{(s)} \equiv \sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} \pi_i^{(s)} = \sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} v^*_{i,r}(t) - \sum_{od \in W} \sum_{t \in T} \sum_{r \in R_{od}} F_r^{(s)}(t) P_r^{(s)}(t). \tag{4.32}$$

Note that the manager needs to know only the total payoff to adjust path capacities.

After generating the demand information, the manager considers all demand information for the current day and past days, $V^{(s+1)} \equiv \{V^{(s)} \cup (\Pi^{(s)}, \mathbf{P}^{(s)})\}$, and adjusts each path capacity by solving the restricted master problem [RMP]. However, this is computationally intensive because the problem [RMP] (i.e., the problem (4.24)) is a large integer programming (IP) problem with one continuous variable. To avoid this, we solve the linear relaxation of the problem [RMP] and obtain an integer solution by rounding off the fractional solution. Such a strategy was suggested by McDaniel and Devine (1977) and has successfully used in various problems (e.g., Cordeau, Soumis, and Desrosiers, 2000). This strategy is suitable for our situation because non-individual variables (path capacities) in the problem [RMP] are control variables of the road manager and can be treated as continuous variables, although the individual variables (allocation of network permits) cannot be treated as continuous. In addition, we should note here that the relaxation of integrality constraints does not affect the convex feasible region Ω_{SD} of the dual sub-problem and that an extreme point can be generated from any integer solution. Thus, the problem with continuous variables $(\tilde{\mathbf{F}}, \tilde{\mathbf{Y}})$ that the road manager needs to solve is given as

$$\max_{\overline{\theta} \geq 0,\ (\tilde{\mathbf{F}}^{(s+1)}, \tilde{\mathbf{Y}}^{(s+1)}) \geq \mathbf{0}} . \overline{\theta} \tag{4.33}$$

subject to

$$\overline{\theta} \leq \Pi^{(s)} + \sum_{od \in W} \sum_{t \in T} \sum_{r \in R_{od}} \tilde{F}_r^{(s+1)}(t) P_r^{(s)}(t) \qquad (\Pi^{(s)}, \mathbf{P}^{(s)}) \in V^{(s+1)} \tag{4.34}$$

$$\tilde{Y}_a^{(s+1)}(t) \leq \mu_a \qquad \forall a \in A,\ \forall t \in T \tag{4.35}$$

$$\tilde{Y}_a^{(s+1)}(t) = \sum_{od \in W} \sum_{r \in R_{od}} \tilde{F}_r^{(s+1)}(t + T_{a,r}) \delta_{a,r(o,d)} \qquad \forall a \in A,\ \forall t \in T. \tag{4.36}$$

The optimal objective value is an upper bound on the maximum social surplus SS^* of the problem [DSO], which is weaker than an upper bound that is produced with the integer programming problem [RMP].

From the optimality conditions of the problem, a path capacity adjustment rule can be derived as

$$\begin{cases} \sum_{a \in A} p_a(t - T_{a,r}) \delta_{a,r(o,d)} = \overline{P}_r(t) & \text{if} \quad \tilde{F}_r^{(s+1)}(t) > 0 \\ \sum_{a \in A} p_a(t - T_{a,r}) \delta_{a,r(o,d)} \geq \overline{P}_r(t) & \text{if} \quad \tilde{F}_r^{(s+1)}(t) = 0 \end{cases} \qquad \forall r \in R_{od},\ \forall t \in T,\ \forall od \in W, \tag{4.37}$$

where the $\overline{\mathbf{P}}$ are the (convex combinations of) bundle prices that produce the weak upper bound (i.e., the constraint (**??**) is bounded), and $\mathbf{p}$ is the Lagrange multiplier for the constraint (4.35). This Lagrange multiplier is interpreted as a permit price for each link that satisfies the following (market clearing) condition:

$$\begin{cases} \tilde{X}_a^{(s+1)}(t) = \mu_a & \text{if} \quad p_a(t) > 0 \\ \tilde{X}_a^{(s+1)}(t) \leq \mu_a & \text{if} \quad p_a(t) = 0 \end{cases} \qquad \forall a \in A,\ \forall t \in T. \tag{4.38}$$

If the path capacity is positive in the path capacity adjustment rule (4.37), the bundle price estimated for the path by means of link permit prices and is equal to the bundle price determined in the auction phase. For a path whose the estimated price exceeds the realized price, the path capacity is zero. This means that no path capacities are allocated to the worthless paths. The integer path capacities $\mathbf{F}^{(s+1)}$ on day $s + 1$ can be obtained by rounding-off all continuous path capacities; i.e., $F_r^{(s+1)}(t) = \lfloor \tilde{F}_r^{(s+1)}(t) \rfloor$.

Stabilizing strategy for Benders decomposition

Although the problem (4.33) is easy to solve, there remains one issue relevant to the convergence rate of the Benders decomposition; i.e., path capacities usually oscillate, which results

in slow convergence (Magnanti and Wong, 1981). To accelerate and stabilize the Benders decomposition, we add "boxstep constraints" (Marsten, Hogan, and Blankenship, 1975) to the above problem (4.33):

$$\tilde{F}_r^{(s)}(t) - \epsilon \leq \tilde{F}_r^{(s+1)}(t) \leq \tilde{F}_r^{(s)}(t) + \epsilon \qquad \forall r \in R_{od},\ \forall t \in T,\ \forall od \in W, \tag{4.39}$$

where ϵ is a boxstep parameter. At each step, the solution $\tilde{\mathbf{F}}^{(s+1)}$ to the master problem is constrained to lie within a box centered on the previous solution $\tilde{\mathbf{F}}^{(s)}$ and so the oscillation is dramatically reduced. Note that the problem including the boxstep constraints does not necessarily produce an upper bound on the maximum social surplus SS^*. Thus, we solve the problem (4.33) to obtain the upper bound $\overline{\theta}$.

4.5.3 Convergence of the day-to-day auction mechanism

We now establish the convergence result of the day-to-day auction mechanism on the basis of the Benders decomposition technique. The standard Benders decomposition algorithm converges to an optimal solution when the strong upper bound obtained by the problem [RMP] is equal to the optimal objective value of the sub-problem (i.e., the social surplus achieved in the auction phase). However, the weak upper bound $\overline{\theta}$ obtained with the proposed mechanism will exceed the maximum value of the social surplus SS^* even if all the extreme points are generated, and thus we cannot use $\overline{\theta}$ as the convergence criterion.

To resolve this problem, we introduce a new convergence criterion $\underline{\theta}$:

$$\underline{\theta} \equiv \min_{(\Pi^{(s)},\mathbf{P}^{(s)})\in V^{(s+1)}} . \Pi^{(s)} + \sum_{t\in I}\sum_{r\in R} F_r^{(s+1)}(t)P_r^{(s)}(t), \tag{4.40}$$

and an update rule of the criterion is

$$\underline{\theta}^{(s+1)} = \min\left\{\underline{\theta}^{(s)}, \underline{\theta}\right\}. \tag{4.41}$$

The criterion $\underline{\theta}$ optimizes (i.e., minimizes) the objective function of the problem [RMP] only with respect to extreme points $(\Pi^{(s)}, \mathbf{P}^{(s)})$ given at the integer path capacities $\mathbf{F}^{(s+1)}$, which results in good convergence properties as shown in the proof of the proposition below. This criterion $\underline{\theta}$ is equal to or less than the strong upper bound since it does not maximize the objective function of the problem [RMP] with respect to the path capacities. Therefore, we

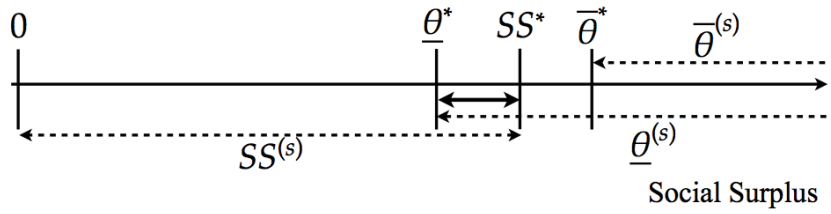


Figure 4.2 Relationship between the convergence criterion, the weak upper bound, the achieved social surplus, and the maximum value of the social surplus

conclude that the permit allocation under the proposed mechanism converges to an approximate dynamic system optimal state when the achieved social surplus $SS^{(s+1)}$ $(= \sum_{od} SS_{od}^{(s+1)})$ in the auction phase is equal or more than the convergence criterion $\underline{\theta}^{(s+1)}$.

Fig.4.2 shows the relationship between the convergence criterion $\underline{\theta}^{(s)}$, the weak upper bound $\overline{\theta}^{(s)}$, the achieved social surplus $SS^{(s)}$, and the maximum social surplus SS^*. The horizontal axis represents the social surplus (or its upper bound) and dotted lines represent the ranges in which the variables can exist. The achieved social surplus $SS^{(s)}$ can exist in the range $[0, SS^*]$. The convergence criterion and the weak upper bound take minimum values $\underline{\theta}^*$ and $\overline{\theta}^*$ when we have all the extreme points.

By using the convergence criterion $\underline{\theta}^{(s)}$, we obtain the value of the social surplus in a range that is represented by the solid arrow in Fig.4.2. The ratio $SS^{(s)}/SS^*$ between the achieved social surplus and the maximum value of the social surplus is confined within in the range

$$\frac{\underline{\theta}^*}{SS^*} \leq \frac{SS^{(s)}}{SS^*} \leq 1. \tag{4.42}$$

Assuming that the ratio between the total number of users Q $(= \sum_{od} |\mathcal{N}_{od}|)$ and the total link capacity $\sum_t \sum_a \mu_a$ is held constant, the range (4.42) converges to zero (i.e., the left-hand side of Eq.(4.42) converges to 1) when the number of users is sufficiently large. This is because the effect of rounding off the continuous path capacities is negligible in that case. In addition, a new extreme point is generated in each auction phase when the achieved social surplus does not satisfy the convergence criterion, so the proposed mechanism can converge in a finite number of steps. Therefore, the following proposition holds.

Proposition 4.2 *Assume that the ratio between the number of users and total link capacity is constant. Then, the day-to-day auction mechanism converges in a finite number of steps, and the value of the social surplus achieved by the mechanism reaches its maximum value when the number of users is large.*

Proof See 4.A for the proof.

4.6 An extended mechanism which obviates path enumeration

The day-to-day auction mechanism presented in the previous sections assumes that the road manager can enumerate all the paths that users may choose. However, it is not necessarily evident how the manager should do so for large-scale networks. To obviate path enumeration, we construct an extended mechanism by introducing a path generation phase into the day-to-day auction mechanism. This consists of applying a column generation procedure to the system optimal allocation problem [DSO]. In the extended mechanism, users generate paths successively, and hence path enumeration is obviated for the manager.

A column generation for a network flow problem considers a problem that has only a subset of the paths of the original problem (i.e., a *restricted master problem*) and paths are generated as needed (Ahuja, Magnanti, and Orlin, 1993). Hence, by considering only a subset of the (dynamic) paths of the problem [DSO], a restricted master problem [C-RMP] is formulated as

$$\max_{(\mathbf{f},\mathbf{y})} . \sum_{od\in W} \sum_{i\in \mathcal{N}_{od}} \sum_{t\in T} \sum_{r\in R_{od}(t)} v_{i,r}(t) f_{i,r}(t) \tag{4.43}$$

subject to

$$\sum_{t\in T} \sum_{r\in R_{od}(t)} f_{i,r}(t) \leq 1 \qquad \forall i \in \mathcal{N}_{od},\ \forall od \in W \tag{4.44}$$

$$\sum_{od\in W} \sum_{i\in \mathcal{N}_{od}} y_{i,a}(t) \leq \mu_a \qquad \forall a \in A,\ \forall t \in T \tag{4.45}$$

$$y_{i,a}(t) = \sum_{t\in T} \sum_{r\in R_{od}(t)} f_{i,r}(t + T_{a,r}) \delta_{a,r(o,d)} \qquad \forall a \in A,\ \forall t \in T,\ \forall i \in \mathcal{N}_{od}\ \forall od \in W \tag{4.46}$$

$$f_{i,r}(t), y_{i,a}(t) \in \{0, 1\} \qquad \forall a \in A,\ \forall r \in R_{od}(t),\ \forall t \in T\ \forall i \in \mathcal{N}_{od},\ \forall od \in W, \tag{4.47}$$

where $R_{od}(t)$ is a subset of paths in destination arrival time period t. Since the problem [C-RMP] and the problem [DSO] have the same optimization problem except for the number of paths, we can solve the problem [C-RMP] through the day-to-day auction mechanism presented in the previous sections.

A new path is generated by solving a *column generation sub-problem* corresponding to the *pricing step* of the simplex algorithm (for the liner relaxation of the problem [C-RMP]). In the standard column generation for a multicommodity flow problem, the sub-problem is given as a shortest path problem for each commodity (Ahuja et al., 1993). Thus, by following the standard theory, our sub-problem is formulated as the following all-or-nothing problem for each user:

$$\pi_i^* \equiv \max_{\mathbf{f}_i \geq \mathbf{0}} . \sum_{t \in T} \sum_{r \in R_{od}} \left[v_{i,r}(t) - \sum_{a \in A} \hat{p}_a(t - T_{a,r}) \delta_{a,r(o,d)} \right] f_{i,r}(t) \tag{4.48}$$

$$\text{subject to} \quad \sum_{t \in T} \sum_{r \in R_{od}} f_{i,r}(t) \leq 1, \tag{4.49}$$

where $\hat{p}_a(t)$ is an optimal Lagrange multiplier for the link capacity constraint (4.45) of the linear relaxation of the restricted master problem [C-RMP], which is interpreted as an optimal link permit price. These link permit prices are obtained at the final path capacity adjustment phase of the day-to-day auction mechanism (see Subsection 4.5.2).

The column generation sub-problem yields a path that maximizes each user payoff for given constant link permit prices $\hat{\mathbf{p}}$. The path is generated if a maximum payoff exceeds the current payoff achieved in the final auction phase of the day-to-day auction mechanism. Specifically, the path is generated if the optimal value of the objective function π_i^* exceeds an optimal Lagrange multiplier $\hat{\pi}_i$ for the constraint (4.44); i.e., $\lambda_i \equiv \pi_i^* - \hat{\pi}_i > 0$. To improve his or her payoff, the user requests that the manager sells the bundle for the path in the auction phase. The road manager receives the requests of all users and adds the paths to the set $R_{od}(t)$ (if the path do not exists in the set). Then the restricted master problem [C-RMP] is again solved through the day-to-day auction mechanism.

The steps in the extended mechanism mentioned above can be summarized as follows:

Step 0: *Initial setting*. Set $\tau = 1$. Determine the initial path set $R_{od}^{(1)}(t)$ for each OD pair at each destination arrival time period.

Step 1: *Day-to-day auction phase*. For fixed path set $R_{od}^{(\tau)}(t)$, the restricted master problem [C-RMP] is solved through the day-to-day auction mechanism (see Section 4.4 and

4.5). The optimal link permit prices $\hat{\mathbf{p}}^{(\tau)}$ are determined at the final path capacity adjustment phase and are announced by the road manager.

Step 2: *Path generation phase*. Each user finds a path by solving the column generation sub-problem and requests the manager to add the path if the maximum payoff π_i^* exceeds the current payoff $\hat{\pi}^{(\tau)}$. If all requested paths exist in path set $R_{od}^{(\tau)}(t)$, then stop. Otherwise, the road manager creates a new path set $R_{od}^{(\tau+1)}(t)$ by adding requested paths to the set $R_{od}^{(\tau)}(t)$. Let $\tau = \tau + 1$. Go to **Step 1**.

The paths are efficiently generated in **Step 2** because the numerous number users generate paths simultaneously. However, the road manager employs a path-adding rule that allows each user to purchase not only paths generated by himself but also those generated by other users of the same OD pair[6], which promotes path generation. The extended mechanism is guaranteed to converge because the number of paths is finite. Furthermore, when the number of users is large, the allocation of network permits achieved under the extended mechanism converges to the optimal one (i.e., the optimal solution of the problem [DSO]) since the gap between the problem [DSO] and the linear relaxation converges to zero (Proposition 4.2).

4.7 Numerical example

We finally show a numerical example to demonstrate the convergence properties of the proposed mechanism in a realistic network. The network that we employ is the Sioux Falls network (LeBlanc et al., 1975) which has 24 nodes and 76 links (Fig.4.3). The physical conditions of each link (i.e., free-flow travel time, capacity), which is based on Han (2003), are summarized in Table 4.1 in 4.B. The network has 528 OD pairs, which was used by (LeBlanc et al., 1975), and the number of users for each OD pair is a quarter of the number provided in Dr. Hillel Bar-Gera's website (http://www.bgu.ac.il/~bargera/tntp/); i.e., the total number of users is 90150. We set time interval for each time period to $\Delta t = 3$ (minute) and the number of time periods to $|T| = 40$. The desired arrival time period for each user is set randomly and the distribution of the desired arrival time periods is shown in Fig.4.4. Under

[6] If we employ the standard column generation procedure, subsets of the paths differ among users because the column generation sub-problem (4.48) is formulated for each user. However, in the auction phase, it will be more natural that the same set of paths are sold for all users of the same OD pair.

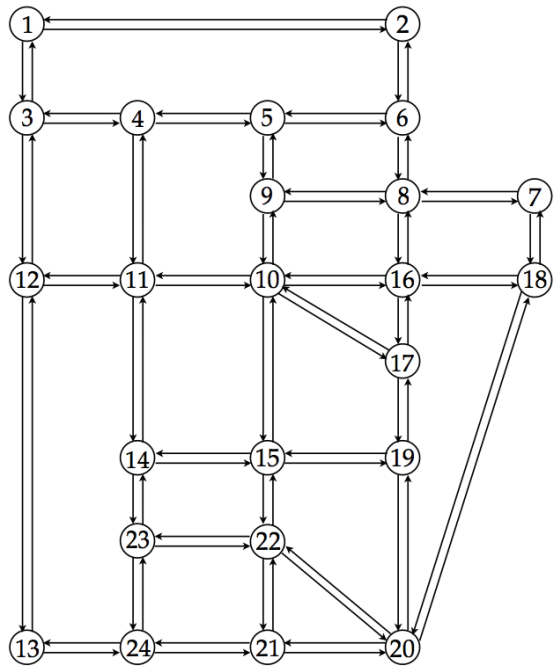


Figure 4.3 Sioux Falls network

this distribution, the network is congested (i.e., almost links have positive permit prices) during peak periods. As the initial path set for each OD pair, we simply choose some shortest paths. A box step parameter $\epsilon = 5$ is chosen. An optimal social surplus is calculated by 10,000 iterations of the proposed mechanism for a sufficiently accurate determination of the maximum one.

Fig.4.5 illustrates the convergence process of the proposed mechanism until the relative error between the achieved social surplus $SS^{(s)}$ and the optimal social surplus is reduced below 0.05%. The horizontal axis represents the number of days, s, and the vertical axis represents the ratio between the achieved social surplus $SS^{(s)}$ on each day and the optimal social surplus. The vertical lines (at day 59, 110, 164, 232, ...) show days at which a day-to-day auction phase (or mechanism) terminated. On such a day, the path generation phase starts. Note that the path set is fixed in each day-to-day auction phase.

By using Fig.4.5, we explain the convergence properties of the first day-to-day auction phase from day 1 to day 59. In this phase, the achieved social surplus $SS^{(s)}$ (the solid black curve) increases as path capacities are adjusted on a day-to-day basis. Conversely, the upper bound of the maximum social surplus $\overline{\theta}^{(s)}$ (the gray curve) for a fixed path set and the

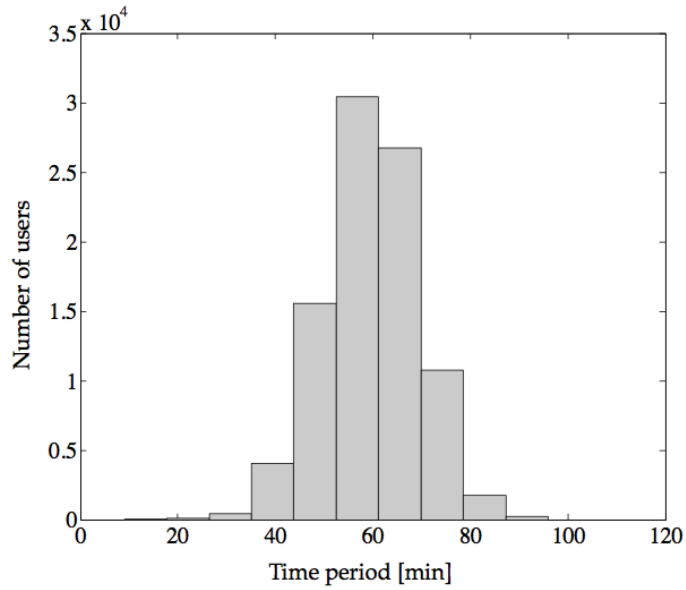


Figure 4.4 Distribution of the desired arrival time

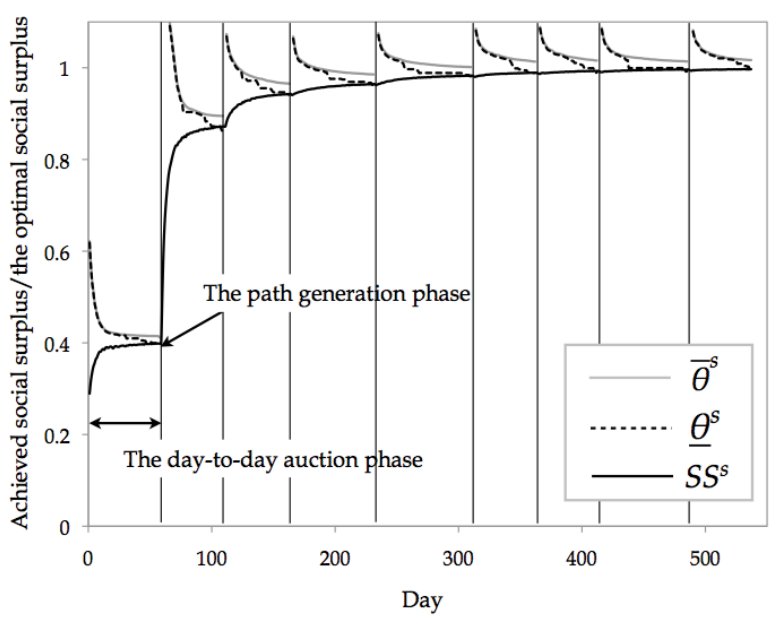


Figure 4.5 Convergence process of the proposed mechanism

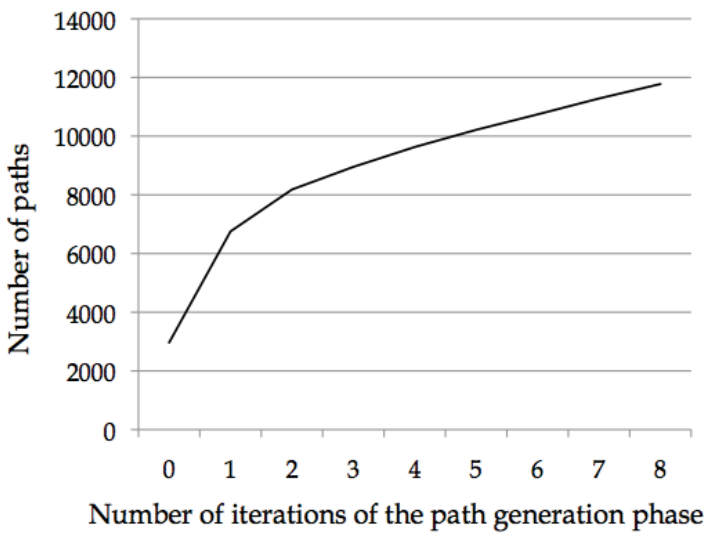


Figure 4.6 Number of paths in each day-to-day auction phase

convergence criterion $\underline{\theta}^{(s)}$ (the black dotted curve) that are obtained in the path capacity adjustment phase decrease monotonically. Eventually, these three values converge to the almost the same value. This means that the allocation of network permits achieved under the day-to-day auction phase converges to the approximate dynamic system optimal allocation for a fixed path set.

After the first day-to-day auction phase terminates (at day 59), the first path generation phase starts. In the path generation phase, each user requests a path to improve his or her payoff based on the current permit prices and payoff realized in the previous day-to-day auction phase. The achieved social surplus increases drastically in the second day-to-day auction phase. This is because a large number of paths is generated in the first path generation phase (see Fig.4.6). We also see from Fig.4.6 that the number of paths generated in each subsequent phase decreases, and then the achieved social surplus reaches close to the optimal value with a small number of iterations of the path generation phase.

4.8 Conclusion

To implement trading markets for the network permits, we proposed an auction mechanism for general networks. We first discussed the impossibility of applying the VCG mechanism to

the trading markets due to NP-hardness. To avoid such computational infeasibility, we constructed a day-to-day auction mechanism that is readily implementable for general networks. We then proved that the proposed mechanism is strategy-proof and the network allocation pattern under the mechanism converges to an approximation of the socially optimal state in the sense that the achieved social surplus reaches its maximum value when the number of users is large. Furthermore, we showed that the proposed mechanism can be extended to mitigate path enumeration by introducing a column generation procedure, and demonstrated its convergence property in a realistic network.

While this chapter focused on managing road transportation networks, the mechanism proposed seems applicable in principle to the management of other transportation networks (e.g., railway and freight networks). For example, freight networks are used by many users who choose routes and departure times so as to maximize their utility as is the case for road transportation networks. In contrast, the behaviors of network managers totally differ; i.e., while a road manager aims to maximize the social surplus, a freight network manager (i.e., a freight company) aims to maximize his or her profit. Nevertheless, managing other transportation networks using the proposed mechanism seems a fruitful topic for future work.

Appendix 4.A Proof of the Proposition 4.2

We first show that a new extreme point is generated in every *auction phase* until the convergence criterion is satisfied. We denote the path capacities at day s by $\mathbf{F}^{(s)}$ and the convergence criterion by $\underline{\theta}^{(s)}$. From the Eq.(4.40), the following holds:

$$\underline{\theta}^{(s)} \leq \Pi^{(s)} + \sum_{od \in W} \sum_{t \in I} \sum_{r \in R} F_r^{(s)}(t) P_r^{(s)}(t) \qquad \forall (\Pi^{(s)}, \mathbf{P}^{(s)}) \in V^{(s)} \tag{4.50}$$

holds. From the duality theorem, the optimal value of the objective function of the sub-problem at day s (i.e., the value of the social surplus achieved by the ascending proxy auction), $SS_{od}^{(s)}$, coincides with the optimal value of the objective function of its dual problem, that is

$$SS^{(s)} = \sum_{od \in W} SS_{od}^{(s)} = \sum_{od \in W} \sum_{i \in \mathcal{N}_{od}} \sum_{t \in T} \sum_{r \in R_{od}} v_{i,r}(t) f_{i,r}^{s*}(t) = \Pi^{s*} + \sum_{od \in W} \sum_{t \in T} \sum_{r \in R_{od}} F_r^{(s)}(t) P_r^{s*}(t), \tag{4.51}$$

where $(\mathbf{f}^{s*}, \Pi^{s*}, \mathbf{P}^{s*})$ is the optimal solution of the sub-problem and its dual problem. We here consider the case that the convergence criterion is not satisfied (i.e., $SS^{(s)} < \underline{\theta}^{(s)}$). Then, the

following relationships are hold:

$$\sum_{od\in W}\sum_{i\in \mathcal{N}_{od}} \pi_i^{s*} + \sum_{od\in W}\sum_{t\in T}\sum_{r\in R_{od}} F_r^{(s)}(t)P_r^{s*}(t) = SS^{(s)}$$
$$< \underline{\theta}^{(s)} \le \Pi^{(s)} + \sum_{od\in W}\sum_{t\in I}\sum_{r\in R} F_r^{(s)}(t)P_r^{(s)}(t) \quad (\forall(\Pi^{(s)}, \mathbf{P}^{(s)}) \in V^{(s)}). \tag{4.52}$$

Hence, $(\Pi^{s*}, \mathbf{P}^{s*}) \neq (\Pi^{(s)}, \mathbf{P}^{(s)}) \in V^{(s)}$ is obtained; i.e., a new extreme point is generated. Since the number of extreme points is finite, we can conclude that the proposed mechanism converges in a finite number of steps.

Next, we show that the ratio $\underline{\theta}^*/SS^*$ in the left-hand side of Eq.(4.42) converges to 1 when the number of users is large (assuming that the ratio between the number of uses and the total link capacity is held constant). In order to show this, we prove that a ratio $\underline{\theta}^*/\overline{\theta}^*$ that is less than $\underline{\theta}^*/SS^*$ converges to 1. We denote the extreme point that minimizes the problem (4.40) by $(\underline{\Pi}, \underline{\mathbf{P}}) \in V$, and we denote the extreme point that produces the weak upper bound $\overline{\theta}^*$ by $(\overline{\Pi}, \overline{\mathbf{P}}) \in V$. Then the gap between $\overline{\theta}^*$ and $\underline{\theta}^*$ is investigated with the following equations:

$$\overline{\theta}^* - \underline{\theta}^* = \left(\overline{\Pi} + \sum_{od\in W}\sum_{t\in I}\sum_{r\in R} \tilde{F}_r^{(s+1)}(t)\overline{P}_r(t)\right) - \left(\underline{\Pi} + \sum_{od\in W}\sum_{t\in I}\sum_{r\in R} F_r(t)\underline{P}_r(t)\right) \tag{4.53}$$
$$\le \left(\underline{\Pi} + \sum_{od\in W}\sum_{t\in I}\sum_{r\in R} \tilde{F}_r^{(s+1)}(t)\underline{P}_r(t)\right) - \left(\underline{\Pi} + \sum_{od\in W}\sum_{t\in I}\sum_{r\in R} F_r(t)\underline{P}_r(t)\right) \tag{4.54}$$
$$< \left(\sum_{od\in W}\sum_{t\in I}\sum_{r\in R} \underline{P}_r(t)\right) = (\text{the number of paths}) \times (\text{average price}). \tag{4.55}$$

The second line represents the fact that the extreme points of minimizing (4.40) and (4.33) are different. The third line follows because the maximum rounded value of each path capacity is 1.

Alternatively, $\overline{\theta}^*$ can be estimated as follows:

$$\overline{\theta}^* \ge SS^* = \sum_{od\in W}\sum_{i\in \mathcal{N}_{od}} v_{i,r}^*(t) = (\text{the number of users}) \times (\text{average winning valuation}) \tag{4.56}$$

where $v_{i,r}^*(t)$ is the winning valuation when the social surplus is maximized. By using the above equations, the relative error between $\overline{\theta}^*$ and $\underline{\theta}^*$ is obtained as follows:

$$\frac{\overline{\theta}^* - \underline{\theta}^*}{\overline{\theta}^*} < \frac{(\text{the number of paths}) \times (\text{average price})}{(\text{the number of users}) \times (\text{average winning valuation})} < \frac{(\text{the number of paths})}{(\text{the number of users})}. \tag{4.57}$$

Since the bundle prices obtained by the ascending proxy auction never exceed the truthful valuation of each user, the final inequality holds. When the number of users is large (i.e., $Q \to \infty$) with the ratio between the number of users and the total link capacity held constant, the relative error converges to zero because the number of paths is constant. Thus, the following equations hold:

$$\lim_{Q\to\infty} \frac{\underline{\theta}^*}{\overline{\theta}^*} = 1 \quad \Rightarrow \quad \lim_{Q\to\infty} \frac{\underline{\theta}^*}{SS^*} = 1 \tag{4.58}$$

Hence, we can conclude that the range (4.42) converges to zero when the number of users is large.

Appendix 4.B Network and OD data

Table 4.1 Physical conditions of links in Sioux Falls network

Link (upstream, downstream)	Free-flow travel time [min]	Capacity [vehicles /min]	Link (upstream, downstream)	Free-flow travel time [min]	Capacity [vehicles /min]
(1, 2) and (2, 1)	9	65	(11, 12) and (12, 11)	3	60
(1, 3) and (3, 1)	3	55	(11, 14) and (14, 11)	6	50
(2, 6) and (6, 2)	3	60	(12, 13) and (13, 12)	9	65
(3, 4) and (4, 3)	3	60	(13, 24) and (24, 13)	3	60
(3, 12) and (12, 3)	6	60	(14, 15) and (15, 14)	3	50
(4, 5) and (5, 4)	3	50	(14, 23) and (23, 14)	3	40
(4, 11) and (11, 4)	6	55	(15, 19) and (19, 15)	3	40
(5, 6) and (6, 5)	3	50	(15, 22) and (22, 15)	3	45
(5, 9) and (9, 5)	3	50	(16, 17) and (17, 16)	3	45
(6, 8) and (8, 6)	3	45	(16, 18) and (18, 16)	3	55
(7, 8) and (8, 7)	3	40	(17, 19) and (19, 17)	3	45
(7, 18) and (18, 7)	3	50	(18, 20) and (20, 18)	12	55
(8, 9) and (9, 8)	3	45	(19, 20) and (20, 19)	6	50
(8, 16) and (16, 8)	3	45	(20, 21) and (21, 20)	3	40
(9, 10) and (10, 9)	3	45	(20, 22) and (22, 20)	6	45
(10, 11) and (11, 10)	3	50	(21, 22) and (22, 21)	3	50
(10, 15) and (15, 10)	6	45	(21, 24) and (24, 21)	3	50
(10, 16) and (16, 10)	3	40	(22, 23) and (23, 22)	3	40
(10, 17) and (17, 10)	3	45	(23, 24) and (24, 23)	3	40

Chapter 5

A trading mechanism for network permits with multiple purchase opportunities

Chapter 4 constructed micro mechanism for implementing trading markets in more spatially general situations than the single bottleneck case (Wada and Akamatsu, 2010). This chapter[1] considers more temporally general situations where network permits for a specific day (a trip day) are sold in multiple period markets (e.g., future markets and spot markets).

The main difference between this chapter and the previous chapters lies in the fact that here we explicitly consider when road users would participate in trading markets or when the markets would end. On the other hand, previous chapter implicitly assumed that all of the users would gather in the markets on the day before making trips. This assumption is reasonable for recurrent trips (e.g., a daily commute) because the users' valuations for the permits can be regarded as constants over time. In contrast, for non-recurrent trips, the users' valuations will depend on when they purchase the permits. A typical example is the case in which a person has multiple travel plans that depend on the purchase periods. The value of the permit generally reflects not the utility of the trip itself but that of the activity associated with the trip, i.e., the users' valuations of the permits change over time. Thus, providing multiple purchase opportunities can be expected to achieve more efficient resource allocation

[1] This chapter is based on joint research with Pengfei Wang, Takashi Akamatsu, and Takeshi Nagae (Wada, Wang, Akamatsu, and Nagae, 2012). A preliminary version presented in *Infrastructure Planning Conference, Japan Society of Civil Engineering*, 2010 (Wang, Akamatsu, and Wada, 2010).

than a single period market.

Under multiple period markets, each road user can be free to choose a purchase period and a network permit for a pre-specified time within the trip day (i.e., a destination arrival time). On the other hand, the road manager has to allocate a bottleneck capacity to these markets as well as allocating to users. As a first step for implementing such markets, we design a dynamic auction mechanism in which the number of permits sold for each market is fixed. This mechanism can determine an efficient permits allocation along with the time sequence if each user bids "net valuations" (valuations minus the option value of deferring purchase) truthfully. It is proved that truthful revelation of net valuations is a dominant strategy for each user, which also guarantees that the market choice of the user is optimal. Then we derive an adjustment rule of the number of permits sold for each market and demonstrate that combining the dynamic auction and the adjustment rule maximizes the social surplus in a finite number of iterations. Finally, we numerically demonstrate that the proposed mechanism works effectively for a dynamic population case under a certain condition.

This chapter is organized as follows. Section 5.1 discusses related works. Section 5.2 outlines the tradable network permits scheme with multiple purchase opportunities. Section 5.3 formulates a system optimal network permits allocation problem. We also present the design framework for a mechanism to implement the scheme by decomposing this problem. Section 5.4 designs a dynamic auction mechanism. Section 5.5 derives an adjustment rule of the number of permits for each market and clarify the properties of the whole mechanism. Section 5.6 investigates the validity of the proposed mechanism for a dynamic population case by numerical experiments. Section 5.7 concludes the chapter.

5.1 Related works

The scheme considered in this chapter corresponds to introducing a reservation system to the conventional tradable network permits scheme. Reservation systems have been widely studied in the field of revenue management for many years (see Talluri and van Ryzin, 2004; Chiang, Chen, and Xu, 2007, for comprehensive reviews of the literature). Moreover, in the transportation field, much research has been performed on the theory and practice of reservation systems (e.g., airline seat reservations). Furthermore, the several researchers have examined reservation systems for better use of road infrastructure (e.g., Akahane and

Kuwahara, 1996; Wong, 1997). Almost all of the above studies have aimed at maximizing revenue or social surplus by market segmentation and discriminatory pricing. However, as previously stated, it is difficult to determine an optimal price because there is an asymmetric information between suppliers (road managers) and buyers (road users).

One of the approaches to resolve the asymmetric information problem is to employ auction mechanisms. Recently, in the field of mechanism design/auction theory, much effort has been put into extending the theory to dynamic settings (Parkes, 2007; Bergemann and Said, 2011; Vohra, 2012). These studies can be classified into two groups: online mechanisms and dynamic mechanisms. The former considers a situation where the population of agents varies over time, but their private information is constant. Parkes and Singh (2004) proposed an *online VCG mechanism* that generalizes the static VCG mechanism to a dynamic population setting and showed that this mechanism achieves the efficient resource allocation. The latter type is related to our study in the sense that these mechanism consider a situation where the population of agents is fixed but their private information changes over time. Bergemann and Välimäki (2010) also generalized the static VCG mechanism to a dynamic setting (*dynamic pivot mechanism*)[2]. They then proved that the mechanism achieves the efficient resource allocation and satisfies the ex post inventive compatibility. However, this mechanism cannot be directly applied to our problem because, in our setting, each buyer wishes to purchase at most one permit within multiple period markets unlike the setting in Bergemann and Välimäki (2010). Moreover, they did not consider the adjustment problem that arises when supplying items for each period. In the field of revenue management, the studies of dynamic auction mechanisms have been performed as an alternative to traditional reservation systems (Vulcano, Van Ryzin, and Maglaras, 2002; Chiang, Chen, and Xu, 2007). Vulcano et al. (2002) proposed a dynamic auction mechanism that determines both the items allocation and the item supply for each period. In this mechanism, the item supply is controlled by setting a threshold: if a bid is less than the threshold, the seller does not accept it. However, this is a revenue maximization mechanism that will not maximize a social surplus. From the above discussion, we conclude that a trading mechanism that combines a dynamic auction and an adjustment rule is a major contribution of this chapter.

[2] Cavallo, Parkes, and Singh (2006) proposed a mechanism similar to the dynamic pivot mechanism.

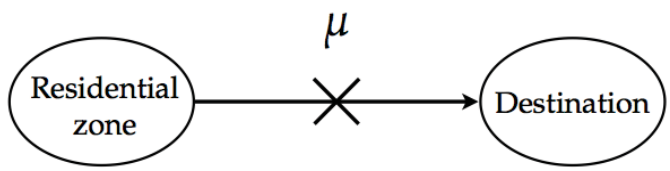


Figure 5.1 Single bottleneck network

5.2 Model

5.2.1 Networks

In this chapter, we consider discrete time dynamic traffic flows on a single bottleneck network where an origin (e.g., residential zones) is connected to a destination (Figure 5.1). All of the road users must pass through a bottleneck to make trips. This bottleneck is represented by a point queue model with constant capacity μ. The time interval to which we assign the dynamic flow is large enough and is divided into small intervals. Each time interval is denoted by $t \in T$. In addition to the aforementioned within-day traffic assignment, this chapter considers users' dynamic decision-making during the periods leading up to a trip day: the problem here involves the determination of when the user decides to take a trip (i.e., an optimal stopping problem). We assume that there are only two periods for users to make a decision, the prior day $m = 0$ and the trip day $m = 1$.

5.2.2 Agents

The road manager aims to alleviate traffic congestion in the network and maximize the social surplus. To achieve this, the manager regulates the traffic flow rates entering each bottleneck in the network using time-dependent network permits. The precise definition and setup of the network permit system with multiple purchase opportunities are described in Subsection 5.2.3.

Each atomic user $i \in \mathcal{N}$ makes, at most, a trip on day $m = 1$ from the origin to the destination in the network. The user chooses a destination arrival time on the trip day during the decision-making period to maximize his utility. A detailed definition of utility is given in Subsection 5.2.4. Under the system of tradable network permits, each user must purchase

a permits corresponding to the user's chosen destination arrival time through two period trading markets.

5.2.3 Tradable network permits with multiple purchase opportunities

We assume that the number of permits issued for the bottleneck in each time is equal to or less than the bottleneck capacity μ. Under this setting, the arrival flow rate at a bottleneck at any time, based on the definition of the scheme, is equal to the number of permits. This implies that we can completely eliminate the occurrence of queuing congestion.

In this chapter, we consider the case in which network permits for the trip day are sold both in the trading market on day $m = 0$ and in the trading market on day $m = 1$ (hereafter, we refer to these as the future market and the spot market respectively). Therefore, the road manager needs to determine the number of permits sold for each market μ_t^m out of the total number of issued permits μ; that is

$$\mu_t^1 + \mu_t^0 \leq \mu \qquad \forall t \in T. \tag{5.1}$$

The permits are put on sale by the road manager. In the trading markets, the prices and the allocation of time-dependent permits are determined through an auction mechanism. Note that we assume that there is no resale and cancellation of the permits. The detailed trading rules are given in Section 5.4.

5.2.4 User valuation and utility

We suppose that each user i has a valuation $v_{i,t}^m$ of destination arrival time t on the trip day; this valuation depends on the purchase period m. As an example of this type of situation, a person can have multiple activity plans that depend on purchase periods: a person will conduct a meeting with a business partner, if he schedules a trip in advance (i.e., $m = 0$); otherwise, he meets with another partner[3]. In this case, the valuation $v_{i,t}^m$ represents the utility of the activity corresponding to purchase period m. We should note that the valuations of each user are private information that cannot be observed by the road manager (or other users).

[3] Another example is a situation where people needs to make a reservation in advance to carry out an activity.

Each user is assumed to have quasi-linear utility $u_{i,t}^m$. Hence, the utility of each user who purchases a permit for destination arrival time t in market m is given by

$$u_{i,t}^m \equiv v_{i,t}^m - p_t^m, \tag{5.2}$$

where p_t^m is the permit purchase cost that determined in an auction.

5.3 System optimal allocation of network permits

In this section, we define the system optimal allocation of network permits to be achieved by a mechanism. Then, by decomposing this problem, we present the design framework for a mechanism to implement the scheme.

5.3.1 System optimal allocation problem

The objective of the mechanism proposed in this chapter is to maximize a social surplus. The social surplus is given as the sum of the users' valuations in every period because the users' valuations in each period are equal to the utility value obtained on the trip day. Thus, we formulate an optimization problem [SO] to determines the system optimal allocation of network permits:

$$SS := \max_{\mathbf{y}^0,\mathbf{y}^1,\mathbf{z}^0,\boldsymbol{\mu}^0,\boldsymbol{\mu}^1} \sum_{i\in\mathcal{N}} \sum_{t\in T} v_{i,t}^0 y_{i,t}^0 + \sum_{i\in\mathcal{N}} \sum_{t\in T} v_{i,t}^1 y_{i,t}^1 \tag{5.3}$$

subject to

$$\mu_t^0 + \mu_t^1 \leq \mu \qquad \forall t \in T \tag{5.4}$$

$$\sum_{i\in\mathcal{N}} y_{i,t}^m \leq \mu_t^m \qquad \forall t \in T,\ \forall m \in \{0,1\} \tag{5.5}$$

$$1 - \sum_{t\in T} y_{i,t}^0 = z_i^0 \qquad \forall i \in \mathcal{N} \tag{5.6}$$

$$\sum_{t\in T} y_{i,t}^1 \leq z_i^0 \qquad \forall i \in \mathcal{N} \tag{5.7}$$

$$y_{i,t}^m, z_i^m \in \{0,1\} \qquad \forall i \in \mathcal{N},\ \forall t \in T,\ \forall m \in \{0,1\} \tag{5.8}$$

$$\mu_t^m \geq 0 \qquad \forall t \in T,\ \forall m \in \{0,1\}. \tag{5.9}$$

where $y_{i,t}^m$ is 1 if a network permit corresponding to destination arrival time t in market m is allocated to user i and zero otherwise. A discrete variable z_i^0 represents whether or not user i has an option to purchase network permits at the end of period $m = 0$.

The problem [SO] finds an efficient permit allocation $\mathbf{y}^m := (y_{i,t}^m)_{\forall i,t}$; the optimal purchase timing for each user $\mathbf{z}^0 := (z_i^0)_{\forall i}$; and the optimal number of permits to sell in each period market $\boldsymbol{\mu}^m := (\mu_t^m)_{\forall t}$. More specifically, the first and second terms of the objective function (5.3) represent the social surplus obtained in the future market and the spot market respectively. The first constraint (5.4) is the condition that the total number of permits sold in these markets does not exceed the bottleneck capacity. The second constraint (5.5) is the capacity (not the bottleneck capacity) constraint for each market. The third and fourth constraints (5.6), (5.7) result in the *unit-demand* condition: each user purchases at most one permit. Constraints (5.8) and (5.9) are the 0–1 integer constraint and nonnegative constraint, respectively.

As is apparent from the above constraint conditions, the problem [SO] is a linear mixed-integer problem. Although the mixed-integer problems are difficult to solve in general, we obtain an optimal integral solution by solving a linear relaxation of the problem [SO] if the bottleneck capacity μ is integer-valued. This is because the constraint matrices satisfy *totally unimodularity* (see Appendix 5.A for the proof). Therefore, $y_{i,t}^m \geq 0$ and $z_i^0 \geq 0$ replace constraints (5.8), assuming that the bottleneck capacity is given as an integer.

5.3.2 Decomposition of the system optimal allocation problem

The problem [SO] optimizes three types of unknown variables, $\mathbf{y}^m$, $\mathbf{z}^0$, and $\boldsymbol{\mu}^m$, in a simultaneous manner. However, such a simultaneous optimization is difficult unless the manager accurately obtains users' private information $(\mathbf{v}^0, \mathbf{v}^1)$. Hence, we decompose the problem into the following two problems by applying the Benders decomposition principle to the problem [SO]:

1. Sub-problem: a problem that determines the allocation of permits, $\mathbf{y}^m$ (and $\mathbf{z}^0$) ,

2. Master problem: a problem that adjusts the number of permits sold in each market, $\boldsymbol{\mu}^m$.

Sub-problem: Network permits allocation problem

Suppose that the number of permits sold in each period market is fixed. Then a network permits allocation problem [$\mathrm{SO}_{\mathrm{sub}}$-P] that maximizes the social surplus is formulated as

$$\max_{\mathbf{y}^0,\mathbf{y}^1,\mathbf{z}^0\geq\mathbf{0}} \sum_{i\in\mathcal{N}}\sum_{t\in T} v^0_{i,t}y^0_{i,t} + \sum_{i\in\mathcal{N}}\sum_{t\in T} v^1_{i,t}y^1_{i,t} \tag{5.10}$$

subject to Eq.(5.5), Eq. (5.6), and Eq.(5.7).

This sub-problem [$\mathrm{SO}_{\mathrm{sub}}$-P] has two meanings. First, it is obvious that its optimal solution is equal to that of the problem [SO] if the number of permits for each market is given appropriately. Second, the sub-problem [$\mathrm{SO}_{\mathrm{sub}}$-P] is equivalent to an optimization problem for a market equilibrium in which each user both chooses both a destination arrival time and a purchase period.

To show the second point more precisely, we consider the Kuhn-Tucker conditions for the sub-problem [$\mathrm{SO}_{\mathrm{sub}}$-P]:

$$\begin{cases} \sum_{i\in\mathcal{N}} y^m_{i,t} = \mu^m_t & \text{if} \quad p^m_t > 0 \\ \sum_{i\in\mathcal{N}} y^m_{i,t} \leq \mu^m_t & \text{if} \quad p^m_t = 0 \end{cases} \qquad \forall t\in T, \forall m\in\{0,1\} \tag{5.11}$$

$$\begin{cases} v^m_{i,t} - p^m_t = \pi^m_i & \text{if} \quad y^m_{i,t} = 1 \\ v^m_{i,t} - p^m_t \leq \pi^m_i & \text{if} \quad y^m_{i,t} = 0 \end{cases} \qquad \forall i\in\mathcal{N},\ \forall t\in T,\ \forall m\in\{0,1\} \tag{5.12}$$

$$\begin{cases} \pi^1_i = \pi^0_i & \text{if} \quad z^0_i = 1 \\ \pi^1_i \leq \pi^0_i & \text{if} \quad z^0_i = 0 \end{cases} \qquad \forall i\in\mathcal{N} \tag{5.13}$$

$$\begin{cases} \sum_{t\in T} y^1_{i,t} - z^0_i = 0 & \text{if} \quad \pi^1_i > 0 \\ \sum_{t\in T} y^1_{i,t} - z^0_i \leq 0 & \text{if} \quad \pi^1_i = 0 \end{cases} \qquad \forall i\in\mathcal{N} \tag{5.14}$$

+ equality constraint (5.6)

where $\mathbf{p}^m := (p^m_t)_{\forall t}$ and $\boldsymbol{\pi}^m := (\pi^m_i)_{\forall i}$ are the optimal Lagerange multipliers for constraints (5.5) and (5.7) respectively. The optimality conditions, (5.11)–(5.14), can be interpreted as the market equilibrium by regarding the Lagrange multipliers $\mathbf{p}^m$ and $\boldsymbol{\pi}^m$ as equilibrium permit prices and option values in market m. Specifically, Eq.(5.11) represents the market-clearing condition, and Eqs.(5.12)–(5.14) are interpreted as the user choice equilibrium (i.e., arrival time and purchase period) conditions when permit prices are given. Hence, the following proposition holds:

Proposition 5.1 *Assume that the number of permits sold in each period market is fixed. We also assume that the trading markets are perfectly competitive. Then, the equilibrium permits allocation pattern that is realized under the tradable network permits with multiple purchase opportunities that maximizes the social surplus defined by* (5.10).

Proof We first confirm the demand-supply equilibrium condition for each destination arrival time in each period corresponding to the optimality condition (5.11). This correspondence is clear if the Lagerange multipliers $(\mathbf{p}^0, \mathbf{p}^1)$ are regarded as permit prices (i.e., competitive equilibrium prices) in the future market and the spot market, respectively.

We then show that the user choice equilibrium conditions are equivalent to the optimality conditions (5.12)–(5.14). For given permit prices, each user determines a destination arrival time and a purchase period so as to maximize his/her utility:

$$\max_{m\in\{0,1,2\}} \max_{t\in T} . \{v_{i,t}^m - p_t^m\} \qquad \forall i \in \mathcal{N}. \tag{5.15}$$

For convenience, we use $m = 2$ to show people that do not purchase any permits; their payoff is zero. At this time, the Lagrange multipliers π_i^0 and π_i^1 can be viewed as optimal value functions of the problem (5.15) in period $m \in \{0, 1\}$:

$$\pi_i^m := \max_{\tau\geq m} \max_{t\in T} . \{v_{i,t}^\tau - p_t^\tau\} \qquad \forall m = \{0, 1\},\ \forall i \in \mathcal{N}. \tag{5.16}$$

By applying the DP (*dynamic programming*) principle, we obtain the optimal decision-making at the beginning of each market. More specifically, the optimal choice pair (m^*, k^*) can be obtained by "backward induction."

We first solve the following choice problem (i.e., Hamilton-Jacobi-Bellman equation) in period $m = 1$ (i.e., the trip day):

$$\pi_i^1 = \max\left\{\max_{t\in T}\left\{v_{i,t}^1 - p_t^1\right\}, \pi_i^2\right\}, \tag{5.17}$$

where $\pi_i^2 = 0$. By using the optimal choice function (5.17), the optimal choice in period $m = 0$ is given by

$$\begin{cases} \max_{t\in T}\left\{v_{i,t}^0 - p_t^0\right\} \geq \pi_i^1 \\ \Leftrightarrow \text{ purchase: } z_i^0 = 0 \text{ and } y_{i,t}^0 > 0,\ \exists t \in T \\ \max_{t\in T}\left\{v_{i,t}^0 - p_t^0\right\} < \pi_i^1 \\ \Leftrightarrow \text{ non-purchase: } z_i^0 = 1 \text{ and } y_{i,t}^0 = 0,\ \forall t \in T. \end{cases} \tag{5.18}$$

Then, we have the choice problem in period $m = 0$:

$$\pi_i^0 = \max\left\{\max_{t\in T}\left\{v_{i,t}^0 - p_t^0\right\}, \pi_i^1\right\}. \tag{5.19}$$

Let us now confirm the equivalence between the user choice equilibrium conditions (5.17) and (5.19) and the optimality conditions (5.12)–(5.14). The equilibrium condition (5.17) in period $m = 1$ can be rewritten as

$$\pi_i^1 \geq \pi_i^2 \tag{5.20}$$

$$\pi_i^1 \geq \max_{t\in T}\{v_{i,t}^1 - p_t^1\}. \tag{5.21}$$

We see that Eq.(5.21) corresponds to the optimality (5.12); Eq.(5.20) corresponds to the optimality condition (5.14). In the same way, the equilibrium condition (5.19) in period $m = 0$ corresponds to the optimality conditions (5.12), (5.13). The above discussion shows that both the equilibrium conditions and the optimality conditions have the exactly the same form. Therefore, equilibrium permits allocation pattern is equal to that obtained by solving the sub-problem [$\mathrm{SO}_{\mathrm{sub}}$-P].

Proposition 5.1 states that the optimal solution of the problem [$\mathrm{SO}_{\mathrm{sub}}$-P] can be achieved as a result of distributed behavior of users. However, in order to hold this proposition, It is necessary to assume that users do not play strategic behaviors that affect prices (e.g., perfectly competitive markets). For example, we suppose that a user manipulates a permit price. This strategic behavior may decrease other users' utility, which results in a failure to achieve the system optimal state. Hence, we have to design a mechanism in which each user has no incentive to exhibit a strategic behavior.

Master problem: Adjustment of the number of permits sold for each period market

The problem of adjusting the number of permits sold for each market is obtained as a Benders master problem. Thus, the problem is formulated by exploiting a dual problem of the sub-problem [$\mathrm{SO}_{\mathrm{sub}}$-P] (the derivation of this problem is shown in Appendix 5.B):

$$\max_{\boldsymbol{\mu}^0,\boldsymbol{\mu}^1\geq\mathbf{0}} . \sum_{t\in T}\mu_t^0 p^0(\boldsymbol{\mu}_t) + \sum_{t\in T}\mu_t^1 p^1(\boldsymbol{\mu}_t) + \pi^0(\boldsymbol{\mu}) \tag{5.22}$$

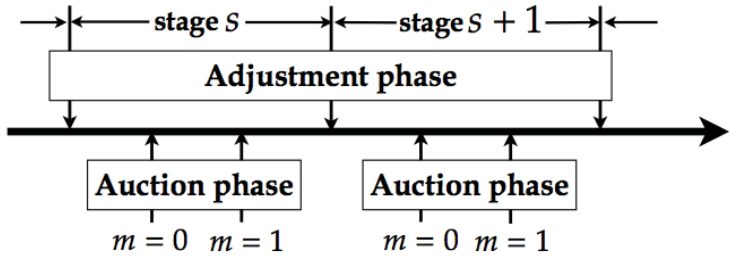


Figure 5.2 Procedures for the proposed mechanism

subject to Eq.(5.4),

$$
\begin{aligned}
&(\mathbf{p}(\boldsymbol{\mu}), \pi^0(\boldsymbol{\mu})) \\
&= \arg\min_{\mathbf{p},\boldsymbol{\pi}\geq\mathbf{0}} \sum_{t\in T} \mu_t^0 p_t^0 + \sum_{t\in T} \mu_t^1 p_t^1 + \pi^0 && (5.23)\\
&\text{subject to} \\
&\pi_i^m \geq v_{i,t}^m - p_t^m \qquad \forall i \in \mathcal{N},\ \forall t \in T,\ \forall m \in \{0,1\} && (5.24)\\
&\pi_i^0 \geq \pi_i^1 \qquad \forall i \in \mathcal{N} && (5.25)\\
&\text{where} \quad \pi^0(\boldsymbol{\mu}) := \sum_{i\in\mathcal{N}} \pi_i^0(\boldsymbol{\mu}), \quad \pi^0 := \sum_{i\in\mathcal{N}} \pi_i^0.
\end{aligned}
$$

where Eqs.(5.23)–(5.25) is the dual problem [$\mathrm{SO}_{\mathrm{sub}}$-D] of the sub-problem [$\mathrm{SO}_{\mathrm{sub}}$-P], and $(\mathbf{p}(\boldsymbol{\mu}), \pi^0(\boldsymbol{\mu}))$ is the optimal solution of the dual problem for a parameter $\boldsymbol{\mu}$; that is, it is an extreme point of the convex feasible region Ω that consists of the constraints (5.24) and (5.25). By using the extreme points, the master problem is finally represented as

$$
\max_{\boldsymbol{\mu}^0,\boldsymbol{\mu}^1\geq\mathbf{0}} \left[\min_{(\mathbf{p}(s),\pi^0(s))\in V(\Omega)} \sum_{t\in T} \mu_t^0 p_t^0(s) + \sum_{t\in T} \mu_t^1 p_t^1(s) + \pi^0(s) \right] \tag{5.26}
$$

subject to Eq.(5.4).

The master problem is equivalent to the problem [SO] if all of the extreme points are known. Because it is difficult to obtain the extreme points in advance, we employ an evolutionary approach of generating an extreme point (or solving the sub-problem).

5.3.3 Design framework of a mechanism for implementing the tradable network permits with multiple purchase opportunities

From the above discussion, in order to implement the scheme, it is necessary to design a mechanism for solving the master problem and the sub-problem iteratively (Figure 5.2). More specifically, we have to design

1. a dynamic auction mechanism to implement the dynamic user choice equilibrium,

2. an adjustment rule for the number of permits sold in each market to converge to the optimal one by using the extreme points.

We call the former the "auction phase" and the latter the "adjustment phase," whereas one iteration of both phases is a "stage." Each stage is denoted by s. In addition, we assume that each user behaves myopically and makes his/her choice so as to maximize the utility defined at each stage s (i.e., a myopic best response strategy). In the following two sections, we concretely design mechanisms and clarify the desired properties.

5.4 Auction phase

Assuming that the number of permits sold in each period market $(\boldsymbol{\mu}^0(s), \boldsymbol{\mu}^1(s))$ is fixed, we showed that the sub-problem $[\mathrm{SO}_{\mathrm{sub}}]$ is equivalent to the market and the user choice equilibrium state (Proposition 5.1). However, the problem simultaneously determines the permits allocation variables of both the future market and the spot market: it cannot represent an actual sequence of the multiple period markets; the network permits allocation in the future market is determined before that in the spot market. Hence, we first show that the problem $[\mathrm{SO}_{\mathrm{sub}}]$ can be decomposed to be consistent with the actual sequence of the markets (or time) under a certain condition. By doing this, we can apply a standard incentive-compatible multi-item auction to each market. Moreover, in this auction, the optimal market choice of each user is guaranteed.

5.4.1 Time decomposition of multiple period markets

Let us rewrite the objective function of the dual sub-problem [SO_{sub}-D]:

$$\sum_{t\in T}\mu_t^0(s)p_t^0+\sum_{i\in\mathcal{N}}(\pi_i^0-\pi_i^1)+\sum_{t\in T}\mu_t^1(s)p_t^1+\sum_{i\in\mathcal{N}}\pi_i^1. \tag{5.27}$$

By using this equation, we can transform the problem [SO_{sub}-D] into the following equivalent bi-level problem:

[SO_{sub}-D^0]

$$\min_{\mathbf{p}^0\geq\mathbf{0},\boldsymbol{\pi}^0}\sum_{t\in T}\mu_t^0(s)p_t^0+\sum_{i\in\mathcal{N}}(\pi_i^0-\pi_i^1) \tag{5.28}$$

subject to

$$\pi_i^0\geq v_{i,t}^0-p_t^0 \qquad \forall i\in\mathcal{N},\ \forall t\in T \tag{5.29}$$

$$\pi_i^0\geq\pi_i^1 \qquad \forall i\in\mathcal{N} \tag{5.30}$$

[SO_{sub}-D^1]

$$\min_{\mathbf{p}^1,\boldsymbol{\pi}^1\geq\mathbf{0}}\sum_{t\in T}\mu_t^1(s)p_t^1+\sum_{i\in\mathcal{N}}\pi_i^1 \tag{5.31}$$

subject to

$$\pi_i^1\geq v_{i,t}^1-p_t^1 \qquad \forall i\in\mathcal{N},\ \forall t\in T. \tag{5.32}$$

The upper level problem [SO_{sub}-D^0] represents the permits allocation problem in the future market; the lower level problem [SO_{sub}-D^1] represents the permits allocation problem in the spot market. From this bi-level problem, we see the following two facts: (i) the spot market can be treated as independent of the future market; (ii) the future market also can be treated independently if the option values π_i^1 are given. In other words, if each user can know his/her own option value in the spot market, the multiple period markets can be decomposed to be consistent with time sequence. From now on, we discuss the auction mechanism, assuming the condition holds.

Let us introduce a new variable $\hat{v}^0_{i,t} := v^0_{i,t} - \pi^1_i$; it represents the"*net valuation*" (i.e., a truthful valuation minus the option value). Then, the problem [SO_{sub}-D^0] is equal to the following problem with a new unknown variable $\hat{\pi}^0_i := \pi^0_i - \pi^1_i$:

$$\min_{\mathbf{p}^0, \hat{\boldsymbol{\pi}}^0 \geq \mathbf{0}} \sum_{t \in T} \mu^0_t(s) p^0_t + \sum_{i \in \mathcal{N}} \hat{\pi}^0_i \tag{5.33}$$

subject to

$$\hat{\pi}^0_i \geq \hat{v}^0_{i,t} - p^0_t \qquad \forall i \in \mathcal{N},\ \forall t \in T. \tag{5.34}$$

This problem has the same form as the problem [SO_{sub}-D^1] that is described by using $\hat{v}^1_{i,t} := v^1_{i,t}$, $\hat{\pi}^1_i := \pi^1_i$. Finally, an independent assignment problem for each period market is given as the primal problem [SO_{sub}-P^m] of the dual problem (5.33):

$$\max_{\mathbf{y}^m \geq \mathbf{0}} \sum_{i \in \mathcal{N}} \sum_{t \in T} \hat{v}^m_{i,t} y^m_{i,t} \tag{5.35}$$

subject to

$$\sum_{i \in \mathcal{N}} y^m_{i,t} \leq \mu^m_t(s) \qquad \forall t \in T \tag{5.36}$$

$$\sum_{t \in T} y^m_{i,t} \leq 1 \qquad \forall i \in \mathcal{N}. \tag{5.37}$$

where the constraint (5.37) is the unit-demand condition. From the above discussion, we find that the sub-problem [SO_{sub}-D] can be solved to be consistent with time sequence if each user reports *net valuations* truthfully in the future market.

5.4.2 Auction mechanism for multiple period markets

Because the problem [SO_{sub}-P^m] is the standard assignment problem, we can apply the various incentive compatible auction mechanisms to it (e.g., the VCG mechanism). Now, let us employ the proxy DGS auction (shown in Chapter 4 and Appendix C) for implementing the multiple period markets. Then, the following proposition holds:

Proposition 5.2 *Assume that the number of permits sold in each period market is fixed. We also assume that each user knows his/her own option value realized in the spot market. Then, the proxy DGS auction mechanism for each period market is strategy-proof and achieves an efficient network permits allocation.*

Proof Each market can be treated independently when each user knows his/her own option value realized in the spot market. Therefore, each market is strategy-proof from Demange et al. (1986) and Parkes and Ungar (2000b). The allocation of the network permits of each market is the optimal solution of the *decomposed* sub-problem [$\mathrm{SO}_{\mathrm{sub}}$-$\mathrm{P}^m$]. On the other hand, the (undecomposed) sub-problem [$\mathrm{SO}_{\mathrm{sub}}$-P] maximizes the social surplus under the condition that the number of permits sold for each market is fixed. Because the undecomposed sub-problem [$\mathrm{SO}_{\mathrm{sub}}$-P] and the decomposed sub-problems [$\mathrm{SO}_{\mathrm{sub}}$-$\mathrm{P}^m$] are equivalent, the network permits allocation achieved by the DGS auction also maximizes the social surplus.

Furthermore, we reveal that the user's market choice is optimal by using the Proposition 5.2. Because the future market is strategy-proof, each user's allocation of permits is given by

$$\begin{cases} \hat{v}^0_{i,t} - p^0_t = \hat{\pi}_i^{\,0} & \text{if} \quad y^0_{i,t} = 1 \\ \hat{v}^0_{i,t} - p^0_t \leq \hat{\pi}_i^{\,0} & \text{if} \quad y^0_{i,t} = 0 \end{cases} \qquad \forall i \in \mathcal{N}. \tag{5.38}$$

Thus, for all users,

$$\begin{aligned} & \hat{\pi}_i^{\,0} = \max\left\{\max_{t\in T}\left\{\hat{v}^0_{i,t} - p^0_t\right\}, 0\right\} \\ \Leftrightarrow\ & \pi^0_i - \pi^1_i = \max\left\{\max_{t\in T}\left\{v^0_{i,t} - \pi^1_i - p^0_t\right\}, 0\right\} \\ \Leftrightarrow\ & \pi^0_i = \max\left\{\max_{t\in T}\left\{v^0_{i,t} - p^0_t\right\}, \pi^1_i\right\} \end{aligned} \tag{5.39}$$

holds. This equation is equal to the optimal market choice condition in period $m = 0$ (i.e., Eq.(5.19)). That is, truthful reporting of the net valuations in the future market simultaneously means choosing a purchase period so as to maximize the utility of each user.

Proposition 5.3 *Assume that each user knows his/her own option value realized in the spot market. Then, the market choices of all of the users that participate in the multiple period markets are optimal.*

5.5 Adjustment phase of the number of permits sold for each period market

5.5.1 Adjustment rule

In the adjustment phase, the road manager generates a new extreme point $(\mathbf{p}(s), \pi^0(s))$ from the information obtained in the multiple period markets, and then determines the number of permits sold in each period market in the next stage. The prices $(\mathbf{p}^0, \mathbf{p}^1)$ can be obtained directly in the auction phase for each period market. The total payoff $\pi^0(s)$, on the other hand, is computed in an indirect way. In the proxy DGS auction, because each user reports true (net) valuations to the proxy agent about the permits that they are interested in, the manager can obtain his/her winning valuations $\hat{v}_{i,t*}^{m*}$ through the agent. By using this information, the manager calculates the total payoff from the duality theorem (see Appendix 5.C for more details):

$$\pi^0(s) = \sum_{i \in \mathcal{N}} \hat{v}_{i,t*}^{m*} - \sum_{t \in T} \mu_t^0(s) p_t^0 - \sum_{t \in T} \mu_t^1(s) p_t^1. \tag{5.40}$$

Note here that the extreme point $(\mathbf{p}(s), \pi^0(s))$ consists of aggregate information.

After generating the extreme point, the road manager considers the set of extreme points until stage s:

$$V'(\Omega) := \left\{ (\mathbf{p}(1), \pi^0(1)), \ldots, (\mathbf{p}(s), \pi^0(s)) \right\} \subseteq V(\Omega) \,.$$

Then, the road manager adjusts the number of permits sold in each period market by solving the following optimization problem:

$$\max_{\boldsymbol{\mu}^0, \boldsymbol{\mu}^1 \geq \mathbf{0}} \left[\min_{(\mathbf{p}(s), \pi^0(s)) \in V'(\Omega)} \sum_{t \in T} \mu_t^0 p_t^0(s) + \sum_{t \in T} \mu_t^1 p_t^1(s) + \pi^0(s) \right] \tag{5.41}$$

subject to Eq.(5.4).

The solution to this problem is the number of permits sold in the each market at the next stage, $\boldsymbol{\mu}(s+1)$. Unlike the master problem (5.26), this problem (5.41) uses a subset of the extreme points $V'(\Omega) \subseteq V(\Omega)$, which produce an upper bound on the optimal value of the problem [SO]. Moreover, the problem can be reduced to the following linear program:

$$\max_{\theta, \boldsymbol{\mu}^0, \boldsymbol{\mu}^1 \geq 0} \theta \tag{5.42}$$

subject to Eq.(5.4)

$$\theta \leq \sum_{t \in T} \mu_t^0 p_t^0(s) + \sum_{t \in T} \mu_t^1 p_t^1(s) + \pi^0(s) \qquad \forall(\mathbf{p}(s), \pi^0(s)) \in V'(\Omega). \tag{5.43}$$

Thus, the problem can be solved in a very efficient way.

5.5.2 Convergence of whole mechanism

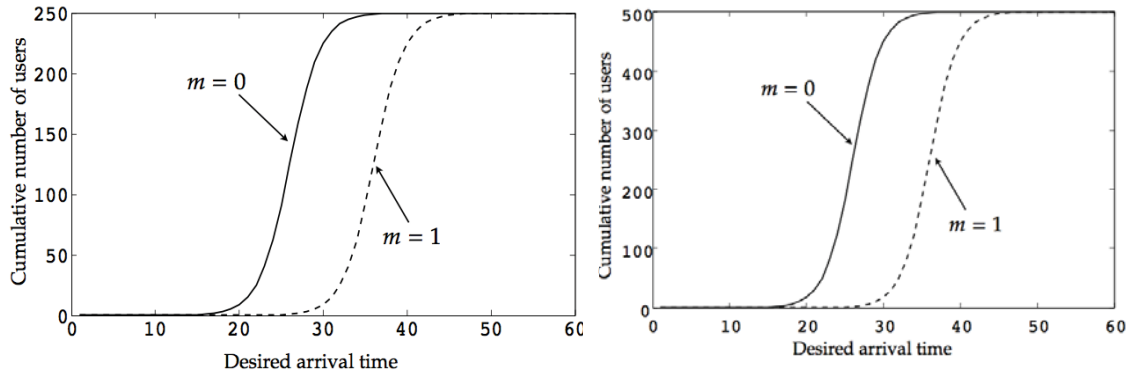
The whole mechanism combining the dynamic auction and adjustment rule corresponds to the Benders decomposition algorithm. The algorithm terminates (i.e., converges to an optimal solution) when the upper bound θ is equal to the optimal objective value of the sub problem (i.e., the social surplus achieved in the auction phase); otherwise, a new adjustment phase begins. Furthermore, a new extreme point is always generated in the auction phase before the procedure terminates, and the set of extreme points is finite. Hence, we have the following convergence result for the whole mechanism.

Proposition 5.4 *The proposed mechanism combining the dynamic auction and adjustment rule achieves the optimal permits allocation pattern in a finite number of iterations.*

Proof Because the decomposed sub-problem [$\mathrm{SO}_{\mathrm{sub}}$-$\mathrm{D}^m$] is bounded (from below), an extreme point always generated at every stage. Therefore, the proposed mechanism corresponds to the Benders decomposition algorithm excluding the step for the case where extreme rays are generated. For a complete proof see, for example, Lasdon (1970).

5.6 Numerical experiments

In the preceding section of this chapter, we described the design of the implementation mechanism for multiple period markets with a *fixed* population (i.e., the same users participate in the markets at every stage s). Nevertheless, concerning the adjustment rule without requiring individual information, the mechanism may work effectively for a *dynamic* population case in which markets participants change over stage but the valuation distribution of the markets participants is fixed. This section examines the validity of the above conjecture by numerical experiments.



Case (i) Case (ii)

Figure 5.3 Desired arrival time distributions

5.6.1 Fixed population case

Before discussing the dynamic population case, we show numerical examples for the fixed population case for clarifying convergence properties (e.g., the speed of convergence) of the proposed mechanism. In the numerical examples, the two cases with different population sizes are considered: (i) 250 users with 60 arrival times and (ii) 500 users with 60 arrival times. The bottleneck capacity μ is 5 vehicles/unit time for the case (i); that is 10 vehicles/unit time for the case (ii). The valuation distribution of the population is given by the following equation:

$$v_{i,t}^{m} = w_i - \max\{\alpha(t_i^m - t), \beta(t - t_i^m)\} \tag{5.44}$$

where w_i represents the trip utility of user i, which is randomly generated from a normal distribution. The second term of the equation represents a linear schedule delay function with identical values for the positive earliness rate α and lateness rate β; t_i^m is the desired arrival time of user i in period m, which is randomly generated from the desired arrival time distribution shown in Figure 5.3. Under these setting, each user's valuations changes over period depend on his/her desired arrival time.

Table 5.1 Number of iterations required to converge (100 samples)

Case (i)		Case (ii)	
	[iterations]		[iterations]
Average (100%)	7.18	Average (100%)	11.70
Average (99%)	4.00	Average (99%)	6.00
Maximum sample (100%)	18	Maximum sample (100%)	22
Minimum sample (100%)	4	Minimum sample (100%)	6

Table 5.1 summarizes the convergence results of 100 samples for each case[4]. According to this table, we find that the social surplus achieved through the auction phase rapidly converges to (close to) the maximum value. Only a small number of iterations are required to achieve 99% of the maximum social surplus, whereas about twice as many are required to reach 100%. To show this more intuitively, the convergence process of the worst case (i.e., the maximum sample for case (ii)) is presented in Figure 5.4. The horizontal axis shows the number of iterations s, and the vertical axis shows the ratio between the achieved social surplus at each stage and the maximum value. Also in this case, the social surplus reaches 99% of the maximum value at six iterations; however, the final convergence is very slow (this phenomenon is well known as the *tailing-off effect*, Lübbecke and Desrosiers, 2005). Note that the effect would be insignificant because the achieved social surplus is high enough. Thus, we conclude that the convergence speed of the proposed mechanism would be fast.

5.6.2 Dynamic population case

We next consider the dynamic population case in which markets participants change between stages but the valuation distribution of the participants is fixed. To estimate the convergence properties, we first show the probability distribution of the maximum social surplus for each case (5000 samples). As indicated in Figure 5.5, the distributions of both cases follow the

[4] To stabilize the convergence process, we here add the box constrains (Martsen, 1975):

$$\mu_t^m(s) - \underline{\gamma} \leq \mu_t^m(s+1) \leq \mu_t^m(s) + \overline{\gamma} \tag{5.45}$$

to the master problem. $\underline{\gamma}, \overline{\gamma}$ are the boxstep parameters.

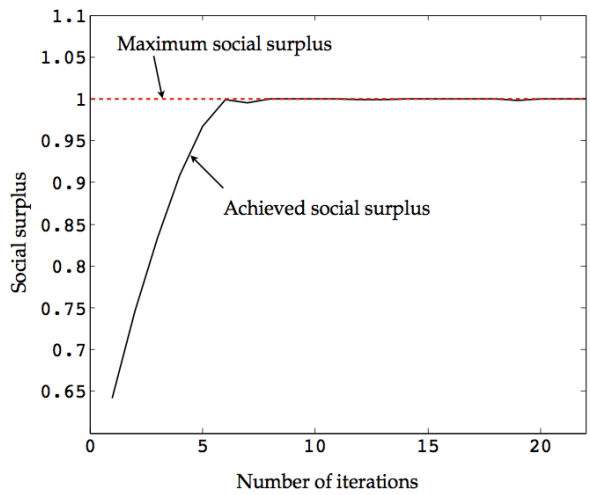
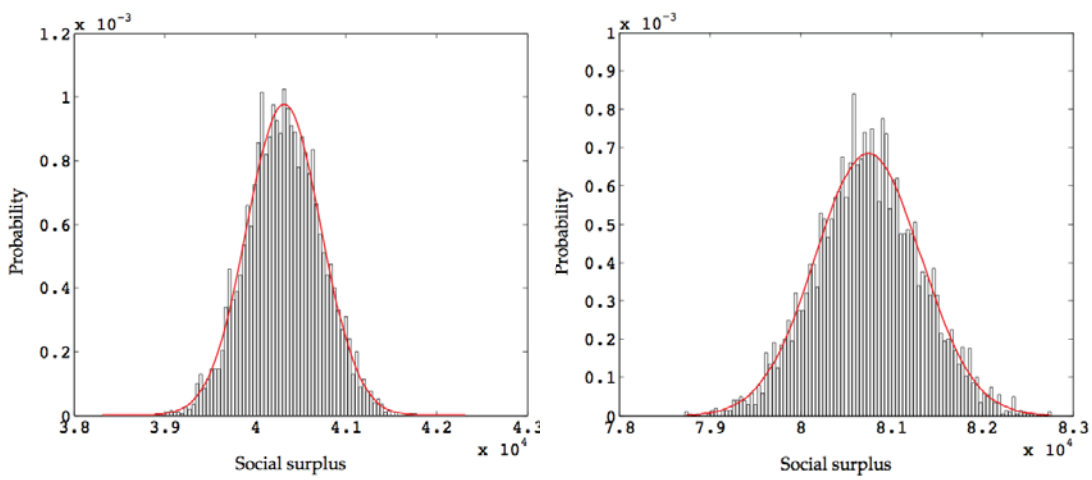
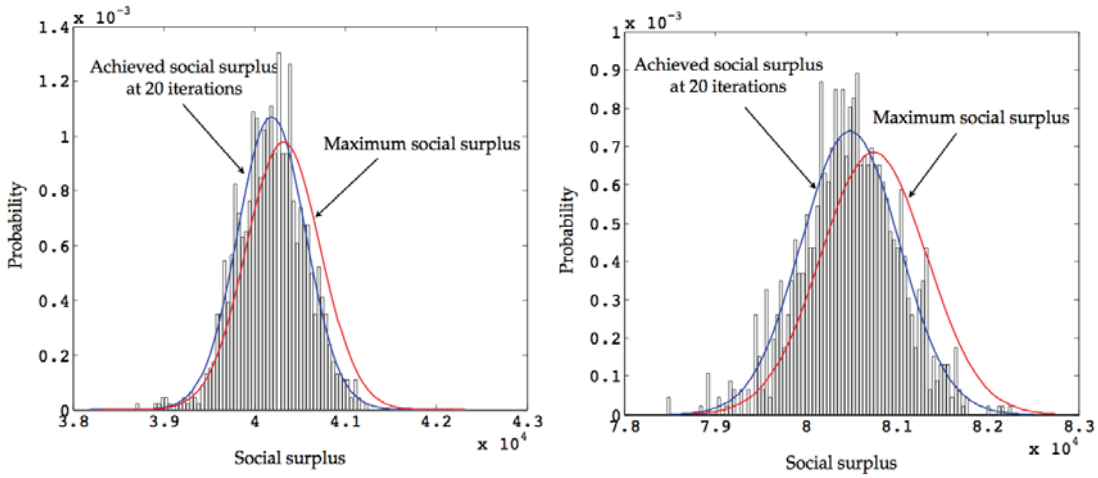


Figure 5.4 Example of convergence process of the proposed mechanism (worst case)



Case (i) Case (ii)

Figure 5.5 Distributions of the maximum social surplus (5000 samples)

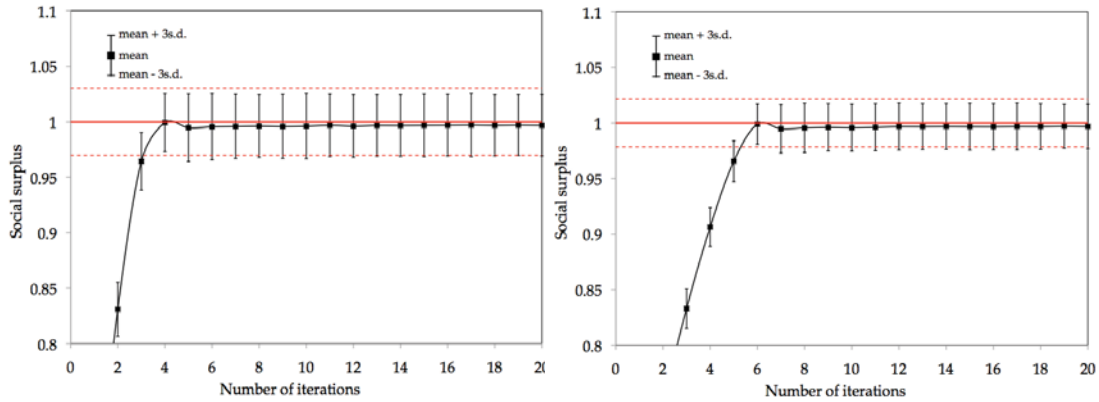


Case (i) Case (ii)

Figure 5.6 Distributions of the achieved social surplus at 20 iterations (1000 samples)

normal distributions (the red lines represent their corresponding normal distributions).

We next show the results of a Monte Carlo simulation of the proposed mechanism. For each case, we generated 1000 sample paths where each user's valuations were randomly chosen from the Eq.(5.44) at each stage s. From the observation in the previous subsection, we set the number of iterations of each sample path at 20. Figure 5.6 depicts the probability distribution of the achieved social surplus at the final stage $s = 20$ for each case. Each blue line represents the normal distribution corresponding to the distribution of the achieved social surplus. From this figure, we can observe that the mean of the achieved social surplus is smaller than that of the maximum social surplus, which means that perfect efficiency is not always obtained. This is because the number of permits sold for each market is optimized by using information available until the current stage and may not be optimal for a new users valuations pattern that arises at the next stage. Nevertheless, the mean of the achieved social surplus reaches 99% of that of the maximum social surplus. It is also worth mentioning that the standard deviation of the achieved social surplus tends to be smaller than that of the maximum social surplus; the lower three sigma levels of both distributions are almost the same. The reason for this is not clear, but the social surplus that consists of a users valuations pattern corresponding to the lower three sigma level may be maximized under any (or many) permit issue pattern. Finally, Figure 5.7 shows that the mean (and three sigma



Case (i) Case (ii)

Figure 5.7 Mean of the achieved social surplus at each iteration (1000 samples)

level) of the achieved social surplus at each iteration. The process of the mean value shows the same tendency as in the case of fixed population. Furthermore, the three sigma levels of the achieved social surplus are almost within that of the maximum social surplus.

From the above discussion, it can be concluded that the proposed mechanism works effectively for the dynamic population case if the valuation distribution of the population does not change. More specifically, the achieved social surplus rapidly converges to close to the maximum value (about 99%), and its standard deviation is small compared to that of the maximum social surplus.

5.7 Conclusion

This chapter considered a situation where network permits for a specific day are sold in multiple period markets and designed a trading mechanism of these markets. We first showed that the system optimal permits allocation for a fixed permit issue pattern is equivalent to the user equilibrium in perfectly competitive markets. This enabled us to decompose the system optimal allocation problem into two sub-problems. We then constructed the mechanism for implementing each sub-problem independently, and proved that the proposed mechanisms have the following desirable properties: (1) the dynamic auction for multiple period markets is strategy-proof and guarantees that the market choice of each user is optimal when each user

knows his/her own option value realized in the market on the trip day; (2) the whole mechanism combining the dynamic auction and the adjustment rule achieves the optimal permits allocation pattern in a finite number of iterations. Finally, we numerically demonstrated that the proposed mechanism works effectively for a dynamic population case.

While it was assumed that each user knows his/her own option value realized in the market on the trip day, the thought behind this assumption is that the user has a prediction formation mechanism based on some learning dynamics. However, it is not obvious what kind of learning dynamics would be suitable. For example, in the field of the evolutionary and learning game theory (e.g., Fudenberg and Levine, 1998; Young, 2004), various learning dynamics were proposed. Therefore, it is necessary to clarify which learning processes encourage an accurate and efficient prediction.

Another important expansion of our model would be to consider the users' dynamic decision-making under uncertainty. Because the proposed mechanism considers important aspects of dynamic allocation problems (i.e., the users' dynamic decision-making and irreversibility of resource allocation), it seems applicable under uncertainty. Thus, generalizing the proposed mechanism to handle uncertainty situations is an important topic for future work.

Appendix 5.A Proof of totally unimodularity of problem [SO]

A totally unimodular (TU) matrix is defined as follows.

Definition 5.1 *An integer matrix* $\mathbf{A}$ *is totally unimodular if any subdeterminant of* $\mathbf{A}$ *is* 0 *or* ± 1.

Then, if a constraint matrix $\mathbf{A}$ is a TU matrix, the following theorem holds:

Theorem 5.1 *Let* $\mathbf{A}$ *be totally unimodular. Then, for any integer vector* $\mathbf{b}$*, extreme points of the following polyhedron:*

$$\{\mathbf{x} : \mathbf{A}\mathbf{x} \leq \mathbf{b}, \mathbf{x} \geq \mathbf{0}\}$$

are integers.

Therefore, a bounded linear program in which the constraint matrix is a TU matrix always produces integer solutions. Well-known problems that have such a constraint matrix are weighted mating problems and network flow problems (e.g., the maximum flow problem, the minimum cost flow problem).

Because the problem [SO] is different from the typical problems, we prove that the constraint matrix of the problem is a TU matrix by using the following sufficient condition (Heller and Tompkins, 1956):

Theorem 5.2 (Heller and Tompkins, 1956) *Let* $\mathbf{A}$ *be a* $0, \pm 1$ *matrix with at most two nonzero entries per column. Then,* $\mathbf{A}$ *is totally unimodular if there is a partition of rows such that*

1. *if two nonzero entries in a column have the same sign, then the rows are partitioned into disjoint sets* T_1, T_2*;*

2. *if the nonzero entries in a column have opposite sign, then the rows are in the same set (i.e.,* T_1 *or* T_2*).*

Let us confirm that the constraint matrices of the problem [SO] satisfy the sufficient condition. We first transpose the unknown variables of the constraints to the left-hand side and partition the constraints as follows:

$$T_1 = \begin{cases} \sum_{i \in \mathcal{N}} y_{i,t}^m - \mu_t^m \leq 0 & \forall t \in T,\ \forall m \in \{0,1\} \\ \mu_t^0 + \mu_t^1 \leq \mu & \forall t \in T \\ 1 - \sum_{t \in T} y_{i,t}^0 - z_i^0 = 0 & \forall i \in \mathcal{N} \\ z_i^0 - \sum_{t \in T} y_{i,t}^1 \leq 0 & \forall i \in \mathcal{N} \end{cases}$$

$$T_2 = \emptyset$$

We let $\mathbf{A}$ be the coefficient matrices of the left-hand side. Then, every entry of $\mathbf{A}$ is 0 or ± 1, and $\mathbf{A}$ has two nonzero entries. In addition, two nonzero entries in every column have opposite signs. Then all of the rows are in T_1; the set T_2 is empty. Thus, the constraint matrix of the problem [SO] is totally unimodular.

Appendix 5.B Derivation of master problem

We first decompose the problem [SO] into the following bi-level problem:

$$\max_{\boldsymbol{\mu}^0,\boldsymbol{\mu}^1\geq\mathbf{0}} . \sum_{i\in\mathcal{N}}\sum_{t\in T} v^0_{i,t}y^0_{i,t}(\boldsymbol{\mu}) + \sum_{i\in\mathcal{N}}\sum_{t\in T} v^1_{i,t}y^1_{i,t}(\boldsymbol{\mu}) \tag{5.46}$$

subject to Eq.(5.4),

$$\begin{aligned}&(\mathbf{y}^0(\boldsymbol{\mu}),\mathbf{y}^1(\boldsymbol{\mu}))\\ &= \arg\max_{\mathbf{y}^0,\mathbf{y}^1,\mathbf{z}^0\geq\mathbf{0}} \sum_{i\in\mathcal{N}}\sum_{t\in T} v^0_{i,t}y^0_{i,t} + \sum_{i\in\mathcal{N}}\sum_{t\in T} v^1_{i,t}y^1_{i,t}\\ &\text{subject to Eq.(5.5), Eq.(5.6), Eq.(5.7).}\end{aligned}$$

where $(\mathbf{y}^0(\boldsymbol{\mu}),\mathbf{y}^1(\boldsymbol{\mu}))$ is an optimal solution of the sub problem for a parameter $\boldsymbol{\mu}$.

From the duality theorem, the optimality value of the objective function of the primal problem [SO_{sub}-P] coincides with that of the dual problem [SO_{sub}-D]:

$$\begin{aligned}Z(\boldsymbol{\mu}) &:= \sum_{i\in\mathcal{N}}\sum_{t\in T} v^0_{i,t}y^0_{i,t}(\boldsymbol{\mu}) + \sum_{i\in\mathcal{N}}\sum_{t\in T} v^1_{i,t}y^1_{i,t}(\boldsymbol{\mu})\\ &= \sum_{t\in T}\mu^0_t p^0(\boldsymbol{\mu}_t) + \sum_{t\in T}\mu^1_t p^1(\boldsymbol{\mu}_t) + \pi^0(\boldsymbol{\mu}).\end{aligned} \tag{5.47}$$

where $\mathbf{p}(\boldsymbol{\mu}),\pi^0(\boldsymbol{\mu})$ is the optimal solution of the dual problem (5.23) for a parameter $\boldsymbol{\mu}$. By using the optimality value function (5.47), the objective function (5.46) is transformed into the function (5.22). Finally, we can obtain the lower level problem of the problem (5.22) by replacing the lower level problem (i.e., problem sub-problem [SO_{sub}-P]) of the problem (5.46) with the dual sub-problem [SO_{sub}-D].

Appendix 5.C Derivation of equation (5.40)

We here derive the total payoff $\pi^0(s)$ by exploiting information, permit prices $(\mathbf{p}^{0*},\mathbf{p}^{1*})$ and wining valuations (or bids) $(\hat{v}^{m*}_{i,t*})_{\forall i}$, which are obtained in the auction phase at each stage s. Note that a single asterisk (*) indicates the optimal value of each variable at each stage (i.e., the value achieved through the auction mechanism).

The social surplus achieved by the auction mechanism is represented as

$$\sum_{m\in\{0,1\}}\sum_{i\in\mathcal{N}}\sum_{t\in T}\hat{v}^m_{i,t}y^{m*}_{i,t} = \sum_{i\in\mathcal{N}}\sum_{t\in T}\hat{v}^{m*}_{i,t^*}. \tag{5.48}$$

From the duality theorem, the optimal value of the objective value of the decomposed sub problem [$\mathrm{SO}_{\mathrm{sub}}$-$\mathrm{P}^m$] coincides with that of the dual problem [$\mathrm{SO}_{\mathrm{sub}}$-$\mathrm{D}^m$]:

$$\sum_{i\in\mathcal{N}}\sum_{t\in T}\hat{v}^m_{i,t}y^{m*}_{i,t} = \sum_{i\in\mathcal{N}}\hat{\pi}^{m*}_i + \sum_{t\in T}\mu^m_t(s)p^{m*}_t. \tag{5.49}$$

By substituting this equation into Eq.(5.48), we have

$$\sum_{i\in\mathcal{N}}\hat{v}^{m^*}_{i,t^*} = \sum_{i\in\mathcal{N}}\hat{\pi}^{0*}_i + \sum_{i\in\mathcal{N}}\hat{\pi}^{1*}_i + \sum_{t\in T}\mu^0_t(s)p^{0*}_t + \sum_{t\in T}\mu^1_t(s)p^{1*}_t. \tag{5.50}$$

We here recall the definitions $\hat{\pi}^0_i := \pi^0_i - \pi^1_i,\ \hat{\pi}^1_i := \pi^1_i$. Then Eq.(5.50) reduces to the Eq. (5.40):

$$\sum_{i\in\mathcal{N}}\hat{v}^{m^*}_{i,t^*} = \pi^0(s) + \sum_{t\in T}\mu^0_t(s)p^{0*}_t + \sum_{t\in T}\mu^1_t(s)p^{1*}_t.$$

Chapter 6

Stochastic convergence of a hybrid scheme of tradable network permits and congestion pricing

Until Chapter 5, we studied the tradable network permits scheme in several situations, considering only queuing congestion. In contrast, this chapter[1] deals with two types of congestion: *queuing* congestion and *flow* congestion. *Queuing* congestion is a negative externality of a queue, while *flow* congestion is a negative externality of the speed decrease arising from an increase in traffic density. Unfortunately, it is difficult to eliminate both congestions by using only tradable network permits scheme, since the mechanisms of two types of congestion are totally different.

To address the problem, we here examine a hybrid scheme of the tradable network permits and congestion pricing. This scheme can simultaneously eliminate both queuing congestion and flow congestion, and does not require detailed user information. Also, Wada and Akamatsu (2010) constructed an evolutionary mechanism of the scheme and demonstrated that traffic flow dynamics arising from the mechanism converge to a system optimal state in the sense that social surplus is maximized.

The above results were obtained in a deterministic situation: agents were assumed to deterministically adjust their behavior in response to the current traffic state. However, agents'

[1] This chapter is based on joint research with Takashi Akamatsu and Takeshi Nagae, presented in *the 15th International Conference of Hong Kong Society for Transportation Studies* (Wada, Akamatsu, and Nagae, 2010).

behavior in practice is stochastic (i.e., they sometimes make sub-optimal choices), and is based on not only the current state of traffic but also experiences of previous trips (i.e., agents learn traffic flow patterns on a day-to-day basis). As such, in the current chapter we dispense with a deterministic model, and instead employ a stochastic learning model. We then show that the evolutionary mechanism of our hybrid scheme can operate robustly in stochastic environments. Specifically, we demonstrate that the stochastic dynamics of the learning process converge to an equilibrium state, and that a traffic flow pattern at equilibrium is efficient in the sense that social surplus is maximized.

In Section 6.1, we describe the preconditions used in the model. In Section 6.2, we define a system optimal state. Section 6.3 presents the framework of the scheme and the user learning model. In Section 6.4, we describe an auction mechanism of the trading markets. Section 6.5 analyzes a stochastic dynamics of the learning process arising from the mechanism. We then demonstrate that the dynamics converges to a system optimal state. Section 6.6 presents our conclusions about the model.

6.1 Model

6.1.1 Networks

This chapter considers a simple road network involving two residential zones and a central business district (CBD), which represents a road system where an urban area is connected with a suburban area (Figure 6.1). Each user commutes from a residential zone to the CBD. This is a basic network in which both queuing congestion and flow congestion exist and interact with each other via users.

We assume that the upstream link has a bottleneck with constant capacity μ (point queue model). In the urban street network, a speed-flow relationship is assumed: the delay of the downstream link means a space-average delay arising in the urban street network. The travel time of the downstream link is given by a strict monotonic function $c(x) : \partial c(x)/\partial x > 0$ where x is the traffic flow per unit of time. We consider time-dependent traffic flow patterns within-day and day-to-day traffic flow dynamics. We then distinguish the day $s \in S$ from the time $t \in T$ within day s.

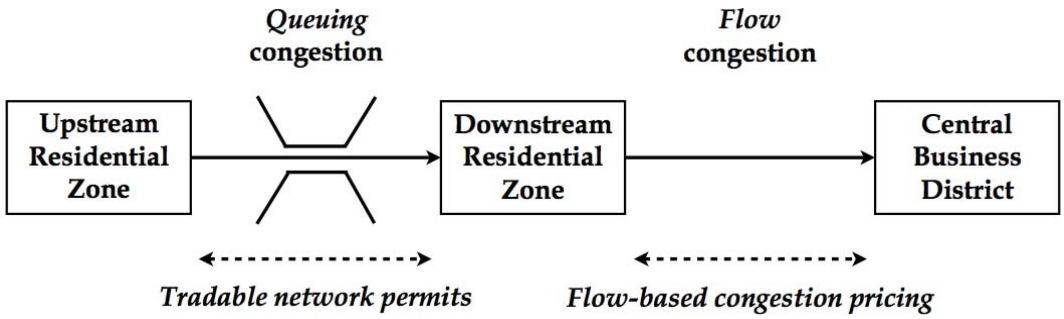


Figure 6.1 Network setting

6.1.2 Agents

The road manager aims to restrain traffic congestion, and to maximize social surplus. To this end, the manager imposes "time-dependent congestion tolls" on users in the downstream link. The manager also regulates the traffic flow entering the upstream link using "time-dependent network permits".

Each user makes a single trip from his or her residential zone to the CBD. The user chooses a destination arrival time based on his or her *prediction* of traffic flow pattern (i.e., users learn traffic flow patterns). Specifically, a downstream user $j \in \mathcal{N}$ chooses a destination arrival time to maximize his or her *predicted* utility. On the other hand, an upstream user $i \in \mathcal{M}$ must purchase a network permit to pass through the upstream bottleneck. This implies that the choice of destination arrival time directly corresponds to purchasing a time-dependent network permit on the trading markets.

Aggregating users' behaviors determines the traffic flow x_t at each time t. The upstream travel demand $Q_{\mathcal{M}}$ and the downstream travel demand $Q_{\mathcal{N}}$ (i.e., trips per day) are given constants.

6.1.3 Transportation demand management schemes

The tradable network permits scheme eliminate queuing congestion, but cannot eliminate flow congestion. We consider a hybrid scheme that combines the tradable network permits with congestion pricing.

Tradable network permits

We assume that the number of permits issued in each unit of time is equal to the traffic capacity of the bottleneck. From the definition of the time-dependent network permits, the inflow rate of the bottleneck is equal to the number of permits issued: hence the inflow rate cannot exceed the traffic capacity, which entails that queuing congestion never occur.

The permits issued for the bottleneck are put on sale by the road manger in a trading market. Each user who would like to pass through the bottleneck must purchase the permit corresponding to user's preferred destination arrival time. In the trading markets, permit prices and the permits allocation are determined through an auction mechanism.

Flow-based congestion pricing

The flow-based congestion pricing (FBCP) scheme relies on the notion of an evolutionary congestion pricing scheme, as proposed by Sandholm (2002). Specifically, a congestion toll $\lambda_t(x_t(s))$ that is imposed on users arriving in the CBD at time t on day s is given by

$$\lambda_t(x_t(s)) = \alpha x_t(s) \frac{\partial c(x_t(s))}{\partial x_t(s)} \qquad \forall t \in T, \tag{6.1}$$

where α is a coefficient that converts travel time to a monetary equivalent, assumed to be the same for all users. In the FBCP scheme the manager sets toll levels based on observed traffic flow alone.

6.1.4 Travel costs and user utility

The transportation cost for a single trip made in the network under the proposed scheme consists of the following costs. The schedule cost is the cost due to the difference between the desired arrival time $\hat{t}$ and the actual arrival time t, which is given by a convex function s_t. To outline the essential aspects of the theory, the desired arrival time is assumed to be the same for all users and is equal to $\hat{t}$. The travel cost on the downstream link is the monetary equivalent of the travel time $c(x_t)$. The congestion toll $\boldsymbol{\lambda}$ is defined in (6.1). The network permit prices $\mathbf{p}$ are determined by an auction mechanism.

A utility that is perceived by each user varies from user to user: each user's utility includes "private information" (e.g., willingness to pay) that is unobservable to other users (we call this information "private utility"). More specifically, each upstream user $i \in \mathcal{M}$ has

stochastic private utility w_t^i, while each downstream user $j \in \mathcal{N}$ has the stochastic private utility w_t^j. The private utilities represent users' bias toward a destination arrival time. The utilities are assumed to follow i.i.d. Gumbel distributions. We can thus define each user's (net) utility as

$$\pi_t^i(x_t, p_t) = w_t^i - h_t(x_t) - p_t \tag{6.2}$$

$$\pi_t^j(x_t) = w_t^j - h_t(x_t) \tag{6.3}$$

$$\text{where} \quad h_t(x_t) \equiv \alpha c(x_t) + s_t + \lambda_t(x_t) \tag{6.4}$$

6.2 System optimal traffic assignment

The hybrid scheme aims to achieve a traffic flow pattern that maximizes a social surplus. To represent a social surplus, we define the users' utility (not net utility) by

$$u_t^i(x_t) \equiv w_t^i - [\alpha c(x_t) + s_t] \tag{6.5}$$

$$u_t^j(x_t) \equiv w_t^j - [\alpha c(x_t) + s_t] \tag{6.6}$$

Note here that this utility does not include user payments to the road manager (the congestion toll and the network permit purchase cost) because the payments are simply income transfers between the users and the road manager. We then formulate an optimization problem [SO] of providing the system optimal traffic assignment:

$$\max_{\mathbf{y} \geq \mathbf{0}} .SS(\mathbf{y}^i, \mathbf{y}^j) \equiv \sum_{t \in T} \sum_{i \in \mathcal{M}} u_t^i(x_t) y_t^i + \sum_{t \in T} \sum_{j \in \mathcal{N}} u_t^j(x_t) y_t^j \tag{6.7}$$

subject to

$$\sum_{t \in T} y_t^i = 1 \qquad \forall i \in \mathcal{M} \tag{6.8}$$

$$\sum_{i \in \mathcal{M}} y_t^i \leq \mu \qquad \forall t \in T \tag{6.9}$$

$$\sum_{t \in T} y_t^j = 1 \qquad \forall j \in \mathcal{N} \tag{6.10}$$

$$x_t = \sum_{i \in \mathcal{M}} y_t^i + \sum_{j \in \mathcal{N}} y_t^j \qquad \forall t \in T \tag{6.11}$$

where y_t is a variable that represents the assignment of each user.

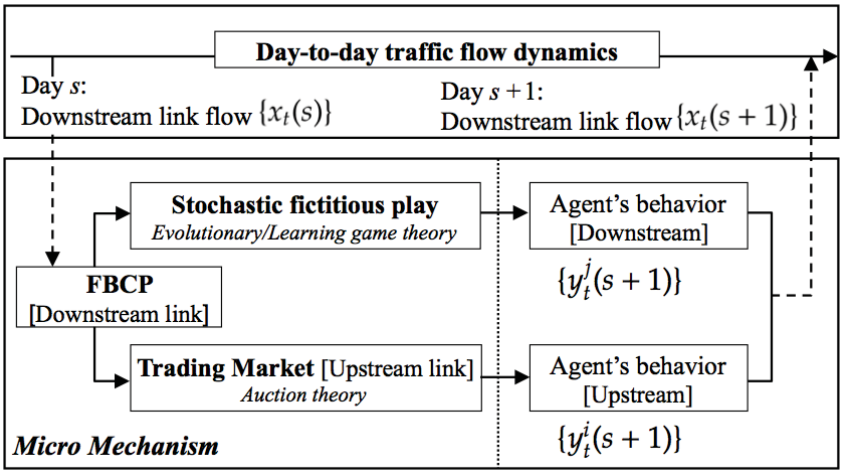


Figure 6.2 Framework of the proposed scheme

This problem is to find individual user assignments $\mathbf{y}^*$ that maximize the social surplus, subject to the physical constraints of flows representing the performance of the network. The (aggregate) optimal traffic flow pattern $\mathbf{x}^*$ is obtained by aggregating the individual assignments. It is, however, almost impossible for the manager to directly obtain solutions of [SO], because the problem includes private utilities w_t^i, w_t^j. To achieve the system optimal traffic assignment, we construct an evolutionary mechanism for the proposed scheme.

6.3 Framework of the hybrid scheme and users' behavior models

6.3.1 Framework of the hybrid scheme

A framework of the hybrid scheme is shown in Figure 6.2. This represents the relationship between the evolutionary mechanism and traffic flow patterns arising from the mechanism. The mechanism of the hybrid scheme consists of the trading rules of the trading markets and a choice model of users' arrival time (i.e., the SFP model), in which users' behaviors are determined by a series of processes. On the other hand, aggregating users' behaviors derives a traffic flow pattern.

To intuitively see how the scheme work under the framework, let us to explain the micro

mechanism using a time-line from day s to day $s+1$. Firstly, the users' trips on day s generate the traffic flow pattern $\mathbf{x}(s)$. The road manager then imposes congestion tolls on users based on the observed traffic flow pattern $\mathbf{x}(s)$. After this, each user chooses a destination arrival time on day $s+1$ with considering information on traffic flow patterns $\{\mathbf{x}(1), \ldots, \mathbf{x}(s)\}$ realized until the current day. More precisely, each upstream user chooses a arrival time through trading permits in the trading markets, while each downstream user chooses arrival time with the stochastic fictitious play (shown in the next subsection). Each user then makes a trip on day $s+1$: the traffic flow pattern $\mathbf{x}(s+1)$ is generated.

6.3.2 Users' behavior models

Learning process

Users are assumed to predict a traffic flow pattern for the day on which they travel (we call this "*prediction*") based information on traffic flow patterns $\{\mathbf{x}(1), \ldots, \mathbf{x}(s)\}$ realized until the current day; each user chooses an arrival time to maximize a "*predicted* utility". The *prediction* for the day $s+1$ is defined by the time average of traffic flow patterns:

$$\overline{\mathbf{x}}(s) = \frac{1}{s}\sum_{s=1}^{s} \mathbf{x}(s) \tag{6.12}$$

$$\text{where} \qquad \mathbf{x}(s) = \sum_{i\in\mathcal{M}} \mathbf{y}^i(s) + \sum_{j\in\mathcal{N}} \mathbf{y}^j(s).$$

Behaviors of upstream users

After making a trip on day s, an upstream user purchases a network permit of day $s+1$ through an auction mechanism. In the auction, each user plays a myopic bidding strategy based on their prediction: truthful valuations that each user has are given by

$$v_t^i(\overline{x}_t(s)) = w_t^i(s) - h_t(\overline{x}_t(s)). \tag{6.13}$$

User i's *predicted* payoff (or net utility) from permit at time t is also given by

$$\overline{\pi}_t^i(s) = v_t^i(\overline{x}_t(s)) - p_t(s) = w_t^i(s) - \Big(h_t(\overline{x}_t(s)) + p_t(s)\Big). \tag{6.14}$$

From the properties of the trading markets shown in the next section, the assignment of the user is determined so as to maximize their predicted payoff: the probability that user i is

assigned time t (i.e., $y_t^i(s+1) = 1$) is given by

$$B_t^i(\overline{x}_t(s), p_t(s)) = \Pr\left(t = \arg\max_{t \in T} \ \overline{\pi}_t^i(s)\right)$$
$$= \frac{\exp[-\eta^i(h_t(\overline{x}_t(s)) + p_t(s))]}{\sum_{t \in T} \exp[-\eta^i(h_t(\overline{x}_t(s)) + p_t(s))]}, \tag{6.15}$$

where η^i is a variance parameter for the stochastic utility w_t^i.

Behaviors of downstream users

We describe the downstream users' behavior using a stochastic fictitious play (SFP) model, as proposed by Fudenberg and Kreps (1993). According to the SFP model, users have *predictions* (or beliefs) about how their opponents will behave; these *predictions* are determined by the time average of past play. Users make their choices after the *predicted* payoffs have been subjected to random shocks. In our model, opponents' behavior is aggregated as a traffic flow pattern: their *predictions* are given by Eq. (6.12). Therefore, downstream user j's *predicted* payoff is given by

$$\overline{\pi}_t^j(s) = w_t^j(t) - h_t(\overline{x}_t(s)). \tag{6.16}$$

After realizing the stochastic utility w_t^j, the user maximizes their *predicted* payoff: the user chooses an arrival time t (i.e., $y_t^j(s+1) = 1$) with probability

$$B_t^j(\overline{x}_t(s)) = \Pr\left(t = \arg\max_{t \in T} \ \overline{\pi}_t^j(s)\right)$$
$$= \frac{\exp[-\eta^j h_t(\overline{x}_t(s))]}{\sum_{t \in T} \exp[-\eta^j h_t(\overline{x}_t(s))]}, \tag{6.17}$$

where η^j is a variance parameter for the stochastic utility w_t^j.

6.4 Auction mechanism for implementing trading markets

Like the previous chapters, the trading markets are implemented by the proxy DGS auction (see also Appendix C). Since the auction is strategy-proof (i.e., no user has an incentive to manipulate the markets), users bid their valuations $v_t^i(\overline{x}_t(s))$ defined in (6.13) truthfully. Also, the auction mechanism achieves allocative efficiency, i.e., the allocation of network permits

on day $s+1$ is determined as an optimal solution of the following assignment problem:

$$\max_{\mathbf{y}^i(s+1)\geq \mathbf{0}} \cdot \sum_{t\in T}\sum_{i\in\mathcal{M}} v_t^i(\overline{x}_t(s))y_t^i(s+1) \tag{6.18}$$

subject to

$$\sum_{t\in T} y_t^i(s+1) = 1 \qquad \forall i \in \mathcal{M} \tag{6.19}$$

$$\sum_{i\in\mathcal{M}} y_t^i(s+1) \leq \mu \qquad \forall t \in T \tag{6.20}$$

Permit prices $\mathbf{p}(s)$ are equivalent to the Lagrange multiplier for the constraints (6.20) and satisfy the market clearing conditions:

$$\begin{cases} \sum_{i\in\mathcal{M}} y_t^i(s+1) = \mu & \text{if} \quad p_t(s) > 0 \\ \sum_{i\in\mathcal{M}} y_t^i(s+1) \leq \mu & \text{if} \quad p_t(s) = 0 \end{cases} \qquad \forall t \in T. \tag{6.21}$$

6.5 Stochastic convergence of day-to-day traffic flow dynamics

Until Section 6.4, considering individual users, we developed a evolutionary mechanism for implementing the hybrid scheme; the mechanism supports users' behaviors $\mathbf{y}$. Note here that the users' behaviors are not equal to the system optimal user assignments $\mathbf{y}^*$ shown in Section 6.2 because each user's *predicted* utility (or payoff) $\overline{\pi}_t(s)$ depends on the *prediction* $\overline{\mathbf{x}}$ about traffic flow pattern.

This section analyzes the stochastic dynamics of the *prediction*. The dynamics depend on changes of the traffic flow pattern $\mathbf{x}$, generated by aggregating users' behaviors $\mathbf{y}$. Subsection 6.5.1 defines the stochastic process and derives the mean dynamics of the process. In Subsection 6.5.2, we show properties of the mean dynamics. We then prove that the original stochastic process converges to the system optimal traffic flow pattern by using stochastic approximation theory (Subsection 6.5.3). Here, when the prediction converges, the traffic flow pattern is equal to it (i.e., $\mathbf{x} = \overline{\mathbf{x}}^*$). Thus, the system optimal user assignments $\mathbf{y}^*$ are achieved though the evolutionary mechanism.

6.5.1 Stochastic dynamics of the prediction of traffic flow pattern

As shown in Subsection 6.3.2, each user's behavior on the day $s+1$ is determined by referring only to the *prediction* on day s. Let us describe the changes of the *prediction* between day s and day $s+1$. From the definition of the *prediction* (6.12), the stochastic dynamics of the *prediction* are given by

$$\overline{\mathbf{x}}(s+1) = \frac{1}{s+1}\Big(\overline{\mathbf{x}} + \mathbf{x}(s+1)\Big). \tag{6.22}$$

A traffic flow pattern $\mathbf{x}(s+1)$ realized on day $s+1$ with probability

$$\begin{aligned}
&\Pr\left(\mathbf{x}(s+1) = \sum_{i\in\mathcal{M}} \mathbf{y}^i(s+1) + \sum_{j\in\mathcal{N}} \mathbf{y}^j(s+1) \mid \overline{\mathbf{x}}(s) = \overline{\mathbf{x}}\right) \\
&= \prod_{i\in\mathcal{M}} \prod_{t:y_t^i(s+1)=1} B_t^i(\overline{\mathbf{x}}(s), \mathbf{p}(t)) \prod_{j\in\mathcal{N}} \prod_{t:y_t^j(s+1)=1} B_t^j(\overline{\mathbf{x}}(s)).
\end{aligned} \tag{6.23}$$

We can then compute expected changes of $\overline{\mathbf{x}}(s)$ by using Eq. (6.23):

$$\begin{aligned}
E\Big(\overline{\mathbf{x}}(s+1) - \overline{\mathbf{x}}(s) \mid \overline{\mathbf{x}}(s) = \overline{\mathbf{x}}\Big) &= \frac{1}{s+1}\left[E\Big(\mathbf{x}(s+1) \mid \overline{\mathbf{x}}(s) = \overline{\mathbf{x}}\Big) - \overline{\mathbf{x}}\right] \\
&= \frac{1}{t+1}\left[\Big(\sum_{i\in\mathcal{M}} \mathbf{B}^i(\overline{\mathbf{x}}(s), \mathbf{p}(t)) + \sum_{j\in\mathcal{N}} \mathbf{B}^j(\overline{\mathbf{x}}(s))\Big) - \overline{\mathbf{x}}\right]
\end{aligned} \tag{6.24}$$

where $\mathbf{B}^i = (B_t^i(\cdot,\cdot))_{t\in T}$, $\mathbf{B}^j = (B_t^j(\cdot))_{t\in T}$. To determine the long-run behavior of the expected changes, allow day s to approach its limiting value (i.e., $s \to \infty$). Then, expected changes of the *prediction* are transformed into the following ordinary differential equation (or mean dynamics):

$$\frac{d\overline{x}_t}{ds} = \left[\sum_{i\in\mathcal{M}} B_t^i(\overline{\mathbf{x}}, \mathbf{p}) + \sum_{j\in\mathcal{N}} B_t^j(\overline{\mathbf{x}})\right] - \overline{x}_t \qquad \forall t \in T. \tag{6.25}$$

6.5.2 Properties of the mean dynamics of the prediction

Rest points of the mean dynamics (6.25) satisfy $d\overline{\mathbf{x}}/ds = \mathbf{0}$. Therefore, the following conditions should be satisfied at the rest points:

$$\overline{x}_t^* = \sum_{i\in\mathcal{M}} B_t^i(\overline{\mathbf{x}}^*, \mathbf{p}^*) + \sum_{j\in\mathcal{N}} B_t^j(\overline{\mathbf{x}}^*) \qquad \forall t \in T \tag{6.26}$$

$$\begin{cases} p_t^* > 0 & \text{if} \quad \sum_{i\in\mathcal{M}} B_t^i(\overline{\mathbf{x}}^*, \mathbf{p}^*) = \mu \\ p_t^* = 0 & \text{if} \quad \sum_{i\in\mathcal{M}} B_t^i(\overline{\mathbf{x}}^*, \mathbf{p}^*) \le \mu \end{cases} \qquad \forall t \in T. \tag{6.27}$$

For the rest points of the mean dynamics, we can obtain the following proposition:

Proposition 6.1 *The rest point of the mean dynamics of the prediction is equal to the system optimal traffic flow pattern* $\mathbf{x}^*$ *defined in Section 6.2.*

Proof See Appendix 6.A for the proof.

Proposition 6.1 can be directly obtained by reformulating [SO] with a deterministic perturbation function (see Hofbauer and Sandholm (2002) more detailed discussion of this function):

$$\begin{aligned}\max_{\mathbf{y}\in\Omega} .SS(\mathbf{y}^i,\mathbf{y}^j) &= -\sum_{i\in\mathcal{M}}\sum_{t\in T}[\alpha c_t(x_t)+s_t]y_t^i + \sum_{i\in\mathcal{M}} H^i(\mathbf{y}^i)\\ &\quad -\sum_{j\in\mathcal{N}}\sum_{t\in T}[\alpha c_t(x_t)+s_t]y_t^j + \sum_{j\in\mathcal{N}} H^j(\mathbf{y}^j)\\ &= -\sum_{t\in T}\int_0^{x_t} h_t(\omega)d\omega + \sum_{i\in\mathcal{M}} H^i(\mathbf{y}^i) + \sum_{j\in\mathcal{N}} H^j(\mathbf{y}^j), \qquad (6.28)\end{aligned}$$

where Ω is the feasible region that satisfies (6.8)–(6.10) and the deterministic perturbation functions are given by the following entropy functions:

$$H^i(\mathbf{y}^i) = -\frac{1}{\eta^i}\sum_{t\in T} y_t^i \ln y_t^i, \quad H^j(\mathbf{y}^j) = -\frac{1}{\eta^j}\sum_{t\in T} y_t^j \ln y_t^j.$$

Since the objective function of the problem is strictly concave, we find that the system optimal traffic assignment is unique: this means that the rest point of the mean dynamics is unique. The above problem is also used to describe the stability of the mean dynamics.

To ensure that the mean dynamics $d\overline{\mathbf{x}}/ds$ globally converge to the rest point, we consider a continuous and differentiable function $SS_L(\overline{\mathbf{x}})$ given by

$$SS_L(\overline{\mathbf{x}}) \equiv -\sum_{t\in T}\int_0^{\bar{x}_t} h_t(\omega)d\omega + \sum_{i\in\mathcal{M}} H^i(\mathbf{y}^i) + \sum_{j\in\mathcal{N}} H^j(\mathbf{y}^j) - SS(\mathbf{y}^{i*},\mathbf{y}^{j*}). \qquad (6.29)$$

Theorem 6.1 *The function* $SS_L(\overline{\mathbf{x}})$ *is a strict Lyapunov function for the mean dynamics.*

Proof See Appendix 6.B for the proof.

Therefore, the following proposition is satisfied for the mean dynamics of the *prediction*:

Proposition 6.2 *Under the hybrid scheme, the mean dynamics of the prediction globally converge to the system optimal state.*

Proof This proposition follows from Proposition 6.1, Theorem 6.1, and Lyapunov stability theory.

6.5.3 Convergence of the stochastic dynamics of the prediction

Proposition 6.2 states that the mean dynamics of the *prediction* globally converge to the system optimal state. However, the original stochastic process (6.22) of the *prediction* jumps past the optimal state with positive probability. Thus, we use stochastic approximate theory to relate the behavior of the stochastic process to the rest point of the mean dynamics. We then obtain the following proposition:

Proposition 6.3 *The stochastic process of the prediction converges to the system optimal state with probability* 1.

Proof Proposition 6.1 and Proposition 6.2 state that the mean dynamics globally converge to the rest point and the rest point is equal to the system optimal state. On the other hand, Theorem 3.3 of Benaïm and Hirsch (1999) and Proposition 5.3 of Benaïm (1999) imply that the stochastic process of the *prediction* converges to a set of the rest points of the mean dynamics. Since the rest point is unique in our model, we conclude that the stochastic process converges to the system optimal state.

From the Proposition 6.3, the stochastic process of the *prediction* converges to the system optimal state $\mathbf{x}^*$, which leads to that the system optimal user assignments $\mathbf{y}^*$ are achieved through the evolutionary mechanism.

6.5.4 Numerical Example

Finally, we show a numerical example to illustrates the convergence process of the day-to-day dynamics of the traffic flow pattern with prediction.

Let the downstream travel time be given as the following BPR type function:

$$c(x_i) = c_0\{1 + a(x_i/\tilde{\mu})^b\} \tag{6.30}$$

where c_0 represents free flow travel time and a, b, and $\tilde{\mu}$ are parameters of the function. We here set $a = 2$ and $b = 5$. The schedule cost function is given by

$$s_t = \max\{e(\hat{t} - t), l(t - \hat{t})\} \tag{6.31}$$

with identical values for the positive earliness rate $e = 25$ (yen/min) and lateness rate $l = 45$ (yen/min); $\hat{t}$ is the desired arrival time. The coefficient that converts travel time to a manetary equivalent is given by $\alpha = 30$ (yen/min).

Table 6.1 Physical parameters on the road network

	Parameter values
The number of the upstream users (M)	2500 vheicles
The number of the downstream users (N)	5000 vheicles
Bottleneck capacity (μ)	50 vheicle/min
BPR parameter ($\tilde{\mu}$)	500
Free flow time on the downstream link (c_0)	15 min

We consider a morning commute; all users have the same desired time $\hat{t} = 8:00$. We set logit parameters $\eta^i = 0.01$, $\eta^j = 0.01$. Physical parameters on the road network are shown in Table 6.1. We also set time interval $\Delta t = 1$ (minitue) and $S = 150$.

Numerical results are shown in Figure 6.3 and Figure 6.4. Figure 6.3 illustrates a sample path of the stochastic process of the aggregate traffic flow pattern per 10 minutes. This process is obtained by aggregating each user behavior. On the other hand, Fig. 6.4 shows the day-to-day dynamics of the total transportation cost[2]; the red line represents the total transportation cost at the socially optimal state (i.e., minimum value of the social transportation cost) and the blue line does the total transportation cost on each day. From these figure, we see that stochastic process of the traffic flow pattern eventually reaches a steady state near the system optimal state that minimizes the total transportation cost.

6.6 Conclusion

Wada and Akamatsu (2010) proposed a hybrid scheme combining tradable network permits and congestion pricing, and demonstrated its efficiency in a deterministic situation. The present study developed an evolutionary mechanism for implementing the hybrid scheme in a stochastic environment, and showed that the mechanism operates robustly with stochastic user behavior. Specifically, we first described an auction mechanism for the trading markets and modeled users' behavior using stochastic learning model (i.e., stochastic fictitious play). We then derive a stochastic dynamics about traffic flow pattern. Finally, we established

[2] Note that the social surplus is maximized when the total transportation cost is minimized in our model.

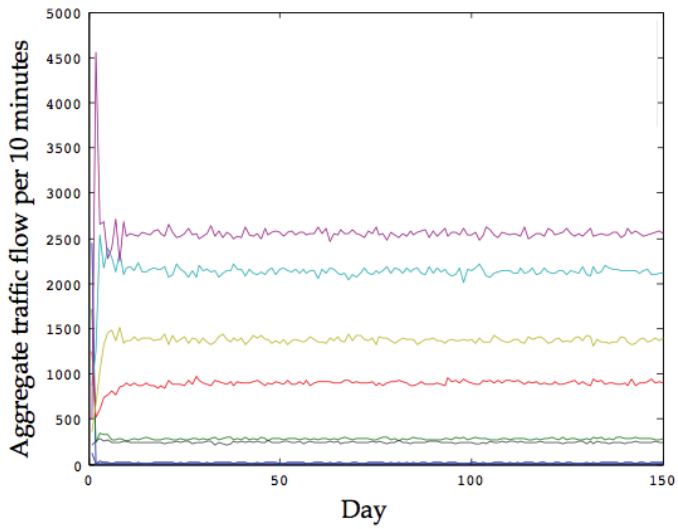


Figure 6.3 Dynamics of the aggregate predictions per 10 minutes

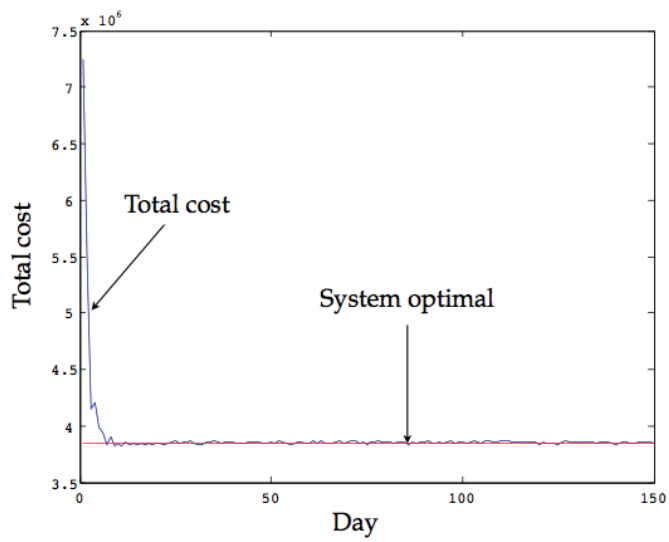


Figure 6.4 Dynamics of the total transportation cost

the following convergence result for the dynamics by relying on stochastic approximation theory. That is, the dynamics converges to a system optimal state in the sense that the social surplus is maximized with probability 1.

Appendix 6.A Proof of the Proposition 6.1

We first consider the Kuhn-Tucker conditions for the system optimal traffic assignment problem [SO]:

$$\begin{cases} y_t^{i*} > 0 & \text{if} \quad \pi^{i*} = w_t^i - h(x_t^*) - p_t^* \\ y_t^{i*} = 0 & \text{if} \quad \pi^{i*} \geq w_t^i - h(x_t^*) - p_t^* \end{cases} \qquad \forall i \in \mathcal{M} \tag{6.32}$$

$$\begin{cases} y_t^{j*} > 0 & \text{if} \quad \pi^{j*} = w_t^j - h(x_t^*) \\ y_t^{j*} = 0 & \text{if} \quad \pi^{j*} \geq w_t^j - h(x_t^*) \end{cases} \qquad \forall j \in \mathcal{N} \tag{6.33}$$

$$\begin{cases} p_t^* > 0 & \text{if} \quad \sum_{i \in \mathcal{M}} y_t^{i*} = \mu \\ p_t^* = 0 & \text{if} \quad \sum_{i \in \mathcal{M}} y_t^{i*} \leq \mu \end{cases} \qquad \forall t \in T \tag{6.34}$$

$$+ \text{ Equality constraints (6.8) and (6.9).} \tag{6.35}$$

where π^i, π^j, and p_t are Lagrange multipliers for the constraints (6.8), (6.9), and (6.10); these are interpreted as the upstream user's net utility, downstream user's net utility, and permit price, respectively.

Eq. (6.32) represents that each upstream user chooses the arrival time that maximizes his or her (net) utility. Therefore, the probability that the user chooses arrival time t is given as

$$\Pr.\left[t = \arg\max_{t \in T}.\{w_t^i - h(x_t^*) - p_t^*\}\right] = B_t^i(\mathbf{x}^*, \mathbf{p}^*). \tag{6.36}$$

Similarly, from the Eq. (6.33), the probability that each downstream user chooses arrival time t is given as

$$\Pr.\left[t = \arg\max_{t \in T}.\{w_t^j - h(x_t^*)\}\right] = B_t^j(\mathbf{x}^*). \tag{6.37}$$

By using these two equation, we have

$$x_t^* = \sum_{i \in \mathcal{M}} B_t^i(\mathbf{x}^*, \mathbf{p}^*) + \sum_{j \in \mathcal{N}} B_t^j(\mathbf{x}^*), \tag{6.38}$$

and Eq. (6.34) is rewritten as the following condition:

$$\begin{cases} p_t^* > 0 & \text{if} \quad \sum_{i\in\mathcal{M}} B_t^i(\mathbf{x}^*, \mathbf{p}^*) = \mu \\ p_t^* = 0 & \text{if} \quad \sum_{i\in\mathcal{M}} B_t^i(\mathbf{x}^*, \mathbf{p}^*) \leq \mu \end{cases} \qquad \forall t \in T. \tag{6.39}$$

It can be easily seen that conditions (6.38) and (6.39) are equivalent to (6.26) and (6.27).

Appendix 6.B Proof of the Theorem 6.1

We show that the function $SS_L(\overline{\mathbf{x}})$ is a strict Lyapunov function for the mean dynamics (6.25). First, we easily see that $SS_L(\overline{\mathbf{x}}) \leq 0$ with equality if and only if $\overline{\mathbf{x}} = \mathbf{x}^*$ (i.e., $\mathbf{y} = \mathbf{y}^*$) (*negative definite*). Next, we show that the following condition:

$$\frac{d}{ds} SS_L(\overline{\mathbf{x}}) = \nabla SS_L(\overline{\mathbf{x}}) \cdot \frac{d\overline{\mathbf{x}}}{ds} > 0 \tag{6.40}$$

is satisfied when $\overline{\mathbf{x}} \neq \mathbf{x}^*$ (i.e., $\mathbf{y} \neq \mathbf{y}^*$).

To obtain this, we consider the following problem [P]:

$$\max_{\mathbf{y}\in\Omega} .\hat{SS}_L(\mathbf{x}) = -\mathbf{h}(\overline{\mathbf{x}}) \cdot \mathbf{x} + \sum_{i\in\mathcal{M}} H^i(\mathbf{y}^i) + \sum_{j\in\mathcal{N}} H^j(\mathbf{y}^j) \tag{6.41}$$

$$\text{where} \quad \mathbf{x} = \sum_{i\in\mathcal{M}} \mathbf{y}^i + \sum_{j\in\mathcal{N}} \mathbf{y}^j$$

The objective function of the problem is the partial linearization of the Lyapunov function, i.e., $\mathbf{h}(\overline{\mathbf{x}})$ is given. Note here that we omit $SS(\mathbf{y}^{i*}, \mathbf{y}^{j*})$ of the Lyapunov function for simplicity. Call this optimal solution $\mathbf{x}'$.

We next define a feasible solution $\overline{\mathbf{x}}(\kappa)$ of the problem [P] (or the original problem [SO]):

$$\overline{\mathbf{x}}(\kappa) \equiv \overline{\mathbf{x}} + \kappa\mathbf{d} = \overline{\mathbf{x}} + \kappa(\mathbf{x}' - \overline{\mathbf{x}}), \tag{6.42}$$

where $\mathbf{d}$ is a direction vector and κ is a step size. If the step size is too small, the change of the value of the Lypunov function is given as

$$\Delta SS_L = \frac{\partial SS_L(\overline{\mathbf{x}}(\kappa))}{\partial \kappa}. \tag{6.43}$$

Here, the only difference between $SS_L(\overline{\mathbf{x}}(\kappa))$ and $\hat{SS}_L(\overline{\mathbf{x}}(\kappa))$ is the first terms of them. Therefore, (6.43) can be evaluated as the change of the value of $\hat{SS}_L(\overline{\mathbf{x}}(\kappa))$:

$$\Delta SS_L = \frac{\partial SS_L(\overline{\mathbf{x}}(\kappa))}{\partial \kappa} = \frac{\partial \hat{SS}_L(\overline{\mathbf{x}}(\kappa))}{\partial \kappa}. \tag{6.44}$$

Meanwhile, since $\hat{SS}_L(\overline{\mathbf{x}}(\kappa))$ is concave,

$$\hat{SS}_L(\overline{\mathbf{x}}(\kappa)) \geq (1-\kappa)\hat{SS}_L(\overline{\mathbf{x}}) + \kappa\hat{SS}_L(\mathbf{x}'). \tag{6.45}$$

Also, since $\mathbf{x}'$ is the optimal solution of the problem [P], $\hat{SS}_L(\mathbf{x}') > \hat{SS}_L(\overline{\mathbf{x}})$. Thus, for any $\kappa \in (0,1)$, the following inequality holds:

$$\hat{SS}_L(\overline{\mathbf{x}}(\kappa)) > \hat{SS}_L(\overline{\mathbf{x}}). \tag{6.46}$$

Further, when $\kappa \to 0+$, this equation means

$$\frac{\partial \hat{SS}_L(\overline{\mathbf{x}}(\kappa))}{\partial \kappa} > 0. \tag{6.47}$$

From the above discussion, by replacing κ with s, we can conclude that

$$\frac{\partial SS_L(\overline{\mathbf{x}}(s))}{\partial s} > 0 \tag{6.48}$$

when $\overline{\mathbf{x}} \neq \mathbf{x}^*$ (i.e., $\mathbf{y} \neq \mathbf{y}^i$).

Chapter 7

Conclusion

In this thesis, we have extended a theory of the tradable network permits scheme in two important directions: (i) to develop a supply side control based on the scheme and (ii) to design implementation mechanisms for the scheme in three different situations. The common objective of the control and mechanisms is to achieve an efficient allocation of network capacity without requiring demand forecasting. To accomplish this objective, we employed an evolutionary approach to achieving an optimal supply level while acquiring demand information sequentially. In the following, the results of this thesis are summarized.

Chapter 3 proposed a distributed signal control policy based on the tradable network permits, while considering a semi-dynamic traffic flow on general networks. The proposed signal control policy has two desirable properties. First, it can determine the green time proportion (i.e., capacity allocation) of each intersection by using only the information permit prices of the intersection; it requires no knowledge of the entire network information (e.g., origin-destination information). Second, an equilibrium traffic assignment under the proposed policy coincides with a system optimal traffic flow pattern that minimizes the total transportation cost in a network. Moreover, we constructed an evolutionary implementation method for the proposed policy and proved that the day-to-day traffic flow dynamics under the scheme converge to the system optimal traffic pattern.

Chapter 4 proposed an implementation mechanism for trading markets of network permits on general networks. Although a naive formulation of the problem of finding a dynamic system optimal allocation of network permits leads to a NP-hard problem, we avoided such computational infeasibility by employing an evolutionary approach. Specifically, we made use of a hybrid mechanism that consistently combines an auction mechanism with a path

capacity control; these are repeated on a day-to-day basis. The former phase involves selling bundles of permits, and the latter phase involves adjustment of the number of bundles of permits, which corresponds to the path capacities. We proved that the proposed mechanism has the following desirable properties: (i) truthful bidding is a dominant strategy for each user on each day and (ii) the permit allocation pattern under the mechanism converges to an approximate dynamic system optimal allocation pattern in the sense that the achieved social surplus reaches its maximum value when the number of users is large. Furthermore, we showed that the proposed mechanism could be extended to obviate path enumeration by introducing a column generation procedure.

Chapter 5 considered a more general situation where network permits for a specific day are sold in multiple period markets. The multiple period markets not only provide a degree of freedom in the purchase of permits but also allow for more efficient resource allocation than a single period market, especially when users' valuations of the permits change over time. Under such circumstances, the road manager needs to allocate a bottleneck capacity to these markets, as well as allocate permits to users. As a first step in implementing these markets, we designed a dynamic auction mechanism in which the number of permits for each market is fixed. This mechanism can determine optimal permit allocation, along with the actual sequence of time if each user truthfully bids "net valuations" (i.e., valuations minus the option value of deferring purchase). It was proved that the truthful revelation of net valuations is a dominant strategy for each user, and that it guarantees that the market choice of the user is optimal. We then derived an adjustment rule of the number of permits sold for each market and demonstrated that combining the dynamic auction and the adjustment rule maximizes the social surplus in a finite number of iterations. Finally, we numerically showed that the proposed mechanism works effectively for a dynamic population case where markets participants change over time.

Chapter 6 developed an evolutionary mechanism for a hybrid scheme of the TNP and congestion pricing, considering multiple negative externalities (i.e., queuing congestion and flow congestion). Specifically, we first described a mechanism consisting of trading rules of the permit markets and users' behaviors expressed by a stochastic learning model. We then derived a stochastic dynamics of the learning process from the mechanism. Finally, we showed that that the stochastic dynamics converges to an equilibrium state, and traffic flow pattern at equilibrium is efficient in the sense that the social surplus is maximized.

This thesis is the first step towards implementing tradable network permits scheme. Therefore, there are several challenges left for future research; the following are but two of them.

1. *Investigating robustness of the proposed mechanisms.* All the implementation mechanisms proposed in this thesis (except Chapter 6) are contingent on the assumption that users behave myopically every day. To investigate the robustness of the mechanisms, a more complete analysis must take into account the day-to-day strategic/learning behaviors of users. In addressing this subject, a full game-theory analysis of individual strategic behaviors might be intractable in our situation, given that the number of users is large. Instead, it would be useful to incorporate the mechanisms with the aggregate dynamics of learning behaviors like Chapter 6. Additionally, analyzing the mechanisms under a stochastic environment is an important topic that should be addressed in future research.

2. *Tradable network permits scheme in the second best situations.* Throughout this study, we considered the first-best situation, in which the road manager can issue network permits for all links. However, in practice, this condition does not always hold, as a road network has a limited number of bottlenecks. Therefore, further investigation of the tradable network permits scheme in second-best situations is needed in order to determine which links should be controlled. To address this issue, we need to connect the tradable network permits scheme to a dynamic traffic assignment (DTA) problem; this is not a trivial problem, because we would face difficulties by virtue of the non-convexity of the DTA problem. Nevertheless, since there is every possibility of extending the scheme's range of application, further exploration on this issue would be a challenging but worthwhile topic for future research.

Appendix A

Benders decomposition

Benders decomposition has been known as an effective approach for mathematical programs including several types of variables (e.g., mixed integer problems). This approach partitions the variables of a problem into two subsets and updates those variables in each subset alternately. Therefore, we can exploit special structures (e.g., network structures) of each type of variables to solve the overall problem.

Consider a mixed integer problem with two types of variables, $\mathbf{x}$ and $\mathbf{y}$:

$$\begin{cases} \max_{\mathbf{x},\mathbf{y}} \ \mathbf{c} \cdot \mathbf{x} + \mathbf{d} \cdot \mathbf{y} \\ \text{s.t.} \ \ \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{y} \leq \mathbf{b} \\ \qquad \mathbf{x} \geq \mathbf{0}, \ \mathbf{y} \in \mathbb{Y}, \end{cases} \tag{A.1}$$

where $\mathbf{x}$ is a m-vector of continuous variables, $\mathbf{y}$ is a n-vector of discrete variables, and $\mathbf{Y}$ is a subset of the integer points in n dimensions. The matrices $\mathbf{A}$ and $\mathbf{B}$ and vectors $\mathbf{c}$, $\mathbf{d}$, and $\mathbf{b}$ have dimensions compatible with those of $\mathbf{x}$ and $\mathbf{y}$.

By fixing variables $\mathbf{y}$, we here formulate a linear program with respect to $\mathbf{x}$:

$$\begin{cases} SP(\mathbf{y}) \equiv \max_{\mathbf{x}} \ \mathbf{c} \cdot \mathbf{x} + \mathbf{d} \cdot \mathbf{y} \\ \text{s.t.} \ \ \mathbf{A}\mathbf{x} \leq \mathbf{b} - \mathbf{B}\mathbf{y} \\ \qquad \mathbf{x} \geq \mathbf{0}. \end{cases} \tag{A.2}$$

We call this "sub-problem." The dual problem of the sub-problem is also formulated as

$$\begin{cases} DSP(\mathbf{y}) \equiv \min_{\mathbf{u}} \ \mathbf{u} \cdot (\mathbf{b} - \mathbf{B}\mathbf{y}) + \mathbf{d} \cdot \mathbf{y} \\ \text{s.t.} \ \ \mathbf{u}\mathbf{A} \geq \mathbf{c} \\ \qquad \mathbf{u} \geq \mathbf{0}, \end{cases} \tag{A.3}$$

where $\mathbf{u}$ are dual variables, and we let S denote the feasible region of $\mathbf{u}$: $S = \{\mathbf{u} \mid \mathbf{uA} \geq \mathbf{c},\ \mathbf{u} \geq \mathbf{0}\}$. It should be noted that the the feasible region S is independent of variables $\mathbf{y}$. If $S = \phi$, from the duality theorem, the primal sub-problem (A.2) (and the original problem) is unbounded or infeasible. On the other hand, if $S \neq \phi$, S is a convex polyhedron. Thus, there are a finite number of extreme points, written $\mathbf{u}_i^p,\ i = 1, \ldots, n^p$, and if S is unbounded, there are a finite number of extreme rays, written $\mathbf{u}_j^l,\ j = 1, \ldots, n^l$. Note here that, for a extreme ray $\mathbf{u}_j^l$, if

$$\mathbf{u}_j^l \cdot (\mathbf{b} - \mathbf{By}) < 0, \tag{A.4}$$

the dual sub-problem (A.3) is unbounded, i.e., (A.2) (and the original problem) is infeasible. Therefore, $\mathbf{y}$ must satisfy

$$\mathbf{u}_j^l \cdot (\mathbf{b} - \mathbf{By}) \geq 0 \qquad \forall j = 1, \ldots, n^l. \tag{A.5}$$

Conversely, if this condition is satisfied, an optimal solution of the dual sub-problem (A.3) is obtained as one of the extreme points.

From the above discussion, if those extreme points and rays are known, the original problem (A.1) can be reformulated as

$$\begin{cases} \max_{\mathbf{y}} \left\{ \min_{\mathbf{u}_i^p}\ \mathbf{u}_i^p \cdot (\mathbf{b} - \mathbf{By}) + \mathbf{d} \cdot \mathbf{y} \right\} \\ \text{s.t.}\ \ \mathbf{u}_j^l \cdot (\mathbf{b} - \mathbf{By}) \geq 0 \ \ \forall j = 1, \ldots, n^l \\ \qquad \mathbf{y} \in \mathbb{Y}. \end{cases} \tag{A.6}$$

But this problem is equivalent to the following:

$$\begin{cases} \max.\ \theta \\ \text{s.t.}\ \ \mathbf{u}_i^p \cdot (\mathbf{b} - \mathbf{By}) + \mathbf{d} \cdot \mathbf{y} \geq \theta \quad \forall i = 1, \ldots, n^p \\ \qquad \mathbf{u}_j^l \cdot (\mathbf{b} - \mathbf{By}) \geq 0 \qquad\qquad \forall j = 1, \ldots, n^l \\ \qquad \mathbf{y} \in \mathbb{Y}. \end{cases} \tag{A.7}$$

This problem is called the "master problem." Denoting $\mathbf{y}^*$ is an optimal solution of this problem. There exists an optimal solution $\mathbf{x}^*$ for fixed $\mathbf{y} = \mathbf{y}^*$. Then a pair $(\mathbf{x}^*, \mathbf{y}^*)$ is an optimal solution of the original problem (A.1).

While we see that the original problem is reduced to the problem (A.7), it is difficult to solve this problem directly. This is because a number of extreme points and rays is generally

too large and cannot be known in advance. However, only a small number of the constraints will be biding at an optimal solution. Hence, we adopt an iterative procedure of generating a new constraint successively. More specifically, we first consider a relaxation problem of (A.7) (restricted master problem) that has a small number of extreme points and rays. This problem produces an upper bound $\overline{\theta}$ on the optimal objective value of the original problem. Then we add a new constraint to the restricted master problem successively by solving the dual sub-problem until the optimal solution is obtained.

Algorithm

Step 0: *Initial setting.* Set the initial $\hat{\mathbf{y}}$ by solving the restricted master problem.

Step 1: *Sub-problem.* For a fixed $\hat{\mathbf{y}}$, solve the dual sub-problem (A.3). If the problem is infeasible then, the original problem has an bounded solution, then stop. If the problem is unbounded, we obtain a new extreme ray $\hat{\mathbf{u}}_j^l$ and go to **Step 2**. Otherwise, we obtain a new extreme point $\hat{\mathbf{u}}_i^p$. Then, if

$$\overline{\theta} = \hat{\mathbf{u}}_i^p \cdot (\mathbf{b} - \mathbf{B}\mathbf{y}) + \mathbf{d} \cdot \hat{\mathbf{y}}, \tag{A.8}$$

a pair of $\hat{\mathbf{y}}$ and $\hat{\mathbf{x}}$, which corresponds to $\hat{\mathbf{u}}_i^p$, is an optimal solution of the original problem. Otherwise, go to **Step 2**.

Step 2: *Restricted master problem.* Add a constraint corresponding to the extreme point $\hat{\mathbf{u}}_i^p$ or extreme ray $\hat{\mathbf{u}}_j^l$ to the restricted master problem. Then we solve a new restricted master problem and obtain a new $\hat{\mathbf{y}}$ and a new upper bound $\overline{\theta}$. Go to **Step 1**. If the restricted master problem is infeasible, so is the original problem, then stop.

Note that a new extreme point or ray is always generated in each Step 1. Therefore, finite convergence of the algorithm follows directly from the finite number of constraints of the problem (A.7) (see, Lasdon, 1970, for a comprehensive review of the Benders decomposition).

Appendix B

Vickrey-Clarke-Groves mechanism

The VCG mechanism is the most celebrated sealed-bid (combinatorial) auction that achieves allocative efficiency and induces truth-telling. This is a generalization of the well known sealed-bid second price auctions proposed by Vickrey (1961). This generalization was made by Clarke (1971) and Groves (1973).

To give a formal definition of this mechanism, let N be the set of bidders and M the set of heterogeneous items. For every subset S of M, bidder $i \in N$ is assumed to have the private valuation $v_i(S) \geq 0$. Each bidder is also assumed to have the following quasi-linear utility:

$$u_i(\mathbf{y}_i, p_i) \equiv \sum_{S \subseteq M} v_i(S) y_i(S) - p_i, \tag{B.1}$$

where p_i is bidder i's payment and $y_i(S) = 1$ if S is allocated to bidder i and zero otherwise.

The VCG mechanism for combinatorial auctions is defined as follows:

1. Bidders simultaneously report sealed bids $\mathbf{v}_i$ giving their value for each possible combination (i.e., bundle) of all items (we here assume that each bidder is truthful).

2. An auctioneer then chooses the allocation of items so as to maximize the sum of the accepted bids (social surplus) by solving the following winner determination problem:

$$V = \max_{\mathbf{y}} \sum_{i \in N} \sum_{S \subseteq M} v_i(S) y_i(S) \tag{B.2}$$

$$\text{s.t.} \quad \sum_{S \ni j} \sum_{i \in N} y_i(s) \leq 1 \qquad \forall j \in M \tag{B.3}$$

$$\sum_{S \subseteq M} y_i(S) \leq 1 \qquad \forall i \in N \tag{B.4}$$

$$y_i(s) \in \{0, 1\} \qquad \forall S \subseteq M, \ \forall i \in N. \tag{B.5}$$

The first constraint (B.3) represents a capacity constraint for each item. The second constraint (B.4) is the condition no bidder receives more than one subset. Call this optimal solution $\mathbf{y}^*$.

3. Payment p_i for bidder i is equal to the decrease in the value of the social surplus by adding him or her to the auction, i.e.,

$$p_i \equiv V_{-i} - \sum_{k \neq i} \sum_{S \subseteq M} v_k(S) y_k^*(S), \tag{B.6}$$

where the index $-i$ represents that bidder i is removed from the auction.

From the definition, if all bidders report their valuations truthfully, the VCG mechanism maximizes the social surplus (allocatively efficient). Then, bidder i's utility is

$$u_i(\mathbf{y}_i^*, p_i) = V - V_{-i}. \tag{B.7}$$

This is sometimes called the *marginal product*.

We next show that the VCG mechanism induces truth-telling. To do this, suppose each bidder reports (possibly untruthful) bids $\mathbf{b}_i$. The auctioneer chooses the allocation of items $\hat{\mathbf{y}}$ by maximizing $\sum_{i \in N} \sum_{S \subseteq M} b_i(S) y_i(S)$. Then, bidder i's utility is

$$\begin{aligned} u_i(\hat{\mathbf{y}}_i, p_i) &= \sum_{S \subseteq M} v_i(S) \hat{y}_i(S) - p_i \\ &= \sum_{S \subseteq M} v_i(S) \hat{y}_i(S) - \left[V_{-i} - \sum_{k \neq i} \sum_{S \subseteq M} b_i(S) \hat{y}_i(S) \right] \\ &= \left[\sum_{S \subseteq M} v_i(S) \hat{y}_i(S) + \sum_{k \neq i} \sum_{S \subseteq M} b_i(S) \hat{y}_i(S) \right] - V_{-i} \end{aligned} \tag{B.8}$$

The second term of the final line is determined independent of bidder i's bids. On the other hand, the first term is always maximized when bidder i reports truthful bids $\mathbf{v}_i$. This is because the first term equals to the optimal objective value of the winner determination problem in that case, i.e., the auctioneer chooses the allocation by maximizing $\sum_{S \subseteq M} v_i(S) y_i(S) + \sum_{k \neq i} \sum_{S \subseteq M} b_k(S) y_k(S)$. Therefore, there is no user has incentive to report untruthful bids.

Theorem B.1 (VCG mechanism) *In the VCG mechanism, truth reporting is a dominant strategy for each bidder. The social surplus is maximized when all bidders report their valuations truthfully.*

Unfortunately, the VCG mechanism requires complex tasks to both the auctioneer and bidders. Specifically, the auctioneer have to solve $|N| + 1$ combinatorial optimization problems *exactly*. It is computational burden when the number of bidders is large. On the other hand, bidders have to report bids for possible combination of all items. This is not just the computationally hard problem but also undesirable in terms of the privacy: bidders are required to reveal their private information more than necessary. See Ausubel and Milgrom (2006) for a comprehensive review of the VCG mechanism.

Appendix C

Primal-dual algorithm

In this appendix, we briefly introduce a primal-dual algorithm corresponding to the (exact) ascending auction proposed by Demange, Gale, and Sotomayor (1986). This algorithm is known as the "Hungarian method" (Kuhn, 1955).

Consider an assignment problem (notations used here are same as in Appendix B):

$$\max_{\mathbf{x}\geq\mathbf{0}} \sum_{i\in N}\sum_{j\in M} v_{ij}y_{ij} \tag{C.1}$$

$$\text{s.t.} \sum_{i\in N} y_{ij} \leq 1 \qquad \forall j \in M \tag{C.2}$$

$$\sum_{j\in M} y_{ij} \leq 1 \qquad \forall i \in N, \tag{C.3}$$

and its dual problem:

$$\min_{(\boldsymbol{\pi},\mathbf{p})\geq\mathbf{0}} \sum_{i\in N} \pi_i + \sum_{j\in M} p_j \tag{C.4}$$

$$\text{s.t. } \pi_i \geq v_{ij} - p_j \qquad \forall i \in N,\ \forall j \in M. \tag{C.5}$$

where $(\boldsymbol{\pi}, \mathbf{p})$ are Lagrange multipliers for constraints (C.8), (C.9), which are interpreted as the bidders' payoffs and item prices.

In the primal-dual algorithm for the assignment problem, we first set an initial feasible solution (i.e., initial price): $\mathbf{p} = 0$. We next define the *admissible set* $D_i(\mathbf{p})$ that satisfies the following condition:

$$D_i(\mathbf{p}) = \{j \mid \pi_i = v_{ij} - p_j\} \qquad \forall i \in N. \tag{C.6}$$

This set corresponds to bidder's demand set in each round.

Next we check the feasibility of primal variables by solving the following restricted primal problem:

$$\max_{\mathbf{x}\geq\mathbf{0}} \sum_{i\in N} \sum_{j\in D_i(\mathbf{p})} y_{ij} \tag{C.7}$$

$$\text{s.t.} \sum_{i\in N} y_{ij} \leq 1 \qquad \forall j \in M \tag{C.8}$$

$$\sum_{j\in M} y_{ij} \leq 1 \qquad \forall i \in N, . \tag{C.9}$$

This problem can be seen as the maxi-flow problem on a certain network. Thus, we adopt the labeling algorithm (Ford-Fulkerson algorithm) to solve this problem. If a feasible solution exists, the solution and the current dual variables are optimal, i.e., all bidders can be allocated an item form his demand set and prices are competitive equilibrium prices.

Otherwise, we have to update dual variables. To achieve minimum competitive equilibrium prices, the DGS auction here chooses a minimal overdemanded set and updates prices in the set. However, finding the overdemanded set is computationally burden in general. For this problem, Sankaran (1994) demonstrated that such prices can be achieved by updating dual variables in a certain overdemanded set. Interestingly, this set is identified by solving the restricted primal problem based on the Ford-Fulkerson algorithm. Thus, we increase values of dual variables in this set by one unit in each round until the optimal solution is obtained.

Algorithm

Step 0: *Initialization.* Set dual variables $\mathbf{p} = \mathbf{0}$.

Step 1: *Checking the primal feasibility.* First, we generate the admissible sets $D_i(\mathbf{p})$ under the current dual variables $\mathbf{p}$. Then, we solve the restricted primal problem using the Ford-Fulkerson algorithm. If a feasible solution exists, the solution and the current dual variables are optimal, then stop. Otherwise, go to **Step 2**.

Step 2: *Updating dual variables.* Increase values of dual variables in the overdemanded set that is identified in **Step 1** by one unit

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