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Abstract

This paper explores the implications of the possible bankruptcies of firms for their own wage schemes and the structure of the labor market using a two-period general equilibrium model. The bankruptcy risk flattens the wage profile of each firm, weakening its incentive effect and thereby making room for the efficiency wage to be used as a supplementary incentive device. This, in turn, stratifies the labor market into the primary market, in which job rationing is observed, and the secondary market, in which job rationing is not observed. A substantial utility differential emerges between those who have found a job in the primary market and those who have not, even though there is no difference in their innate abilities. This differential widens as bankruptcies become more likely. Moreover, the dual structure of the labor market renders the employment size in the primary market too small to attain a social optimum.

Keywords: wage profile, efficiency wages, dual labor markets.

JEL classification: J31, J41, J65

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1 Introduction

Motivating hired workers is one of the most important tasks assigned to those in charge of business management. The difficulty in accomplishing this task depends on various factors, but when workers' performance is either directly or indirectly observable, an employer can motivate workers by paying wages according to their observed performance. This system is simple and effective, and thus has been used in many workplaces. The piece rate system applied to insurance agents and taxi drivers is a well-known example of this practice; others include the systems of seniority wage and severance pay applied to salaried workers. As Lazear (1979, 1981) argued, in effect, the latter systems force workers to deposit part of the wages they earn in the early stages of their career with the employer, with the condition that it will be refunded if their performance is satisfactory.

For these systems to work well, some conditions must be satisfied. First, workers' performance must be observable by the employer since these systems make the wage payment dependent on it. Their performance need not be observable at any moment, but must become observable by the end of the employment period. Second, the employer must not go bankrupt. When the employer goes bankrupt with some positive probability, the workers inescapably bear the risk of not receiving their wages, which seriously weakens the motivating effect of these systems. If one of these conditions is not satisfied, neither the seniority wage system nor the severance pay system can effectively motivate workers any longer. In such a situation, the employer is forced either to implement an additional incentive device or to switch from these systems to a totally different one.

This paper explores the implications of the possible bankruptcies of firms for their own wage schemes and the structure of the labor market in the framework of a two-period general equilibrium model. In this model, the economy consists of "capital good" and "consumption good" sectors. In the capital good sector, firms produce the capital good from labor services on a one-to-one basis. They can observe the performance of hired workers without cost, thus paying a competitive wage only to workers who have delivered the desired performance. In the consumption good sector, firms produce the consumption good from the capital good and labor services subject to the following technological constraints. First, they must input labor services in the first period, although the output is obtained in the second period. Second, they cannot observe the performance of hired workers until the second period. Third, a substantial percentage of the firms that have hired workers cannot output the consumption good in the second period; however, in the first period, no one knows which firms will become unproductive in the next period. The third constraint means that with a positive probability, each firm in the consumption good sector goes bankrupt, and thus, its hired workers cannot receive their wages. No insurance against this "bankruptcy

risk” is sold in this economy.

The presence of the bankruptcy risk gives special weight to the second constraint that the performance of hired workers becomes known with a one-period lag. When the risk is absent, this constraint does not matter to firms in the consumption good sector since they can choose to pay the entire amount of wages in the second period, thereby motivating hired workers. In contrast, when the risk is present, it poses the following agency problem to these firms: they are forced to “flatten” the wage profile—to pay a portion of the wages in the first period—because their workers desire to save it to protect themselves against the bankruptcy risk. Under the second constraint, however, this payment gives a moral hazard incentive to the workers. Since their performance is not observable in the first period, the wage payment in that period must be made unconditionally. They may make bad use of it, working not for their original employer but for a firm in the capital good sector, thereby receiving wage revenues from both employers. Of course, such behavior is known to the original employer in the second period, and thus, the cheating workers cannot receive severance pay, that is, the wage otherwise paid in the second period. However, if the lifetime utility attained by this cheating behavior is higher than that by honest behavior, the workers choose to cheat their original employer. To prevent this moral hazard, the firms must pay a sufficient amount of severance pay, thereby keeping the lifetime utility attained by honest behavior higher than that by cheating behavior.

Such responses by the firms in the consumption good sector cause a segmentation of the labor market. The bankruptcy risk induces them to pay more wages, which makes the lifetime utility of their workers far higher than that of those working in the capital good sector while reducing the total employment of the consumption good sector. As a result, jobs in the consumption good sector are rationed among applicants, and those who fail to obtain these jobs must then work for firms producing the capital good. Put another way, the labor market is stratified into the “primary” market, where job rationing is observed, and the “secondary” market, where job rationing is not observed. The utility differential between those who have found jobs in the consumption good sector and those who have not widens as bankruptcies become more likely. It is also shown that the dual structure of the labor market reduces the total employment of the consumption good sector to an inefficiently low level. Given the utility differential mentioned above, the hiring decision of a firm in that sector has a substantial effect on the lifetime utility of the hired worker since, if not hired by this firm, she has no choice but to work in the capital good sector, accepting a low level of lifetime utility. However, no firms in the consumption good sector consider this external effect when making a hiring decision, which renders their employment size too small to attain a social optimum.

It must be emphasized that the model of this paper bears characteristics

of both the incentive theory of wage profiles developed by such authors as Lazear (1979, 1981) and the efficiency wage theory developed by such authors as Shapiro and Stiglitz (1984); these have been viewed as opposing theories on motivating workers in the literature. On the one hand, the model of this paper yields the result that severance pay is used to motivate workers, which is a characteristic of the incentive theory of wage profiles. On the other hand, it yields the result that the total wage payment by a firm in the consumption good sector substantially exceeds the market-clearing level, which is a characteristic of the efficiency wage theory. The bankruptcy risk unites within the model the characteristics of these two distinct theories in such a manner that it flattens the wage profile, weakening its incentive effect and thereby making room for the efficiency wage to be used as a supplementary incentive device. Such an effect of the bankruptcy risk immunizes the present model against the criticism of Carmichael (1985). He criticizes the efficiency wage theory for neglecting the possibility that workers can buy a job in the primary labor market by paying an “entrance fee.” However, such an offer is never accepted by firms in the present model since it is equivalent to reducing the wage payment in the first period, which seriously weakens the morale of workers.

It should also be noted that the flatter wage profile (which, in the model of this paper, results from an increased bankruptcy risk) is not merely a theoretical possibility, but a reality for some economies. For example, Hamaaki, Hori, Maeda, and Murata (2012) analyze a dataset of the wage structure in Japan and find that the age–wage profile has been gradually flattening since the burst of the Japanese economic bubble in the 1990s, especially for employees in the middle or final phase of their career. Hori and Iwamoto (2012) confirm this finding using a different dataset.¹ Behind their findings, the mechanism stated above might play a role because there is no doubt that after experiencing the burst of that bubble, increasingly fewer Japanese businesspeople can have good future prospects.

The rest of this paper is organized as follows. Section 2 presents the model. Section 3 considers an optimal wage scheme that solves the agency problem for firms in the consumption good sector. Section 4 examines the general equilibrium of this model using analytical and numerical methods. Section 5 adduces a Pareto-improving policy to establish an allocative inefficiency of the general equilibrium. Section 6 concludes.

¹Kambourov and Manovskii (2009) and Kong and Ravikumar (2012, 2013) also report that in the US economy, workers who have recently entered the workforce experience a slower increase in wages than those who entered it many decades ago, although they investigate not only the wage profiles of those who are guaranteed lifelong employment but also the earnings profiles of those who switch jobs several times during their careers.

2 The Model

Time is discrete. The economy operates during two periods. At the beginning of the first period, a continuum of identical two-period-lived workers of measure one are born. For these workers, the first and second periods are, respectively, their young and old ages. They are endowed with one unit of indivisible labor service in the first period and want to consume in the second period. Their lifetime utility depends solely on their old-age consumption, which is formalized as

$$U = \log C, \quad (1)$$

where U and C denote, respectively, the *ex-post* level of lifetime utility and the amount of old-age consumption. Thus, in the first period, every worker hires out the endowed labor service and saves all the received income for her old age.

In both periods, a single kind of final good is competitively produced. The final good can be consumed in the same period in which it is produced. If it is carried over to the next period, it can be also used as “physical capital.” Under the assumption that workers consume only in the second period, all of the final good produced in the first period is carried over to the second period and used as physical capital, whereas all of the good produced in the second period is consumed within that period. Thus, we call the final good produced in the first period the “capital good,” and that produced in the second period the “consumption good.”

There are two technologies available for the production of the final good. The first technology turns labor services into the final good in the first period on a one-to-one basis. It is formulated as

$$Y^S = L^S, \quad (2)$$

where Y^S and L^S denote, respectively, the output of the final good and the input of labor services. Firms using this technology can monitor the performance of hired workers without cost, so hired workers receive wages equal to the marginal product of labor:

$$w^S = 1. \quad (3)$$

These firms constitute the “capital good sector” of this economy.

The second technology turns capital and labor services into the final good in the second period. It is formulated as

$$Y^P = AK^\alpha (L^P)^{1-\alpha}, \quad (4)$$

where Y^P denotes the output of the final good in the second period, A the total factor productivity, K the input of physical capital, L^P the input of

labor services, and α a constant satisfying $\alpha \in (0, 1)$. We assume that the input of labor services must be done in the first period, while that of physical capital must be done in the second period, which differentiates (4) from the normal type of Cobb–Douglas technology that assumes the simultaneous input of capital and labor services. We make the following assumptions about firms that use this technology, which constitute the “consumption good sector” of this economy. First, the input level of physical capital that a firm can choose is either 0 or 1, which ensures that firms yielding the final good in the second period are of the same size. It is also assumed that these firms can procure physical capital at a price of r in the competitive market that opens in the second period and that physical capital fully depreciates once inputted into the production process. Second, firms using this technology cannot know the performance of hired workers until the second period. This implies that if a firm and a worker have written a contract in which wage payments are made in both periods, then the payment in the first period must be made unconditionally. Third, a percentage θ ($\in (0, 1]$) of firms using this technology find at the beginning of the second period that their total factor productivities take a unitary value, that is, $A = 1$, while the others find themselves unproductive, that is, $A = 0$ (although, in the first period, no one knows which firms become unproductive in the next period). In the following, we express the state of a firm that turns out to be unproductive in the second period as “bankruptcy.” The possibility of bankruptcy forces the workers hired by such firms to bear the risk of not receiving the portion of their wages that is supposed to be paid in the second period. We call this the “bankruptcy risk,” and we assume that no insurance against the bankruptcy risk is sold in this economy.²

Next, consider a young worker who has received a certain amount of the final good in exchange for the endowed labor service. She can save for her old age by exercising either of the following two options. The first option is to carry the received good over to the next period and then to sell it as the capital good in the competitive market. The gross rate of return on this option is r . The second option is to lend it at the gross interest rate of R to firms in the consumption good sector, which need the final good for first-period wage payments. Although each firm in the consumption good sector goes bankrupt and defaults on its debts with probability $1 - \theta$, the worker can secure the gross rate of return on this option at θR by sufficiently diversifying among borrowers. As a result of the interest arbitrage behavior,

²Some readers may think that this assumption is not justifiable because the bankruptcy risk is idiosyncratic and thus insurable, at least in theory. However, there are some conceivable situations in which this assumption can be justified. For example, when insurance companies cannot at all track the working records of insured workers, they never provide insurance against the bankruptcy risk since they cannot determine who should receive insurance money.

these rates of return are equal in equilibrium:

$$r = \theta R. \quad (5)$$

Thus, if this worker is hired by a firm in the capital good sector, then her lifetime utility is given by

$$U^S = \log rw^S = \log r. \quad (6)$$

If she is hired by a firm in the consumption good sector, then her (expected) lifetime utility is given by

$$U^P = \theta \log(rw_1^P + w_2^P) + (1 - \theta) \log rw_1^P \quad (7)$$

where w_1^P and w_2^P denote, respectively, the wage paid in the first period and that paid in the second period (i.e., severance pay).

When the bankruptcy risk is present, that is, $\theta < 1$, both w_1^P and w_2^P take a positive value, which can be explained as follows. In such a case, workers hired by a firm in the consumption good sector eagerly desire wage payment in the first period. Although they are forced to bear the risk of not receiving severance pay with probability $1 - \theta$, no insurance against this risk is provided in this economy. All they can do to protect themselves against this risk is to save the wage paid in the first period, thereby securing their old-age consumption in the case of bankruptcy. Put another way, if a firm in the consumption good sector decides to make no wage payment in the first period, that is, $w_1^P = 0$, then the expected utility of a worker employed by this firm becomes a negative infinity, that is, $U^P = -\infty$ (See (7)). Since the lifetime utility of a worker employed in the capital good sector is strictly higher than a negative infinity, that is, $U^S = \log r > -\infty$ (See (6)), this firm finds no job applicant as long as it maintains a zero wage payment in the first period. For a firm in the consumption good sector, it is not optimal to pay the whole amount of wage in the first period either. As stated above, the wage payment in the first period is unconditionally made since the firm cannot observe the performance of hired workers in that period. Thus, if the labor contract stipulates that all of the wage must be paid in the first period, hired workers have an incentive to work not for their original employer, but for a firm in the capital good sector, and to thus receive wages from both employers. To prevent such a moral hazard, some portion of wages must be paid in the second period. We will explore these points further in the next section.

3 Optimal Design of the Wage Scheme

As pointed out in the previous section, the wage payment to a worker employed in the consumption good sector is usually made in installments. This

section examines how the levels of payment in the first period, w_1^P , and the second period, w_2^P , are determined.

Let us first consider the problem of moral hazard on the part of hired workers. As described in the previous section, workers hired by a firm producing the consumption good have an incentive to make bad use of the contract stipulation that the wage payment in the first period must be made unconditionally. Specifically, they work not for their original employer but for a firm producing the capital good, thereby receiving not only the wage payment from the original employer, w_1^P , but also extra income from the firm in the capital good sector, $w^S = 1$. Of course, such behavior becomes known to the original employer in the second period, and thus, severance pay is not paid to the cheating workers. However, by saving this income, the cheating workers can attain a lifetime utility equal to

$$U^M = \log r(1 + w_1^P). \quad (8)$$

If this utility dominates that attained by honest behavior, U^P , all workers choose to cheat their original employer. In other words, firms in the consumption good sector must design their wage scheme in such a manner that $U^P \geq U^M$ or, equivalently,

$$\theta \log(rw_1^P + w_2^P) + (1 - \theta) \log rw_1^P \geq \log r(1 + w_1^P). \quad (9)$$

Otherwise, they cannot prevent the moral hazard of their hired workers.

Given the interest rate R and the market price of the capital good r , firms in the consumption good sector maximize their profit subject to (9) by optimally choosing the number of workers, L^P , the wage paid in the first period, w_1^P , and severance pay, w_2^P . This problem can be formulated as

$$\begin{aligned} \max_{L^P, w_1^P, w_2^P} & (L^P)^{1-\alpha} - r - (Rw_1^P + w_2^P)L^P \\ \text{s.t.} & \theta \log(rw_1^P + w_2^P) + (1 - \theta) \log rw_1^P \geq \log r(1 + w_1^P). \end{aligned} \quad (10)$$

In solving this problem, we should bear the following two points in mind. First, the above formulation does not reflect the possibility of bankruptcy. Firms need not consider this possibility because in the case of bankruptcy, they earn zero profit, being exempted from the repayment of debts because of limited liability. Second, in the formulation of (10), the input of physical capital is preset at $K = 1$. Indeed, it is conceivable that profit takes a negative value for any possible combination of L^P , w_1^P , and w_2^P , and thus, productive firms may decide to set the input of physical capital at zero. However, this cannot be an equilibrium phenomenon because it means zero production of the consumption good.

The first-order conditions of (10) are derived as

$$(1 - \alpha)(L^P)^{-\alpha} - R w_1^P - w_2^P = 0, \quad (11)$$

$$-R L^P + \lambda \left[\theta \frac{r}{r w_1^P + w_2^P} + (1 - \theta) \frac{r}{r w_1^P} - \frac{r}{r(1 + w_1^P)} \right] \leq 0, \quad (12)$$

$$\left\{ -R L^P + \lambda \left[\theta \frac{r}{r w_1^P + w_2^P} + (1 - \theta) \frac{r}{r w_1^P} - \frac{r}{r(1 + w_1^P)} \right] \right\} w_1^P = 0, \quad (13)$$

$$-L^P + \lambda \theta / (r w_1^P + w_2^P) = 0, \quad (14)$$

where λ is the Lagrangian multiplier associated with the incentive compatibility constraint (9).³ The optimal employment size and optimal wage scheme are obtained by solving (9), (11)–(14) with respect to L^P , w_1^P , w_2^P , and λ .

Conditions (9), (11)–(14) suggest that the optimal wage scheme varies depending on whether or not the bankruptcy risk is present. When the bankruptcy risk is absent, that is, $\theta = 1$, conditions (9), (11)–(14) are easily solved as

$$w_1^P = 0, \quad w_2^P = r, \quad L^P = [(1 - \alpha)/r]^{1/\alpha}, \quad (15)$$

which imply that the whole amount of the wage is paid in the second period. This result is not surprising since the wage payment in the first period is made as insurance against the bankruptcy risk. In this case, the lifetime utility of a worker employed in the consumption good sector is derived as $U^P = \log r$, and thus, there is no difference in lifetime utility between the workers employed in the capital good sector and those working in the consumption good sector. In contrast, when the bankruptcy risk is present, that is, $\theta < 1$, w_1^P never takes a zero value because if $w_1^P = 0$, the incentive compatibility constraint (9) does not hold. In this case, the workers employed in the consumption good sector are better off than those working in the capital good sector. As implied by (9), their lifetime utilities are equal to $\log r(1 + w_1^P)$, which is strictly higher than $U^S = \log r$. Naturally, all of the workers desire to be hired by a firm producing the consumption good, although this desire is fulfilled for only a limited number of workers. In practice, jobs in the consumption good sector are rationed among applicants, and those who fail to obtain these jobs must then work for firms producing the capital good.⁴ In other words, the presence of the bankruptcy

³The first-order condition with respect to w_1^P must be given by (12) and (13) because the optimal choice of w_1^P can be a corner solution, that is, zero, in the absence of the bankruptcy risk. No such possibility arises with L^P or w_2^P .

⁴In contrast to the argument by Carmichael (1985), the workers in this economy cannot buy a job in the consumption good sector by paying an “entrance fee.” In this model, the payment of an entrance fee is equivalent to the reduction of the wage payment in the first period, w_1^P , which makes it difficult for the incentive compatibility constraint (9) to hold. Thus, every firm in that sector rejects such an offer.

risk stratifies the labor market of this economy into the “primary” market, where job rationing is observed, and the “secondary” market, where job rationing is not observed.

4 General Equilibrium

To define the general equilibrium of this economy, we need to evaluate the first-order conditions for the firms in the consumption good sector from an economy-wide perspective. Let l denote the percentage of workers who are hired by firms in the consumption good sector. Using this notation, we can express the capital–labor ratio of the consumption good sector as $(1 - l)/\theta l$. Since all of the firms in this sector are identical, their capital–labor ratios coincide with that value:

$$1/L^P = (1 - l)/\theta l. \quad (16)$$

Using (5) and (16), we can rewrite (11)–(14) as

$$(1 - \alpha)[(1 - l)/\theta l]^\alpha - (r/\theta)w_1^P - w_2^P = 0, \quad (17)$$

$$w_2^P(1 - \theta - \theta w_1^P) - r(w_1^P)^2 = 0. \quad (18)$$

Then, we define the general equilibrium of this economy as follows.

Definition 1. *Given the values of α and θ , the general equilibrium of this economy is a combination of w_1^P , w_2^P , l , and r that satisfies (9), (17), (18), and*

$$\alpha[(1 - l)/\theta l]^{\alpha-1} - r = 0. \quad (19)$$

Condition (19) requires that the marginal product of physical capital is equal to its market value in equilibrium. This equalization is attained by free entry to the consumption good sector. As long as the marginal product of physical capital dominates its price, new entry to this sector continues since it is profitable to set up a consumption good-producing firm. Such continuous entry decreases the level of l , narrowing the difference between the marginal product and the price to zero.⁵

We examine the general equilibrium of this economy by taking the following steps. First, we analyze a case in which the bankruptcy risk is absent. This case serves as a benchmark in analyzing the cases in which the bankruptcy risk is present. Then, we move to risky cases in which each firm in the consumption good sector goes bankrupt with some positive probability, focusing on the responses of the equilibrium values of w_1^P , w_2^P , l , and r to a change in θ .

⁵As the consumption good sector contains increasingly more firms, increasingly fewer workers are hired by that sector. This paradoxical result can be explained by the fact that an increase in the number of consumption good-producing firms necessitates the increased production of physical capital, and thus, employment increases in the capital good sector.

4.1 The Riskless Case

When the bankruptcy risk is absent, that is, $\theta = 1$, firms in the consumption good sector set their wage scheme as $w_1^P = 0$ and $w_2^P = r$, as seen in the previous section. Under this wage scheme, conditions (17) and (19) jointly produce

$$\alpha[(1-l)/l]^{\alpha-1} = (1-\alpha)[(1-l)/l]^\alpha$$

or, equivalently,

$$l = 1 - \alpha. \quad (20)$$

By substituting (20) into (19), we obtain

$$r = w_2^P = \alpha^\alpha(1-\alpha)^{1-\alpha}. \quad (21)$$

In this case, there is no difference in lifetime utility among workers, that is, $U^P = U^S = \log \alpha^\alpha(1-\alpha)^{1-\alpha}$.

4.2 Risky Cases

When the bankruptcy risk is present, that is, $\theta < 1$, the general equilibrium can be obtained through the following procedure. Equations (9) and (18) produce

$$(1-\theta)^\theta w_1^P (1+w_1^P)^{\theta-1} / (1-\theta-\theta w_1^P)^\theta = 1, \quad (22)$$

which uniquely determines the equilibrium value of w_1^P .⁶ On the other hand, equations (17) and (19) produce

$$\frac{1-l}{\theta l} = \frac{\alpha}{1-\alpha} \cdot \frac{1-\theta}{\theta} \cdot \frac{w_1^P}{1-\theta-\theta w_1^P}. \quad (23)$$

By substituting the equilibrium value of w_1^P into (23), we can obtain the value of l , which, combined with (19), determines the value of r . Finally, by substituting the equilibrium values of w_1^P and r into (18), we can obtain the value of w_2^P .

Proposition 1. *Let $w_1^P(\theta)$, $l(\alpha, \theta)$, and $r(\alpha, \theta)$ denote the equilibrium values of w_1^P , l , and r , respectively. Then, for $\forall(\alpha, \theta) \in (0, 1) \times (0, 1)$,*

$$dw_1^P/d\theta < 0, \quad \partial l/\partial\theta > 0, \quad \partial r/\partial\theta > 0, \quad (24)$$

⁶The uniqueness of the equilibrium value of w_1^P is established by the fact that the LHS of (22) is an increasing function of $w_1^P \in [0, (1-\theta)/\theta)$, which takes a zero value at $w_1^P = 0$ and approaches a positive infinity as $w_1^P \rightarrow (1-\theta)/\theta$.

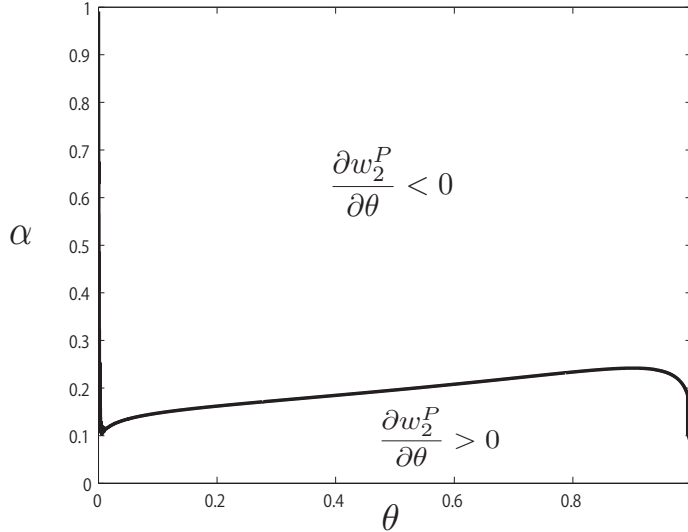


Figure 1: The sign of $\partial w_2^P / \partial \theta$

with

$$\lim_{\theta \rightarrow 0} w_1^P(\theta) = +\infty, \quad \lim_{\theta \rightarrow 1} w_1^P(\theta) = 0, \quad (25)$$

$$\lim_{\theta \rightarrow 0} l(\alpha, \theta) = 0, \quad \lim_{\theta \rightarrow 1} l(\alpha, \theta) = 1 - \alpha, \quad (26)$$

$$\lim_{\theta \rightarrow 0} r(\alpha, \theta) = 0, \quad \lim_{\theta \rightarrow 1} r(\alpha, \theta) = \alpha^\alpha (1 - \alpha)^{1 - \alpha}. \quad (27)$$

Proof. See Appendix. \square

This proposition asserts that the equilibrium value of w_1^P is decreasing with θ from a positive infinity to zero, and that those of l and r are increasing with θ from zero to $1 - \alpha$ and $\alpha^\alpha (1 - \alpha)^{1 - \alpha}$, respectively. As θ takes a smaller value or, equivalently, the production of the consumption good becomes riskier, firms producing that good choose to hire fewer workers and pay them more in the first period. A smaller value of θ also lowers the price of the capital good since the employment reduction in the consumption good sector necessarily leads to the same scale of employment expansion in the capital good sector, and thus to the increased production of that good.

While the signs of $dw_1^P/d\theta$, $\partial l/\partial\theta$, and $\partial r/\partial\theta$ are determined as in (24), that of $\partial w_2^P/\partial\theta$ may take either a positive or negative value depending on α and θ , as illustrated in Figure 1. The line drawn in that figure, on which $\partial w_2^P/\partial\theta$ takes a zero value, divides the unit square of the possible combinations of α and θ into two regions: w_2^P is decreasing with θ in the upper region and increasing with θ in the lower region. This contrast between the

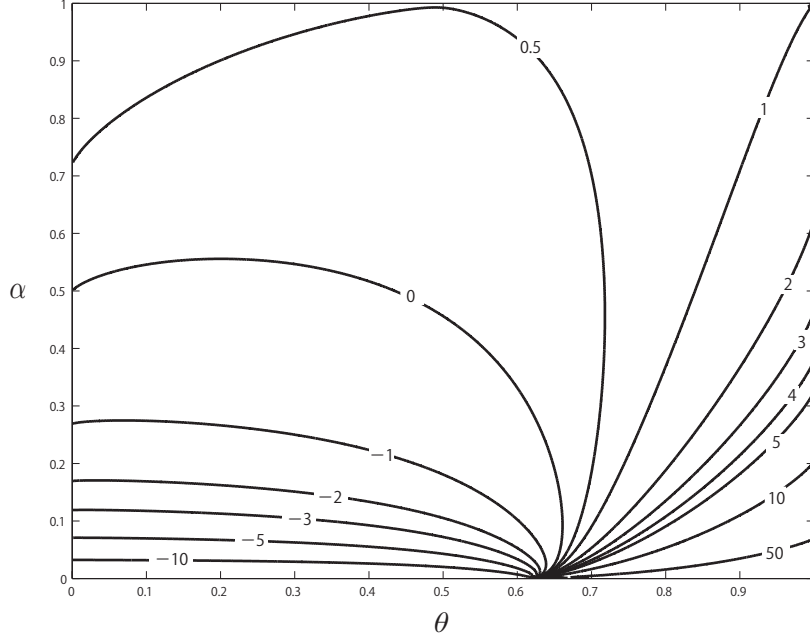


Figure 2: Contours for wage profiles

two regions can be explained by the differential effects of a given increase in θ on U^P and U^M . As shown in Proposition 1, such an increase in θ lowers the level of w_1^P and raises that of r , thereby changing the values of U^P and U^M in such a manner that their equality cannot be maintained unless the level of w_2^P is changed. Because the level of w_2^P is set to restore that equality, it is decreased to reduce the value of U^P if the increase in θ makes U^P larger than U^M , and it is increased if the increase in θ makes U^P smaller than U^M . The first type of response is observed in the upper region of Figure 1, and the second type is observed in the lower region.

Numerical methods also enable us to examine how a wage profile for the workers employed in the consumption good sector, which is measured by the difference between the wages paid in the first and second periods—that is, $w_2^P - w_1^P$ —is determined for a given pair of α and θ . Figure 2 depicts the contours for the wage profiles, each of which consists of combinations of α and θ that produce a given value of $w_2^P - w_1^P$. Specifically, the contour n consists of combinations of α and θ that support $w_2^P - w_1^P = n$. As shown in Figure 2, the wage profiles for the workers employed in the consumption good sector are front-loading—that is, $w_2^P - w_1^P < 0$ —when both α and θ are small enough, and they are back-loading—that is, $w_2^P - w_1^P > 0$ —in other cases. The value of $w_2^P - w_1^P$ is not always increasing with θ , but

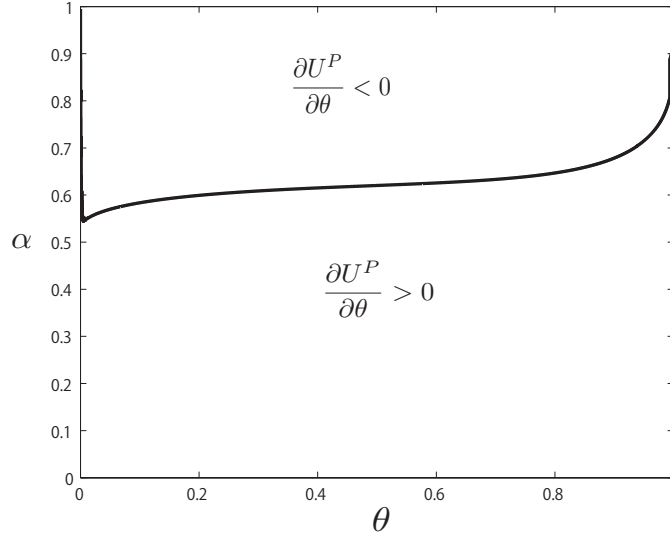


Figure 3: The sign of $\partial U^P / \partial \theta$

when θ is as large as $\theta > 0.5$, a small increase in θ leads to an increased value of $w_2^P - w_1^P$ for any value of α . Put another way, if the firms in the consumption good sector perceive a small reduction in θ from such a high level, they offer a flatter wage profile to their job applicants.

Proposition 2. *Let $U^P(\alpha, \theta)$ and $U^S(\alpha, \theta)$ denote, respectively, the equilibrium values of U^P and U^S . Then, for $\forall(\alpha, \theta) \in (0, 1) \times (0, 1)$, the following are true:*

$$\partial U^S / \partial \theta > \max[\partial U^P / \partial \theta, 0], \quad U^P(\alpha, \theta) > U^S(\alpha, \theta). \quad (28)$$

Proof. See Appendix. □

This proposition asserts that when the bankruptcy risk is present, job rationing occurs at the entrance of the consumption good sector, and thus, the workers employed in the consumption good sector are better off than those working in the capital good sector.

Proposition 2 also implies that the bankruptcy risk differentially affects the welfare of the workers employed in the capital good sector and that of those working in the consumption good sector. Specifically, a smaller value of θ invariably worsens the welfare of those working in the capital good sector while worsening less (or even improving in some cases) the welfare of those working in the consumption good sector, as illustrated in Figure 3. This result is paradoxical since the bankruptcy risk is more detrimental to those working in the capital good sector, who seem less subject to its

influence, than to those working in the consumption good sector, who seem more subject to its influence. This paradox can be explained by the fact that the risk has two opposing effects on the welfare of those working in the consumption good sector, but only a negative effect on the welfare of those working in the capital good sector. When the risk is present, the firms in the consumption good sector are compelled to pay a portion of the wage in the first period. This gives a moral hazard incentive to their employed workers, which, in turn, induces these firms to offer a sufficient amount of severance pay to prevent it. Undoubtedly, this sequence contributes to improving the welfare of those working in the consumption good sector, although it reduces that sector's total employment. This reduction expands employment in the capital good sector on the same scale, increasing the production of that good and lowering its market price. The lowered price of the capital good contributes to the worsening of the welfare of all workers since it implies a decreased rate of return on their savings. Thus, those working in the capital good sector are more heavily damaged by the bankruptcy risk than those working in the consumption good sector.

5 A Pareto-improving Policy

Our final task is to demonstrate that the general equilibrium of this economy fails to attain a constrained efficient allocation by adducing a Pareto-improving policy.

In the previous section, we saw that an increase in the bankruptcy risk may improve the welfare of those working in the consumption good sector while invariably worsening the welfare of those working in the capital good sector. However, on the balance, an increase in the bankruptcy risk negatively affects economic welfare, which is measured by the average value of lifetime utilities, $lU^P + (1-l)U^S$. This measure coincides with the expected value of the lifetime utility each worker estimates before entering the labor market. For any value of α , this measure is increasing with θ , implying that the *ex-ante* economic welfare deteriorates as bankruptcies become more and more likely.

Proposition 3. *Let $l(\alpha, \theta)$, $U^P(\alpha, \theta)$, and $U^S(\alpha, \theta)$ be defined as in Propositions 1 and 2. Then, for $\forall(\alpha, \theta) \in (0, 1) \times (0, 1)$, the following is true:*

$$\frac{\partial}{\partial \theta} \{l(\alpha, \theta)U^P(\alpha, \theta) + [1 - l(\alpha, \theta)]U^S(\alpha, \theta)\} > 0. \quad (29)$$

Proof. See Appendix. □

We must note that part of the observed deterioration in economic welfare is unavoidable. This deterioration reflects the reduced production of the consumption good, which is caused by an increase in the bankruptcy

risk. Note that the aggregate production of the consumption good can be expressed as $(1-l)^\alpha(\theta l)^{1-\alpha}$. As is obvious from this expression, a decrease in θ reduces the aggregate output for any given value of l . We call this the “direct” effect. In addition, it reduces the aggregate output by lowering the level of l . As is easily verified, the aggregate output is maximized at $l = 1 - \alpha$, although the realized level of l cannot take this value unless $\theta = 1$. As θ takes a smaller value, the realized level of l deviates from $1 - \alpha$, which effectively reduces the aggregate output. We call this the “indirect” effect. Of these effects, the direct one is obviously unavoidable. Part of the indirect effect is also unavoidable since it arises from the necessity of providing hired workers with a chance of self-insurance and preventing their moral hazard. Therefore, the question is whether the realized level of l is too low to be justified by these necessities. The answer is in the affirmative because the following policy can increase the employment of the consumption good sector, thereby improving economic welfare.

Suppose that the government decides to subsidize firms in the consumption good sector by s per worker. If this subsidy is financed by the lump-sum tax on the consumption good-producing firms, then the subsidy rate, s , and the lump-sum tax rate, t , must satisfy

$$sL^P = t, \quad (30)$$

where L^P denotes the number of workers per firm. Under this policy, the profit of a firm can be written as

$$(L^P)^{1-\alpha} - r - (Rw_1^P + w_2^P - s)L^P - t,$$

which modifies the first-order condition (11) as

$$(1 - \alpha)(L^P)^{-\alpha} - Rw_1^P - w_2^P + s = 0 \quad (31)$$

and leaves the other conditions unchanged. This implies that the policy is neutral to the equilibrium value of w_1^P since it is obtained from the unchanged conditions (18) and (22). In equilibrium, $r = \theta R$ and $L^P = \theta l / (1 - l)$, so (31) can be reduced to

$$(1 - \alpha)[(1 - l)/\theta l]^\alpha = (r/\theta)w_1^P + w_2^P - s. \quad (32)$$

The policy also modifies the equilibrium condition (19) as

$$\alpha[(1 - l)/\theta l]^{\alpha-1} = r + t. \quad (33)$$

Using $k \equiv (1 - l)/\theta l$, we can rewrite (30), (32), and (33) as, respectively,

$$t = s/k, \quad (34)$$

$$(1 - \alpha)k^\alpha = (r/\theta)w_1^P + w_2^P - s, \quad (35)$$

$$\alpha k^{\alpha-1} = r + s/k. \quad (36)$$

Combined with (18), conditions (35) and (36) produce

$$(1 - \alpha)k^\alpha/s = B/[\alpha B - (1 - \alpha)k] - 1, \quad (37)$$

$$r = k^\alpha/(B + k), \quad (38)$$

where $B \equiv (1 - \theta)w_1^P/\theta(1 - \theta - \theta w_1^P)$. We can treat B as a constant since the equilibrium value of w_1^P depends solely on θ .

When the value of s is sufficiently close to zero, equation (37) has two distinct roots, $k^*(s)$ and $k^{**}(s)(> k^*(s))$. As is easily verified, these roots satisfy

$$0 < k^*(s) < k^{**}(s) < [\alpha/(1 - \alpha)]B, \quad (39)$$

$$dk^*/ds > 0, \quad \lim_{s \rightarrow 0} k^*(s) = 0, \quad (40)$$

$$dk^{**}/ds < 0, \quad \lim_{s \rightarrow 0} k^{**}(s) = [\alpha/(1 - \alpha)]B, \quad (41)$$

where $[\alpha/(1 - \alpha)]B$ is the capital-labor ratio when $s = 0$ (See (23)). Which value of k is chosen in equilibrium depends crucially on the value of t . That is, if t is set as $t = s/k^*(s)$, then $k^*(s)$ is realized. However, if t is set as $t = s/k^{**}(s)$, then $k^{**}(s)$ is realized. In what follows, we assume that t is set as $t = s/k^{**}(s)$ because it is preferred by all workers.⁷

When $t = s/k^{**}(s)$, the average value of lifetime utilities can be written as follows:

$$\begin{aligned} lU^P + (1 - l)U^S &= lU^M + (1 - l)U^S \\ &= l \log r(1 + w_1^P) + (1 - l) \log r \\ &= \log r + l \log(1 + w_1^P) \\ &= \log \left(\frac{(k^{**}(s))^\alpha}{B + k^{**}(s)} \right) + \frac{1}{\theta k^{**}(s) + 1} \log(1 + w_1^P). \end{aligned}$$

By differentiating this with respect to s and then evaluating it at $s = 0$, we can obtain

$$\frac{d}{ds} [lU^P + (1 - l)U^S] \Big|_{s=0} = - \frac{\theta}{(\theta k^{**}(s) + 1)^2} \log(1 + w_1^P) \frac{dk^{**}}{ds} > 0,$$

which implies that the government can improve economic welfare by slightly subsidizing firms in the consumption good sector, and thus, the *laissez-faire* equilibrium fails to attain a constrained efficient allocation. The above policy has two conflicting effects on the average value of lifetime utilities. On the one hand, it lowers the capital good price, thereby reducing the values of U^P and U^S . This contributes to decreasing the average value of lifetime utilities.

⁷Workers are better off when $k^{**}(s)$ is realized than when $k^*(s)$ is realized. This is because both $U^P (= \log r(1 + w_1^P))$ and $U^S (= \log r)$ are increasing with r and because under this policy, r is increasing with $k \in (0, [\alpha/(1 - \alpha)]B)$ (See (38)).

On the other hand, it augments employment in the consumption good sector, thereby enhancing the probability of a worker being hired by that sector. This contributes to increasing the average value of lifetime utilities. When the subsidy rate, s , is sufficiently close to zero, the former negative effect becomes negligible, while the latter positive effect remains substantial. As implied by (38), the level of r is maximized at $k = [\alpha/(1 - \alpha)]B$. Since $k^{**}(s)$ satisfies (41), a sufficiently small subsidy hardly changes the level of r , while it enhances the level of l in the first order. This is why this policy can increase the average value of lifetime utilities and thus improve economic welfare.

By resorting to a numerical method, we can compute the optimal levels of tax and subsidy and their effects on employment and economic welfare. In the form of contours, Figures 4 and 5 summarize, respectively, the levels of tax (i.e., t) and subsidy (i.e., s) that maximize the average value of lifetime utilities for possible pairs of α and θ . In Figure 4, contours are drawn in increments of 0.01; the figure suggests that the optimal tax rate is decreasing with θ unless α takes a sufficiently small value. When α is sufficiently small, the optimal tax rate is not a monotone function of θ . It is increasing in some domains of θ and decreasing in others. In Figure 5, contours are drawn in increments of 1 because the optimal subsidy rate can take a broad range of positive values: it becomes as small as 0.0001 when $\alpha = 0.99$ and $\theta = 0.01$, and as large as 292.38 when $\alpha = 0.01$ and $\theta = 0.99$. As suggested by these contours, the optimal subsidy rate is decreasing with α and increasing with θ .

The effects of the policy with the optimally chosen tax and subsidy rates are summarized in Figures 6 and 7, which respectively report the population of workers who are additionally employed in the consumption good sector and the increments of the average value of the lifetime utilities attained by that policy. As shown in these figures, the policy has the biggest effect when both α and θ take a sufficiently small value. To understand why, we need to recall Proposition 1—(26) in particular—which has demonstrated that the potential number of workers to be hired in the consumption good sector, $1 - \alpha$, increases as α takes a smaller value, and that the actual number of workers hired in that sector, $l(\alpha, \theta)$, decreases as θ takes a smaller value. These imply that when both α and θ are sufficiently small, there is great room for improvement in the economy since the actual employment in the consumption good sector is far below that sector's employment capacity, and the existence of such a large gap makes it possible for strong effects to be observed in these cases. Conversely, when either α or θ is sufficiently large, there is very limited room for improvement. When α is sufficiently large, the employment capacity of the consumption good sector is very small. When θ is sufficiently large, the actual employment in the consumption good sector is extremely close to capacity. In both cases, the policy cannot be very effective because there is little room for a sizable increase in employment in

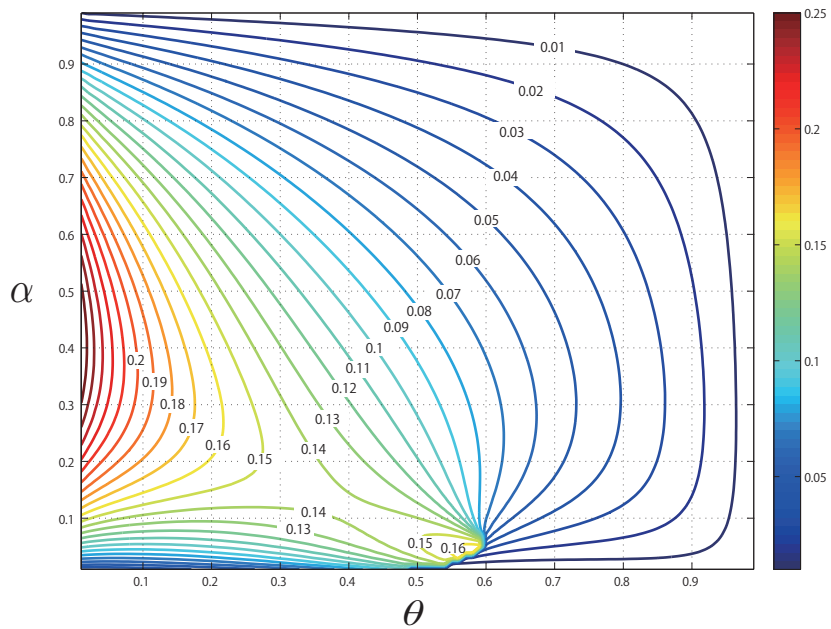


Figure 4: Optimal levels of tax (t)

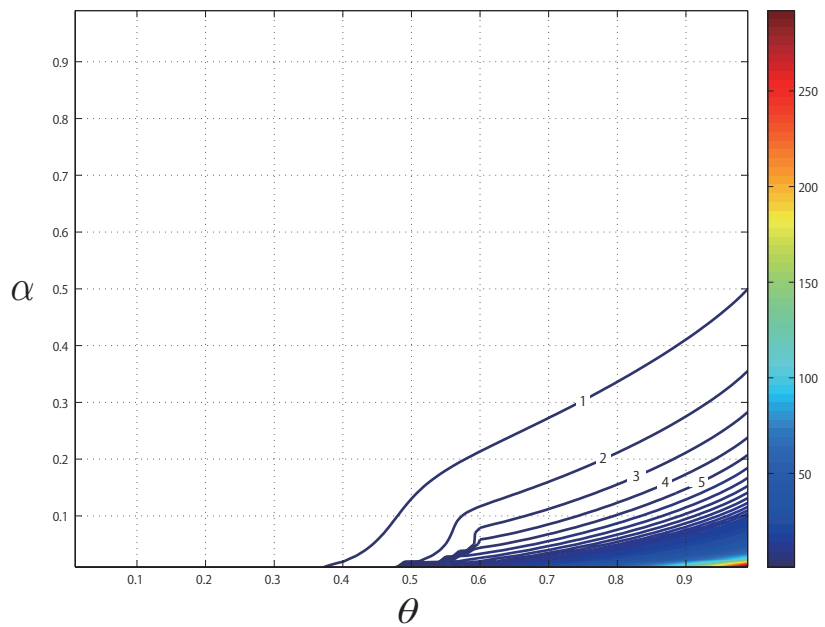


Figure 5: Optimal levels of subsidy (s)

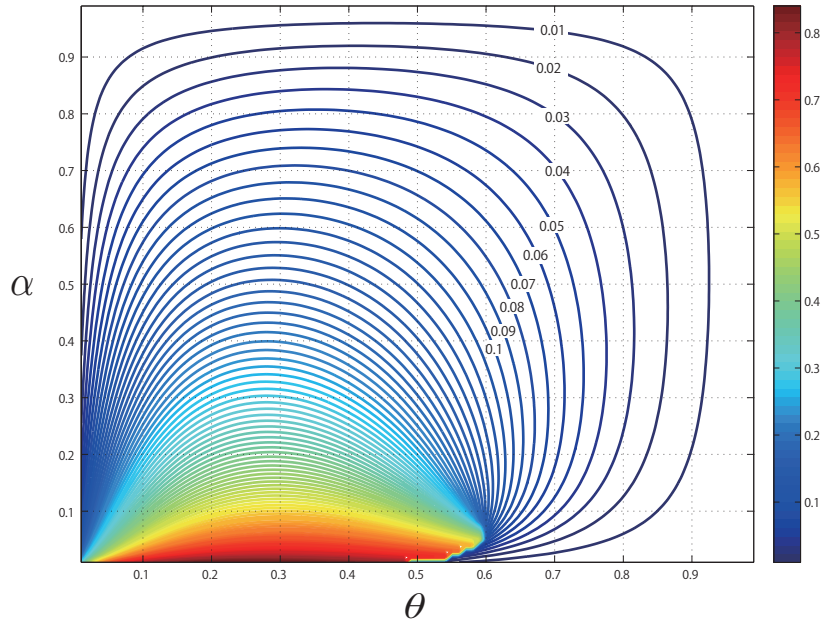


Figure 6: Increased employment in the consumption good sector

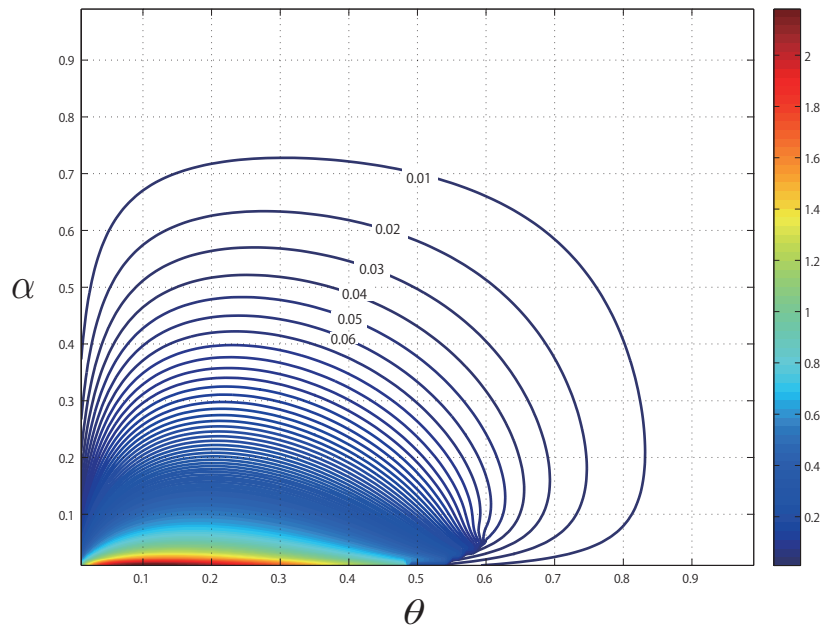


Figure 7: Welfare gains from the policy

the consumption good sector.

The argument presented in this section also reveals what makes the equilibrium value of l so small under *laissez-faire* policy. As a result of the positive spillover effect arising from the dual structure of the labor market, when there is a utility differential between those working in the capital good sector and those working in the consumption good sector, the decision of a firm in the consumption good sector to hire an additional worker unambiguously contributes to raising the average value of lifetime utilities. Nevertheless, no firms in the consumption good sector take into account this marginal contribution to economy-wide welfare when making their hiring decisions because it brings zero benefits to them. Such a gap between private and social gains from additional employment renders the total employment of that sector too small to attain a constrained efficient allocation. The policy considered here does a good job of filling that gap by subsidizing the firms in the consumption good sector.

6 Conclusion

This paper demonstrated that the possible bankruptcies of firms can have a substantial effect on not only their own wage schemes but also on the structure of the labor market. The bankruptcy risk flattens the wage profile of each firm, weakening its incentive effect, and thereby making room for the efficiency wage to be used as a supplementary incentive device. The use of the efficiency wage, in turn, stratifies the labor market into the primary market, in which job rationing is observed, and the secondary market, in which job rationing is not observed. As a result, a substantial utility differential emerges between those who have found a job in the primary market and those who have not, although there is no difference in their innate abilities. This differential widens as bankruptcies become more likely. In addition, the dual structure of the labor market renders the employment size of the primary market too small to attain a social optimum.

The results of this paper have a wide range of applications. For instance, they give an explanation for the observed decline in youth employment in Japan. As reported by Genda (2003), after the bursting of the economic bubble in the early 1990's, large companies in Japan became reluctant to expand and hire, resulting in a substantial decline in youth employment. While Genda identifies diminishing chances for youths to acquire skills as well as severe legal restrictions on dismissal as major causes of their displacement from decent jobs, the results obtained here suggest that an increased possibility of bankruptcy and informational asymmetry have jointly made hiring a new worker more costly and have thereby reduced job opportunities for youths. The results may also explain why large companies, the jobs of which constitute the primary labor market, choose not to substitute the

defined contribution system for the defined benefit system, but rather provide them together for their employees. As reported by Papke, Petersen, and Poterba (1996), most of the American companies that adopted a 401(k) plan, a defined contribution type of corporate pension in the United States, also continued to implement their pre-existing defined benefit pension plans. From the viewpoint of the incentive theory of wage profiles, it is puzzling that some companies that seem to have a strong incentive to maintain lifetime employment provided defined contribution pensions for their employees. Since this is equivalent to an upfront payment of severance pay, according to this theory, it negatively affects the morale of tenured workers.⁸ One possible explanation is that these companies provided the defined contribution plan for their employees as insurance against the possible bankruptcy of their defined benefit plan while motivating them with high wages. This explanation is consistent with the observation of size–wage differential.

Appendix

Proof of Proposition 1

Proof. Equation (22) can be rewritten as

$$\left(\frac{1}{w_1^P} - \frac{\theta}{1-\theta}\right)^{\theta/(1-\theta)} \left(\frac{1}{w_1^P} + 1\right) = 1$$

or

$$(\Omega - \Theta)^\Theta (\Omega + 1) = 1, \quad (42)$$

where $\Omega \equiv 1/w_1^P$ and $\Theta \equiv \theta/(1-\theta)$. First and most importantly, we need to demonstrate that (42) implicitly defines the function $\Omega(\Theta) : \mathbf{R}^+ \rightarrow \mathbf{R}$ and that $\Omega(\Theta)$ has the following properties:

$$0 < \Omega(\Theta) - \Theta < 1, \quad (43)$$

$$(\Omega(\Theta) - \Theta)' = \Omega'(\Theta) - 1 > 0, \quad (44)$$

$$\lim_{\Theta \rightarrow 0} (\Omega(\Theta) - \Theta) = \lim_{\Theta \rightarrow 0} \Omega(\Theta) = 0, \quad (45)$$

$$\lim_{\Theta \rightarrow +\infty} (\Omega(\Theta) - \Theta) = 1. \quad (46)$$

Let Θ take a given positive value. Then, the LHS of (42) becomes zero when $\Omega = \Theta$, and $\Theta + 2$ when $\Omega = \Theta + 1$. Because it is continuously

⁸A similar situation has also been observed in Japan. In July 2002, Toyota Motor Corporation, one of Japan's leading manufacturers, decided to implement defined contribution pensions together with defined benefit pensions by diverting part of the retirement benefit toward contributions to this system. Toyota's decision is puzzling since it seems to contradict the corporate policy of maintaining lifetime employment, the importance of which Toyota's top management has repeatedly emphasized.

increasing with Ω , these facts imply that for any value of Θ , the value of Ω is uniquely determined between Θ and $\Theta + 1$, establishing (43). To establish (44), differentiate both sides of (42) with respect to Θ . Then, we obtain

$$\log(\Omega - \Theta) + \Theta \frac{\Omega'(\Theta) - 1}{\Omega - \Theta} + \frac{\Omega'(\Theta)}{\Omega + 1} = 0,$$

which can be reduced to

$$\Omega'(\Theta) = \frac{-\log(\Omega - \Theta) + \frac{\Theta}{\Omega - \Theta}}{\frac{\Theta}{\Omega - \Theta} + \frac{1}{\Omega + 1}} > 0$$

or, equivalently,

$$\Omega'(\Theta) - 1 = \frac{-\log(\Omega - \Theta) - \frac{1}{\Omega + 1}}{\frac{\Theta}{\Omega - \Theta} + \frac{1}{\Omega + 1}} = \frac{\frac{1}{\Theta} \log(\Omega + 1) - \frac{1}{\Omega + 1}}{\frac{\Theta}{\Omega - \Theta} + \frac{1}{\Omega + 1}}, \quad (47)$$

the second equality of which is obtained from (42). Because (43) implies that the denominator of the RHS of (47) is positive, the sign of $\Omega'(\Theta) - 1$ coincides with that of the numerator of the RHS of (47). Since that numerator is increasing with Ω , we can say that the sign of $\Omega'(\Theta) - 1$ is positive if the following inequality is true:

$$\left[\frac{1}{\Theta} \log(\Omega + 1) - \frac{1}{\Omega + 1} \right] \Big|_{\Omega = \Theta} = \frac{1}{\Theta} \log(\Theta + 1) - \frac{1}{\Theta + 1} > 0. \quad (48)$$

To prove (48), define function f as

$$f(x) = \log(x + 1) - \frac{x}{x + 1}.$$

This function satisfies $f(x) > 0$ for $\forall x > 0$ because $f(0) = 0$ and because $f'(x) = x/(x + 1)^2 > 0$ for $\forall x > 0$. Using function f , we can rewrite the LHS of (48) as $f(\Theta)/\Theta$. Since $\Theta > 0$, this means that (48) is true, thus establishing (44). Properties (43) and (44) jointly imply that there exist limit values of $\Omega(\Theta) - \Theta$ when $\Theta \rightarrow 0$ and when $\Theta \rightarrow +\infty$, and that

$$0 \leq \lim_{\Theta \rightarrow 0} (\Omega(\Theta) - \Theta) < \lim_{\Theta \rightarrow +\infty} (\Omega(\Theta) - \Theta) \leq 1.$$

To establish (45), suppose that

$$\lim_{\Theta \rightarrow 0} (\Omega(\Theta) - \Theta) = \lim_{\Theta \rightarrow 0} \Omega(\Theta) > 0.$$

Then, when $\Theta \rightarrow 0$, (42) is simplified as

$$1 = \underbrace{\left[\lim_{\Theta \rightarrow 0} (\Omega(\Theta) - \Theta)^\Theta \right]}_1 \left(\lim_{\Theta \rightarrow 0} \Omega(\Theta) + 1 \right) = \lim_{\Theta \rightarrow 0} \Omega(\Theta) + 1,$$

which implies that $\lim_{\Theta \rightarrow 0} \Omega(\Theta) = 0$, contradicting our initial hypothesis. To establish (46), rewrite (42) as

$$\Omega(\Theta) - \Theta = (\Omega(\Theta) + 1)^{-1/\Theta}.$$

Combined with (43), this equation implies that

$$(\Theta + 2)^{-1/\Theta} < \Omega(\Theta) - \Theta < (\Theta + 1)^{-1/\Theta}.$$

Thus, our desired result will be obtained if we can show that

$$\lim_{x \rightarrow +\infty} (x + a)^{-1/x} = 1$$

or, equivalently,

$$\lim_{x \rightarrow +\infty} -(1/x) \log(x + a) = 0, \quad (49)$$

where a is a given positive number. Let b be another positive fixed number. Then, for sufficiently large values of x , the following inequalities must hold:

$$0 \geq -\frac{\log(x + a)}{x} \geq -\frac{1}{x} \left[\frac{x - b}{a + b} + \log(a + b) \right],$$

which suggests that

$$0 \geq \lim_{x \rightarrow +\infty} -\frac{\log(x + a)}{x} \geq \lim_{x \rightarrow +\infty} -\frac{1}{x} \left[\frac{x - b}{a + b} + \log(a + b) \right] = -\frac{1}{a + b}.$$

This condition requires that (49) be true. Otherwise, the above condition cannot be true for sufficiently large values of b .

Now that (43)–(46) have been established, we can use them to prove (24)–(27). From the defining equations of Ω and Θ , we can obtain

$$\frac{dw_1^P}{d\theta} = \frac{d}{d\Theta} \left(\frac{1}{\Omega(\Theta)} \right) \cdot \frac{d\Theta}{d\theta} = -\frac{\Omega'(\Theta)}{\Omega^2} \cdot \frac{1}{(1 - \theta)^2} < 0, \quad (50)$$

which is the first condition of (24). In addition, (45) and (46) imply, respectively,

$$\lim_{\theta \rightarrow 0} w_1^P = \lim_{\Theta \rightarrow 0} (1/\Omega(\Theta)) = +\infty$$

and

$$\lim_{\theta \rightarrow 1} w_1^P = \lim_{\Theta \rightarrow +\infty} (1/\Omega(\Theta)) = \lim_{\Theta \rightarrow +\infty} (1/(\Theta + 1)) = 0,$$

both of which constitute (25). Using Θ and $\Omega(\Theta)$, we can rewrite (23) as

$$\frac{1 - l}{l} = \frac{\alpha}{1 - \alpha} \cdot \frac{1}{\Omega(\Theta) - \Theta}. \quad (51)$$

Differentiating both sides of (51) with respect to θ yields

$$-\frac{1}{l^2} \cdot \frac{\partial l}{\partial \theta} = \frac{\alpha}{1-\alpha} \cdot \frac{-\Omega'(\Theta) + 1}{(\Omega(\Theta) - \Theta)^2} \cdot \frac{d\Theta}{d\theta}$$

or

$$\frac{\partial l}{\partial \theta} = \frac{\alpha}{1-\alpha} \cdot \frac{l^2}{(1-\theta)^2} \cdot \frac{\Omega'(\Theta) - 1}{(\Omega(\Theta) - \Theta)^2} > 0,$$

which is the second condition of (24). In addition, (26) is established by (45), (46) and (51). Specifically, (45) implies that

$$\lim_{\Theta \rightarrow 0} \frac{\alpha}{1-\alpha} \cdot \frac{1}{\Omega(\Theta) - \Theta} = +\infty,$$

which, combined with (51), means that $\lim_{\theta \rightarrow 0} l(\alpha, \theta) = 0$. Likewise, (46) implies that

$$\lim_{\Theta \rightarrow +\infty} \frac{\alpha}{1-\alpha} \cdot \frac{1}{\Omega(\Theta) - \Theta} = \frac{\alpha}{1-\alpha},$$

which, combined with (51), means that $\lim_{\theta \rightarrow 1} l(\alpha, \theta) = 1 - \alpha$. We can rewrite (19) as

$$r = \alpha^\alpha (1-\alpha)^{1-\alpha} \left[\frac{1+\Theta}{\Theta(\Omega(\Theta) - \Theta)} \right]^{\alpha-1}. \quad (52)$$

Differentiating both sides of (52) with respect to θ yields

$$\begin{aligned} \frac{\partial r / \partial \theta}{r} &= -(1-\alpha) \left[\frac{1}{1+\Theta} - \frac{1}{\Theta} - \frac{\Omega' - 1}{\Omega - \Theta} \right] \frac{d\Theta}{d\theta} \\ &= (1-\alpha) \left[\frac{1}{\Theta(1+\Theta)} + \frac{\Omega' - 1}{\Omega - \Theta} \right] \frac{1}{(1-\theta)^2} > 0, \end{aligned} \quad (53)$$

which is the third condition of (24). In addition, (27) is established by (45), (46) and (52). Specifically, (45) implies that

$$\lim_{\Theta \rightarrow 0} \frac{1+\Theta}{\Theta(\Omega(\Theta) - \Theta)} = +\infty,$$

which, combined with (52), means that $\lim_{\theta \rightarrow 0} r(\alpha, \theta) = 0$. Likewise, (46) implies that

$$\lim_{\Theta \rightarrow +\infty} \frac{1+\Theta}{\Theta(\Omega(\Theta) - \Theta)} = 1,$$

which, combined with (52), means that $\lim_{\theta \rightarrow 1} r(\alpha, \theta) = \alpha^\alpha (1-\alpha)^{1-\alpha}$. \square

Proof of Proposition 2

Proof. To establish this proposition, we need only recall that

$$U^S = \log r(\alpha, \theta)$$

and that, in equilibrium,

$$\begin{aligned} U^P - U^S &= U^M - U^S \\ &= \log r(\alpha, \theta)(1 + w_1^P(\theta)) - \log r(\alpha, \theta) \\ &= \log(1 + w_1^P(\theta)). \end{aligned}$$

The first condition implies that

$$\frac{\partial U^S}{\partial \theta} = \frac{\partial r / \partial \theta}{r(\alpha, \theta)} > 0.$$

The second condition implies that

$$\frac{\partial}{\partial \theta}(U^P - U^S) = \frac{\partial U^P}{\partial \theta} - \frac{\partial U^S}{\partial \theta} = \frac{dw_1^P / d\theta}{1 + w_1^P(\theta)} < 0.$$

These jointly mean that

$$\partial U^S / \partial \theta > \max[\partial U^P / \partial \theta, 0]$$

and that the value of $U^P - U^S$ is decreasing with θ . Moreover, when $\theta = 1$,

$$U^P - U^S = \log(1 + w_1^P(1)) = \log 1 = 0.$$

Therefore, we can conclude that $U^P - U^S > 0$ for any value of $\theta \in (0, 1)$. \square

Proof of Proposition 3

Proof. The average value of lifetime utilities can be expressed as

$$\begin{aligned} lU^P + (1-l)U^S &= U^S + l(U^P - U^S) \\ &= \log r(\alpha, \theta) + l(\alpha, \theta) \log(1 + w_1^P(\theta)). \end{aligned}$$

Because $0 < l(\alpha, \theta) < 1 - \alpha$, we can obtain the following inequalities:

$$A(\alpha, \theta) < lU^P + (1-l)U^S < B(\alpha, \theta), \quad (54)$$

where A and B are defined as

$$\begin{aligned} A(\alpha, \theta) &\equiv \log r(\alpha, \theta); \\ B(\alpha, \theta) &\equiv \log r(\alpha, \theta) + (1 - \alpha) \log(1 + w_1^P(\theta)). \end{aligned}$$

As is easily verified,

$$\frac{\partial A}{\partial \theta} = \frac{\partial r / \partial \theta}{r} > 0,$$

and, using Ω and Θ , both of which are defined in the proof of Proposition 1, we can show that

$$\begin{aligned} \frac{\partial B}{\partial \theta} &= \frac{\partial r / \partial \theta}{r} + (1 - \alpha) \frac{dw_1^P / d\theta}{1 + w_1^P} \\ &= \frac{1 - \alpha}{(1 - \theta)^2} \left[\frac{1}{\Theta(1 + \Theta)} + \frac{\Omega' - 1}{\Omega - \Theta} - \frac{\Omega'}{\Omega(1 + \Omega)} \right] \\ &= \frac{1 - \alpha}{(1 - \theta)^2} \left[\frac{1}{\Theta(1 + \Theta)} - \frac{1}{\Omega(1 + \Omega)} + \frac{\Omega' - 1}{\Omega - \Theta} \left(1 - \frac{\Omega - \Theta}{\Omega(1 + \Omega)} \right) \right] > 0, \end{aligned}$$

the second equality of which is obtained from (50) and (53). Define $q(\alpha, \theta)$ as $q(\alpha, \theta) \equiv l(\alpha, \theta) / (1 - \alpha)$. By construction, $q(\alpha, \theta)$ is an increasing function of θ , with $\lim_{\theta \rightarrow 0} q(\alpha, \theta) = 0$ and $\lim_{\theta \rightarrow 1} q(\alpha, \theta) = 1$. It is easy to show that

$$\frac{\partial q}{\partial \theta} = \frac{\partial l / \partial \theta}{1 - \alpha} > 0.$$

Using $q(\alpha, \theta)$, we can rewrite $lU^P + (1 - l)U^S$ as

$$lU^P + (1 - l)U^S = [1 - q(\alpha, \theta)]A(\alpha, \theta) + q(\alpha, \theta)B(\alpha, \theta).$$

By differentiating it with respect to θ , we can obtain

$$\frac{\partial}{\partial \theta} [lU^P + (1 - l)U^S] = (1 - q) \frac{\partial A}{\partial \theta} + q \frac{\partial B}{\partial \theta} + (B - A) \frac{\partial q}{\partial \theta} > 0,$$

which implies that the average value of lifetime utilities is an increasing function of θ . \square

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