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TWO PSYCHOPHYSICAL POWER FUNCTIONS OF FORCE OF HANDGRIP

By

SHINYA YOSHIDA (吉田信彌)

(*Department of Psychology, Tohoku University, Sendai*)

The discussed two power functions were $Y=kX^n$ and $Y=k(X-X_0)^n$ where Y is the sensory magnitude and X is the intensity of the stimulus. By mathematical considerations, the basis of these functions were made clear. The basic principle of the former function is that "equal stimulus ratios produce equal sensation ratios" and that of the latter is that "equal increased stimulus ratios produce equal sensation ratios". These principles can be tested directly in the normal plots of the standard stimulus and the comparative stimulus obtained by the method of fractionation and/or multiplication. A new interpretation of X_0 was attempted in terms of this method. X_0 shows the stimulus value near which two comparative stimuli in fractionation and/or multiplication approximate each other and it shows the standard value when we nominally restrict the range the psychophysical function covers in the form of $X>X_0$. Experiments of force of handgrip were performed and twofold power functions were confirmed.

Two Psychophysical Power Functions

As well known S.S. Stevens insisted the psychophysical function can be described as

$$Y = kX^n \tag{1}$$

where Y is the intensity of sensation (the psychological magnitude) and X is the intensity of stimulus (the physical magnitude), and k is an arbitrary constant which depends on the unit of measurement. The exponent n is a constant determined by stimulus conditions, ranging from 0.33 for brightness to 3.5 for electric shock. The function (1) has a convenient characteristics that the function is represented in log-log coordinates by a straight line whose slope is n.

Stevens, S.S. (1959) further generalized the psychophysical function in the form of

$$Y = k(X-X_0)^n \tag{2}$$

where X_0 is "a constant value corresponding to 'threshold' " (Stevens, S.S. 1960). The value of X_0 is "usually negligible" and if $X_0=0$, the psychophysical function (2) coincides with (1). However, in the experiments of tactile vibration on the arm (Stevens, S.S. 1959), warmth and cold on the forearm (Steven, J.C. & Stevens, S.S. 1960) and brightness near threshold (Marks & Stevens 1965, Stevens, J.C. & Stevens, S.S. 1963), the value of X_0 could not be neglected and the function (2) was validated.

The graphic relation in log-log coordinates between the two power functions is represented in Fig. 1. That is, if the linear function (1) is slid in the direction of $+X_0$,

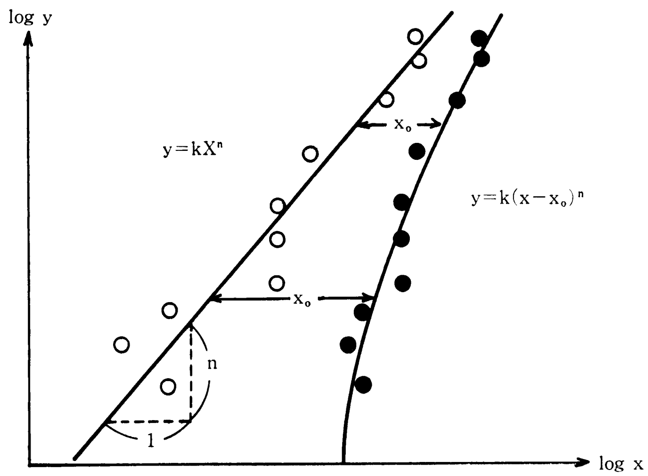

Fig. 1. Two power functions.

then we have the curvilinear function (2). It is often difficult to determine which function should be hypothesized in the log-log plot of the data (X, Y) for it is both possible in the Fig. 1 to hypothesize the straghtline function (1) and the curvilinear function (2) on the same data (X, Y). When the function (2) is fitted, how is X_0 derived ?

The same psychophysical function as (2) was proposed by Ekman (1958). X_0 was still "the absolute threshold" there. But many psychophysicists differ in the interpretation of X_0.

According to Stevens, X_0 is a "threshold" but differs from the threshold which is called in the classical psychophysics. "It is not necessarily the threshold as measured in some arbitrary manner under arbitrary conditions. Rather, it should probably be thought of as the 'effective' threshold that obtains at the time and under the conditions of the experiment in which the magnitude scale is determined. Needless to say, this effective threshold cannot be measured very precisely. Consequently, it becomes expedient to take as the value of X_0 the constant value whose subtraction from the stimulus values succeeds in rectifying the log-log plot of the magnitude function." (Stevens, S.S. 1960)

It seemed reasonable to interpret X_0 as threshold in the experiments of Stevens *et al.* mentioned above. But Corso (1963) critisized "effective threshold" saying that it was chosen too expediently and should be given in terms of experimental operations.

If we attempt to determine "effective threshold" operationally, the threshold and the magnitude scale must be given at the same time in one experiment. Ekman and Gustafsson (1968) attempted such an experiment. But when they subtracted the experimentally determined X_0 from the stimulus values, they found two straght lines in the log-log plot, which differed in the stimulus ranges. It seems so far that only Gescheider and Wright (1968) succeeded in determining the value of X_0 operationally and validating Stevens' law in the form of the function (2).

Several psychologists, however, disagree with the "threshold" interpretation.

Eisler and Ottander (1963) wrote "the parameter X_0 has not much to do with the absolute threshold" and that their results may "throw some light on the problem of the time-order error, an effect that seems to be closely connected with hysteresis." Jones and Woskow (1966), however, explained those results by the adaptation level theory.

Tanaka and Nakatani (1966) argued for Fagot that it is not an adequate interpretation that the parameter X_0 represents a threshold because X_0 sometimes takes negative values and that a negative threshold is impossible.

In the equation (2), if $X_0<0$ and $X=0$, then $Y>0$. That means some sensory magnitude exists while the stimulus does not exist. Ekman (1959) tentatively interpreted it as "a certain amount of sensory noise, resulting from spontaneous nervous activity". But McCallum and Goldberg (1975) thought that Ekman's explanation was not satisfactory and viewed X_0 as a mathematical artifact.

As Poulton (1968) pointed out, the introduction of X_0 results in having three constants in the psychophysical function. Extremely speaking, any given data (X, Y) will fit the equation (2) by arbitrary selection of three constant values. If three or more constants are permitted to involve, many other forms of psychophysical functions will be supposed, as exemplified by Ekman (1959, 1961), Marks & Stevens (1968) and McCallum & Goldberg (1975). In this paper, however, the author will pick up the two power functions which are the most general and popular and discuss them in detail in the following sections.

On the function: $Y=kX^n$

The basic principle of the power function (1) is that "equal stimulus ratios produce equal sensation ratios." (Stevens, S.S. 1957) That is, if the ratio of the two stimuli, X_i and X_j, is constant —

$$\frac{X_i}{X_j}=a\,,$$

then the corresponding ratio of the two sensory magnitudes, Y_i and Y_j, is constant —

$$\frac{Y_i}{Y_j}=p\,.$$

In doubling method, where $p=2$,

$$X_i=aX_j \qquad (3).$$

That is, the subjectively doubled stimulus X_i is a linear function of the standard stimulus X_j which goes through the origin.

Now if the sensory magnitude Y corresponds to the stimulus X, then the doubled sensory magnitude $2Y$ corresponds to the stimulus aX by the equation (3). Doubling it further, one get $4Y$ which corresponds to a^2X. The corresponding relation of X

and Y and their logarithm is viewed as follows:

$\log Y+3\log 2$	$8Y \cdots\cdots a^3X$	$\log X+3\log a$
$\log Y+2\log 2$	$4Y \cdots\cdots a^2X$	$\log X+2\log a$
$\log Y+\log 2$	$2Y \cdots\cdots aX$	$\log X+\log a$
$\log Y$	$Y \cdots\cdots X$	$\log X$
$\log Y-\log 2$	$1/2\ Y \cdots\cdots a^{-1}X$	$\log X-\log a$
$\log Y-2\log 2$	$1/4\ Y \cdots\cdots a^{-2}X$	$\log X-2\log a$
$\log Y-3\log 2$	$1/8\ Y \cdots\cdots a^{-3}X$	$\log X-3\log a$
$\vdots$	$\vdots \quad \vdots$	$\vdots$
$\log Y+(k+1)\log 2$	$2^{k+1}Y \cdots a^{k+1}X$	$\log X+(k+1)\log a$
$\log Y+k\log 2$	$2^kY \cdots\cdots a^kX$	$\log X+k\log a$

Thus when the stimulus value varies from a^kX to $a^{k+1}X$, the corresponding sensory magnitude varies from 2^kY to $2^{k+1}Y$. Define $\mathbf{X}=\log X$ and $\mathbf{Y}=\log Y$. Now when the stimulus varies from $\mathbf{X}+k\log a$ to $\mathbf{X}+(k+1)\log a$, and the sensory magnitude varies from $\mathbf{Y}+k\log 2$ to $\mathbf{Y}+(k+1)\log 2$, then its increased rate $\log 2/\log a$ is constant despite the value of k (cf. Fig. 2). Therefore, if

$$n=\frac{\log 2}{\log a},$$

then

$$\mathbf{Y}=n\mathbf{X}+K$$

$$\log Y=n\log X+K$$

$$Y=KX^n.$$

Accordingly, if that "equal stimulus ratios produce equal sensation ratios," is validated in the form of the equation (3), the psychophysical power function (1) is verified. That

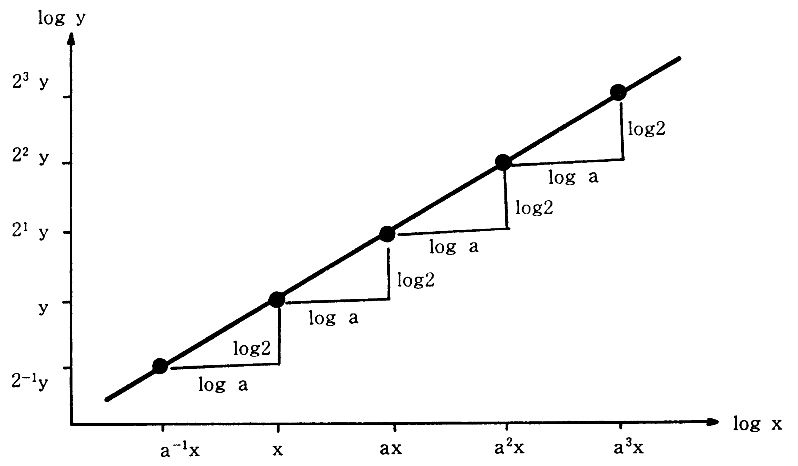

Fig. 2. If $X_i=aX_j$ then $Y=kX^n$.

is, if the stimilus ratios a which corresponds to the sensation ratio p is determined, the exponent n of the psychophysical function (1) will be given, as Stevens (1957) wrote, by

$$n = \frac{\log p}{\log a}\,. \qquad (4)$$

Conversely, if the equation (1) is confirmed and X_p produces the sensory magnitude pY, where p is a constant which fractionates or multiplies the sensory magnitude Y, then

$$Y = kX^n \qquad (1)$$

$$pY = kX_p{}^n \qquad (1)'.$$

Combining (1) and (1)′, we get

$$pX^n = X_p{}^n$$

$$X_p = p^{1/n}X\,.$$

Let $A = p^{1/n}$, then

$$X_p = AX$$

$$n = \frac{\log p}{\log a}\,,$$

and so we have the equations (3) and (4). Therefore, it is concluded that if the equation (1) is confirmed, that "equal stimulus ratios produce equal sensation ratios" is validated.

The most popular way of determining the exponent n is a graphic solution. It is 1) to plot the data (X, Y) obtained by magnitude estimation and production in log-log coordinates, and 2) to identify the value of n with the slope of the straight line on the graph. On the other hand the algebraic solution is 1) to plot the data (X_i, X_j) obtained by p fractionation and/or multiplication in normal coordinates, 2) to identify the value of a with the slope of the line which goes through the origin, (to confirm the equation (3)) and 3) to derive the exponent n from the equation (4).

Few psychologists take this algebraic solution but for two reasons it is necessary to test the hypothesis that "equal stimulus ratios produce equal sensation ratios" by confirming the equation (3). First, although the hypothesis is *mathematically* validated, if the function (1) is confirmed, it should be validated by psychological procedures in a direct form. Secondly, as pointed out before, it is difficult to determine which function (1) or (2) should be chosen only by graphic solution.

A few other algebraic procedures taken will be introduced next and compared with the method discussed here.

Log-log plot of (X_i, X_j). One way of confirming that "equal stimulus ratios produce equal sensation ratios" is by log-log plotting the data (X_j, X_j) and identifying the slope of the line with about 1.0, as exemplified in the experiments of Guilford &

Digman's (1954) lifted weight and Stevens & Mack's (1959) force of handgrip. The difference lies in the log-log plot or the normal plot of the data (X_i, X_j) obtained by the halving method. When the physical units are logarithmic this solution may be practical, but when normal like ones of weight and force the normal plotting is less cumbersome for the procedure of logarithmic version.

However, if the slope does not approximate 1.0 in the log-log plot, what psychophysical function will be derived ? In the normal plot, even if the line does not go through the origin, the function (2) will be derived as will be demonstrated later.

Teghtsoonian's equation Teghtsoonian, R. (1971, 1973) who discussed the problem of range effects, proposed the exponent of Stevens' function (1) is given as

$$n = \frac{\log R_y}{\log R_x}$$

where R_x is the ratio of the strongest to the weakest stimulus intensity and R_y is the ratio of the corresponding sensory magnitudes. As a matter of fact, however, this is never "a revision of Stevens' law" but just a rewriting of the equation (4) in an incomplete form where $p \fallingdotseq R_y$ and $a \fallingdotseq R_x$.

In most experiments the exponent is derived by graphic solution, where n, the slope of the line is determined by *several* (at least more than three) pairs of the data (X, Y). But in Teghtsoonian's equation, n is determined basically by only two pairs of (X, Y) — the strongest stimulus-sensation pair and the weakest one. Since X-Y pairs do not get on a line completely but distribute in probability along it, it is natural that the difference occurs between n derived by several X-Y pairs and n' derived by just two X-Y pairs. (Fig. 3.)

That is the reason why Teghtsoonian (1973), when he analyzed the value of n given by the psychophysical experiments, had to rewrite his equation as

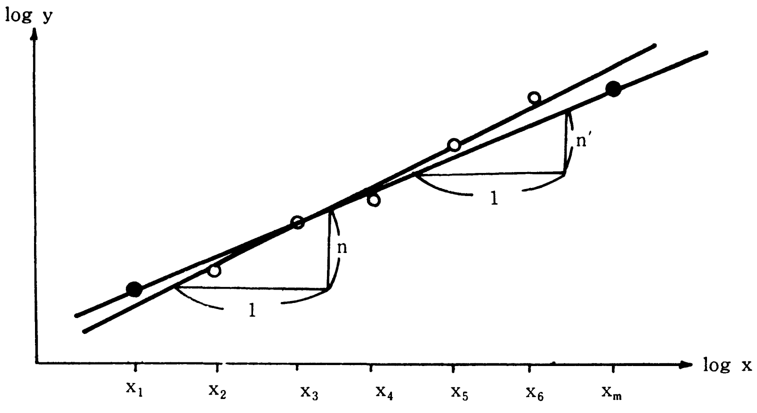

Fig. 3. The difference occurs if n is derived from all the X–Y pairs (filled and unfilled circles) while n' is derived from just two X–Y pairs (filled circles).

$$\log R_y = n \log R_x + K\,.$$

The adding constant K denotes the difference explained above, and does not bear any psychological significance as he expected. As it would be better to determine the value of n by more than two stimulus-sensation pairs, it does not seem that Teghtsoonian's equation implies a new contribution.

His distinctive discovery was the invariance of R_y in most psychophysical experiments. In magnitude estimation

$$\log R_y = 1.53\,,$$

that is, the ratio of the strongest to the weakest sensory magnitude is about 34. However, does this fact denote the invariance of subjective or judgemental range? No, it denotes special favour of stimulus range employed in Stevens' experiments. Take an example of the force of handgrip, (Stevens & Mack, 1959) the stimulus range is limited to 2 kg to 18 kg. Why did they not extend the range to 30 kg or 50 kg? If so, the value of R_y would have been larger as the value of R_x becomes larger.

On the function: $Y = k(X-X_0)^n$

It is Ekman (1958, 1961) who derived from the psychophysical power function (2) that

$$X_p = aX + b \tag{5}$$

where X_p is the stimulus value whose sensory magnitude is p times as large as that of the standard stimulus value X and

$$n = \frac{\log p}{\log a} \qquad X_0 = \frac{b}{1-a}\,. \tag{6}$$

Now we will follow the converse of Ekman's derivation. Suppose the equation (5) is confirmed by means of p fractionation (or multiplication), then we have

$$X_p = aX + b \tag{5}$$

where $b \neq 0$. Note that the equation (5) is identified with (3) if $b=0$.

When $a \neq 1$, the linear function (5) goes through the point $(b/1-a,\ b/1-a)$ where $X_p = X$. Therefore, the equation (5) can be expressed as

$$X_p - \frac{b}{1-a} = a\left(X - \frac{b}{1-a}\right).$$

Let $\mathbf{X} = X - b/(1-a)$, then we obtain

$$\mathbf{X}p = a\mathbf{X}$$

By the same way as we derived the equation (1) from the equation (3), we can obtain

$$\log Y = \frac{\log p}{\log a} \log \mathbf{X} + K \qquad (\mathbf{X} > 0)\,.$$

Let $n = \dfrac{\log p}{\log a}$ and $X_0 = \dfrac{b}{1-a}$, then

$$\log Y = n \log (X - X_0) + K \qquad X > X_0 .$$

Therefore

$$Y = k(X - X_0)^n \qquad X > X_0 \tag{2}$$

When $a=1$, the psychophysical function is derived from $X_p = X + b$ as

$$\log Y = mX + k \qquad m = \frac{\log p}{b} . \tag{7}$$

To summarize the discussions briefly, in the normal plot of the standard stimulus X and the comparative stimulus X_p obtained by fractionation and/or multiplication if

$$X_p = aX$$

that is, if equal stimulus ratios produce equal sensation ratios, then the psychophysical function is described as

$$Y = kX^n .$$

But if

$$X_p = aX + b \qquad (a \neq 1,\ b \neq 0)$$

that is, if equal increased stimulus ratios produce equal sensation ratios, then the psychophysical function is

$$Y = k(X - X_0)^n .$$

Therefore when to chose which to hypothesize the function (1) or (2) is difficult by graphic solution, the choice can be made in terms of which equation (3) or (5) is confirmed in fractionation and/or multiplciation, that is, whether or not the line goes through the origin in X–X_p normal coordinates.

As reviewed in the introductory part of this paper, the interpretation of X_0 is always the problem in the psychophysical function (2). But these mathematical discussions will reveal the character of X_0.

In the equation

$$X_p = aX + b \tag{5}$$

if $X = \dfrac{b}{1-a} (= X_0)$, then $X_p = \dfrac{b}{1-a} (= X_0)$.

It follows that the standard stimulus X equals the comparative stimulus X_p which produces the sensory magnitude p times as large as that of the X. As a matter of fact it is rare that the standard stimulus equals the comparative stimulus by fractionation or multiplication. Rather it would be right to say that the standard stimulus approximates the comparative stimulus near the value of X_0.

Accordingly, if X_0 is absolute threshold value, it is natural that the standard stimulus should approximate the comparative stimulus, where two stimuli are difficult to detect and distinguish. And it is not always near the threshold where the standard stimulus approximates the comparative. If one would prefer the term "threshold", it may be termed "threshold of fractionation or multiplication".

When X_0 takes negative value, it follows that the stimulus value which equals the standard to the compararive stimulus is negative. That means that such a stimulus value cannot be taken in p fractionation or multiplication. This may be rather realistic. When X_0 is negative, $Y>0$ in the function (2). Some psychologists speculated and doubted about it, but what the psychophysical function intends is the description of the changing way of sensory intensity and does not deal with the absolute value of sensory magnitude Y.

In fractionation and multiplication the standard stimulus and the comparative are constantly in a relation of larger or smaller. And in the equation (5)
In case of $1-a>0$,

$$X_p > X \text{ if } X < \frac{b}{1-a} = X_0;$$

$$X_p < X \text{ if } X > \frac{b}{1-a} = X_0.$$

In case of $1-a<0$,

$$X_p < X \text{ if } X < \frac{b}{1-a} = X_0;$$

$$X_p > X \text{ if } X > \frac{b}{1-a} = X_0.$$

It is seen that X_0 is a critical value which reverses the larger or smaller relationships between X and X_0. For example, in the halving method where $p=1/2$, the relationships between the standard stimulus X_s and the halved stimulus X_h is

if $X > X_0$, then $X_s > X_h$.
But if $X < X_0$, then $X_s < X_h$.

That denotes that subjective halving of 5 kg is 8 kg. And in terms of the function (2), if $X<X_0$, we have to solve the power of negative value. To exclude such a contradiction, X_0 suggests the standard value when we restrict the stimulus range that the psychophysical function covers in the form of $X>X_0$.

In conclusion X_0 is mathematically the value where the standard stimulus X equals the comparative stimulus X_p, and is the critical value which reverses the larger or smaller relationships of X and X_p. Actually or realistically it shows the stimulus value near which two compared stimuli in fractionation and/or multiplication approximate each other and is the standard value when we restrict the range that the psychophysical function covers in the form of $X>X_0$.

Experiments of Force of Handgrip

Stevens, J.C. and Mack, J.D. (1959) proposed the psychophysical function of the force of handgrip as

$$Y = kX^{1.7}.$$

They confirmed that "equal stimulus ratios produce equal sensation ratios" in the log-log plot of the two compared stimuli obtained by the method of halving and doubling and derived the exponent by graphic solution from the data (X, Y) obtained by magnitude production and estimation. The stimulus range employed, however, was limited to from 2 kg. to 18 kg. Why don't we extend the stimulus range to the maximum force of handgrip ? Applying the halving method to wider range, we will test the psychophysical power law of the force of handgrip.

Experiment I. Halving Production

Subject: Four undergraduates (male) whose maximum forces were more than 50 kg.

Procedure: The standard forces were taken so that they would distribute at random from 0 kg. to the maximum force. Subjects exerted the standard force first and then extented the force whose perceptual magnitude was half that of the former force. After halving, subjects were asked to rate every halving judgement, that is, A for "successful halving" or "judged with certainty", B for "so so" or "OK", and C for "failure" or "have no idea". About eighty or even one hundred pairs of the standard force X_s and the halved force X_h were obtained in a 1-hour experiment during which the subject often took a sufficient rest. Every subject performed the experiment three or four times on different days. Totally more than 250 pairs of X_s and X_h were obtained for each subject. Only the pairs rated as A were picked up as available data.

Data processing: All the A-responses (X_s, X_h) obtained during the experiments were plotted in X_s–X_h normal coordinates for each subject respectively (Individual data). In order to inspect the range difference two lines were fitted by the method of least squares. One line was fitted on the X_s–X_h response pairs which were distributed over the whole range from 0 kg. to the maximum. The other was on the pairs over the range from 0 kg. to 20 kg. which Stevens and Mack dealt with. By seeing whether or not the fitted lines go very near the origin, we will determine which equation (3) or (5) that is, which psychophysical function (1) or (2) should be valid to hypothesize.

Results: Results were represented in Fig. 4. The fitted lines and the derived functions were as follows:

The whole range from 0 kg. to the maximum.

K.S. $X_h = 0.54X_s + 2.24$ ($r = .963$) $\longrightarrow Y = k(X-4.8)^{1.1}$

I.K. $X_h = 0.44X_s + 3.18$ ($r = .918$) $\longrightarrow Y = k(X-5.7)^{0.8}$

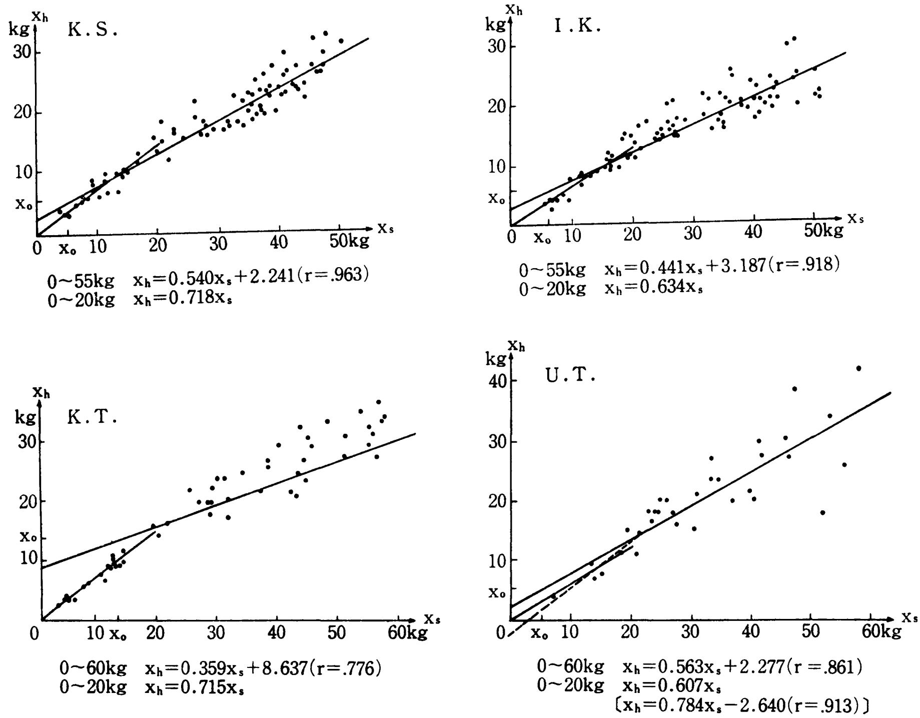

Fig. 4. Halving production: Results of Experiment I.

K.T. $X_h=0.35X_s+8.63$ ($r=.776$) ⟶ ....*

U.T. $X_h=0.56X_s+2.27$ ($r=.861$) ⟶ $Y=k(X-5.2)^{1.2}$

As the fitted lines for the smaller range go very near the origin, it seemed valid to hypothesize the equation (3) and derive the psychophysical function (1) by the equation (4).

The smaller range from 0 kg. to 20 kg.

K.S. $X_h = 0.71X_s$ ⟶ $Y = \mathrm{k}X^{2.1}$

($X_h=0.71X_s+0.11$ $r=.931$)**

I.K. $X_h = 0.63X_s$ ⟶ $Y = kX^{1.5}$

($X_h=0.66X_s-0.42$ $r=.933$)

K.T. $X_h = 0.71X_s$ ⟶ $Y = kX^{2.1}$

($X_h=0.76X_s-0.60$ $r=.965$)

U.T. $X_h = 0.60X_s$ ⟶ $Y=kX^{1.4}$

($X_h=0.78X_s-2.64$ $r=.913$ ⟶ $Y=k(X+12.2)^{2.9}$)

* Since the coefficient of correlation was not so high, the psychophysical function was not derived. However, for the results of K.T. the coefficients obtained in daily experiments were high. (more than .95)

** The functions in the parentheses were the original fitted lines fitfed by the method of least squares. The intercepts were less than 1.0 except that of U.T.

It was thus found that the principle meaning that "equal stimulus ratios produce equal sensation ratios" was valid for the smaller range from 0 kg. to 20 kg. as Stevens and Mack confirmed. But in the wider range this principle did not seem valid and the power law was confirmed in the form of the function (2).

Experiment II Magnitude Production

As Tanaka and Nakatani (1966) pointed out, it is dangerous to derive psychophysical functions only by the halving method because any constant error may be contained in halving judgements. A comparison was made between the exponents derived by the halving method and those determined by magnitude production.

Subject: The same subjects that participated in Experiment I.

Procedure: In the Experiment II–1 the standard stimulus was taken to be about 5 to 8 kg. so that the stimulus range would not be over 20 kg. In Experiment II–2 the standard was about 15 to 20 kg. so that the range would be wider. Both experiments were by the method of magnitude production without modulus. The assigned numbers for sensory magnitude were 3, 6, 10, 20, and 30, such that the Experiment II–1 was the same as in magnitude production of Stevens & Mack (1959).

Data processing: The exponents were derived by graphic solution. According to the results of in the smaller range Experiment I, the psychophysical function (1) was hypothesized in the smaller range, while the function (2) was hypothesized in the wider range. Therefore, the exponent in the Experiment II–1 was identified with the slope of the line in the log-log plot of the data (X, Y). In Experiment II–2 the results of Experiment I was applied to determine the value of X_0 and the exponent was identified with the slope in the log-log plot of (X–X_0, Y).

Results: The exponents obtained were compared in Table 1.

Table 1. The exponents derived in Experiment I and the exponents derived in Experiment II.

	The smaller range		The wider range		X_0(kg)
	Exp I	Exp II–1	Exp I	Exp II–2	
K.S.	2.1	1.2	1.1	1.0	4.8
I.K.	1.5	1.1	0.85	0.85	5.7
K.T.	2.1	1.9	—	—	—
U.T.	1.4 (2.9)†	1.2 (3.3)†	1.2	1.3	5.2

† The exponent when $X_h=0.78X_s-2.64$ ($X_0=-12.2$) is hypothesized.

In general the exponents seemed to coincide well except that of K.S. in the smaller range.

Experiment III Halving Production in the Wider Range

Experiment III was performed as the number of subjects in wide-ranged experiments was not enough to get a general conclusion.

Subject: Thirteen undergraduates. (Male)

Procedure: Halving production. The standard stimuli were four degrees of forces, weak (light), moderate (neither weak nor strong), strong, and maximum. Subjects chose 8 to 12 kg. for weak, 15 to 22 kg. for moderate, about 30 kg. for strong, and maximum for 40 to 65 kg. as a result. First the experimenter called, for instance, "moderate", then the subject exerted the "moderate" standard force and halved its perceptual magnitude. The order of the standard stimuli presented was at random. Totally about 16 pairs of the standard force X_s and the halved force X_h were obtained for each subject and were all plotted in normal coordinates respectively (Individual data processing). The line was fitted by the method of least squares and the psychophysical function was derived from the equation (6).

Results: The derived values of n and X_0 were graphed in Fig. 5, the functions obtained on each day for one subject in Experiment I being added.

In the wider range most values of n and X_0 gathered in the range where $0.4<n<1.2$, and 4 kg.$<X_0<$10 kg. despite Stevens and Mack's proposal that n was about 1.7 and X_0 was negligible (suppose $1.2<n<2.5$, $-4<X_0<4$, the values in the dotted frame in Fig. 5.).

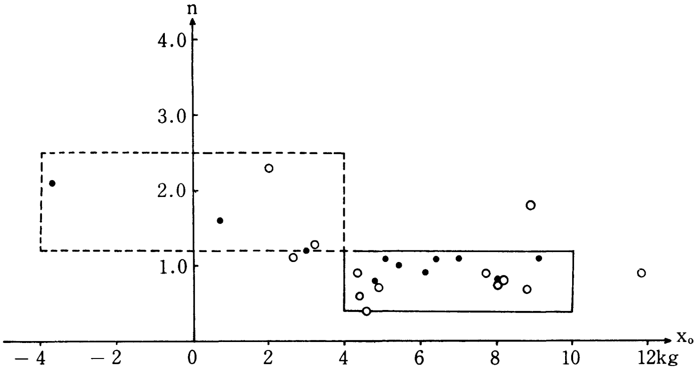

Fig. 5. In the function $Y=k(X-X_0)^n$, the values of n are plotted in the ordinate and those of X_0 are in the abscissa. *Unfilled circle*: functions obtained in Experiment III *Filled circles*: functions obtained on each day in Experiment I.

Conclusion of Exps. I, II and III

Twofold power functions were confirmed in the experiments of the force of hand-grip.

In the smaller range less than 20 kg.

$$Y = kX^n \quad (8)$$

$$1.2 < n < 2.5$$

In the wider range to the maximum force

$$Y = k(X - X_0)^n \qquad X > X_0$$
$$0.4 < n < 1.2 \tag{9}$$
$$4 < X_0 < 10 \text{ (kg.)}$$

Accordingly, if one avails oneself of the power law of the force of handgrip to cross-modal experiments like those of Stevens, J.C., Mack, J.D. & Stevens, S.S. (1960) and Rule & Markley (1971), it becomes more practical to restrict the force range employed to less than 20 kg., where the additive constant X_0 is negligible and the function (1) can be reasonably hypothesized. But if one wants the power law which covers the wider operating dynamic range, the psychophysical function is described in the form of (9).

It is thus necessary to see in what range that "equal stimulus ratios produce equal sensation ratios" is valid. Further investigations will be needed to see whether these range differences suggest a different underlying mechanism of the force of handgrip.

As far as X_0 is concerned, subjects reported the forces near X_0 were "very weak" or "a feeling of touch rather than force" and that halving judgements became most difficult when the standard forces were less than X_0 or maximum. The standard force and the halved force approximate each other near X_0, for as the absolute values in physical scales become lower, the difference between two compared forces becomes smaller.

According to mathematical considerations, X_0 is the critical value which reverses the larger or smaller relationships of X_s and X_h. But analyzing the obtained pairs of X_s-X_h in the whole halving experiments, it was found almost always

$$X_s > X_h$$

even if $X > X_h$. The reverse relationships $X_s < X_h$ was not obtained in the present experiment though it does not seem impossible. And although the force less than X_0 is "very weak" for subjects, it is still a realistic stimulus value, as Stevens & Mack took it as the standard in magnitude production and estimation. Therefore the restriction $X > X_0$ seems nominal.

Conclusion

The two psychophysical power functions

$$Y = kX^n \tag{1}$$

and

$$Y = k(X - X_0)^n \tag{2}$$

were discussed. In terms of the stimulus X and its comparative stimulus X_p whose sensory magnitude is p times as large as that of X, if the psychophysical function (1) is confirmed then

$$X_p = aX \tag{3}$$

$$n = \frac{\log p}{\log a}. \tag{4}$$

If the function (2) is confirmed then

$$X_p = aX + b \qquad a \neq 1,\ b \neq 0 \tag{5}$$

$$n = \frac{\log p}{\log a} \qquad X_0 = \frac{b}{1-a}. \tag{6}$$

And their converses are both possible. It is necessary to test that "equal stimulus ratios produce equal sensation ratios", in the form of the equation (3) directly and to see the range where it is valid.

According to mathematical considerations, X_0 is the value when $X_p = X$ in the equation (5) and is the critical value which reverses the larger or smaller relationships of X_p and X. But actually it shows the stimulus value near which two compared stimuli, X_p and X, approximate each other and also the standard value when we nominally restrict the stimulus range that the psychophysical function covers in the form of $X > X_0$.

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