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# Bandwidth Allocation in ATM Networks: Heuristic Approach

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# Abstract

In this paper, we present a heuristic approach to analyze a boundary of network bandwidth allocated to the source(s). This approach can be applied for Call Ad-mission Control(CAC) in ATM Networks. Our approach applies two characteristic functions, a time  $\epsilon$ quantile function to characterize the source behavior and a function to characterize a maximum amount of network bandwidth served by the multiplexer. These two functions are computed independently and when used simultaneously, it allows us to obtain a new and useful notion of the statistical bandwidth allocation. Moreover, we demonstrate the use of our approach on stochastic and deterministic sources. For the deterministic source, we apply our approach to the source characterized by Dual Leaky Bucket-based traffic descriptor. Its upper bound on bandwidth requirement can be easily obtained for performing a CAC function in real time while providing a significant improvement of network utilization when compared to the peak rate-based bandwidth allocation.

# 1. Introduction

Generally, when a new connection request is received at the ATM networks, the Call Admission Control (CAC) function is performed to decide whether to accept or reject the call. A call is accepted if the network has enough resources (bandwidth) to provide the Quality of Service (QoS) requirements without affecting the QoS provided to existing connections[1, 2]. Accordingly, bandwidth allocation is necessary for dealing with determining the amount of bandwidth required by a new connection for the network to provide the required QoS.

Bandwidth allocation generally is classified into two types, a deterministic and a statistical bandwidth allocation. In deterministic bandwidth allocation, each connection is allocated by its peak bandwidth. This causes an inefficient bandwidth allocation for bursty connections. On the other hands, the amount of bandwidth allocated to a bursty connection in statistical bandwidth allocation is in between its average and peak rate. Therefore, the sum of peak rates of connections multiplexed onto a link can be greater than the link bandwidth if the sum of their statistical bandwidths is less than or equal to the offered link bandwidth. In general, statistical bandwidth allocation allows more connections to be multiplexed in the network than the deterministic, thereby, allowing better utilization of network resources.

From a practical viewpoint, the statistical bandwidth allocation has some difficulties in implementation because the statistical bandwidth of a connection does not only depend on its own stochastic characteristics, but also the characteristics of the existing connections in the network. Several approaches have been proposed for the statistical bandwidth allocation [5, 6, 7, 8]. The well recognized is the equivalent capacity approach where an amount of bandwidth required by a source(s) is estimated from queueing problems[5, 6, 9]. Consider a single source input to a finite capacity queue. The equivalent capacity of the source is equal to the service rate of the queue that achieves a desired cell loss probability. However, there are some evidences for the the inaccuracy of equivalent capacity[9]. Moreover, another problem of the equivalent capacity approach is that an equivalent capacity of the source(s) could not be simply determined particularly when it can not be calculated analytically. Specifically, for a given the buffer capacity value, we have to find by trial and error the minimum value of service rate such that the loss probability encountered by source is less or equal to the required QoS. This iterative process can be very cumbersome because each iteration (i.e., for each value of service rate to try) will involve in general a lengthy simulation to obtain the loss probability.

In this paper, we provide an alternative approach in order to simply determine bandwidth allocation. We have adapted and extended the heuristic framework from source policing[4] to statistical bandwidth allocation. The basic concept is to compute an efficient characterization of the sources and multiplexer independently but whose use in conjunction leads to a new and useful definition for bandwidth allocation. In addition, we illustrate the use of our approach on the stochastic source and deterministic source. For the deterministic source, we consider the worst-case source passing through Dual Leaky Bucket(DLB). Accordingly, we have obtained the useful formulae of bandwidth allocation for real-time computation of bandwidth allocation.

# 2. Heuristic Approach



### Figure 1. System Model

To present the prominent concept of our approach, we consider the system as shown in Fig.1. A multiplexer is modeled as a finite queue of capacity L (including one in server) served by a single server with first-in-first out (FIFO) service discipline. The service time  $\mu$  is constant, equal to the time it takes to transmit an ATM cell. Let us assume that the multiplexer provides a cell loss ratio(CLR) less than a given value  $\epsilon$ for the current traffic from the existing calls. The new call is accepted, if CLR would be also less than  $\epsilon$  for the total traffic carried by the Multiplexer. In the discussion below,  $\mu$  is taken as unity ( $\mu = 1$ ), which means that time is measured in slots with a length equal to a cell transmission time ((53 × 8)/link speed(bps)).

In order to determine bandwidth required by the source(s), we provide a heuristic approach which uses two functions, a time  $\epsilon$ -quantile function to characterize the source behavior and a function to bound the amount of bandwidth served by the multiplexer.

# 2.1. Source Characterization: Time $\epsilon$ -Quantile Function

To characterize the behavior of source, we apply the time  $\epsilon$ -quantile function associated with the arrival process[3]. Let  $a[t_1, t_2]$  denote the number of arrivals (cells) from the source in the interarrival  $[t_1, t_2]$ . The time- $\epsilon$  quantile function corresponding to  $\{a[s, s + T]\}, \forall s, T$ , is defined as below.

**Definition:** Let  $A_{\epsilon}(T)$  denote the  $\epsilon$ -quantile of  $a[s, s + T], \forall s, T$  i.e.,

$$A_{\epsilon}(T) \geq \min\{m : Prob.(a[s, s+T] > m) \leq \epsilon\}.(1)$$

Specifically,  $A_{\epsilon}(T)$  specifies the maximum number of arrivals from source during an interval of length T slots for a given probability  $(1 - \epsilon)$  where  $\epsilon$  is related to the required QoS.

For the deterministic source,  $A_{\epsilon}(T)$  in Eq.1 is adapted to the following definition.

$$A_{\epsilon}(T) \geq (1-\epsilon)a(s,s+T), \forall s,T$$
 (2)

It is noted that the advantage of the characterization of the source by time  $\epsilon$ -quantile function is that although the computation of  $A_{\epsilon}(T)$  is possible analytically for simple source models only, very efficient techniques exist for empirical estimators of  $A_{\epsilon}(T)[4, 12]$ .

# 2.2. Multiplexer Characterization

In the following, we characterize a maximum network bandwidth which can be served by multiplexer in any interval of time. Let  $m[t_1, t_2]$  denote a number of cells served by the multiplexer in the intervals  $[t_1, t_2]$ . The maximum number of cells served by the multiplexer is defined as follows.

**Definition:** Let M(T) denote the maximum number of cells served by the multiplexer during an interval of length T slots i.e.,

$$M[T] \ge m[s, s+T], \forall s, T.$$
(3)

According to the system model, M(T) function corresponding to the FIFO-multiplexer is given as follows.

$$M(T) = T \tag{4}$$

# 2.3. New Definition of Bandwidth Allocation

In this subsection, we illustrate how to apply these two functions,  $A_{\epsilon}(T)$  and M(T) to obtain a new definition of bandwidth allocation.

To guarantee the source characterized by  $A_{\epsilon}(T)$  the QoS requirement specified by loss probability  $\epsilon$ , the multiplexer must satisfy the following condition.

$$A_{\epsilon}(T) \leq B(T), \forall T.$$
 (5)  
where  $B(T) = M(T) + L - 1$ 

Eq.5 can be expressed in the following form:

$$c_{\epsilon}(T) \equiv \frac{A_{\epsilon}(T)}{B(T)} \le 1.$$
(6)

According to Eq.6, an amount of bandwidth required by the source(s) is defined as follows.

**Definition:** Let  $c_{\epsilon}$  denote the amount of bandwidth required by a source(s) corresponding to  $\epsilon$  i.e.,

$$c_{\epsilon} \equiv max_{\forall T} (\frac{A_{\epsilon}(T)}{B(T)}). \tag{7}$$

In the case of n multiplexed sources, the required bandwidth can also determined by  $c_{\epsilon}$  but  $A_{\epsilon}(T)$  of the aggregated traffic from these n sources need to be used. The computation of  $A_{\epsilon}(T)$  could be complicated. However, we may simply calculate the bandwidth requirement of the n multiplexed sources as follows.

**Definition:** Let  $C_{\epsilon}$  define the upper bound on bandwidth requirement of n multiplexed sources i.e.,

$$C_{\epsilon} = max_{\forall T} \frac{\sum_{i=1}^{n} A_{\epsilon}^{i}(T)}{B(T)}.$$
(8)

where  $A_{\epsilon}^{i}(T)$  is  $A_{\epsilon}(T)$  of the connection *i*.

 $C_{\epsilon}$  in Eq.8 sometimes overestimates the actual bandwidth requirement of the multiplexed n sources. However, it is simple and useful from a practical viewpoint while providing a significant improvement of network utilization compared to the peak rate-based allocation as will be shown in Numerical Results.

By the principle of induction, we easily find that

$$max_{\forall T} \frac{\sum_{i=1}^{n} A_{\epsilon}^{i}(T)}{B(T)} \leq \sum_{i=1}^{n} max_{\forall T} \frac{A_{\epsilon}^{i}(T)}{B(T)}$$
(9)

for  $n \in N, N = \{1, 2, ...\}.$ 

Thus, the relationship between  $C_{\epsilon}$  and  $c_{\epsilon}$  is obtained as follows.

$$C_{\epsilon} \le \sum_{i=1}^{n} c_{\epsilon}^{i} \tag{10}$$

# 3. Applications of Heuristic Approach

One of the advantages of heuristic approach is that the bandwidth allocation can be simply determined because  $A_{\epsilon}(T)$  and B(T) are computed independently. We first compute (in case of source models) or measure (in case of real sources like the video VBR source) the  $A_{\epsilon}(T)$  and then find  $\max_{\forall T} \frac{A_{\epsilon}(T)}{B(T)}$ . In this section, we illustrate the use of our approach to stochastic and deterministic source model.

#### 3.1. Stochastic Source Model

To illustrate the use of our analysis on stochastic source, we consider here a Bernoulli source. A Bernoulli source is completely specified by the probability p that the source emits a cell in a slot[13].

Let  $f_i(x)$  denote the probability that x cells from the call *i* arrive at the queue during interval of T slots. The time  $\epsilon$ -quantile function of a Bernoulli source *i* with parameter  $p_i$  is simply given by

$$A_{\epsilon}(T) = \min\{m : \sum_{k=m+1}^{T} f_i(k) \le \epsilon\}$$
(11)

$$= \min\{m: I_{p_i}(m+1, T-m) \le \epsilon\}$$
(12)

where  $f_i(x) = \frac{T!}{x!(T-x)!} p_i^x (1-p_i)^{T-x}$  and  $I_{p_i}(a,b), a, b > 0$ , is an incomplete beta function[13] expressed by

$$I_{p_i}(a,b) = \frac{\int_0^{p_i} t^{a-1} (1-t)^{b-1} dt}{\int_0^1 t^{a-1} (1-t)^{b-1} dt}.$$
(13)

Consequently,  $c_{\epsilon}$  and  $C_{\epsilon}$  are expressed as follows.

$$c_{\epsilon} = max_{\forall T} \frac{min\{m: I_{p_{\epsilon}}(m+1, T-m) \le \epsilon\}}{T+L-1}$$
(14)

$$C_{\epsilon} = \max_{\forall T} \sum_{i=1}^{n} \frac{\min\{m : I_{p_i}(m+1, T-m) \le \epsilon\}}{T+L-1}$$
(15)

Note that  $C_{\epsilon}$  in Eq.15 is the upper bound on bandwidth requirement. We can calculate the actual statistical bandwidth required by the aggregate traffic from  $c_{\epsilon}$  which uses  $A_{\epsilon}(T)$  of the aggregate traffic. For example, the actual  $A_{\epsilon}(T)$  of the aggregate traffic from n multiplexed Bernoulli sources is expressed by

$$A_{\epsilon}(T) = \min\{m : \sum_{k=m+1}^{nT} f_1 \star f_2 \star \dots \star f_n(k) \leq \epsilon\} (16)$$

where operator  $\star$  denotes the convolution of the form  $f \star g(k) = \sum_{l} f(l)g(k-l)$ .

# 3.1.1. Numerical Examples

Fig.2 shows the behavior of  $c_{\epsilon}(T)$  of a Bernoulli source with parameter p and QoS parameter  $\epsilon$  for given buffer size L. It is noted that  $c_{\epsilon}(T)$  is sensitive to p while relatively insensitive to  $\epsilon$ . In addition, the increase in L will result in decrease of  $c_{\epsilon}(T)$  in short-term whereas negligible decrease in the long term.



Figure 2. Behavior of Bandwidth Requirement of a Bernoulli Source wrt. T

Here we show the numerical results of  $C_{\epsilon}$  (Eq.15) of two multiplexed Bernoulli sources, p = 0.03 and p = 0.3. Table 1 shows the results of  $c_{\epsilon}$  and  $C_{\epsilon}$  of a Bernoulli source(s) for various L and  $\epsilon$ . It is noted that the sum of individual  $c_{\epsilon}$  is greater than  $C_{\epsilon}$  that can be explained by Eq.10.

Table 1. Bandwidth Requirement ofBernoulli source(s)

ε	L	$c_{(p=0.03)}$	$c_{(p=0.3)}$	$\sum c$	C
$10^{-3}$	30	0.08928	0.35384	0.44313	0.42424
$10^{-3}$	50	0.06862	0.33273	0.40100	0.38738
$10^{-6}$	30	0.14285	0.41269	0.55555	0.53000
$10^{-6}$	50	0.10465	0.37162	0.47627	0.45664

# 3.2. Deterministic Source Model

To illustrate the use of our approach on deterministic source model, we consider the system in Fig. 3, where every source is enforced by a Dual Leaky Bucket before entering the network.



Figure 3. System Model

The DLB is constructed with two leaky buckets (LB) in a series where a (single) LB is considered as a discrete-state LB version. The discrete-state LB algorithm (LB in the following) used in this study conforms to the device initially introduced in [11]. The LB characterized by parameters (I, K) can be explained as follows. A copy of arriving cells is put into a queue of finite capacity K (including one in the server). This queue is served by a server with constant service time equal to I. If no space is available for the copy when the cell arrives, the cell is discarded. Note that the LB only deals with copies of actual cells: a conforming cell incurs no additional delay in it. The DLB consists of a Peak Cell Rate (PCR)-LB with parameters  $(I_p, K_p)$ and a Sustainable Cell Rate (SCR)-LB with parameters  $(I_s, K_s)$ , where  $I_s$ , the inverse of SCR<sup>1</sup>, is larger than  $I_p$ , the inverse of PCR. Note that  $I_p \ge 1$  and  $I_s > I_p$  are normalized with respect to  $\mu$ . Let us as-sume that the  $K_p = 1$  so that the inter-cell spacing of conforming streams is constrained to be at most  $I_p^2$ .

The Maximum Burst Size (MBS)<sup>3</sup> is calculated by  $MBS(I_p, I_s, K_s) = \lfloor \frac{I_sK_s-1}{I_s-I_p} \rfloor$  where  $\lfloor x \rfloor$  is the largest integer smaller than or equal to x. The set of three parameters  $(I_p, I_s, K_s)$  (or equivalently,  $(I_p, I_s, MBS)$ ) form a traffic descriptor which is negotiated in a contract between the user and the network during the call setup phase. To guarantee QoS the existing connections, it is necessary to allocate network bandwidth for all possible cell traffic that may be originated from the connections and will pass through UPC. Therefore,  $A_{\epsilon}(T)$  of worst case source passing through UPC is used for bandwidth allocation.

According to the LB(I,K) specification, the maximum number of cells permitted by DLB during T slots is given by  $\lceil T/I \rceil + K - 1$ , where  $\lceil x \rceil$  is the smallest

integer greater than or equal to x [4]. Thus,  $A_{\epsilon}(T)$  of worst case source passing through LB is given by

$$A_{\epsilon}^{LB}(T) = (1 - \epsilon)(\lceil T/I \rceil + K - 1).$$
(17)

Since DLB consists of two LBs,  $A_{\epsilon}(T)$  of DLB $(I_p, I_s, K_s)$  is given as follows.

$$A_{\epsilon}^{DLB}(T) = (1 - \epsilon)min(\lceil T/I_p \rceil, \lceil T/I_s \rceil + K_s - 1).$$
(18)

Therefore, we can determine the  $c_{\epsilon}$  of the source characterized by  $\text{DLB}(I_v, I_s, K_s)$  as follows.

$$c_{\epsilon} = max_{\forall T}(1-\epsilon)\frac{min(\lceil T/I_{p}\rceil, \lceil T/I_{s}\rceil + K_{s}-1)}{T+L-1} (19)$$

 $c_{\epsilon}$  in Eq.19 can be derived from the following formulae (See proof in Appendix).

$$c_{\epsilon} = \begin{cases} (1-\epsilon)max(\frac{1}{I_{p}} + \frac{I_{p}-L}{I_{p}L}, \frac{1}{I_{s}} + \frac{K_{s}-\frac{L}{I_{s}}}{I_{s}(\lceil\frac{T_{s}}{I_{s}}\rceil)+L}) & \text{if } L \leq I_{p} \\ (1-\epsilon)max(\frac{1}{I_{p}} + \frac{1-\frac{L}{I_{p}}}{I_{p}(\lceil\frac{T_{s}}{I_{p}}\rceil-1)+L}, \frac{1}{I_{s}} + \frac{K_{s}-\frac{L}{I_{s}}}{I_{s}(\lceil\frac{T_{s}}{I_{s}}\rceil)+L}) \\ & \text{if } I_{p} < L < I_{s}K_{s} \\ (1-\epsilon)\frac{1}{I_{s}} & \text{otherwise} \end{cases}$$
(20)

In the case of n sources, the bandwidth requirement  $C_\epsilon$  is given by

$$C_{\epsilon} = max_{\forall T}(1-\epsilon)\sum_{i=1}^{n} \frac{min(\lceil T/I_{p}^{i}\rceil, \lceil T/I_{s}^{i}\rceil + K_{s}^{i} - 1)}{T+L-1}.$$
(21)

Since the calculation of  $C_\epsilon$  in Eq.21 is rather complicated, we provide the following approximation.

$$C_{\epsilon} \approx min\{\hat{C}_{\epsilon}, \sum_{i=1}^{n} c_{\epsilon}^{i}\}$$
 (22)

where 
$$\hat{C}_{\epsilon} = (1-\epsilon)max_{(T\in S)} \frac{\sum_{i=1}^{n} (\hat{A}_{\epsilon}^{i}(T)+1)}{B(T)}$$
 (23)

$$\hat{A}_{\epsilon}^{i}(T) = min(T/I_{p}^{i}, T/I_{s}^{i} + K_{s}^{i} - 1)$$
(24)

$$S = \{T_1, T_2, \cdots, T_n\}$$
 (25)

$$T_{i} = \frac{I_{s}^{i}I_{p}^{i}(K_{s}^{i}-1)}{I_{s}^{i}-I_{p}^{i}}$$
(26)

This approximation is based on two observations.

1.  $C_{\epsilon} \leq \sum_{i=1}^{n} c_{\epsilon}^{i}$ . 2.  $min([T/I_{p}], [T/I_{s}] + K_{s} - 1) < min(T/I_{p}, T/I_{s} + K_{s} - 1) + 1$ .

<sup>&</sup>lt;sup>1</sup> SCR is an upper bound on the mean cell rate.

<sup>&</sup>lt;sup>2</sup> Values of  $K_p > 1$  allow for Cell Delay Variation (CDV) at the customer premises.

<sup>&</sup>lt;sup>3</sup> MBS is the maximum number of back-to-back cells that can be sent at the peak cell rate  $(1/I_p)$  without violation of sustainable cell rate  $(1/I_s)$ .

### **3.2.1.** Numerical Examples

In Fig.4, we show the numerical results of  $c_{\epsilon}$  with respect to L by varying  $I_p$ ,  $I_s$ , and  $K_s$ , respectively. By setting  $\epsilon$  equal to  $10^{-6}$  and 0,  $c_{\epsilon}$  has the following observations. First,  $c_{\epsilon}$  is less sensitive to  $\epsilon$  values. Secondly,  $c_{\epsilon}$  decreases with the increase of L and is equal to a long-term value  $((1 - \epsilon)\frac{1}{I_s})$  if L is greater than  $K_s \times I_s$ . Finally,  $c_{\epsilon}$  increases with the decrease of  $I_p$ and  $I_s$ , while it increases with the increase of  $K_s$ .



Figure 4. Effect of  $(I_p, I_s, I_m)$  to  $c_e$ 

We now apply  $C_{\epsilon}$  in Eq.22 for performing real-time CAC function. The new connection characterized by  $(I_{p_{n+1}}, I_{s_{n+1}}, K_{s_{n+1}})$  will be accepted if the bandwidth required by n + 1 connection does not exceed limited link load  $\rho, (\rho \leq 1)$ .

According to the system model in Fig.3, we show the performance of CAC function through numerical examples. For presentation purpose, the calls handled by the multiplexer are classified into F classes. In this paper, we limit F to 2. Every call in the same class iis enforced by a DLB with identical  $(I_p, I_s, K_s)$  values. The system parameters are chosen as follows. We set the required cell loss ratios at multiplexer  $\epsilon$  equal to 0 and the limited network load  $\rho$  equal to 1. We consider the admission control of two classes where class 1 and class 2 traffic are characterized by a set of parameter values (20,100,80) and (10,50,40) respectively.

Let us define  $n_i$  as the number of admitted calls of class i, i=1,2,...,F. An admission region is defined as the set of all combinations of calls from F classes  $(n_1, n_2, ..., n_F)$  for which the required  $\epsilon$  is achievable. In the numerical results given below, we obtain the outermost boundary of the regions. The admission regions obtained for the upper bound-based CAC scheme are shown in Fig. 5, where L is 500, 1000, 2000, or 8000. The results show that the upper bound-based CAC scheme provides the admission region larger than peak rate-based CAC does. This suggests that the upper bound-based CAC is more effective compared to the peak rate-based CAC.



Figure 5. Admission regions

We also show the sensitivity of the upper boundbased CAC scheme to changes in buffer size L. Assuming that only single class i(i = 1, 2) of calls are transported, we obtain the maximum number of admitted calls as a function of the buffer size. The buffer size is increased from 1 to 10,000, while the required CLR is fixed at  $\epsilon = 0$ . As shown in Fig. 6, the larger the L, the larger the maximum number of calls that can be admitted.



Figure 6. Maximum admitted calls vs.L

# 4. Conclusions

In this paper, we have introduced a heuristic approach in order to determine an amount of network bandwidth allocated to a source(s). Our approach applies two efficient functions: a time  $\epsilon$ -quantile function associated with source and a function associated with the multiplexer. These two functions are computed independently and when used simultaneously, it allows us to obtain a new and useful notion for bandwidth allocation. In addition, we demonstrate the application of our approach to real-time Call Admission Control when source is characterized by Dual Leaky Bucket based traffic parameters. Numerical results indicate that significant improvement of network utilization can be achieved when compared to the peak ratebased bandwidth allocation.

# A. Proof of Eq.19

$$c_{\epsilon} = (1-\epsilon)max_{\forall T}\frac{min(\lceil \frac{T}{I_p} \rceil, \lceil \frac{T}{I_s} \rceil + K_s - 1)}{T+L-1}$$
(27)

where  $I_p < I_s$  and  $K_s \ge 1$ 

Define 
$$\lceil \frac{T}{I_p} \rceil = X_p, \qquad \lceil \frac{T}{I_s} \rceil = X_s$$
 (28)

where  $X_p, X_s$  are the integer numbers.

Hence 
$$(X_p - 1)I_p + \alpha_p = (X_s - 1)I_s + \alpha_s$$
 (29)

where  $1 \leq \alpha_p \leq I_p$  and  $1 \leq \alpha_s \leq I_s$ .

Define  $T_x$  be the maximum T that gives  $\left\lceil \frac{T}{I_p} \right\rceil = \left\lceil \frac{T}{I_s} \right\rceil + K_s - 1$ 

1. If  $T \leq T_x$ , Eq.27 can be replaced by

$$c_{\epsilon} = (1 - \epsilon) max_{1 \le T \le T_x} \frac{\left\lceil \frac{T}{I_p} \right\rceil}{T + L - 1}$$
(30)

$$= (1-\epsilon)max_{1 \le X_p \le \lceil \frac{T_x}{I_p} \rceil} \frac{X_p}{I_p(X_p-1) + \alpha_p + L - 1} (31)$$

$$= (1-\epsilon)max_{1 \le X_{p} \le \lceil \frac{T_{x}}{I_{p}} \rceil} \frac{1}{I_{p}} + \frac{1-\frac{1}{L_{p}}}{I_{p}(X_{p}-1)+L}$$
(32)

$$= \begin{cases} (1-\epsilon)(\frac{1}{I_{p}}+\frac{p}{I_{p}L}) & \text{if } I_{p}-L > 0\\ (1-\epsilon)(\frac{1}{I_{p}}+\frac{1-\frac{L}{I_{p}}}{I_{p}(\lceil\frac{T_{p}}{I_{p}}\rceil-1)+L}) & \text{if } I_{p}-L \le 0 \end{cases}$$
(33)

2. If  $T > T_x$ , Eq.27 can be replaced by

$$c_{\epsilon} = (1-\epsilon)max_{T>T_{x}} \frac{\left\lceil \frac{T}{I_{s}} \right\rceil + K_{s} - 1}{T+L-1}$$
(34)

$$= (1-\epsilon)max_{X_s>\lceil \frac{T_x}{T_s}\rceil} \frac{X_s + K_s - 1}{I_s(X_s - 1) + \alpha_p + L - 1}$$
(35)

$$= (1-\epsilon)max_{X_s > \lceil \frac{T_x}{T_s} \rceil} \frac{1}{I_s} + \frac{K_s - \frac{L_s}{I_s}}{I_s(X_s - 1) + L}$$
(36)

$$= \begin{cases} (1-\epsilon)(\frac{1}{I_s} + \frac{K_s - \frac{T_s}{I_s}}{I_s(\lceil \frac{T_s}{I_s}\rceil) + L}) & \text{if } I_s K_s - L > 0\\ \frac{(1-\epsilon)}{I_s} & \text{if } I_s K_s - L \le 0 \end{cases} (37)$$

From Eq. 33 and 37, we finally obtain Eq.20, where  $\left\lceil \frac{T_x}{T_p} \right\rceil$  and  $\left\lceil \frac{T_x}{T_p} \right\rceil$  is computed by

$$\left[\frac{T_x}{I_p}\right] = \left\lfloor \frac{I_s K_s - 1}{I_s - I_p} \right\rfloor \tag{38}$$

$$\left[\frac{T_x}{I_s}\right] = \left[\frac{I_p(K_s - 2) + I_s + I_p - 1}{I_s - I_p}\right].$$
 (39)

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