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Recently we have considered[1] the detectability or the detection efficiency of two types of resonance curve, i.e., the Lorentz and the dispersion curves in relation with a study of hyperfine interactions.[2] From this consideration it follows that the amplitude of a dispersion curve should be easier to detect than that of a Lorentz curve, provided the expected amplitude is the same for the two curves. The consideration was, however, a limited one because only the amplitude and the background level were assumed to be extracted from a measured resonance curve (two-parameter analysis). Here we remove this limitation and consider the analysis of a resonance in terms of a Lorentz or a dispersion curve having full parameters, i.e., amplitude a, position p, width w and background level b (four-parameter analysis).

The Lorentz curve $f_L(x)$ and the dispersion curve $f_D(x)$ are expressed, respectively, as

$$f_L(x) = \frac{aw^2}{(x-p)^2+w^2} + b \quad \text{and} \quad f_D(x) = \frac{aw(x-p)}{(x-p)^2+w^2} + b, \tag{1}$$

where x represents the energy abscissa. Linearization of eq.(1) leads to

$$\left.\begin{aligned} f_L(x) &\simeq \frac{1}{1+x^2}\cdot a_1 + \frac{2x}{(1+x^2)^2}\cdot a_2 + \frac{2x^2}{(1+x^2)^2}\cdot a_3 + a_4 \\ \text{and} \quad & \\ f_D(x) &\simeq \frac{x}{1+x^2}\cdot a_1 + \frac{x^2-1}{(1+x^2)^2}\cdot a_2 + \frac{x(x^2-1)}{(1+x^2)^2}\cdot a_3 + a_4 \,. \end{aligned}\right\} \tag{2}$$

In eq.(2) $a_1 \equiv a$, $a_2 \equiv \delta p$, $a_3 \equiv \delta w$ and $a_4 \equiv b$, and the initial position and width are taken to be $p_0 = 0$ and $w_0 = 1$, respectively, without loss of generality. We assume[1] that the background level is very large and that the energy points of measurement are equidistant and symmetric; $\Delta x = h$ and $-X \leqq x_i \leqq X$. On the basis of eq.(2) we define the F matrix of a least-squares fitting as $F_{i\mu} = f_\mu(x_i)$, e.q., $f_2(x) = 2x/(1+x^2)^2$ for the Lorentz curve etc. We then approximate F^tF by integrals[1], i.e.,

$$(F^tF)_{\mu\nu} = \sum_i F_{i\mu}F_{i\nu} \simeq \frac{1}{h}\int_{-X}^{X} f_\mu(x)f_\nu(x)dx \,. \tag{3}$$

The error matrix Σ of the fitting parameters a_μ's is obtained by inverting F^tF ; $\Sigma = (F^tF)^{-1}$. Finally the detection efficiency of the μ-th parameter a_μ is defined by

$$E_\mu = \frac{h}{2X}(\Sigma_{\mu\mu})^{-1} \,. \tag{4}$$

We calculated E_μ's as a function of X using the analytical expressions obtained from the integral approximation of eq.(3), e.q.,

$$(E_1)_D = \frac{(T-U_1)(\frac{1}{2}T-\frac{3}{2}U_1+\frac{7}{3}U_2-\frac{4}{3}U_3) - (\frac{1}{2}T-\frac{3}{2}U_1+U_2)^2}{2(\frac{1}{2}T-\frac{3}{2}U_1+\frac{7}{3}U_2-\frac{4}{3}U_3)}$$

with

$$T \equiv (\tan^{-1}X)/X \qquad \text{and} \qquad U_n \equiv (1+X^2)^{-n} , \qquad (5)$$

where D denotes the dispersion curve; eq.(5) is an extension of eq.(4) of ref. 1. We also computed E_μ's directly on the basis of the summation of eq.(3). The results from the two methods are in agreement except for a small difference due to a finite value of h. The results for the amplitude ($E_a \equiv E_1$) and the position ($E_p \equiv E_2$) from the integral method are shown in Fig. 1 in comparison with those from the two-parameter analysis (E_a).

From Fig. 1 we see that the detection efficiency of the amplitude of the Lorentz curve $(E_a)_L$ is always larger than that of the dispersion curve $(E_a)_D$, similarly as in the two-parameter analysis, but the difference between the two types of resonance becomes smaller in the present case of four parameters. On the contrary, the detection efficiency of the resonance position is larger for the Lorentz curve; $(E_p)_L \gtrsim (E_p)_D$. It is further noted that $(E_w)_D \gtrsim (E_w)_L$, i.e., the dispersion curve can be employed to advantage in determining the width of a resonance. Considering such a situation, and also taking into account the assumptions underlying the present analysis, we should conclude that measurements of both the Lorentz and the dispersion types of resonance are desirable in the study of level-crossing in hyperfine interactions[2); such measurements should also clarify the misalignment problem[3) in a level-crossing experiment.

Another feature seen from Fig. 1 is that the position and value of the maximum of E_a remains the same irrespectively of the number of parameters employed in the analysis. This property can be proven directly by considering the behaviour of the present least-squares algorithm when the number of fitting parameters is changed.

References

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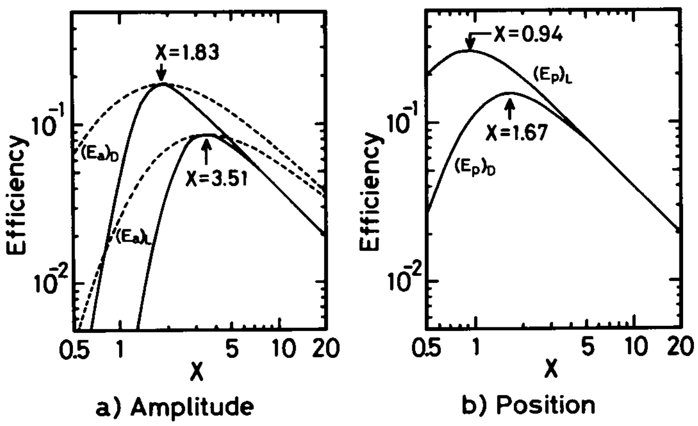

Fig. 1. Detection efficiency of a) amplitude and b) position of a Lorentz (denoted by L) or a dispersion (D) types of resonance as a function of half the width of measurement X in unit of FWHM of the Lorentz curve; see eq.(4) in the text. The position of maximum of the efficiency is indicated. In a) the results from the two-parameter analysis of ref. 1 are shown as broken curves for comparison.