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著者	Rodriguez M., Ishii K., Yamazaki H., Matsuyama S., Kikuchi Y., Oishi Y., Aihara T., Yamaguchi T., Watanuki S., Itoh M., Orihara H.
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Rodriguez M., Ishii K., Yamazaki H., Matsuyama S., Kikuchi Y., Oishi Y., Aihara T., Yamaguchi T., Watanuki S., Itoh M.* Orihara H.**

Quantum Science and Energy Engineering, Tohoku University
*Cyclotron and Radioisotope Center**

Introduction

Positron Emission Tomography (PET) requires accurate attenuation correction (AC) to obtain good quantitative results[1]. The imaged object attenuates photons traveling within an LOR. This attenuation has several detrimental effects: overall loss of counts, leading to higher image noise, image non uniformity due to the higher attenuation of photons from some body regions than other and distortions due to the differential attenuation of photons from a particular source location as a function of angle. Overall losses of counts in the body due to attenuation can be as high as 85% for a moderately large body, but the loss varies substantially with body size, resulting in varying noise levels[2]. Two general approaches are currently used to correct for attenuation: calculated attenuation and measured attenuation[3]. The former case, assumes that the outer body or organ (segmentation) contour is known and the attenuation properties within this contour are constant. The outer contour can be determined from a reconstructed image without attenuation correction[4]. The measured attenuation is performed using one additional scan on the patient with an external radiation source. This transmission scan is used to determine the attenuation along each LOR. Those methods have their advantages and disadvantages. The calculated attenuation is easy to apply and do not require a second scan but the homogeneous attenuator assumption leads to serious errors in some types of scans. Usually for brain images, it gives good results. The measured attenuation can measure the attenuation discontinuities, which is desirable for heterogeneous attenuator as the case in whole body or thorax images. Measured attenuation introduces statistical errors since another set of measurements is required. High counting can reduce this noise but it will imply either larger scanning time or higher radiation dose. This research aims to develop a method for determining the attenuation coefficient within the imaged object using the single event counts measured by each radiation detector in the PET system. Here we will present the conceptual development of the method.

Theory and description of the method

A PET system measures the coincidence events originated from the annihilation positron-electron within the subject. However, there are many events that do not have the conditions to be considered as coincidence events and the data acquisition system rejects them. Several PET systems can measure the single event counts since they are useful to correct for accidental coincidences. However, the single counts on each detector also carry useful information about the attenuation within the subject. Laymon and Turkington[5)] developed a method for PET and SPECT to calculate attenuation correction factors using single events. However, this method is not feasible for a ring PET since it requires septa for every single detector in the PET system. Our goal is to extract the information about attenuation that the detected single counts brings. Let us consider a ring detector array for PET measurements as shown in the figure 1. The system is formed by d=[1,...,D] detector pairs counting n_d coincidence events each one. Each detector pair d has two detectors: d_1 and d_2. The two dimensional image can be discretized into b=[1,...,B] pixels. Each pixel has activity λ_b which might be different from pixel to pixel. Each detector counts N single events. The coincidence events counted by each pair d can be written as:

$$n_d = \sum_{b=1}^{B} \lambda_b C_{db} exp(-\sum_{b'=1}^{B} \mu_{b'} l_{db'}) \qquad (1)$$

where $\mu_{b'}$ is the attenuation coefficient for each pixel b within the strip defined by the pair d. $\ell_{db'}$ is the distance within the pixel b' that a photon should travel in the way to one of the pair d's detector. C_{db} is the geometrical probability that the pair of photons emitted in the pixel b will be detected by the pair d. The fact that a coincidence event is actually two photons traveling at opposite directions results in the same attenuation factor for any coincidence event regardless the pixel where the photon pair is emitted. Therefore, equation (1) can be written as:

$$n_d \exp(\sum_{b'=1}^{B} \mu_{b'} l_{db'}) = \sum_{b=1}^{B} \lambda_b C_{db} \qquad (2)$$

Each detector will have a single count defined as:

$$N_{d_1} = \sum_{b=1}^{B} \lambda_b S_{d_1 b} \exp(-\sum_{b'=1}^{B} \mu_b r_{d_1 b'}) \text{ and } N_{d_2} = \sum_{b=1}^{B} \lambda_b S_{d_2 b} \exp(-\sum_{b'=1}^{B} \mu_b r_{d_2 b'}) \quad (3)$$

where $S_{d_1 b}$ and $S_{d_2 b}$ are the geometrical probability that photon emitted in the pixel b will detected by the detectors d_1 and d_2 respectively. The parameters $r_{d_1 b'}$ and $r_{d_2 b'}$ are the distance that a photon should travel within the pixel b' on its way to the respective detector either d_1 or d_2. Our method proposes to mix the coincidence event information with the respective detectors single counts to obtain the attenuation coefficient for each pixel.

The equation (2) can be written as:

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \cdots \\ \lambda_B \end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & \vdots & C_{1B} \\ C_{21} & C_{22} & C_{23} & \vdots & C_{2B} \\ C_{31} & C_{32} & C_{33} & \vdots & C_{3B} \\ \cdots & \cdots & \cdots & \vdots & \cdots \\ C_{D1} & C_{D2} & C_{D3} & \vdots & C_{DB} \end{bmatrix}^{-1} \begin{bmatrix} n_1 \exp(\sum_{b'=1}^{B} \mu_b l_{1b'}) \\ n_2 \exp(\sum_{b'=1}^{B} \mu_b l_{2b'}) \\ n_3 \exp(\sum_{b'=1}^{B} \mu_b l_{3b'}) \\ \cdots \\ n_D \exp(\sum_{b'=1}^{B} \mu_b l_{Db'}) \end{bmatrix} \quad (4)$$

This system will have a solution only if $D \geq B$. In other words, the amount of LORs should be at least equal to the amount of image elements. The equation (4) is known as discrete back projection.

The equation (3) is as follow:

$$\begin{bmatrix} N_{1_1} \\ N_{1_2} \\ N_{2_1} \\ \cdots \\ N_{D_2} \end{bmatrix} = \begin{bmatrix} S_{1_1 1} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{1_1 b'}) & S_{1_1 2} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{1_1 b'}) & S_{1_1 3} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{1_1 b'}) & \vdots & S_{1_1 B} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{1_1 b'}) \\ S_{1_2 1} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{1_2 b'}) & S_{1_2 2} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{1_2 b'}) & S_{1_2 3} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{1_2 b'}) & \vdots & S_{1_2 B} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{1_2 b'}) \\ S_{2_1 1} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{2_1 b'}) & S_{2_1 2} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{2_1 b'}) & S_{2_1 3} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{2_1 b'}) & \vdots & S_{2_1 B} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{2_1 b'}) \\ \cdots & \cdots & \cdots & \vdots & \cdots \\ S_{D_2 1} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{D_2 b'}) & S_{D_2 2} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{D_2 b'}) & S_{D_2 3} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{D_2 b'}) & \vdots & S_{D_2 B} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{D_2 b'}) \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \cdots \\ \lambda_B \end{bmatrix} \quad (5)$$

The vector $[\lambda_b]$ in the equation (5) can be substituted with its value defined by equation (4). It will result in the equation:

$$\begin{bmatrix} N_{1_1} \\ N_{1_2} \\ N_{2_1} \\ \cdots \\ N_{D_2} \end{bmatrix} = \begin{bmatrix} S_{1_1 1} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{1_1 b'}) & S_{1_1 2} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{1_1 b'}) & S_{1_1 3} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{1_1 b'}) & \vdots & S_{1_1 B} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{1_1 b'}) \\ S_{1_2 1} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{1_2 b'}) & S_{1_2 2} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{1_2 b'}) & S_{1_2 3} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{1_2 b'}) & \vdots & S_{1_2 B} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{1_2 b'}) \\ S_{2_1 1} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{2_1 b'}) & S_{2_1 2} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{2_1 b'}) & S_{2_1 3} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{2_1 b'}) & \vdots & S_{2_1 B} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{2_1 b'}) \\ \cdots & \cdots & \cdots & \vdots & \cdots \\ S_{D_2 1} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{D_2 b'}) & S_{D_2 2} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{D_2 b'}) & S_{D_2 3} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{D_2 b'}) & \vdots & S_{D_2 B} \exp(-\sum_{b'=1}^{B} \mu_{b'} r_{D_2 b'}) \end{bmatrix} \begin{bmatrix} C_{11} & C_{12} & C_{13} & \vdots & C_{1B} \\ C_{21} & C_{22} & C_{23} & \vdots & C_{2B} \\ C_{31} & C_{32} & C_{33} & \vdots & C_{3B} \\ C_{D1} & C_{D2} & C_{D3} & \vdots & C_{DB} \end{bmatrix}^{-1} \begin{bmatrix} n_1 \exp(\sum_{b'=1}^{B} \mu_{b'} l_{1b'}) \\ n_2 \exp(\sum_{b'=1}^{B} \mu_{b'} l_{2b'}) \\ n_3 \exp(\sum_{b'=1}^{B} \mu_{b'} l_{3b'}) \\ \cdots \\ n_D \exp(\sum_{b'=1}^{B} \mu_{b'} l_{Db'}) \end{bmatrix} \quad (6)$$

Finally, the product between the single counts for each pair d is obtained by the following operation:

$$\begin{bmatrix} N_{1_1} \\ N_{1_2} \\ N_{2_1} \\ \cdots \\ N_{D_2} \end{bmatrix} \begin{bmatrix} N_{1_1} \\ N_{1_2} \\ N_{2_1} \\ \cdots \\ N_{D_2} \end{bmatrix}^T \qquad (7)$$

The result of this operation will be a set of equations where the unknown variables are the attenuation coefficients μ_b for each image element.

The PT-711 (Time of Flight type PET system) located at CYRIC permits single counting. This method will be tested with the data acquired in that system. In order to test this method, several measurements with different attenuators are required. We plan to use data without attenuator (air), uniform attenuator and a heterogeneous attenuator obtained from a phantom. The attenuation coefficients obtained with this method will be compared with the real values to test the validity of this method. The method does not include dead time corrections. Therefore, we expect the method will have limitations when high activity is injected to the patient.

Conclusion

The theoretical fundamentals for a method to calculate the attenuation coefficient of the imaged subject are presented in this report. A well performance of this method will reduce the patient radiation exposure since the transmission scan will not be necessary. In addition to the lower radiation dose, the noise introduced by the transmission measurements is eliminated. The single count detected by each individual detector is statistically good hence, the attenuation coefficients are expected to show good accuracy.

References

1) Huang S. et al, J. Nucl Med **22** (1981) 627-637.
2) Harkness B., Comparison of currently available PET scanners, Society of Nuclear Medicine, 47th annual meeting, June 2000.
3) Turkington T., J Nucl Med Technol **29** (2001) 1-8.
4) Mizuta T. et al, Attenuation Correction in 3D-PET, CYRIC annual report, 165-167, 1998.
5) Laymon C. and Turkington T., IEEE Trans Med Imag, **18** (1999) 1194-1200.

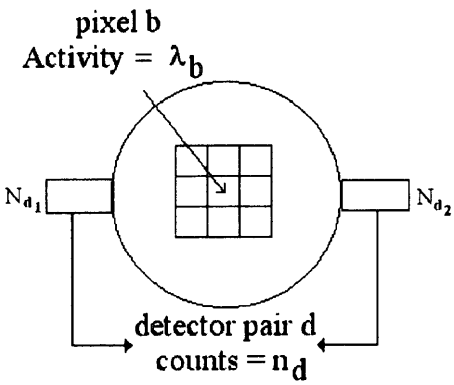

Fig. 1. Discretized 2D image viewed by a pair of radiation detectors.