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Mechanical analogue of active Josephson transmission line

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The mechanical analogue of the sine-Gordon equation, which includes the effects of loss and the effect of a distributed bias source is constructed. The mechanical analogue itself is a active line transmitting kinks which are a kind of the solitons given by the sine-Gordon equation. The various vortex interactions on an active Josephson transmission line can be demonstrated vividly by using the mechanical line. Our distributed bias source which consists of a series of air nozzles blowing against each of aluminum disks is able to supply energy to kinks on the line. The analogue gives a quick and new physical insight into the highly nonlinear behavior.

I. INTRODUCTION

There has been much interest in the analysis and design of active transmission lines because of their potential application for electronics. The long Josephson junction[1] transmission line consists of two superconductor strips separated by an oxide layer which is thin enough to permit coupling of the superconducting wave functions or tunneling of superconducting electrons.[2,3] On the active Josephson junction transmission line, pulses which contain vortices propagate down the Josephson line, being supplied kinetic energy by a bias Josephson current.[4–6]

Scott and Johnson have studied the motion of vortices propagating down the Josephson active line on a computer.[7,8] Scott has constructed a mechanical analogue of the Josephson strip line which is not equipped with a bias source and vividly demonstrated the vortices traveling in the Josephson line on the mechanical analogue.[3,7] The analogy of the Josephson coupling energy between two superconductors and a rigid pendulum in a gravitational field has been used by many others[9–14,20] as an aid in visualizing some of the phenomena associated with the Josephson effect. Sullivan and Zimmerman have constructed precision mechanical analogues of a small-area Josephson junction and a superconducting quantum interference device, and showed the great potential for the use of mechanical analogues in understanding Josephson devices.[14]

To make the Josephson line work actively, it is necessary to bias dc Josephson current flowing all over the transmission line across the layer isolating two superconductors.[7] The dc Josephson current source is an analogue of a constant torque applied to the pendulum axle on the mechanical line.[13,14] Sullivan and Zimmerman applied a constant torque by a rapidly spinning bar magnet weakly coupled to the aluminum disk of the mechanical analogue.[14]

We have constructed a mechanical analogue of the Josephson transmission line equipped with a bias source to study pulse propagation and collision on the active Josephson transmission line, and to understand vortex behavior. We applied a constant torque by blowing air against each edge of the aluminum disks. Figure 1 is a photograph of our mechanical analogue.

In Sec. II we discuss the identifications between the Josephson transmission line and the mechanical analogue, and construction of the mechanical analogue. In Sec. III we illustrate and discuss how pulses propagate down the mechanical analogue as a function of the applied torque and how pulses with the same or opposite screw senses interact on the mechanical analogue line.

II. MECHANICAL ANALOGUE OF THE JOSEPHSON TRANSMISSION LINE

A Josephson transmission line is described by the following nonlinear partial differential equation[3,7]:

$$\frac{\Phi_0}{2\pi L}\frac{\delta^2\theta}{\delta x^2}-\frac{\Phi_0 C}{2\pi}\frac{\delta^2\theta}{\delta t^2}=J_c\sin\theta, \qquad (1)$$

where $\Phi_0=h/2e$ is the flux quantum, L and C are the series inductance and the shunt capacitance per unit length of the line, respectively, J_c is a constant giving the maximum current per unit length that the junction line will pass, and θ is the quantum-mechanical phase difference across the junction. If one considers the effect of losses which are due to a conduction current of normal electrons across the barrier, and a distributed bias source which could supply energy to flux quantum pulses through the Lorentz force, Eq. (1) becomes[7]

$$\frac{\Phi_0}{2\pi L}\frac{\delta^2\theta}{\delta x^2}-\frac{\Phi_0 C}{2\pi}\frac{\delta^2\theta}{\delta t^2}-g\frac{\Phi_0}{2\pi}\frac{\delta\theta}{\delta t}=J_c\sin\theta-j_B, \qquad (2)$$

where g and j_B are the shunt conductance and the distributed bias source per unit length of the line, respectively. If distances are measured in units of $\lambda_J=(\Phi_0/2\pi L J_c)^{1/2}$ and time in units of $\tau_J=(\Phi_0 C/2\pi J_c)^{1/2}$, then Eq. (2) is normalized to[3,7,15]

$$\frac{\delta^2\theta}{\delta x^2}-\frac{\delta^2\theta}{\delta t^2}-\Gamma\frac{\delta\theta}{\delta t}=\sin\theta-\gamma, \qquad (3)$$

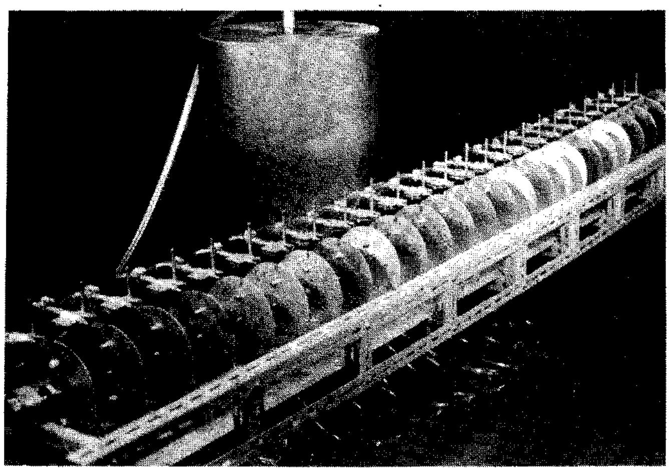

FIG. 1. The mechanical analogue of the Josephson active transmission line.

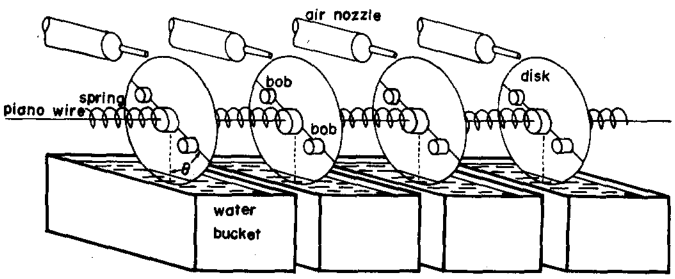

FIG. 2. The mechanical analogue of the Josephson active transmission line. Air passed through a nozzle is blown against each edge of aluminum disks. This is the constant torque analogue of a constant current source. The pair of masses which are fastened on each disk symmetrically are used as pendulum bobs. By varying the difference in mass while keeping the sum constant, we could vary the effective pendulum mass without altering the moment of inertia.

where $\Gamma = g(\Phi_0/2\pi J_c C)^{1/2}$ and $\gamma = j_B/J_c$.

Our mechanical analogue of the Josephson transmission line shown in Figs. 1 and 2 consists of a series of aluminum disks (18 cm in diameter, 1 mm in thickness) connected by steel springs and supported horizontally on a taut length of piano wire. And each disk has a pair of different weight pendulum bobs fastened symmetrically a distance l ($\simeq 5$ cm) from the center. Rotational motion of each disk is damped by liquid viscosity which is an analogue of shunt conductance of a Josephson line since the power dissipation is proportional to $\dot{\theta}^2$.

The torque, which is the analogue of the bias current source, is applied to each pendulum axle by blowing air through a nozzle against each edge of the aluminum disks. The angular displacement (Θ) of the pendulum from the free-hanging position is the analogue of the quantum-mechanical phase difference (θ) across the junction. The mechanical transmission line is described by the following difference differential equation[3,7,14,15]:

$$I\frac{d^2\Theta_i}{dt^2} + k\frac{d\Theta_i}{dt} = K(\Theta_{i+1} - 2\Theta_i + \Theta_{i-1}) - mgl\sin\Theta_i + T_B, \qquad (4)$$

where I is the moment of inertia of a single pendulum plus a disk, k is the damping constant of a single disk, K is the torque constant of a section of spring between two pendula, $mgl\sin\Theta_i$ is the gravitational restoring torque of the ith pendulum, and T_B is the applied torque to a single pendulum.

Taking the distance between two pendula to be Δx (11.5 cm in our mechanical analogue), Eq. (4) can be written[3,7,15]

$$K\Delta x \frac{\Theta_{i+1} - 2\Theta_i + \Theta_{i-1}}{\Delta x^2} - \frac{I}{\Delta x}\frac{d^2\Theta_i}{dt^2} - \frac{k}{\Delta x}\frac{d\Theta_i}{dt} = \frac{mgl}{\Delta x}\sin\Theta_i - \frac{T_B}{\Delta x}. \qquad (5)$$

When distances are measured in units of $\lambda_0 = (K/mgl)^{1/2}\Delta x$, time in units of $\tau_0 = (I/mgl)^{1/2}$, and waves vary slightly over a distance Δx, Eq. (5) can be written[3,7,15]

$$\frac{\delta^2\Theta}{\delta x^2} - \frac{\delta^2\Theta}{\delta t^2} - \Gamma_m\frac{\delta\Theta}{\delta t} = \sin\Theta - \gamma_m, \qquad (6)$$

where $\Gamma_m = k/(Imgl)^{1/2}$ and $\gamma_m = T_B/mgl$. Equations (3) and (6) are written in the same form. We can then translate the Josephson transmission line directly to the mechanical one.

On our mechanical line, the losses damping the motion of disks can be varied by adjusting the liquid level and the torque applied to the disks by adjusting output pressure of an air compressor. Pulses are supplied to

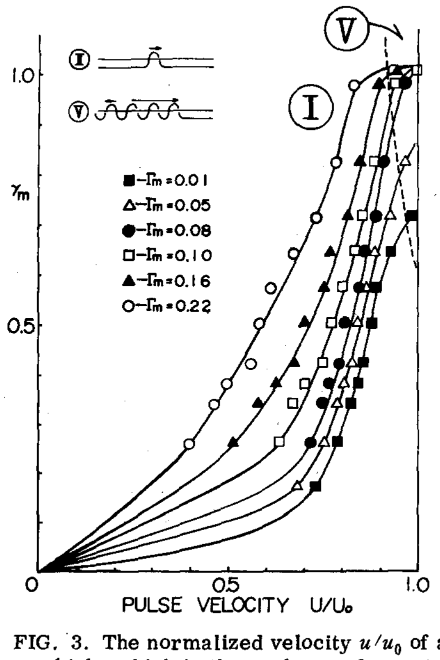

FIG. 3. The normalized velocity u/u_0 of a pulse containing one kink, which is the analogue of a vortex in the Josephson junction or a soliton given by the sine-Gordon equation, vs the normalized applied torque γ_m for various values of the dimensionless parameter Γ_m.

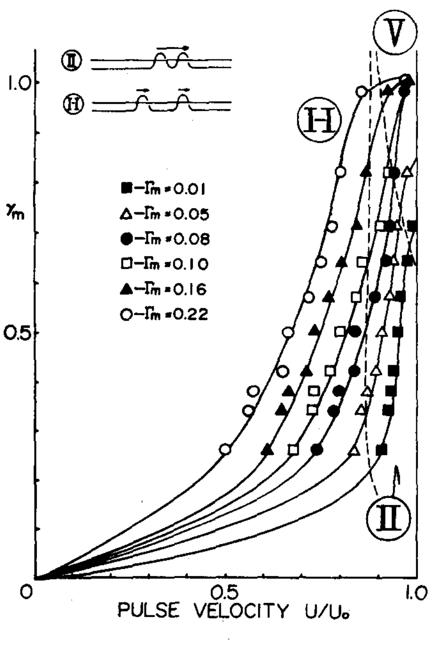

FIG. 4. The normalized velocity u/u_0 of a pulse containing two kinks vs the normalized applied torque γ_m for various values of the dimensionless parameter Γ_m.

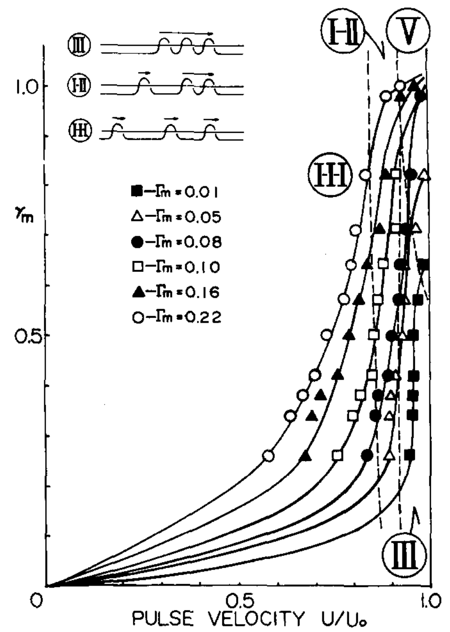

FIG. 5. The normalized velocity u/u_0 of a pulse containing three kinks vs the normalized applied torque γ_m for various values of the dimensionless parameter Γ_m.

the mechanical line by turning a disk at the end of the line, by hand. The mechanical line used is 2.76 m in length and has 25 disks. We use a pendulum mass (m) of 2 g which is the difference in mass between the weights of 12- and 10-g bobs. For our system, the limiting velocity u_0 is 0.215 m/sec, $\lambda_0 = 0.125$ m, and $\tau_0 = 0.58$ sec. To determine the values of Γ_m, the time constants are measured by making the effective pendulum mass $m = 0(11 - 11) = 0$ g without changing the moment of inertia.[13]

III. EXPERIMENTAL RESULTS AND DISCUSSION

Pulses which contain vortices can propagate down the Josephson line.[4] When one vortex passes a point on the line, the quantum-mechanical phase difference between two superconductors at the point changes by 2π.[1] One kink[16] on the mechanical line corresponds to one vortex on the Josephson line. Experimental results for the normalized velocity u/u_0 of a pulse containing one kink, two kinks, and three kinks on the mechanical line vs the normalized applied torque γ_m as a function of the dimensionless parameter Γ_m are shown in Figs. 3–5, respectively. It can be seen from these figures that the pulse velocity increases with increasing applied torque and decreasing Γ_m. For a certain region denoted by V where the pulse nears the limiting velocity in these figures, many pairs of kinks with opposite screw senses are created behind the transmitting pulse, and all disks begin to rotate uniformly in the direction of the applied torque after the transmitting pulse has arrived at the other end of the mechanical line.

At a region denoted by I in Fig. 3, one pulse transmitted propagates down steadily and stably. At a region denoted by II in Fig. 4, two kinks which bunched together rather closely propagate down. At region I-I, a pulse containing two kinks bunched together splits up into two pulses containing one kink each. At a region denoted by III, three kinks bunched together propagate down. At a region I-II, three kinks split up into two kinks bunched together and one kink which becomes detached from the trailing pulse edge. At a region I-I-I, a transmitting pulse containing three kinks bunched together splits up into three pulses containing one kink each. The velocity of a pulse containing bunched kinks increases with an increase in the number of the bunched kinks under the condition of the same values of γ_m and Γ_m.

The interaction between kinks with the same screw sense is always repulsive in a static state on the mechanical line. But in a dynamic state, same sense kinks applied a torque that can be bunched at a certain high-velocity region. The kink-kink interaction in this case is attractive. The attractive force is caused by a mutual coupling of kinetic energy of kinks. When bunched kinks propagate down, the leading kink is helped with its propagation by the other following kinks. Even in the attracitve state of kinks the interaction is opposed by the repulsive force caused by the potential energy of kinks. Therefore if the kinetic energy of the kinks is smaller than a critical value, the first n kinks bunched together split up into $n-1$ kinks bunched together and one kink which becomes detached from trailing pulse edge. And successively bunched kinks scatter their kinks behind them one by one until the mutual coupling of kinetic energy of residual bunched kinks is larger than its own critical value with which the bunched kinks can propagate down the line steadily.

Experimental results for the collisions of kinks with the opposite screw senses and the opposite propagating directions on the mechanical line are shown in Fig. 6. All these experiments are performed for the value $\Gamma_m = 0.22$. Figure 6(a) shows four kinds of states of kinks before collisions, and symbols shown in Fig. 6(b) denote the states of kinks after collisions. The results of the collision of kinks are given in Fig. 6(c) corresponding to the denoted numbers in Fig. 6(a) as a function of

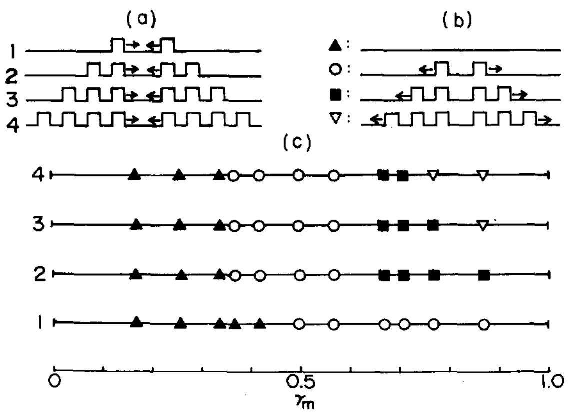

FIG. 6. The collisions of kinks with opposite screw sense and opposite propagating directions as a function of the normalized applied torque γ_m for the value of $\Gamma_m = 0.22$. Four kinds of states of kinks before collisions are denoted by 1, 2, 3, and 4 in (a). The states of kinks after collisions are denoted by symbols shown in (b).

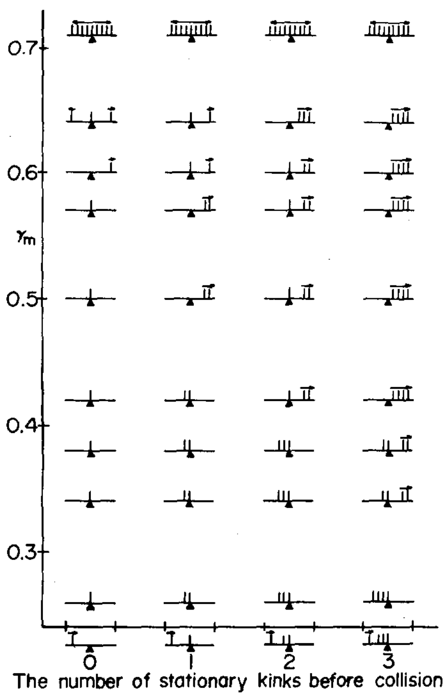

FIG. 7. The propagations of kinks and the collisions between stationary kinks and a transmitting kink with the same screw sense on the line with a local inhomogeneous part, as a function of γ_m. ▲ denotes the inhomogeneous part, the solid line denotes the mechanical line, | denotes a stationary kink, and ⌉ denotes a kink propagating in the direction of the arrow.

the normalized applied torque γ_m. As can be seen from this figure, for small γ_m, kinks annihilate each other, but for the values of further increased γ_m, some leading kinks annihilate each other and the other following kinks pass through without mutual destruction. For large γ_m, all kinks pass through each other without mutual destruction.

If kinks with the opposite screw sense collide and annihilate each other, their energy is radiated onto the line and dissipated by some loss factors contained in the line. When kinks with the opposite screw sense collide on the line having no dissipative effects, the kinks must always pass through without mutual destruction. If two kinks with the opposite screw sense collide and overlap each other, all the energy of the kinks is converted into the kinetic energy of the disks and pendula. This kinetic energy creates two kinks with the opposite screw sense and the opposite propagating directions. A part of this kinetic energy is dissipated by the dynamic loss factor of the line. If the dissipated energy is too great, the kinetic energy cannot create the kinks again. But kinks which have enough kinetic energy compared with the dissipated energy before collision can pass through each other. For the collisions between two groups of plural kinks, the leading kinks give the following kinks some amount of their kinetic energy if the following kinks come on at suitable intervals.

It is possible to arrange a collision between a moving kink and a stationary kink with the same screw sense when the mechanical line has no applied torque. Then the moving kink stops and the stationary kink moves as if they were billiard balls.[7] For the line with applied torque, a kink can be made to stand still by making a local inhomogeneous part on the line or by applying inverse directional torque partially. Figures 7 and 8 show the experimental results of collisions and propagations of kinks on the line with a local inhomogeneous part, as a function of γ_m. To make an inhomogeneous part we change the effective pendulum bob mass of one disk placed at about the middle part of the line from 2 to 11 g (=16.5 g − 5.5 g) without changing the moment of inertia (16.5 g + 5.5 g = 22 g). Experiments are performed at $\Gamma_m = 0.01$ for the homogeneous regions of the line and at $\Gamma_m = 0.004$ for the inhomogeneous part. The inhomogeneous part can keep one or two kinks from propagating up to $\gamma_m = 0.98$. Three and four kinks are kept up to $\gamma_m = 0.64$ and 0.50, respectively. In Figs. 7 and 8 black triangles, horizontal lines, vertical bars, and vertical bars with an arrow denote the inhomogeneous parts, the mechanical lines, the stationary kinks, and the moving kinks propagating in the direction of the arrow, respectively. In Fig. 7, one kink is transmitted to the right from the left side of the line. If the kinetic energy of the moving kink is insufficient to pass through the inhomogeneous part, the kink is held up at the inhomogeneous part on the line. If the kinetic energy of the kink is increased by increasing γ_m, the kink can pass through the inhomogeneous part. When a transmitting kink collides with stationary kinks of the same screw sense which have been held up at the inhomogeneous part, some pieces of kinks can pass through the inhomogeneous part by a relatively low kinetic energy of the transmitting kink. If γ_m is increased, it does not necessarily follow that the number of kinks passing through the inhomogeneous part increase. In some cases at which the transmitting kink has high kinetic energy, the kinetic energy creates some pairs of kinks with opposite screw senses or part of the kinetic energy

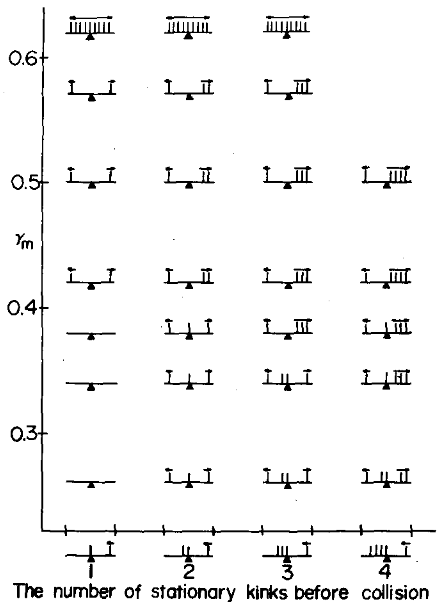

FIG. 8. The collisions between stationary kinks and a transmitting kink with the opposite screw senses on the line with a local inhomogeneous part, as a function of γ_m. ▲ denotes the inhomogeneous part, the solid line denotes the mechanical line, | denotes a stationary kink, and ⌉ denotes a kink propagating in the direction of the arrow.

is consumed by a reflection, at the inhomogeneous part. If γ_m is further increased to obtain a higher kinetic energy of the transmitting kink, many pairs of kinks with opposite screw sense are created and all disks rotate in the direction of the applied torque.

In the case of Fig. 8, one kink is transmitted to the left from the right side of the line. The transmitting kink collides with stationary kinks of the opposite screw sense which have been held up at the inhomogeneous part. If the transmitting kink has small kinetic energy, the kink is held up at the inhomogeneous part. Then kinks with the opposite screw sense are held up on either side of the inhomogeneous part. As shown in this figure, if the transmitting kink having further increased kinetic energy collides with one stationary kink, the kinks annihilate each other. But if sufficient kinetic energy is supplied to the transmitting kink, both transmitting and stationary kinks pass through each other and propagate in the opposite direction of the line. When plural kinks held up at the inhomogeneous part collide with the transmitting kink, some pieces of stationary kinks can pass through the inhomogeneous part and propagate to the right. It is possible to pass all stationary kinks through the inhomogeneous part and propagate in the opposite direction of the transmitting kink, by using the transmitting kink having sufficient kinetic energy. When a transmitting kink, which has been accelerated on a part of the line with applied torque and has sufficient kinetic energy, collides with a stationary kink of the opposite screw sense on the other part of the line without applied torque, the kinks pass through each other and the stationary kink begins to propagate in the opposite direction of the transmitting kink. On the line having the effects of loss, the collision between kinks having the same screw sense or the opposite screw sense to each other causes the kinks to share their energy among themselves.

IV. CONCLUSION

A mechanical analogue of the sine-Gordon equation,[16] which includes the effects of loss and the effect of a distributed bias source, is constructed. The mechanical analogue itself is an active line transmitting kinks which are a kind of the solitons given by the sine-Gordon equation.[17–19] The various behaviors of vortices on a Josephson line can be demonstrated on the mechanical line. Our distributed bias source which consists of a series of air nozzles blowing against each edge of the aluminum disks is able to supply energy to kinks on the line. It is possible to transmit a kink at any speed under the limiting velocity by adjusting the normalized applied torque γ_m which is dependent upon the bias source. If the kink speed approaches the limiting velocity, it is observed that many pairs of kinks with opposite screw sense to each other are created behind the transmitting kink, and finally all disks are rotated in the direction of the applied torque. The transmission of coupled kinks which consist of bunched plural kinks can be faster than the transmission of kinks separately. It may be due to the fact that coupled kinks more easily acquire energy from the bias source than separated kinks under the high-speed condition.

When collisions between kinks and antikinks with opposite propagating directions are carried out on the line, the states of kinks after collisions are divided into three cases: (i) all kinks annihilate each other, (ii) some leading kinks annihilate each other and the other following kinks pass through each other, and (iii) all kinks pass through each other, depending upon the effects of loss on the line, and the kinetic energy, number, and mutual relations of colliding kinks. A transmitting kink can be trapped at a inhomogeneous part which is made by increasing the effective pendulum bob mass of one disk. Kinks trapped at the inhomogeneous part can be released and transmitted by colliding a kink of the same screw sense. Trapped kinks are annihilated or released and transmitted by colliding with a kink of the opposite screw sense. An inhomogeneous part which has nearly the same characteristics with the above mentioned for kinks can be made by applying a torque in the opposite direction partially on the line. Whenever kinks with the same or opposite screw senses collide with each other on the line having the effects of loss, the kinks having higher energy give a fraction of the energy to the kinks having lower energy. This mechanical line can be used to investigate functions of the Josephson line in using the computer elements such as the gate, the counter, the shift register,[20,21] etc., and can also be used as logic devices.

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