Direct observation of the current oscillation in a dc SQUID

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$S = 2500\ V^{-1}$ for each contact because the voltage is divided among the four contacts. This is close to the theoretical value of $q/k_b T = 2760\ V^{-1}$ at 4.2 K.

Finally, it should be pointed out that the contact resistance was quite high ($\approx 20\ k\Omega$); this is believed to be due to a surface oxide present on the GaAs.

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Direct observation of the current oscillation in a dc SQUID

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The current waveforms of the self-resetting oscillation in a dc superconducting quantum interference device (SQUID) are observed by means of a Josephson sampling system. The frequency of the oscillation is around 60 GHz. The time period of the self-resetting oscillation is experimentally measured as a function of the bias current to the SQUID. Theoretical and numerical analyses are presented for the self-resetting oscillation in the dc SQUID.

The Josephson sampler was first used to measure the influence of the internal superconducting quantum interference device (SQUID) dynamics on switching between voltage states by Faris and Pedersen.[1] Wolf *et al.* measured the switching transitions of the two-junction interferometer over its whole vortex boundary, including vortex-to-vortex transitions by using a Josephson sampler with 2.1 ps resolution.[2] But no one has yet observed the oscillatory waveform of a two-junction interferometer, or a single junction which may be interesting in terms of the dynamic influence, on many device applications,[3] as well as the chaotic behavior of the Josephson junction itself.[4] However, these oscillation frequencies of a single junction or the SQUID itself are too high to be measured even by using a Josephson sampler.

In this communication, we present the first real time observations of the oscillatory behavior in externally shunted dc SQUID's. The oscillation frequency is lowered enough by a shunt resistance to be measured by use of a Josephson sampler. We also compare the experimental observations with the results of a theoretical analysis and a numerical computer simulation.

A dc SQUID and a Josephson sampling circuit were fabricated on a Nb ground plane using Pb-alloy planar-junction technology for evaporation, liftoff, and rf plasma oxidation. The minimum linewidth was 5 μm. Figure 1(a) is a photograph of the sample. It consists of a dc SQUID under investigation and a sampling circuit whose equivalent circuit is shown in Fig. 1(b). The SQUID is a symmetric two-junction interferometer connected through resistor R_c to a single-junction comparator. The connecting resistor R_c is also the load resistance for the SQUID. The SQUID is externally shunted by R_c when the comparator junction is in the zero-voltage state. Figure 1(c) shows the current voltage (I-V) characteristics of the SQUID. With increasing bias current I_b, the SQUID switches from the zero-voltage state to a voltage state on the resistive branch, and finally switches to the gap voltage when the critical current of the comparator junction is exceeded. The basic operation of a sampling system is basically the same as the one reported previously.[1] Three currents are applied to the comparator junction shown in Fig. 1(b): (1) the signal which is to be sampled, (2) a sampling pulse produced with an adjustable delay with respect to the signal, and (3) a dc current. The comparator is part of a feedback loop which adjusts the dc current so that the maximum height of the sum of the three currents is exactly at the threshold of the comparator. Since the threshold level and the amplitude of the sampling pulse are constant, the dc current will follow the signal current at the time of the sampling pulse. The repetition rate of the feedback loop is 10 kHz in our system. Both the SQUID under investigation and the SQUID as the sampling pulse generator are triggered from a zero-voltage state to a voltage state by control current pulses with approximately 500 ps rising and falling edges. The bias current for the SQUID under investigation was maintained at a fixed value during each repetitive cycle. The sweep speed of the mechanical variable delay (air line) of our system is 1.2 ps/s.[5] The maximum delay time is about 800 ps.

Figure 2(a) shows a typical example of the time evolution of the oscillatory behavior (self-resetting oscillation) of the SQUID. The circuit parameters of the SQUID under investigation shown in the left-hand side of Fig. 1(b) are the following: the critical current of the junction (J_1 or J_2) $I_0 = 400\,\mu$A, the loop inductance of the SQUID $L = 1.2$ pH, the damping resistance $R_d = 5.7\ \Omega$, the capacitance of the junction $C_J = 1.12$ pF, and the connecting resistance $R_c = 0.18\ \Omega$. The junction capacitance and the loop inductance

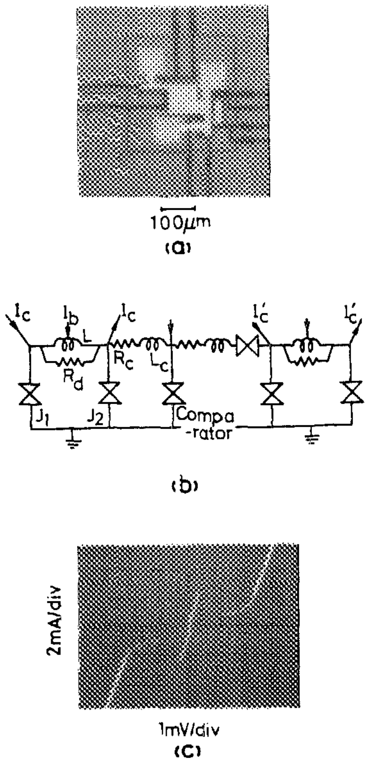

FIG. 1. (a) Photograph of the dc SQUID and the Josephson sampling circuit which has a 5-μm minimum linewidth; (b) equivalent circuit of (a); (c) current-voltage characteristics of the heavily damped dc SQUID which corresponds to the left-hand side dc SQUID shown in the equivalent circuit (b). The curve is locally nonlinear. A slight reduction of the critical current in the I-V characteristics is due to the leakage flux.

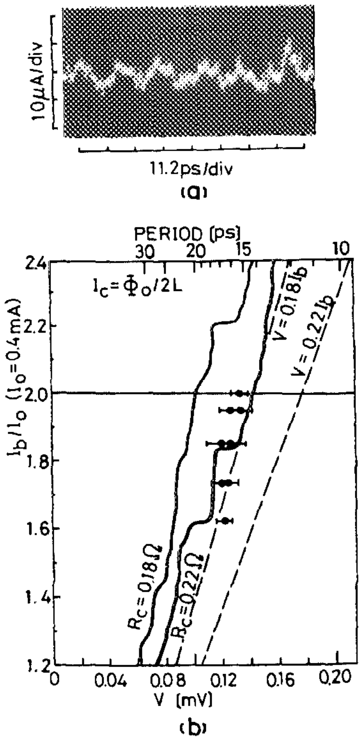

FIG. 2. (a) Experimentally obtained example of the time evolution of the oscillatory behavior of the dc SQUID; (b) solid circles are the experimentally obtained time period of the self-resetting oscillation as a function of the bias current. The voltage scale is calculated from the relation 2 eV $= hf$. Solid lines show the numerically calculated I-V characteristics for the two values of connecting resistance. The complicated nature of the curves originates from the chaotic behavior of the SQUID. Broken lines corresponding to Ohm's law for two values of resistances are also shown for reference.

of the SQUID have been estimated from the measurements of an interference pattern and the resonance induced step in the I-V characteristics of another monitor SQUID on the same chip. The resistance has also been estimated from the measurement of a monitor resistor on the same chip. The frequency and the peak-to-peak height of the self-resetting oscillation shown in Fig. 2(a) are 59.5 GHz and 14 μA, respectively. The frequency of the self-resetting oscillation depends on the value of the bias current to the SQUID. Figure 2(b) shows the time period of self-resetting oscillation as a function of the bias current which is denoted by the solid circles. The result shown in Fig. 2(b) has been obtained from the measurement of the same SQUID as the one used to measure the I-V characteristics shown in Fig. 1(c) as well as the oscillation waveform shown in Fig. 2(a). Figure 2(b) also shows the dc voltages calculated from the relation 2 eV $= hf$ where f is the frequency of the self-resetting oscillation. The average value of the resistance estimated from the resistive branch in the I-V characteristics shown in Fig. 1(c) is about 0.22 Ω, which is nearly equal to the value of the connecting resistance ($R_c = 0.18\ \Omega$) estimated from the monitor resistor. The control current amplitude I_c is $\Phi_0/2L$ for the sampling measurement, while $I_c = 0$ for the measurement of I-V characteristics.

In order to investigate the dependence of an oscillation time period on a bias current, a connecting resistance, and a control current, we analyze theoretically the usual two-junction interferometer model, under an assumption of symmetric loading. The SQUID with symmetric loading by a heavy damping resistance does not show any chaotic behavior, because the system has only two degrees of freedom. On the other hand, an asymmetrically loaded system as shown in Fig. 1(b) has at least three degrees of freedom, and may exhibit a complicated behavior. But an averaged periodic behavior may be explained by a symmetric loading model. We introduce the following new variables:

$$\chi = (\phi_1 + \phi_2)/2, \quad \psi = (\phi_1 - \phi_2)/2, \tag{1}$$

where ϕ_1 and ϕ_2 are the phase differences across junctions 1 and 2, respectively, χ represents the average phase of the junctions, whereas ψ represents the internal degree of freedom which distinguishes an interferometer from a point junction. The loop inductance L is fed by a control current I_c across its ends. To the center of the inductance is fed a bias current source I_b. We obtain the following equations describing the dynamics of ψ and χ for the dc SQUID:

$$\beta_c \beta \frac{d^2\psi}{d(t/\Gamma)^2} + \beta \frac{d\psi}{d(t/\Gamma)} + \beta \cos\chi \sin\psi = -2\psi + \beta\gamma_c, \tag{2}$$

$$\beta_c \frac{d^2\chi}{d(t/\Gamma)^2} + \frac{d\chi}{d(t/\Gamma)} + \cos\psi \sin\chi = \frac{\gamma}{2}, \tag{3}$$

where t is measured in units of $\tau_J = (\hbar C_J/2eI_0)^{1/2}$, $\beta_c = 2eI_0C_J/\hbar G^2$, $\Gamma = \beta_c^{-1/2}$, $\beta = 2eI_0L/\hbar$, $\gamma_c = I_c/I_0$, $\gamma = I_b/I_0$. Here G ($= G_J + G_{\text{load}}/2$, $G_{\text{load}} = 1/R_c \gg G_J$) is the total shunt conductance of the junction. For heavily damped junctions $\beta_c \ll 1$, the first terms in Eqs. (2) and (3) can be neglected. If we further assume the inductance to be small ($\beta \ll 1$), the conductance G_J to be linear, the bias current to be constant, and the control current γ_c to be large enough[6] compared to $1/\beta$, then Eqs. (2) and (3) can be integrated to yield

$$\psi = \beta \frac{\gamma_c}{2}, \tag{4}$$

$$\tau = \frac{2\Gamma}{\sqrt{(\gamma/2)^2 - \cos^2\psi}} \left(\frac{\tan^{-1}(\gamma/2)\tan(\chi/2) - \cos\psi}{\sqrt{(\gamma/2)^2 - \cos^2\psi}} \right)_0^{2\pi}$$
$$= \frac{2\pi\Gamma}{\sqrt{(\gamma/2)^2 - \cos^2\psi}}, \tag{5}$$

therefore,

$$V = (1/G)\sqrt{(I_b/2)^2 - (I_0 \cos\psi)^2}, \tag{6}$$

where τ is the time period of the self-resetting oscillation. The time period τ is proportional to the normalized conductance Γ, and it is a monotonically decreasing function of the bias current. When $I_c = \Phi_0/2L$, Eq. (6) is the simple Ohm's law shown in Fig. 2(b). The model explains that the slope obtained by the sampling measurement ($I_c \neq 0$) of the oscillation is greater than the slope from the I-V characteristics ($I_c = 0$) for a given value of I_b, but it does not explain the fact that the former slope is greater than R_c^{-1}. The calculated time period is close to the observed time period. The model is, however, not accurate for $LI_c = \Phi_0/2$, when the amplitude I_m of the oscillation is zero, because the relation $I_m = I_0 \cos\psi$ is obtained from Eq. (3).

We have done the following numerical simulation in order to investigate the I-V characteristics of a more realistic system. Figure 2(b) shows the numerically simulated I-V characteristics for a realistic circuit of a resistively shunted SQUID including a connecting inductance L_c and a damping resistance R_d shown in Fig. 1(b). Here V is obtained numerically from the time average of $d\chi/dt$, and χ is the mean value of the phase difference of the two junctions. The circuit parameters used for the simulation are the following: $LI_0/\Phi_0 = L_c I_0/\Phi_0 = 0.23$, the normalized damping conductance $\Gamma_d = (\hbar/2eI_0C_J)^{1/2}/R_d = 0.15$, the normalized connecting conductance $\Gamma_c = (\hbar/2eI_0C_J)^{1/2}/R_c = 4.8$ or 3.9, which corresponds to $R_c = 0.18\ \Omega$ or $0.22\ \Omega$, respectively, the normalized junction normal conductance $\Gamma_N = (\hbar/2eI_0C_J)^{1/2}/R_{NN} = 0.23$ where $R_{NN} = 3.7\ \Omega$ is the normal resistance of the junction which was obtained experimentally, and the ratio of the subgap conductance to the junction normal conductance $\Gamma_J/\Gamma_N = 0.1$. The numerical simulation shows that the average slope of I-V characteristics for the control current $I_c = \Phi_0/2L$ is greater than that obtained from Eq. (6), and that the I-V characteristics of the SQUID are locally nonlinear and full of complex structure which is probably related to the chaotic dynamics of the SQUID. This is expected for a system asymmetrically loaded by a connecting resistance R_c, because there are at least three degrees of freedom even though R_c is very small. The experimental result for the period of oscillation agrees with the numerical result for $R_c = 0.22\ \Omega$, except for the fine complex nonlinear structure. The small steplike structures are sensitive to the slight variation of the circuit parameters in a simulation. One should notice that the resistive branch shown in Fig. 1(c) is also observed to be locally nonlinear, and that it has small steplike structures. The observed waveform shown in Fig. 2(a) is something noisy and not sinusoidal which may be related to a chaotic behavior.

In conclusion, we have observed directly for the first time the waveforms of the self-resetting oscillation in the dc SQUID by using the Josephson sampling system. The frequency of the oscillation is around 60 GHz, and increases with increasing bias current to the SQUID. The experimental result for the period of oscillation agrees quantitatively with the numerical result for the I-V characteristics of a SQUID, except for the fine complex nonlinear structure. The period of the self-resetting oscillation as a function of a total conductance, a bias current, and a control current of a shunted dc SQUID is analyzed theoretically in a heavy damping and small self-inductance limit. The experimentally observed period is close to the analytical estimation. The overall behavior of the self-resetting oscillation is in qualitative and quantitative agreement with numerical and theoretical analyses.

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