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Analysis of Timing and Amplitude Jitter due to Intrachannel Dispersion-Managed Pulse Interactions

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***Abstract*—Transmission impairments due to nonlinear intrachannel crosstalk in strongly dispersion-managed systems are investigated. Analytical expressions to estimate timing and amplitude jitter due to intrachannel pulse interactions are provided. Timing jitter is found to be a dominant limiting factor for small values of map strength, whereas amplitude jitter is responsible for system performance degradation especially for large map strength. The analysis agrees with direct numerical simulations of the full system.**

***Index Terms*—Dispersion management, optical fiber communication, optical fiber nonlinearity.**

I. Introduction

NONLINEAR intrachannel crosstalk between adjacent pulses is one of the major limiting factors in dispersion-managed return-to-zero (RZ) pulse transmission with channel bit rate over 40 Gb/s and beyond [1], [2]. Strong overlap of the neighboring pulses due to large pulsewidth breathing within a dispersion management period induces nonlinear mixing and leads to serious transmission penalties such as timing jitter and amplitude fluctuations. Timing jitter arises from intrachannel cross-phase modulation (XPM), where interacting pulses shift the mean frequency of a signal, resulting in a shift in its temporal position through fiber dispersion [3]–[5]. On the other hand, intrachannel four-wave mixing (FWM) generates a new component at the location $t_k = -t_l + t_m + t_n$ in the presence of three pulses centered at t_l, t_m, and t_n, yielding ghost pulse generation at t_k when the bit is zero and amplitude fluctuation when t_k coincides with an occupied bit slot [6]–[8].

In this letter, we provide analytical formulae to estimate timing and amplitude jitter induced by intrachannel pulse interactions in a transmission system with strong periodic dispersion management (an analytical model for constant and strong dispersion fibers without periodic dispersion compensation was discussed in [9]). To compute jitter, a statistical analysis is applied to the formulae for timing shifts and energy transfer obtained in [5], [8]. The analytical model developed here is used to evaluate system performance for a wide range of dispersion map strengths.

II. Theory

Propagation of optical pulses in dispersion-managed fibers in the presence of loss and amplification is described by the perturbed nonlinear Schrödinger equation. We introduce dimensionless variables $t = t_{\rm ret}/t_*$, $z = z_{\rm lab}/z_*$, $u = E/\sqrt{gP_*}$, $D = k''/k''_*$ with the characteristic parameters denoted by the subscript $*$, where $t_{\rm ret}$ and $z_{\rm lab}$ are the retarded time and the propagation distance, respectively, and E denotes the slowly varying envelope of the optical field. With a particular choice of t_*, P_* and $z_* = z_{\rm NL} \equiv 1/\nu P_*$ where ν is the nonlinear coefficient (which yields $k''_* = -t_*^2/z_{\rm NL}$), we have

$$i\frac{\partial u}{\partial z} + \frac{D(z)}{2}\frac{\partial^2 u}{\partial t^2} + g(z)|u|^2 u = 0. \tag{1}$$

The functions $D(z)$ and $g(z)$ describe the dispersion variation of the fiber and the variation of power due to loss and amplification, respectively, which are both periodic with period z_a. As is standard, $g(z)$ is given by $g(z) = g_0\exp[-2\Gamma(z - nz_a)]$, $nz_a \le z < (n+1)z_a$, where $g_0 = 2\Gamma z_a/[1 - \exp(-2\Gamma z_a)]$ and Γ is the dimensionless loss coefficient.

The timing shift and energy change of a signal centered at $t = 0$ due to nonlinear intrachannel interactions with adjacent pulses can be analyzed by writing $u = u_0 + u_l + u_m + u_n$ in (1), where u_k denotes a pulse centered at $t = kT$ with $k = l,\ m,\ n$ integers and T the bit interval. Substituting this into (1), we find the equation which describes the evolution of u_0 perturbed by one of the nonlinear terms $u_l^* u_m u_n$

$$i\frac{\partial u_0}{\partial z} + \frac{D(z)}{2}\frac{\partial^2 u_0}{\partial t^2} + g(z)|u_0|^2 u_0 = -g(z)u_l^* u_m u_n. \tag{2}$$

Among the perturbation terms on the right hand side of (2), phase-independent terms $|u_n|^2 u_0 (n \neq 0)$ bring about a timing shift of u_0 due to intrachannel XPM with all the other nonzero bits u_n [5], whereas the phase-dependent terms $u_l^* u_m u_n (l = m + n; m, n \neq 0)$ are responsible for the energy change of u_0 as a result of intrachannel FWM among nonzero bits u_l, u_m and u_n [8]. It is convenient to introduce the following integrals in order to calculate the timing shifts and energy exchange: the energy $W_0 = \int_{-\infty}^{\infty} |u_0|^2 dt$ and the mean temporal position $t_0 = \int_{-\infty}^{\infty} t|u_0|^2 dt / W_0$.

Explicit formulae to compute the timing shift at the chirp free points (Δt_0) and the energy change (ΔW_0) are obtained by as-

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suming that average disperion is negligible and a signal u_k is periodic in z. When u_k is written as a Gaussian pulse of the form

$$u_k(z,t) = \frac{\alpha}{\sqrt{2\pi\xi(z)}} \exp\left[-\frac{(t-kT)^2}{2\xi(z)}\right], \ \xi(z) = \beta + iC(z) \tag{3}$$

where β is constant and $C(z) = \int_0^z D(z')dz'$, we find the equations for total amount of timing shift Δt_0 and energy change ΔW_0 [5], [8]

$$\Delta t_0(z) = \sum_{n=-N/2}^{N/2} b_n \bar{P}_n z \tag{4}$$

$$\bar{P}_n = -\frac{\alpha^2 \beta n T}{\sqrt{2}\pi z_a} \int_0^{z_a} \frac{C(z)g(z)}{|\xi(z)|^3} \exp\left(-\frac{\beta n^2 T^2}{2|\xi(z)|^2}\right) dz \tag{5}$$

$$\Delta W_0(z) = \sum_{n=-N/2}^{N/2} \sum_{m=-N/2}^{N/2} b_{m+n} b_m b_n \bar{Q}_{m,n} z, \tag{6}$$

$$\bar{Q}_{m,n} = -\frac{\alpha^4}{2\pi^2 z_a}\sqrt{\frac{\pi}{2\beta}} \cdot \int_0^{z_a} \mathrm{Im}\left[\frac{g(z)}{|\xi(z)|} \cdot \exp\left(-\frac{(m^2+n^2)\beta + 2imnC(z)}{2|\xi(z)|^2} T^2\right)\right] dz. \tag{7}$$

where b_k represents encoded binary data of the bit at $t = kT$ which takes either one or zero and N is the total number of the interacting pulses.

Because of a random sequence of bits b_k, Δt_0 and ΔW_0 are random variables and thus (4) and (6) allow us to compute the mean value and the variance of Δt_0 and ΔW_0, which yields timing and amplitude jitter. Timing and amplitude jitter are given by the variance of the mean temporal position of pulses and the normalized energy variance, respectively

$$\sigma_t^2 = \langle t_0^2 \rangle - \langle t_0 \rangle^2, \rho^2 \equiv \frac{\sigma_W^2}{W_0^2} = \frac{\langle W_0^2 \rangle - \langle W_0 \rangle^2}{W_0^2}. \tag{8}$$

Since b_k takes the value one and zero with probability 1/2, from (4) and (6) we have

$$\sigma_t^2 = \left\langle (\Delta t_0)^2 \right\rangle - \langle \Delta t_0 \rangle^2 = \frac{z^2}{4} \sum_n \bar{P}_n^2 \tag{9}$$

$$\begin{aligned}\rho^2 =& \frac{\left\langle (\Delta W_0)^2 \right\rangle - \langle \Delta W_0 \rangle^2}{W_0^2} \\ =& \frac{z^2}{W_0^2}\left[\frac{3}{16}\sum_n \left(\bar{Q}_{n,n}^2 + \bar{Q}_{n,n}\bar{Q}_{2n,2n}\right) + \frac{7}{64}\sum_{\substack{m,n \\ m\neq n}} \bar{Q}_{m,n}^2 + \sum_{\substack{n_1,m_2,n_2 \\ m_2\neq n_2}} p_1 \bar{Q}_{n_1,n_1}\bar{Q}_{m_2,n_2} + \sum_{\substack{m_1,n_1,m_2,n_2 \\ m_1\neq n_1, m_2\neq n_2}} p_2 \bar{Q}_{m_1,n_1}\bar{Q}_{m_2,n_2}\right]\end{aligned} \tag{10}$$

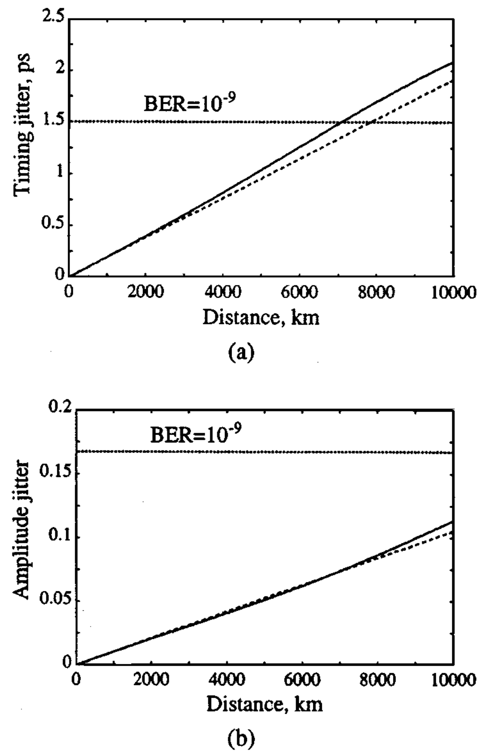

Fig. 1. (a) Timing jitter σ_t and (b) amplitude jitter ρ versus transmission distance for $s = 30$. The solid lines are results of direct numerical simulation of (1) with $2^8 - 1$ PRBS bit pattern and the dashed lines are the analytical results obtained from (9) and (10) with (5) and (7).

where all sums are taken from $-N/2$ to $N/2$ and $p_1 = 3/16$ or 1/16 if for each combination (n_1, m_2, n_2) there are three or four distinct elements, respectively, among the sequence $\{n_1, 2n_1, m_2, n_2, m_2 + n_2\}$ and $p_2 = 3/64$ or 1/64 if for each (m_1, n_1, m_2, n_2) there are four or five distinct elements, respectively, among $\{m_1, n_1, m_1 + n_1, m_2, n_2, m_2 + n_2\}$.

III. Numerical Results

In order to verify the analysis above, we compare the obtained analytical result with direct numerical simulation of (1). The parameters used in the calculations are as follows: $\alpha = \sqrt{2\pi}$, $\beta = 1.0$, $T = 8.3$, $\Gamma = 10$, and $z_a = 0.125$. With the choice of $t_* = 3$ ps, $\nu = 2.5\ \mathrm{W}^{-1}\mathrm{km}^{-1}$ and $P_* = 1$ mW (i.e., $z_{\mathrm{NL}} = 400$ km and $k''_* = -2.25 \times 10^{-2}\ \mathrm{ps}^2/\mathrm{km}$), they correspond to the full-width at half-maximum (FWHM) $\tau_{\mathrm{FWHM}} =$ 5 ps (minimum), the initial peak power 2.31 mW, the bit interval $t_{\mathrm{bit}} = 25$ ps (i.e., the bit rate 40 Gb/s), the fiber loss 0.22 dB/km and period 50 km. The dispersion map is given by a symmetric two-step profle composed of fibers having positive and negative dispersion ($\pm\Delta$) with equal length $z_a/2$. The dimensionless map strength for this profile is defined as $s = \Delta z_a/4$. The number of interacing pulses are estimated as $N = (2/M)\sqrt{1 + (s/\beta)^2}$ where $M = t_{\mathrm{bit}}/\tau_{\mathrm{FWHM}} = 5$ [6].

Fig. 1 shows plots of (a) timing jitter σ_t and (b) amplitude jitter ρ versus transmission distance when $s = 30$, corresponding to $k'' = \pm 21.6$ ps/km. As predicted from the model, both timing and amplitude jitter grow linearly with respect to distance. As a simple estimate, in order to achieve the bit error rate $< 10^{-9}$, σ_t, and ρ must satisfy the condition

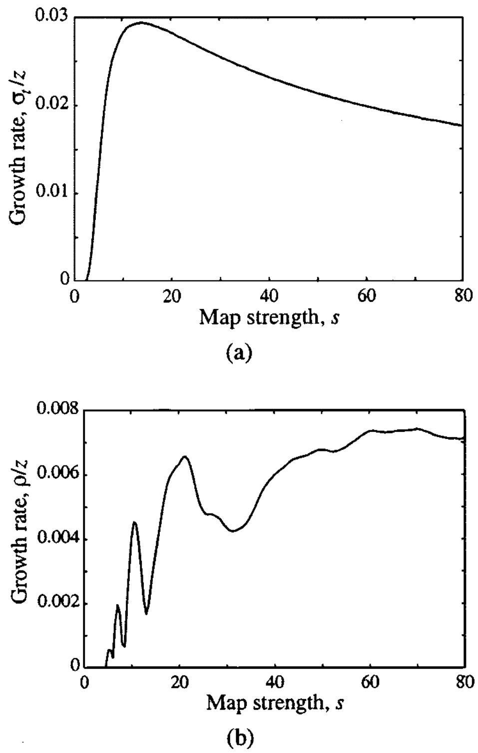

Fig. 2. Normalized growth rate of (a) timing jitter σ_t/z and (b) amplitude jitter ρ/z versus map strength s, obtained from (9) and (10) with (5) and (7) calculated numerically.

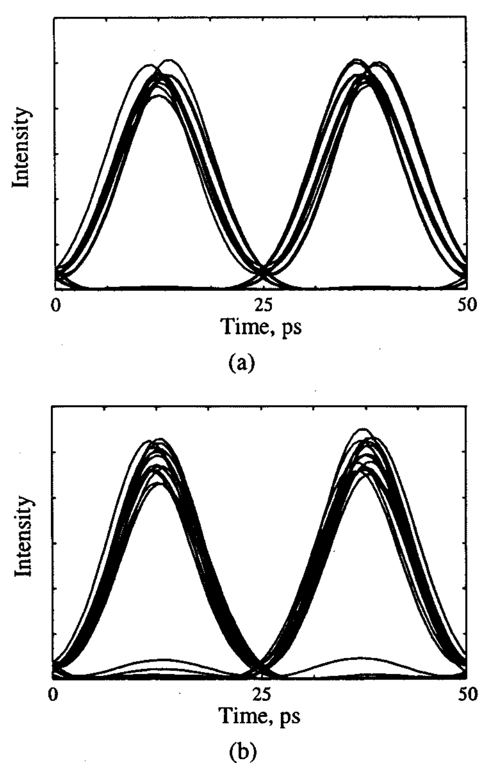

Fig. 3. Eye diagrams after 5000-km transmission when (a) $s = 10$ and (b) $s = 80$.

$\sigma_t < 0.06\, t_{\mathrm{bit}} = 1.5$ ps and $\rho < 1/6 = 0.167$ by assuming that t_0 and W_0 follow Gaussian statistics [10]. The threshold required for error free transmission is also plotted in this figure.

The analytical model allows us to study how system performance is limited by intrachannel pulse interactions depending on the value of map strength. Fig. 2 shows the normalized growth rate of (a) timing and (b) amplitude jitter for various values of map strength obtained from (9) and (10) with (5) and (7). Both timing and amplitude jitter takes the largest value for moderate values of $s(\sim 15\text{–}20)$. For larger s, timing jitter decreases whereas amplitude jitter still remains to be a dominant cause of transmission penalty. This is also seen in Fig. 3, where we plot eye diagrams when (a) $s = 10$ and (b) $s = 80$ after 5000-km transmission. Major degradation is caused by timing jitter when $s = 10$, whereas for $s = 80$ large amplitude fluctuation of nonzero bits and ghost pulse generation at zero bits are observed, resulting in a main source of eye closure.

IV. Conclusion

We have studied system impact of nonlinear intrachannel interactions on high-speed dispersion-managed RZ pulse transmission. Explicit formulae to estimate timing and amplitude jitter caused by intrachannel crosstalk have been presented. Timing and amplitude jitter grows linearly with respect to distance. Based on the analytical model, we found that timing jitter is a major limitation in system performance for smaller values of map strength, whereas for larger map strength amplitude jitter causes significant transmission impairments. Timing and amplitude jitter are expected to be suppressed considerably by employing distributed Raman amplification with the same value of path-average power [11].

References

[1] R. J. Essiambre, B. Mikkelsen, and G. Raybon, "Intra-channel cross-phase modulation and four-wave mixing in high-speed TDM systems," *Electron. Lett.*, vol. 35, pp. 1576–1578, 1999.
[2] P. V. Mamyshev and N. A. Mamysheva, "Pulse-overlapped dispersion-managed data transmission and intrachannel four-wave mixing," *Opt. Lett.*, vol. 24, pp. 1454–1456, 1999.
[3] A. Mecozzi, C. B. Clausen, and M. Shtaif, "Analysis of intrachannel nonlinear effects in highly dispersed optical pulse transmission," *IEEE Photon. Technol. Lett.*, vol. 12, pp. 392–394, Apr. 2000.
[4] J. Martensson, A. Berntson, M. Westlund, A. Danielsson, P. Johannisson, D. Anderson, and M. Lisak, "Timing jitter owing to intrachannel pulse interactions in dispersion-managed transmission systems," *Opt. Lett.*, vol. 26, pp. 55–57, 2001.
[5] M. J. Ablowitz and T. Hirooka, "Intra-channel pulse interactions in strongly dispersion-managed systems: timing shifts," *Opt. Lett.*, vol. 26, pp. 1846–1848, 2001.
[6] ——, "Resonant nonlinear intrachannel interactions in strongly dispersion-managed transmission systems," *Opt. Lett.*, vol. 25, pp. 1750–1752, 2000.
[7] S. Kumar, "Intrachannel four-wave mixing in dispersion managed RZ systems," *IEEE Photon. Technol. Lett.*, vol. 13, pp. 800–802, Aug. 2001.
[8] M. J. Ablowitz and T. Hirooka, "Intra-channel pulse interactions in strongly dispersion-managed systems: energy transfer," *Opt. Lett*, vol. 27, pp. 203–205, 2002.
[9] A. Mecozzi, C. B. Clausen, and M. Shtaif, "System impact of intra-channel nonlinear effects in highly dispersed optical pulse transmission," *IEEE Photon. Technol. Lett.*, vol. 12, pp. 1633–1635, Dec. 2000.
[10] E. Iannone, F. Matera, A. Mecozzi, and M. Settembre, *Nonlinear Optical Communication Networks*. New York: Wiley, 1998.
[11] T. Hirooka and M. J. Ablowitz, "Suppression of intra-channel dispersion-managed pulse interactions by distributed amplification," *IEEE Photon. Technol. Lett.*, vol. 14, pp. 316–318, Mar. 2002.