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Numerical Evaluation of Natural Frequency Change of a Ferromagnetic Plate due to Magnetic Stiffness Effect

T. Takagi and J. Tani
Institute of Fluid Science, Tohoku University, Katahira 2-1-1, Aoba-ku, Sendai 980-77, Japan

P. Ruatto, B. Krevet and L. V. Boccaccini
Forschungszentrum Karlsruhe, Postfach 3640, Karlsruhe D76021, Germany

Abstract - **This paper describes the numerical simulation of the dynamic behavior of a ferromagnetic thin plate vibration in a magnetic field. The behavior of a ferromagnetic cantilevered plate, which is set in the direction of uniform magnetic field, shows that its natural frequency increases with the external magnetic induction. Two methods were applied to evaluate this phenomenon. The magnetic field of the deformed shape has been analyzed by 3D analysis and 1D approximation. The natural frequencies were evaluated by the energy method in 3D case and by a transient response in 1D case. The results show that the natural frequency change can be explained by the magnetic stiffness effect caused by magnetization including its saturation.**

I. INTRODUCTION

The interaction of magnetic fields and mechanical deformation of a ferromagnetic material has been studied by some researchers [1]-[4]. Most of the analytical and experimental studies were concerned with the investigation under saturation field [1]-[3]. In such studies the deformation due to magnetomechanical interaction is proportional to the square of the applied field. Recently, a ferromagnetic material has been considered for use in high magnetic field beyond the saturation field as a first wall of a magnetic fusion reactor [5]. If a high field, beyond the saturation field, is applied, strange mechanical behavior has been observed [4].

Until now, studies of magnetomechanical behavior of a ferromagnetic material have been mainly focused on magnetoelastic instability of a soft ferromagnetic plate. The ferromagnetic plate placed normal to an incident magnetic field is known to buckle at a certain magnetic induction [1]-[3]. Moon and Pao [1] first made an experiment on the magnetoelastic buckling of a ferromagnetic plate and showed the variation of magnetic buckling field has 3/2 power of the ratio of length to thickness. Because of a big discrepancy between the theoretical prediction and experimental results shown in their paper, there has been some research to explain it [2], [3].

The critical field of magnetoelastic buckling must be considered. However the magnetomechanical vibration problem is a common one and more important for structural design of high magnetic field devices. Some of the authors have already presented an experimental study on magnetic damping and stiffness effects occurring in a ferromagnetic plate in a high magnetic field [6]. But the quantitative description of the dynamic behavior of a ferromagnetic plate has not been given till now, though ferromagnetic material attracts the attention of researchers and engineers who work towards a magnetic fusion reactor.

The objective of this paper is to show a method for the evaluation of natural frequency change of ferromagnetic plates under strong magnetic field. The experimental and analytical model treated here is similar to the benchmark model of TEAM problem 16 [7]. In the case of the problem 16 the plate material is copper and the coupling between magnetic field and the plate vibration causes only the magnetic damping effect. But in this study the vibration of a ferromagnetic plate causes the natural vibration change as well as the magnetic damping effect.

II. EXPERIMENT

A part of experimental results has been presented in [6]. Therefore the authors describe the experiment briefly. A ferritic steel : 8%Cr-2%W-0.2%V-0.04%Ta-Fe (hereafter referred to as F82H) was chosen as the ferromagnetic material and copper was chosen as the non-ferromagnetic material. F82H is a reduced activation ferritic steel and is a candidate for the first wall material of a fusion reactor [5].

In this experiment four kinds of test pieces were made from F82H and copper as shown in Table I. Plates with 0.29mm thickness and 0.3mm thickness are referred to as 0.3t plates and plates with 0.5mm as 0.5t. A plate is set under a uniform static field and a time-changing coil field as shown in Fig. 1. The material constants are shown in Table II. The saturation magnetic induction of F82H is about 1.9T and the relative permeability is around 12. An external uniform field was applied using a superconducting magnet at the Institute for Materials Research, Tohoku University. The maximum available magnetic induction is 8T and the effective magnetic field volume is 22 cm (diameter) × 40cm (length).

The eddy current induced by the transient coil field causes bending deflection by the interaction with the external field. The deflection of the plate was measured by a laser Doppler sensor and non-induced type strain gauges. A pulse coil was made from 27-turn copper wire. The maximum coil current was around 500A with a half period width of 2ms. Outer and inner diameters of this solenoid coil were 22.0 mm and 20.0 mm respectively. The coil height was 24.2 mm.

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T. Takagi, e-mail takagi@ifs.tohoku.ac.jp, fax +81-22-217-5248.

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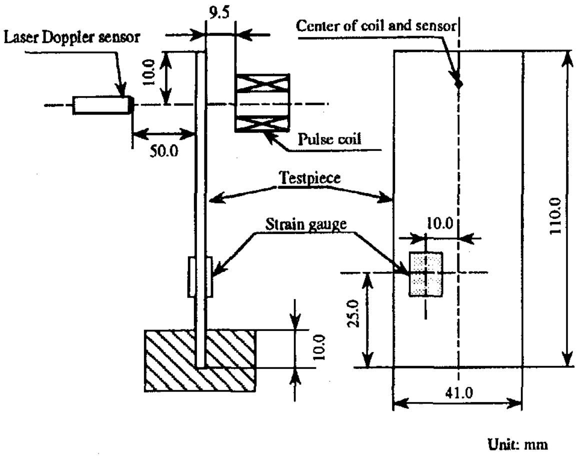

Fig. 1. Experimental setup of a ferromagnetic cantilevered plate.

TABLE I
TEST PIECES MADE OF F82H AND COPPER

Item	Material	Length (mm)	Width (mm)	Thickness (mm)
1	F82H	110	41	0.29
2	F82H	110	41	0.5
3	Cu	110	40	0.3
4	Cu	110	40	0.5

TABLE II
MATERIAL CONSTANTS OF F82H AND COPPER

Item	F82H	Cu
Density	$7.8\text{x}10^3$ (kgm^{-3})	$8.9\text{x}10^3$
Conductivity	$2.3\text{x}10^6$ (Sm^{-1})	$5.8\text{x}10^7$
E	$2.0\text{x}10^{11}$ (Pa)	$1.1\text{x}10^{11}$
v	0.3	0.34

III. NUMERICAL METHOD

A. 3D Analysis (3D)

The transient eddy current analysis program AENEAS has been developed in order to apply it to the dynamic behavior analysis of ferromagnetic structures of future magnetic fusion reactors like DEMO [8]. The program solves the transient eddy current problem for a conducting body Ω with a ferromagnetic subregion Ω_M. The governing equation is written by the following integral formulation,

$$\eta(r)\boldsymbol{J}(r,t)+\frac{\mu_0}{4\pi}\int_{\Omega}\frac{1}{|r-r'|}\frac{\partial \boldsymbol{J}(r',t)}{\partial t}dv'=$$
$$-\frac{\partial \boldsymbol{A}_e(r,t)}{\partial t}-\nabla\phi(r,t)-\frac{\mu_0}{4\pi}\int_{\Omega_M}\frac{\partial \boldsymbol{M}(r',t)}{\partial t}\times\nabla\frac{1}{|r-r'|}dv' \qquad (1)$$

where $\boldsymbol{\eta}$ is the electrical resistivity tensor, $\boldsymbol{J}$ the current density, $\boldsymbol{A}_e$ the vector potential corresponding to the externally applied field, ϕ the scalar electric potential, M the magnetization and the primes distinguish the source points from the field ones.

The formulation (1) is derived from the corresponding one of an already existing code for non-magnetic materials [9] adding a term that takes into account the contribution of the magnetization density to the magnetic vector potential.

As the new unknown M is introduced, the relation between magnetization M and magnetic field intensity H is considered to solve the problem.

$$M(r,t)=\chi(|H(r,t)|)H(r,t) \qquad (2)$$

where χ is the magnetic susceptibility and a function of the magnitude of H calculated by the Biot-Savart law.

To compute magnetization, an initial distribution of M is needed. It is determined by solving the magnetostatic problem given by (2) and the Biot-Savart law iteratively. At the i-step of the iterative procedure the susceptibility matrix is evaluated utilizing the net field and successive relaxation is applied. This iterative procedure is repeated until each component of $\{M\}$ satisfies a convergence condition.

Force and torque due to magnetization can be given as [10]:

$$F=\int_{\Omega_M}(\nabla\times M)\times B_0 dv+\int_{\partial\Omega_M}(M\times n)\times B_0 dS \qquad (3)$$

$$C=\int_{\Omega_M}r\times\left[(\nabla\times M)\times B_0\right]dv+\int_{\partial\Omega_M}r\times\left[(M\times n)\times B_0\right]dS \qquad (4)$$

where $\boldsymbol{B}_0$ is applied magnetic induction. An algorithm to obtain the force and torque has been developed utilizing equivalent magnetizing currents as an extension of an existing technique by T. Kabashima [11].

The dynamic response of a ferromagnetic plate can be calculated by solving the equation of motion step by step as well as the magnetic field equation. In the case of 3D analysis it needs much calculation time to follow this procedure. In order to spare the time the authors assumed the following:

(i) The plate vibrates with the first mode.
(ii) Energy dissipation due to the damping of the structure as well as the magnetic one is neglected, since measured damping coefficient was smaller than a copper case.
(iii)The transient magnetic field due to the pulse coil disappears rapidly and only the static magnetic field exists after then.

From the above assumption (ii) the authors of this paper solve the magnetic induction considering the nonlinearity of magnetic property, but not considering the vibration induced current. Though the program developed here can solve eddy currents in a ferromagnetic plate, eddy currents do not have to be calculated from the above assumption (ii). Hence the magnetostatic analysis was performed, which is one of functions of the program developed here.

Concerning the natural frequency of a plate, the value of it can be evaluated from the energy method under the assumptions of (i) and (ii),

$$T_{\max}=U_{\max}+W_{\max} \qquad (5)$$

where T, U and W are kinetic, elastic and magnetic energies, and a subscript max means the maximum value.

B. Beam Approximation (1D)

Static magnetic field in a ferromagnetic plate is obtained from the boundary conditions of $\boldsymbol{B}$ and $\boldsymbol{H}$.

From the continuity of $\boldsymbol{B}$, the absolute value of magnetic induction in a ferromagnetic plate is given as,

$$|B_0| = |B_{0_e}|\frac{\cos\beta_e}{\cos\beta} \tag{6}$$

where B_0, B_{0e}, β and β_e are defined in Fig. 2.

Since the continuity condition of magnetic field intensity gives the value of β, the absolute value of magnetization can be obtained by the following expression.

$$|M| = \frac{\chi}{\mu_0(1+\chi)}|B_{0_e}|\frac{\cos\beta_e}{\cos\beta} \tag{7}$$

In the case of the beam approximation the vibration of a plate is calculated by the time integration of the equation of motion considering the magnetic torque. In order to obtain the consistent solution 5-10 iterations of deflection and magnetization calculations are repeated.

IV. RESULTS AND DISCUSSION

As shown in (3) and (4), the outer products of magnetic induction and magnetization give force and torque, which might be almost proportional to displacement or deflection angle. This force or torque might act as a kind of magnetic stiffness and cause the natural frequency to change under the external field.

In the calculation of the magnetic field the nonlinear relation (2) of magnetization was considered by using Akima's fitting [12] in the 3D case as shown in Fig. 3. The magnetization curve of F82H was measured by NKK corporation for the same material as the test pieces. For the beam approximation magnetic permeability is treated as constant under the saturation magnetic induction B_{sat}, and $\mu_0 M$ is B_{sat} when $\mu_0 M$ is over B_{sat}. For the beam approximation firstly external force which simulates the Lorentz force induced by a pulse coil field was applied, and the dynamic response was obtained. From this result the natural frequency was evaluated using the FFT (fast Fourier transform) analysis.

In the 3D case a 0.3t F82H plate was divided into 480 (20(length)×8(width)×3(thickness)) finite elements while a 0.5t plate was divided into 800(20×8×5) elements. A part of the mesh division around the edge is shown for a 0.3t plate in Fig. 4. The number of elements was determined to give the sufficiently accurate solution for this static analysis. In the case of the beam approximation 22 beam elements were used.

Numerical results using the beam approximation (1D) and 3D analysis (3D) as well as experimental ones regarding natural frequencies are summarized in Fig. 5. As shown in Fig. 5 the natural frequency for F82H plates increases as external field does. Both numerical and experimental results show that the natural frequency changes in the case of F82H plates while very little change was shown in copper plates. The results from 3D are closer to the experimental values but still smaller. The discrepancy might be attributed to a little ambiguity of the material properties, such as magnetic permeability, saturation magnetic induction, elastic properties, etc. In low field less than 0.2T the curves have different shape. This difference might be due to the negligence of magnetic induction caused by eddy current.

The authors made three assumptions for the analysis. Assumption (iii) is reasonable because of a short duration of a pulse coil current. From the measured power spectrum the authors found that only bending mode vibration occurs. Hence assumption (i) is also valid. In order to verify assumption (ii), the natural frequency of a 0.3t copper plate was evaluated using the full coupling and MMD methods [13]. From Table III the authors could not find the natural frequency change both in experiment and analysis caused by the magnetic damping effect. Since the damping ratio for a F82H plate is much smaller (ς=0.024 at B=1.0T obtained in experiment), it can be concluded magnetic damping effect has little influence on the frequency change.

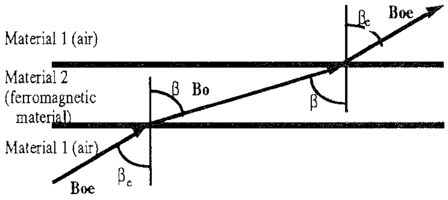

Fig. 2. Schematic figure of magnetic induction

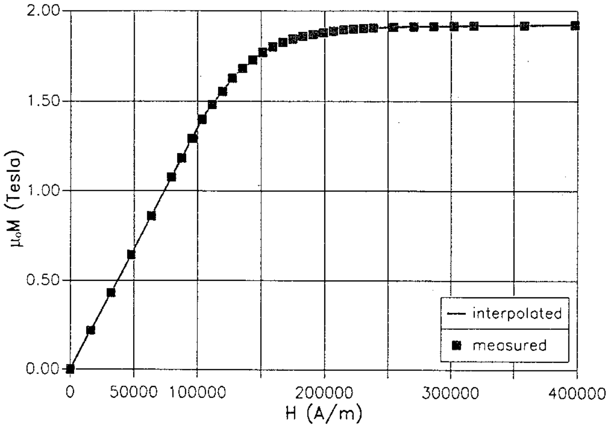

Fig. 3. Magnetization curve of F82H and the fitting by Akima's method

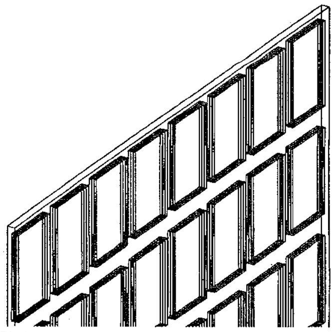

Fig. 4. Finite element meshes around edge region for 0.3t F82H plate.

TABLE III

NATURAL FREQUENCIES AND CRITICAL MAGNETIC VISCOUS DAMPING RATIOS OF 0.3t COPPER PLATE OBTAINED IN EXPERIMENT AND NUMERICAL ANALYSIS. N/A MEANS THAT THE VALUE OF FREQUENCY CANNOT BE OBTAINED IN FFT ANALYSIS BECAUSE OF FAST DAMPING.

Item	B=0.0T	B=0.1T	B=0.2T	B=0.3T	B=0.4T
f by experiment	18.5	18.5	18.5	18.5	17.5
f by analysis	17.5	17.5	17.5	17.3	N/A
ζ by experiment	0.0	0.011	0.048	0.11	0.15
ζ by analysis	0.0	0.013	0.054	0.12	0.21

Both in 3D and 1D analyses the saturation of magnetization was considered. Fig. 6 shows that if the saturation is not considered the obtained frequencies become larger than those obtained by considering the saturation. Since the relative magnetic permeability of F82H is 12, the material would be saturated with the external field of 0.2T. This means that the calculation without considering the saturation overestimates the magnetic stiffness effect.

V. CONCLUSIONS

The authors applied (i) 3D analysis and (ii) beam approximation to evaluate natural frequency changes of ferromagnetic plates under magnetic fields.

(i) The magnetization at deformed shape was found to make the natural frequency change and cause the magnetic stiffness effect.

(ii) Both methods almost agreed with experimental results. The beam approximation method is applicable to a simple model treated here. The 3D method would be more general and accurate in complicated structures.

(iii) Though the magnetic damping and magnetic stiffness effects occur at the same time in the present model, the natural frequency change can be evaluated independent of the damping effect.

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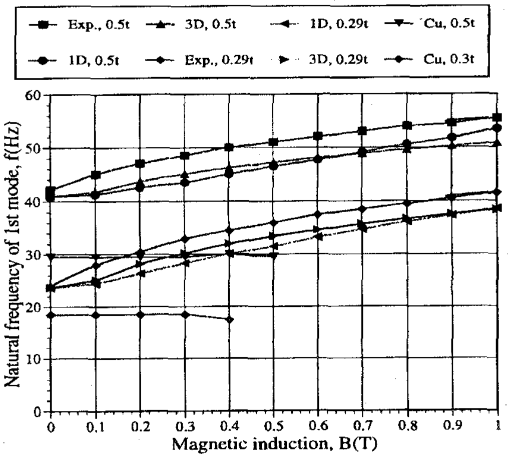

Fig. 5. Magnetic stiffness effect caused by magnetization considering the saturation

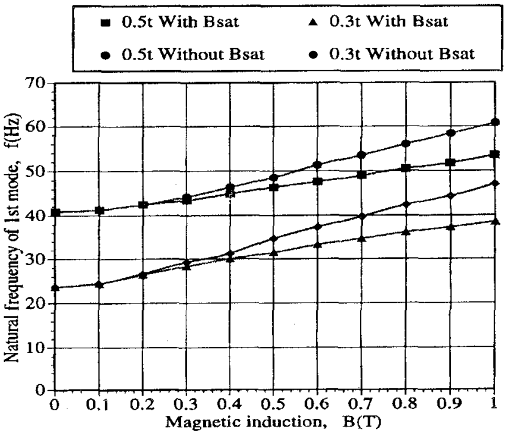

Fig. 6. Magnetic stiffness effects caused by magnetization considering the saturation and not considering the saturation