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Analysis and Experiment of Dynamic Deflection of a Thin Plate with a Coupling Effect

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Abstract — **This paper describes computational analyses and an experiment of thin plate deflection in magnetic field. The authors show the governing equation for the eddy current and the deflection analyses of a thin plate considering the coupling effect between the eddy current and the deflection under non-uniform magnetic field. A computer code considering the effect was developed based on a thin plate approximation. The code was applied to the FELIX cantilevered beam (without torsion) and the results (current density, deflection and bending strain) coincided quite well with the experimental results. The authors performed a plate deflection experiment (with torsion) and the code was also applied to this experiment. The comparison between numerical and experimental results verified the analysis method and the code.**

I. INTRODUCTION

When impulsive and/or transient magnetic fields are applied to electrically conducting structure, induced current causes electromagnetic force due to the interaction with magnetic field. This Lorentz force deflects thin structures. But when additional eddy currents due to the movement of a vibrating structure are induced under external magnetic field, the Lorentz force sometimes acts as a damping force and makes the displacement smaller. This force can be evaluated if we consider the coupling effect between eddy current and the deflection of structure. This coupling occurs when the vibrating structure intercepts additional magnetic flux.

The coupling effect of beam under uniform transient field was investigated by T.Q. Hua [1], L.R. Turner [2], D.W. Wessenburger [3], T. Morisue [4], Z. Ren [5], H. Tsuboi [6] and so on. These works treated rigid-body movement and elastic beam deflection under uniform time-changing field. In this paper, the authors treat the elastic thin plate deflection with and without torsion under time-changing coil field. The governing equation of the eddy current and the plate deflection considering the coupling effect is derived. A computer code to calculate eddy current, deflection and bending strain is developed and applied to the purely bending experimental results [7] in order to verify the code. A deflection experiment with torsion is carried out, and the computer code is also applied to this experiment. The problem treated here is considered an extension of TEAM Workshop problem No. 12, "Copper cantilever vibration problem" [7].

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II. NUMERICAL ANALYSIS METHOD

A. Eddy Current Analysis

A thin elastic isotropic homogeneous plate with electrical conductivity σ and magnetic permeability μ_o is set under magnetic field. From Faraday's and Ohm's laws, the following equation is obtained using current vector potential $\boldsymbol{T}$,

$$\mathrm{rot}\,(\mathrm{rot}\,\boldsymbol{T}) = -\sigma\frac{\partial \boldsymbol{B}}{\partial t} + \sigma\ \mathrm{rot}\,(\boldsymbol{u}\times\boldsymbol{B}) \tag{1}$$

where $\boldsymbol{u}$ is deflection velocity vector.

Assuming that the deflection is small and a plate is thin enough compared with skin depth, we can finally obtain the governing equation of eddy current analysis considering the coupling effect as follows:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = \sigma\frac{\partial}{\partial t}(B_{oz}+B_{ez}) - \sigma C \tag{2}$$

$$C = B_{ox}\frac{\partial^2 w}{\partial x\partial t} + B_{oy}\frac{\partial^2 w}{\partial y\partial t} - \frac{\partial B_{oz}}{\partial z}\frac{\partial w}{\partial t} \tag{3}$$

where w is lateral displacement and B_{ez} is obtained from Biot-Savart's law.

$$B_{ez} = \frac{\mu_o h}{4\pi}\iint_S \frac{\frac{\partial T}{\partial x}(x-x') + \frac{\partial T}{\partial y}(y-y')}{\{(x-x')^2+(y-y')^2\}^{3/2}}\,dx'\,dy' \tag{4}$$

Eq.(3) shows the coupling effect between eddy current and deflection. If the external field is uniform, only the first two terms of (3) should be evaluated. The equation for the uniform field case was derived by T. Morisue [4]. Therefore (2) and (3) are the generalized expression of Morisue's equation. When non-uniform field like coil field is treated, all terms of (3) must be considered.

B. Deflection Analysis

On the basis of the small bending theory of thin plate, plate deflection is analyzed. The governing equation for the deflection of the thin plate considering Lorentz force is expressed as follows:

$$D\left(\frac{\partial^4 w}{\partial x^4} + 2\frac{\partial^4 w}{\partial x^2 \partial y^2} + \frac{\partial^4 w}{\partial y^4}\right) + \rho h\frac{\partial^2 w}{\partial t^2} = h\,B_y j_x - h\,B_x j_y \quad (5)$$

where D, h and ρ are bending rigidity, plate thickness and mass density, respectively. Coupling with eddy current calculation using triangular finite elements, the authors calculate the plate deflection based on the finite element method with nonconforming triangular elements. Same finite element mesh division is used in both eddy current and deflection calculations.

In the deflection analysis each node has three components w, $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$. These three components are assumed to change linearly in each time step and calculated by using a modal analysis method with Duhamel's integral [8].

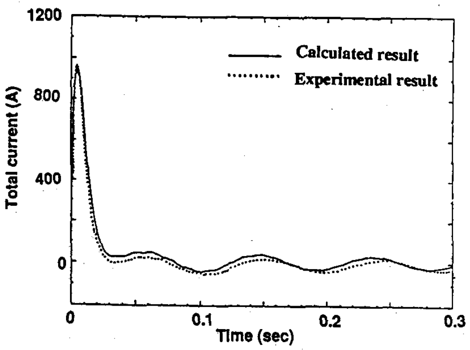
A computer code was developed based on the described method. Three terms of (3) can be evaluated by differentiating analytically w, $\frac{\partial w}{\partial x}$ and $\frac{\partial w}{\partial y}$ with respect to time using the Duhamel's integral obtained for each time step. The solution considering the coupling effect was obtained by an iterative method in the step.

III. EXPERIMENT

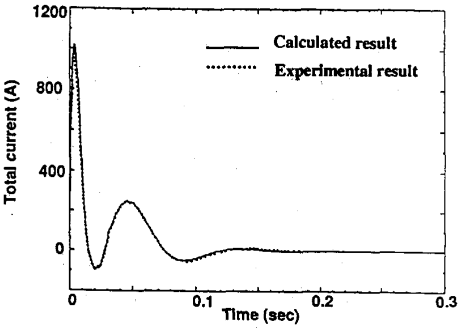
A copper plate is set under a uniform time-constant field and a time-changing coil field. Eddy current induced by the transient coil field causes torsional movement as well as bending deflection by interaction with the external uniform field. Length, width and thickness of the plate are 115, 40 and 0.3 mm. The deflection of the plate is measured by an optical non-contact displacement sensor. A pulse coil is made from 27-turn copper wires. Outer and inner diameters of this solenoid coil are 22.0 and 20.0 mm. The coil height is 24.6 mm. The distance between the plate and the coil is 9.5 mm.

IV. RESULTS AND DISCUSSIONS

The code developed here was first applied to "Copper cantilever vibration problem" [7] in order to verify the code. A copper rectangular beam rigidly clamped at one end is placed in time-changing field (B_z) and static uniform magnetic field (B_x). Four natural vibration modes were used in numerical analysis. Numbers of nodes and elements were 85 and 128. Mode No. 3 is a torsional mode but did not appear in the results. The results for total circulating eddy current in the case of B_x = 0.2 T and 0.7 T are shown in Fig. 1 for the present numerical analysis (solid one) and measurement (broken line) by L.R. Turner. Both results agreed quite well and showed very small difference. Deflection and bending strain are also shown in Figs. 2 and 3 for Bx=0.7 T. Numerical results (solid line) agreed well with experimental results broken line), but there are some difference of frequency. This may be attributed to the neglect of structural damping in the analysis. These comparison confirmed the validity of the method and the code.



(a) 0.2 Tesla



(b) 0.7 Tesla

Fig. 1 Total circulating current

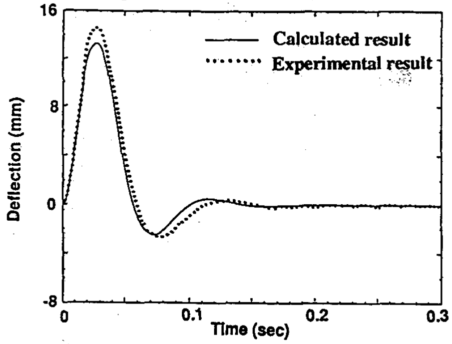


Fig. 2 Deflection at free end (Bx = 0.7 T)

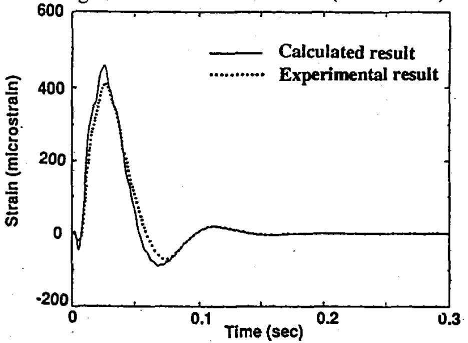


Fig. 3 Strain at clamped end (Bx = 0.7 T)

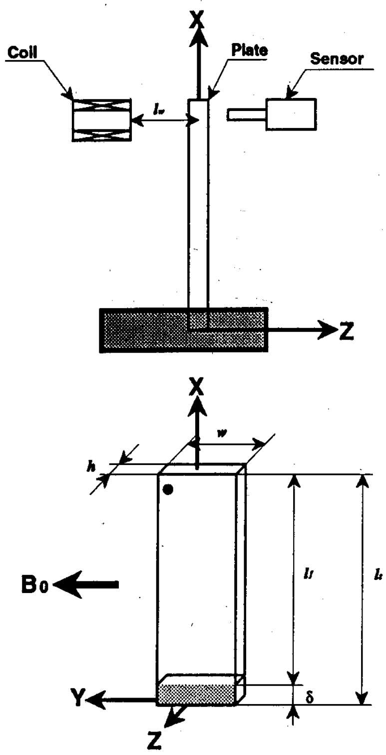


Fig. 4 Experimental model

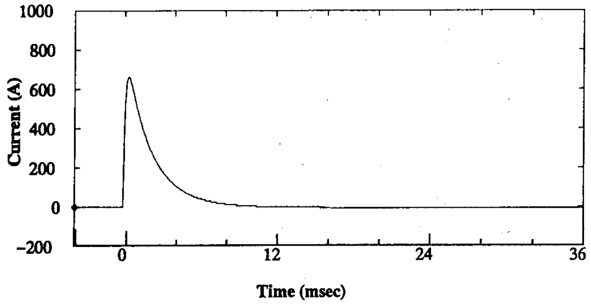


Fig. 5 Typical current profile in the experiment

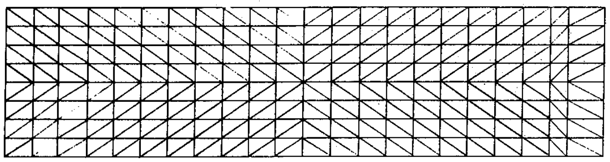
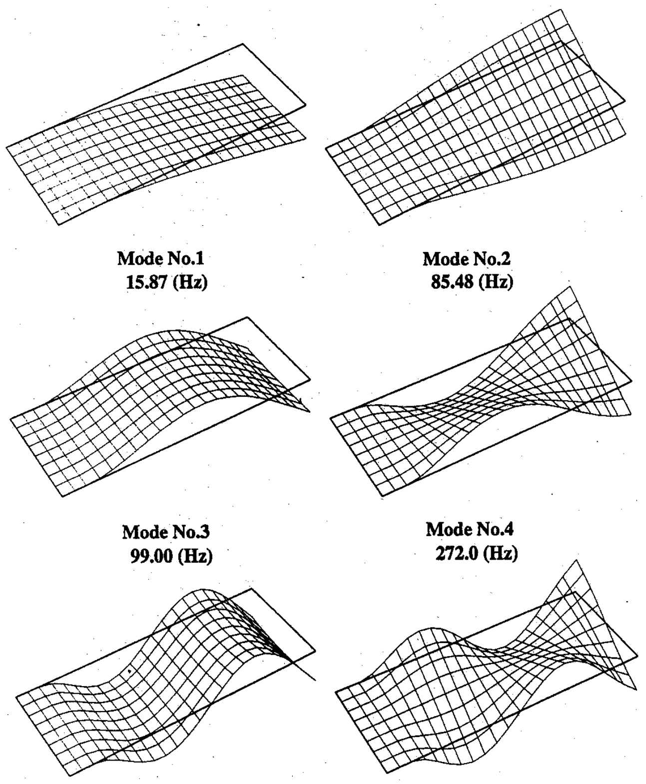


Fig. 6 Mesh division



Mode No.1
15.87 (Hz)

Mode No.2
85.48 (Hz)

Mode No.3
99.00 (Hz)

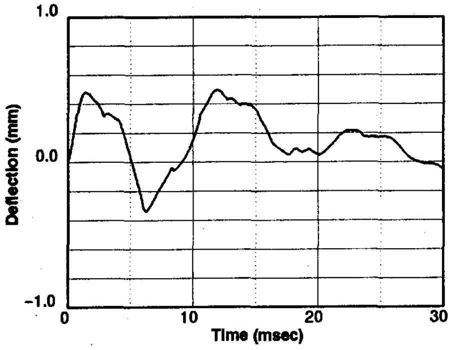
Mode No.4
272.0 (Hz)

Mode No.5
279.3 (Hz)

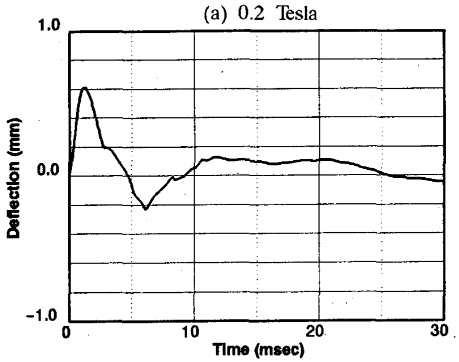
Mode No.6
503.5 (Hz)

Fig. 7 Natural vibration modes and natural frequencies

An experiment with torsional deflection was carried out (Fig.4). A copper cantilevered plate was set under uniform field (B_y) and a transient coil field. The coordinates of the coil center are (105, 7.5). A current profile of a coil current with maximum current of 662 A is shown in Fig. 5. Mesh division (207 nodes, 352 triangular elements) used for the FEM calculation is shown in Fig. 6. In this analysis six vibration modes were used (Fig. 7). Modes No. 2, 4 and 6 are torsional movements. Fig. 8 shows the experimental results of the plate dynamic behaviors for $B_x = 0.2$ T and 0.4 T. The location of the point is (108, 15). Because of the magnetic damping for the second mode (torsional movement), the deflection in the case of $B_y = 0.4$ T attenuated faster than that in the case of $B_y = 0.2$ T. Numerical results which correspond to the cases of Fig. 8 are shown in Fig. 9. Displacement in numerical analysis almost agreed that of experiment. Good agreement was also obtained in terms of frequency. Fig. 10 shows the numerical result ($B_y = 0.4$ T) when the coupling was not considered. The behavior of the plate in this figure was different from the measurement. From these results the authors could conclude that the methods developed here are applicable to the plate deflection with torsional movement under a strong magnetic field.

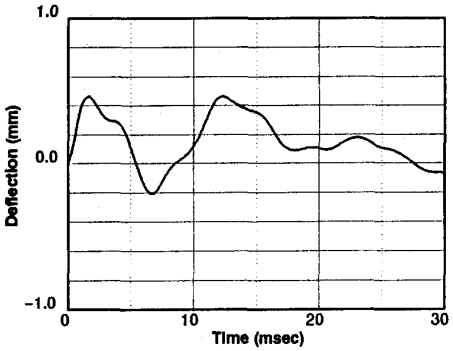


(a) 0.2 Tesla

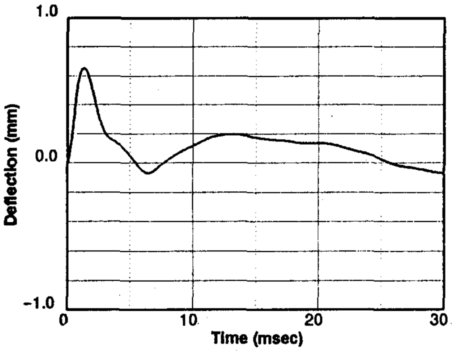


(b) 0.4 Tesla

Fig. 8 Experimental results of deflection



(a) 0.2 Tesla



(b) 0.4 Tesla

Fig. 9 Numerical result of deflection

V. CONCLUSIONS

Numerical analysis method for the dynamic behavior of an elastic thin plate considering the coupling effect between magnetic field and plate deflection has been developed.

The method was applied to "Copper cantilevered vibration problem." The results about current, deflection and strain agreed with measured ones.

A deflection experiment with torsional movement was carried out and the method was applied to this experiment. The analytical results almost agreed with measured ones in terms of displacement and frequency.

All calculation was performed using CRAY-YMP8 of the Supercomputer Center of the Institute of Fluid Science, Tohoku University.

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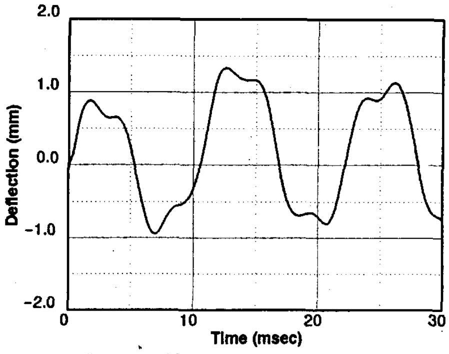

Fig. 10 Numerical result of deflection without coupling($Bx = 0.4$ T)