Iterative Solution Technique for 3-D Eddy Current Analysis Using T-Method

著者	高木　敏行
journal or publication title	IEEE Transactions on Magnetics
volume	24
number	6
page range	2682-2684
year	1988
URL	http://hdl.handle.net/10097/47882

doi: 10.1109/20.92212

ITERATIVE SOLUTION TECHNIQUE FOR 3-D EDDY CURRENT ANALYSIS USING T-METHOD

T. Takagi, T. Sugiura, K. Miyata, S. Norimatsu,
K. Okamura and K. Miya

Nuclear Engineering Research Laboratory
The Faculty of Engineering
The University of Tokyo
Tokai, Ibaraki, 319-11, JAPAN

This paper describes a numerical analysis technique for 3-D eddy current analysis using T-method. The technique is based on an iterative solution method using two matrices. These two matrices are generated by splitting a complex dense matrix which appears in AC problem. Numerical results for a simple problem showed the validity of an over-relaxation method in the iteration procedure. An application to eddy current testing verified the technique's applicability to practical problems.

Introduction

Typical methods for the eddy current analysis are A-φ method and T-Ω method. Both methods require variables in space as well as in a conductor. Some of the authors have already proposed the T-method [1,2] where a scalar potential Ω of the T-Ω method is not included.

Advantages of the T-method are summarized as follows,

(1) The number of variables is reduced very much because one kind of variable (T) is defined.

(2) Mesh division of space is not needed, because current vector potential could be defined in an analysis region.

But the method has disadvantage that a large core memory is needed due to a dense matrix.

The purpose of this paper is to develop numerical analysis technique to overcome the disadvantage by using iterative solution technique. Bettess [3] has proposed an economical solution technique for boundary integral matrices. Bettes reduced calculation time by an iterative solution technique using two matrices splitted from the fully dense matrix. In this paper we present the method reducing core memory needed for calculation by making use of the characteristics of a matrix which appears in T-method coupled with finite element method.

Basic Equations for T-method

Current vector potential T is defined by

$$J = \nabla \times T \quad , \tag{1}$$

which automatically satisfies the conservation of current, $\nabla \cdot J = 0$. From Faraday, Ampere and Ohm laws, we find the following equation,

$$\frac{1}{\sigma} \nabla \times \nabla \times T = -(\dot{B}e + \dot{B}o) \tag{2}$$

where Bo = applied field,
Be = magnetic field due to eddy current,
• = time derivative,
σ = electrical conductivity.

We can find the relation between Be and T as follows [2],

$$Be = \mu T + \frac{\mu}{4\pi} \int_s Tn \, \nabla \frac{1}{R} \, dS \tag{3}$$

where μ = magnetic permeability, Tn = normal component of T on surface. Introducing eq.(3) into eq.(2) we obtain the governing equation for eddy current analysis.

$$\frac{1}{\sigma} \nabla \times \nabla \times T + \mu \dot{T} + \frac{\mu}{4\pi} \int_s \dot{T}n \, \nabla \frac{1}{R} \, dS = -\dot{B}o \tag{4}$$

In this study we deal with an A.C. problem so that time dependent variables, T and Bo, can be expressed as follows,

$$T = T_a \, e^{j\omega t}$$

$$B = B_a \, e^{j\omega t}$$

where $j^2 = -1$, ω = angular frequency. Introducing these equations into eq.(4), we find,

$$\frac{1}{\sigma} \nabla \times \nabla \times T + j\omega\mu T + \frac{j\omega\mu}{4\pi} \int_s Tn \, \nabla \frac{1}{R} \, dS = -j\omega \, Bo \tag{5}$$

where a subscript "a" is omitted for brevity.

Iterative Solution Technique

The Galerkin method is applied to eq.(5). The obtained coefficient matrix are complex, unsymmetric and dense matrix. Direct matrix equation solution procedure such as Gaussian elimination has been commonly used [2].

In this iteration technique, we split the matrix into two matrices. One matrix corresponds to the first and second terms of eq.(5). It is complex, symmetric and band matrix and is here called S matrix. The other corresponds to the third term, "non-local term." It is imaginary, unsymmetric and dense matrix and is here called N matrix. The matrix is not fully densed. (i,j) component of N matrix is not equal to zero when j corresponds to normal components of surface nodes.

The current vector potential is obtained as the solution of the following equation.

$$\{[S] + [N]\}\{T\} = \{Bo\} \tag{6}$$

where {T} = nodal vector of T (unknowns)
{Bo} = nodal vector corresponding to right hand side of eq.(5).

The following equation expresses an iteration technique here.

$$[S]\{T^{(n)}\} = \{Bo\} - [N]\{T^{(n-1)}\} \tag{7}$$

where (n) = n-th iteration.

$\{T^{(n)}\}$ converges when spectral norm of $[S]^{-1}[N]$ satisfies the following inequality.

$$\|[S]^{-1}[N]\| < 1 \tag{8}$$

When $\|[S]^{-1}[N]\| \ll 1$, $\{T^{(n)}\}$ converges very rapidly. When $\|[S]^{-1}[N]\| \approx 1$, $\{T^{(n)}\}$ converges very slowly and this sometimes generates errors due to machine epsilon.

We can accelerate convergence by over-relaxation technique like SOR method. Here three methods are proposed and the current vector potentials in n-th iteration can be obtained by the following equation,

$$[P]\{T^{(n)}\} = \{f^{(n)}\} \tag{9}$$

$$\text{where } \{f^{(n)}\} = \alpha[\{Bo\} - [Q]\{T^{(n-1)}\}] + (1-\alpha)\{f^{(n-1)}\}, \tag{10}$$

α = relaxation factor.

Three methods are as follows.

Method 1

$$[P] = [S], \quad [Q] = [N] \tag{11}$$

Method 2

$$[P] = [S]+[Nb], \quad [Q] = [N]-[Nb] \tag{12}$$

where [Nb] = band matrix which has components of N matrix. (same band width as S matrix)

Method 3

$$[P] = \{[S]+[Nd]\}, \quad [Q] = [N]-[Nd] \tag{13}$$

where [Nd] = diagonal matrix of N matrix.

We could guess that the following inequalities hold.

0018-9464/88/1100-2682$01.00©1988 IEEE

$$||[P]^{-1}[Q]||_{\text{Method 2}} < ||[P]^{-1}[Q]||_{\text{Method 3}} < ||[P]^{-1}[Q]||_{\text{Method 1}} \quad (14)$$

Therefore the method 2 would show the rapid convergence characteristics.

Numerical Results

Simple Test Problem

The problem selected for the comparison of three methods is shown in Fig.1. Frequency dependencies relaxation factor and number of iterations until the convergence criteria was satisfied were also investigated. Square thin plate was divided into 32 1st order isoparametric elements. Total number of nodes, unknowns and half band width are 75, 225 and 96, respectively. The external magnetic field was perpendicularly applied to the plate.

In Figs.2 and 3 shown are convergence characteristics. The method 2 converged more rapidly than the method 1 as expected. Figs.4 and 5 shows relaxation factor dependency of number of iterations until the convergence criteria was satisfied. The following convergence criteria was used in this calculation.

$$\left| \frac{\Delta T_{max}}{T_{max}} \right| = \left| \frac{(T^{(n)}-T^{(n-1)})_{max}}{T_{max}} \right| < 10^{-7} \quad (15)$$

The optimal values of α in the method 2 was smaller than those of methods 1 and 3. As the frequency increases, the optimal values of α had the tendency to increase. Figs.6 and 7 show frequency dependencies of relaxation factor and the number of iterations for optimal value of α. Optimal values of α did not change over the frequency of 500 Hz. Number of iterations until the convergence criteria was satisfied for the optimal value of α did not change much over the frequency of 500 Hz.

An Application to Eddy Current Testing

In Fig.8 is shown the half of a block with a rectangular prism defect. A mesh division using 20 node isoparametric element is also shown. A probe is a cylinder with an inducting solenoid and two smaller receptive coils inside. The prove moves parallel to the plane of the defect. This model is same as that of Bossavit's [4]. Fig.9 shows a distribution of current vector potential on surface when the coil locates at 80mm distant from the center of defect. Steep gradient of the distribution means the concentration of eddy current.

We calculated four cases as shown in Table 1. Those include one case using the conventional approach and three cases using the present method. In Fig.10 is shown an obtained Lissajou's figure for case No.4. The shape shows the characteristic of the size and depth of the defect. Table 1 gives the number of elements, nodes, unknowns, memory and CPU time. A scalar computer HITAC M682 was used for case No.1 and a supercomputer HITAC S820 was used for other cases.

Conclusions

An iterative solution technique for 3-D eddy current analysis using T-method was developed.

The technique was applied to two eddy current problems and the following results were obtained.

(1) For the simple case, over-relaxation was effective for frequency over 500 Hz. Addition of matrix components of non-local matrix to local matrix reduced the number of iterations.

(2) For the case of eddy current testing, the technique developed here reduced the core memory and enabled us to apply the eddy current code to the practical case.

References

[1] K. Miya et.al.,"Three dimensional analysis of an eddy current by the T-method," in Proceddings of the IUTAM Symposium, Tokyo, October, 1986, North-Holland, pp.183-189.

[2] K. Miya and H. Hashizume,"Application of T-method to A.C. problem based on boundary element method," IEEE Transactions on Magnetics, Vol.24, pp.134-137, January 1988.

[3] J.A. Bettess,"Economical solution technique for boundary integral matrices, "International Journal for Numerical Methods in Engineering, Vol.19, pp.1073-1077, 1983.

[4] A. Bossavit and J.C. Verite,"The "TRIFOU" code: solving the 3-D eddy-currents problem by using H as state variable, "IEEE Transactions on Magnetics, Vol.19, pp.2465-2470, November 1983.

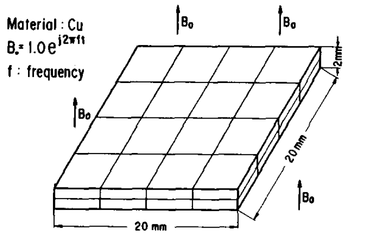

Fig.1 Test problem

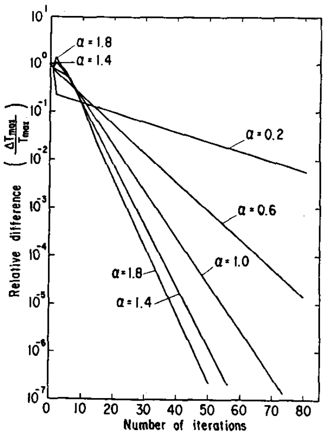

Fig.2 Convergence of the method 1 (f=500Hz)

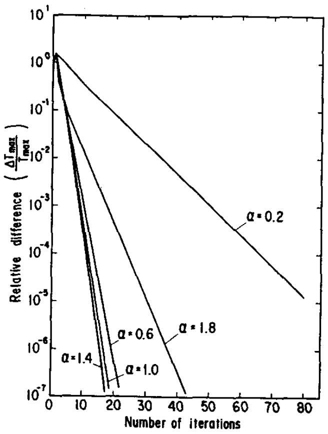

Fig.3 Convergence of the method 2 (f=500Hz)

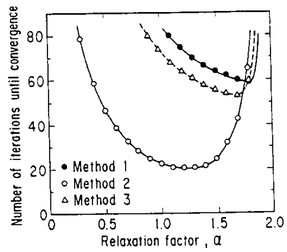

Fig.4 Number of iterations until convergence (f=500Hz)

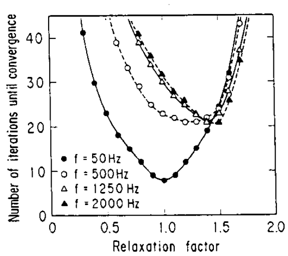

Fig.5 Number of iterations until convergence (method 2)

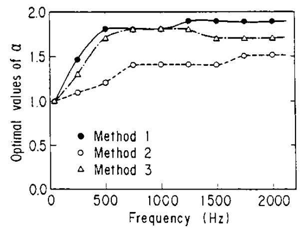

Fig.6 Frequency dependency of relaxation factor

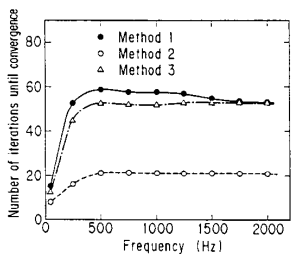

Fig.7 Frequency dependency of number of iterations

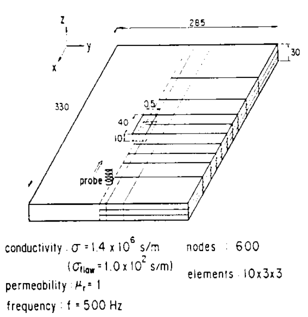

Fig.8 Mesh division for NDT model

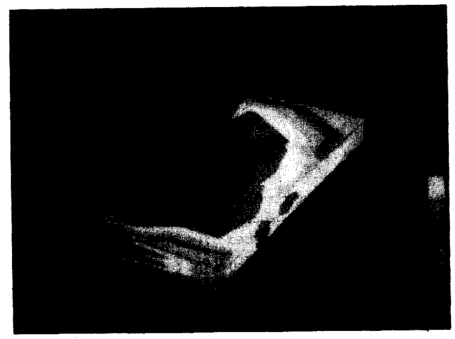

Fig.9 Distribution of vector potential

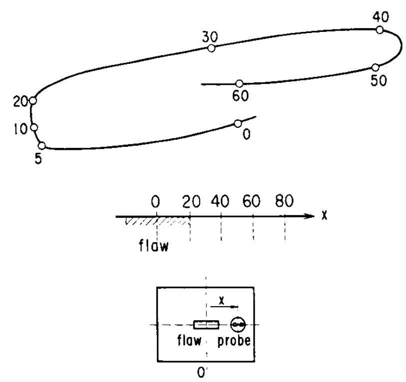

Fig.10 Numerical result of NDT signal trajectory

Table 1 Core momory and CPU time for different mesh divisions

Item \ Case	1	2	3	4
Nodes	600	600	690	1169
Elements	90	90	108	200
Nodes on surface	416	416	452	662
Elements on surface	108	108	123	180
Unknowns	1800	1800	2070	3507
Half band width	588	588	264	387
Computer	M682	S820	S820	S820
Method	direct	iteration	iteration	iteration
Core memory (Mbyte)	45	66	31	76
CPU time (sec)				
①Matrix generation	140	31	40	99
②LU decomposition	534	4	2	5
③Back substitution	25	304	206	503
④total	699	339	248	607