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Dynamical Properties of the Frustrated Ising Models and the Relation with the Ising Percolation Models

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Characteristic properties of three kinds of dynamics (evolution on continuous time, heat bath method and Metropolis method) are discussed in the Ising model with three spins as a simple example by solving the master equation directly. The key problem to consider the relationship between dynamical properties such as "damage spreading" and properties of the corresponding Ising percolation model are pointed out.
KEYWORDS: frustration, damage spreading, Ising percolation, dynamical transition

1. Introduction

Equibrium properties of ordered phases in the Ising system with random interaction such as the $\pm J$ model have been studied by many physicist from many aspects with several methods (numerically and analytically) and the distinct properties from those of uniform systems has been made clear. Dynamical properties of such systems are very difficult to study due to the existence of the long time tail of relaxation and the vague distinction of the dynamical properties from the equibrium ones.

Dynamical properties in paramagnetic (P) phase of the $\pm J$ model are distinct whether $T > T_c^{(pure)}$ or not. Slow relaxation appears in the phase $T < T_c^{(pure)}$ which is called Griffith phase. The slow relaxation is caused by the existence of arbitrary large ferro-like cluster with arbitrary long relaxation time. The idea of cluster is not so clear in frustrated models but clear in dilute ones. Therefore the slow relaxation is not a characteristic property of frustrated models.

On the other hand, a kind of dynamical property which is called "damage spreading" has been a key to characterize the dynamical properties of frustrated models[1-4].There is a characteristic temperature T_d at which the property of damage spreading change drastically. In general, the temperature T_d is located in P phase in frustrated models and it means $T_d > T_c$. It is noted that the temperature T_d appears even in regular (not random) system such as an antiferromagnetic model on the triangular lattice.

The Ising percolation model is constructed from the original Ising spin model by interpreting local Boltzmann weight $e^{-2\beta|J|}$ as the probability of connection[5](explained more correctly in section 3.) In non-frustrated cases, the probability with which two given spins connect with each other in the Ising percolation model corresponds to the correlation function in the original Ising model. It means that the Ising percolation picture has the same information as that in the original model. In frustrated models, the Ising percolation picture has more information than the equibrium quantities of the original Ising model. For example, mean percolation cluster size which is a typical quantity in the percolation model shows a divergence at the temperature T_p which locates in P phase in the original Ising model[6-8].There is no singularity in the equibrium quantities in the Ising model at T_p. It is reported that the relation $T_p \simeq T_c$ holds in several frustrated models[6-8]. It is not clear whether the relation is always true or not. If it is true, the physical reason of the relation is not clear. Before discussing the theoritical reason, it must be noted that the property "damage spreading" depends on the practical construction of the dynamics in the present stage. It seems that the construction of practical dynamics in computer simulation has developed in pursuiting the speed of the convergence to the equibrium state. It is necessary to reconsider what is a physically reasonable dynamics even in the transient process so as to discuss damage spreading as a property which is not artificial but physical.

Characteristic properties of three kinds of dynamics (evolution on continuous time, heat bath method and Metropolis method) are studied in the present article by solving the master equation of the stochastic process directly. A Speculation in section 5 suggests probability of connection in the Ising percolation model reflects a kind of dynamical property in the original Ising model.

2. Damage spreading

The idea of "damage spreading" is conventionally introduced in the following way. Hamming distance $d_H(\{\sigma_i\},\{\sigma_i'\})$ between two spin configulations $\{\sigma_i\},\{\sigma_i'\}$ is defined as follows:

$$d_H(\{\sigma_i\},\{\sigma_i'\}) = \sharp(\{i|\sigma_i \neq \sigma_i'\}), \tag{1}$$

where $\sharp(A)$ denotes a number of the member of the set A. The time evolutions are observed using common random forces for two different initial spin configurations $\{\sigma_i^{(1)}(t=0)\},\{\sigma_i^{(2)}(t=0)\}$. Then the following property holds.

$$\begin{aligned} {}^{\exists}T > 0, d_H(\{\sigma_i^{(1)}(T)\},\{\sigma_i^{(1)}(T)\}) = 0 \Longrightarrow \\ {}^{\forall}t > T, d_H(\{\sigma_i^{(1)}(t)\},\{\sigma_i^{(1)}(t)\}) = 0 \end{aligned} \tag{2}$$

Survival property $P(t)$ for two given initial states is defined followingly.

$$P(t) = Prob(d_H(\{\sigma_i^{(1)}(t)\}, \{\sigma_i^{(1)}(t)\}) \neq 0) \quad (3)$$

The basic idea of "damage spreading" is to study the long time limits $\lim_{t\to\infty} P(t)$ and $\lim_{t\to\infty} D(t)$. Especially the transition temperature T_d from $P(\infty) = 0$ to $P(\infty) > 0$ is a very relevant quantity. It is reported that several frustrated systems have a transition temperature which satisfies the inequalities $T_d > 0$ and $T_d > T_c$. The data are shown in succeeding section.

3. The Ising percolation model

The Ising percolation model is costructed from the Ising model whose Hamiltonian is

$$\mathcal{H} = -\sum_{\langle i,j\rangle} J_{ij}\sigma_i\sigma_j. \quad (4)$$

The Ising percolation model has two kinds of variables which are spin variables and the states of linking. The weight for updating of each state is also divided into two parts which are Boltzmann weight of spin states and the weight for linking states. Boltzmann weight for a given spin state $\{\sigma_i\}$ is

$$W_s(\{\sigma_i\}) = e^{-\beta\mathcal{H}}. \quad (5)$$

Linking proberbility p_{ij} is defined as

$$p_{ij} = \begin{cases} e^{-2\beta|J_{ij}|} & \text{for } J_{ij}\sigma_i\sigma_j > 0 \\ 0 & \text{for } J_{ij} = 0 \text{ or } J_{ij}\sigma_i\sigma_j < 0 \end{cases}. \quad (6)$$

The weight W_l for linking states is expressed using p_{ij} followingly.

$$W_l = \prod_{\langle ij\rangle} p_{ij} \quad (7)$$

The total weight W is

$$W = W_s W_l. \quad (8)$$

Then the following relation between the quantities of the Ising model and the Ising percolation model holds rigorously.

$$\langle\sigma_i\sigma_j\rangle = P_+ - P_-, \quad (9)$$

where

$$P_\pm = Prob(\ \sigma_i \text{ and } \sigma_j \text{ are connected and } \sigma_i\sigma_j = \pm 1). \quad (10)$$

Especially for ferromagnetic cases,

$$\langle\sigma_i\sigma_j\rangle = P_+. \quad (11)$$

It leads to the following results by taking the limit $|\mathbf{r}_i - \mathbf{r}_j| \to \infty$.

$$\text{Long range order} \lim_{|\mathbf{r}_i-\mathbf{r}_j|\to\infty} \langle\sigma_i\sigma_j\rangle \neq 0. \Longleftrightarrow$$
$$\text{The system is in percolated phase.} \quad (12)$$

The relations (11) and (12) illustrate Ising percolation model has the same informations as those of the Ising model in ferromagnetic cases. The properties of non-frustrated models are similarly discussed by reducing to ferromagnetic models after appropriate gauge transformation.

On the othe hand, the relation (9) shows that the Ising percolation model has more information than those of the corresponding Ising model in frustrated cases[9].For example, the condition $P_+ = P_- = 0$ (non-percolated phase) leads to the existence of the long range order but the inverse statement is not true. The present result is expressed in the form $T_p \geq T_c$ where T_p denotes a percolation transition temperature. Samples of already obtained results for T_d and T_p are listed below.

(a) The 2D $\pm J$ model

$$T_c^{(sg)} = 0 < T_p{}^{7)} \simeq T_d{}^{1)} \simeq 1.82 < T_c^{(ferro)} \leq 2.26 \quad (13)$$

(b) The 3D $\pm J$ model

$$T_c^{(sg)} \simeq T_d^{(1)\,2)} \simeq 1.8 < T_d^{(2)\,2)} \simeq T_p{}^{6)} \simeq 3.95 < T_c^{(ferro)} \simeq 4.51 \quad (14)$$

(c) AF model on the triangular lattice

$$T_c^{(AF)} = 0 < T_p \simeq T_d \simeq 2.6^{\,8)} < T_c^{(ferro)} \simeq 3.60 \quad (15)$$

$T_d^{(1)}$ and $T_d^{(2)}$ in the case of 3D $\pm J$ model denote two kinds of dynamical critical temperature.

(a) $T > T_d^{(2)}$
$d_H(t \to \infty)/N = 0$. N is a total number of the spins.

(b) $T_d^{(1)} < T < T_d^{(2)}$
$d_H(t \to \infty)/N = D_\infty > 0$. The value D_∞ is independent of initial two configurations.

(c) $T < T_d^{(1)}$
$d_H(t \to \infty)/N = D_\infty > 0$. The value D_∞ depends on two initial configurations.

It is noted that the relations $T_d > T_c$, $T_d < T_c^{(ferro)}$ and $T_d \simeq T_p$ hold in any case. The reason of the first relation is discussed previously. It is not clear whether the second and third relations always hold or not. It seems that the coincidence of T_d and T_p in three cases suggests some relation of mechanism between dynamical phenomena of the Ising model and equibrium properties of the Ising percolation model.

4. Various constructions of dynamics

Damage spreading depends on practical construction of dynamics by definition. The naturality of each practical dynamics must be checked in a physical sense in order to confirm the fact that damage spreading is caused by some physical reason.

Dynamics on continuous time is reasonable as a standard dynamics compared with other ones on discrete time. The first condition of a proper dynamics is that the distribution function converges to the canonical distribution in the limit $t \to \infty$. The first condition is satisfied by imposing following two subconditions.

- Evolution process is ergodic. It means that each states makes effects on other states within finite time. (This condition is very delicate in infinite systems.)
- The process has a property called "detailed ballance". (Additional comments on this condition are shown in constructing dynamics on discrete time.)

In the standard dynamics, each spin evolves by watching the states of the nearest neighbour spins, because spatial correlations arrives only via the nearest neighbour states. These arguments propose to adopt following dynamics as a reasonable standard dynamics. $D(\{\sigma_i\}|\{\sigma_i'\})$ is an infinitesimal operator for the time evolution of the stocastic process.

$$D(\{\sigma_i\}|\{\sigma_i'\}) = \begin{cases} p & \text{for } d_H(\{\sigma_i\},\{\sigma_i'\}) = 1 \\ q & \text{for } d_H(\{\sigma_i\},\{\sigma_i'\}) = 0 \\ 0 & \text{otherwise} \end{cases} \quad (16)$$

$$p = \gamma e^{-\frac{\beta}{2}\Delta E} \ (\gamma \text{ is some constant.}) \quad (17)$$

$$\Delta E = \mathcal{H}(\{\sigma_i'\}) - \mathcal{H}(\{\sigma_i\}) \quad (18)$$

q is determined satisfying the relation

$$\sum_{\{\sigma_i'\}} D(\{\sigma_i\}|\{\sigma_i'\}) = 0. \quad (19)$$

Then the distribution function $\mathbf{p}(t)$ at the time t is given as the form:

$$\mathbf{p}(t) = e^{tD}\mathbf{p}(0) \quad (20)$$

The infinitesimal operator D has only real nonpositive eigenvalues $\lambda \leq 0$ and only one stationary mode $\lambda = 0$. It means that the process has no osccilatinig modes of ralaxation. The present result proved mathematically has also a physical meaning. The reason of no osccilating modes is that effects of one spin on surroundings feed back to itself without delay and the feed back prevents the overrun of relaxation.

Let me consider two widely used dynamics on discrete time (heat bath method and Metropolis method) and only one spin flip process in the elementary process for the comparison with the case of continuous time. The elementary process for each dynamics is constructed followingly.

(a) Heat bath method

$$P_j(\{\sigma_i\}|\{\sigma_i'\}) = \begin{cases} \frac{1}{1+e^{-2\beta J}} & \text{for } \{\sigma_i\} = \{\sigma_i'\} \\ \frac{1}{1+e^{2\beta J}} & \text{for } \sigma_i = \sigma_i' (i \neq j) \\ & \text{and } \sigma_j = -\sigma_j' \\ 0 & \text{otherwise} \end{cases} \quad (21)$$

(b) Metropolis method

$$P_j(\{\sigma_i\}|\{\sigma_i'\}) = \begin{cases} 1 - e^{-2\beta\Delta E} & \text{for } \{\sigma_i\} = \{\sigma_i'\} \text{ and } \Delta E > 0 \\ 1/2 & \text{for } \{\sigma_i\} = \{\sigma_i'\} \text{ and } \Delta E = 0 \\ e^{-2\beta\Delta E} & \text{for } \sigma_i = \sigma_i'(i \neq j),\ \sigma_j = -\sigma_j' \text{ and } \Delta E > 0 \\ 1/2 & \text{for } \sigma_i = \sigma_i'(i \neq j),\ \sigma_j = -\sigma_j' \text{ and } \Delta E = 0 \\ 0 & \text{otherwise} \end{cases} \quad (22)$$

Total processes are constructed from the elementary processes followingly.

$$P = P_{j(n)}P_{j(n-1)} \cdots P_{j(2)}P_{j(1)} \quad (23)$$

There are various constructions with a selection of the series $j(m)$. It is noted that the series $j(m)$ must include all integers I $(1 \leq I \leq N)$ in order to ensure the ergodicity. Though each elementary process satisfies the condition of detailed ballance, the total process does not always satisfy the condition. Nevertheless the present construction of the total process ensures convergence to the proper final distribution. In genaral, the operator P has non-real eigenvalues due to the uncommutability of the elementary processes P_j. (Random independent choice of the spin $j(m)$ recovers the property of real eigenvalues for the total process.)

5. Analysis of a system with three spins

In the present section, One very simple system with only three spins is analyzed by applying the dynamics constructed in the previous section and the characteristic properties of each dynamics are made clear.

The Hamiltonian of the system with three spins is

$$\mathcal{H} = -J(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1) \quad (24)$$

Dynamics on continuous time and with heat bath method are studied for the ferro- and antiferromagnetic case and dynamics with Metropolis method is studied only for the antiferromagnetic case.

(a) Continuous time

Infinitesimal operator D of evolution for the present Hamiltonian is constructed by applying the general expression (17). D is easily diagonalized exactly using the spin inversion symmetry $(\{\sigma_i\} \to \{-\sigma_i\})$ and traslational symmetry $(1 \to 2 \to 3 \to 1)$ of the Hamiltonian $\mathcal{H}$. The eigen-states are classified as symmetric or antisymmetric modes under the spin inversion whose relaxation times are τ^S and τ^{AS} respectively. It is noted that the modes whose total magnetization $M = \sum_{i=1}^{3}\sigma_i$ relaxes are antisymmetric. Symmetric modes are a stationary mode (canonical distribution) and the modes whose relaxation times are τ_1^S (translationally invariant mode), τ_2^S and τ_3^S. Relaxation times of the antisymmetric modes are $\tau_{1\pm}^{AS}$ (translationally invariant),

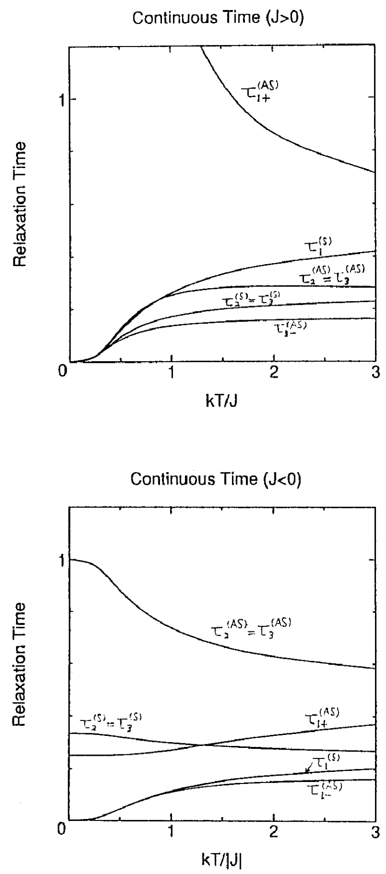

Figure 1: Relaxation times of evolution on continuous time

τ_2^{AS} and τ_3^{AS}. The relaxation time of each mode is expressed as an analytical form. Instead of listing explicit forms, the order of length of relaxation times and the behavior of $\tau's$ near $T = 0$ are listed.

○ $J > 0$

$$\tau_{1-}^{AS} < \tau_2^S = \tau_3^S \begin{array}{l} < \tau_2^{AS} = \tau_3^{AS} < \tau_1^S \ (\alpha > 3) \\ < \tau_1^S \ < \tau_2^{AS} = \tau_3^{AS} \ (1 < \alpha < 3) \end{array}$$

$$< \tau_{1+}^{AS} \tag{25}$$

○ $J < 0$

$$\tau_{1-}^{AS} < \tau_1^S \begin{array}{l} < \tau_2^S = \tau_3^S < \tau_{1+}^{AS} \ (2\sqrt{3} - 3 < \alpha < 1) \\ < \tau_{1+}^{AS} < \tau_2^S = \tau_3^S \ (0 < \alpha < 2\sqrt{3} - 3) \end{array}$$

$$< \tau_2^{AS} = \tau_3^{AS} \tag{26}$$

$$\alpha = e^{\beta J}$$

• $T \to 0$

○ $J > 0 \ (\alpha \to \infty)$

$$\tau_1^S \sim \alpha^{-1} - 3\alpha^{-3}, \ \tau_2^S = \tau_3^S \sim \alpha^{-1} - 3\alpha^{-2}$$

$$\tau_{1+}^{AS} \sim \frac{\alpha^2}{12} + \frac{\alpha}{9}, \ \tau_{1-}^{AS} \sim \alpha^{-1} - 4\alpha^{-2},$$

$$\tau_2^{AS} = \tau_3^{AS} \sim \alpha^{-1} - \alpha^{-2} \tag{27}$$

○ $J < 0 \ (\alpha \to 0)$

$$\tau_1^S \sim \frac{\alpha}{3} - \frac{\alpha^2}{9}, \ \tau_2^S = \tau_3^S \sim \frac{1}{3} - \frac{\alpha}{9},$$

$$\tau_{1+}^{AS} \sim \frac{1}{4} + \frac{\alpha}{3}, \ \tau_{1-}^{AS} \sim \frac{\alpha}{3} - \frac{4\alpha^2}{9},$$

$$\tau_2^{AS} = \tau_3^{AS} \sim 1 - \alpha \tag{28}$$

$$\alpha = e^{\beta J} \tag{29}$$

Relaxation times $\tau's$ behave as in fig.1. The characters of evolution on continuous time are summerized followingly.

- Only the mode with the longest relaxation time τ_{1+}^{AS} diverges at $T \to 0$ for the ferromagnetic case. Because the mode relaxes total magnetization M, relaxation time of M diverges at $T \to 0$ reflecting symmetry breaking of the ground states.
- Relaxation time for any mode does not diverge at $T \to 0$ for the antiferromagnetic case reflecting the relaxation between the ground states with a finite relaxation time due to the zero energy barrior between the ground states.
- No oscillation modes exist as shown in the general theory in section 4.

Using the results as those of a standard dynamics, characters of the dynamics on disrete time are analyzed in the succeeding parts.

(b) Heat bath method

The operator of evolution P is constructed by applying the general formula (21) in section 4. The series $j(m)$ in the relation (23) is taken as $j(m) = m$ regularly to see the effect of uncommutability of elementary processes. The total process P is constructed as the form:

$$P = P_3 P_2 P_1 \tag{30}$$

The operator P has a spin inversion symmetry and no translational symmetry. It follows that each eigenmode is symmetric or antisymmetric under the spin inversion and its eigen-value is not real in general. It is noted that the eigenvalues of the total process P can take complex values even if the elementary processes P_i have only real eigenvalues. It means that some eigenmodes can have an oscillating property. The operator P is easily diagonalized exactly. Relevant properties obtained from the results are listed here. We put the eigenvalues of P in the form $\Lambda_i = \lambda_i^3$ for a technical reason and show the results for $\lambda's$. One stationary mode whose eigen-value $\lambda_0 = 1$ corresponds to the canonical distribution. The results for other modes are given followingly.

$$\lambda_2^S = \lambda_3^S = \lambda_2^{AS} = \lambda_3^{AS} = 0 \tag{31}$$

○ $J > 0$

$$\lambda_{1-}^{AS} < 0 < \lambda_1^S < \lambda_{1+}^{AS}$$

$$|\lambda_1^S| < |\lambda_{1-}^{AS}| < |\lambda_{1+}^{AS}| \tag{32}$$

○ $J < 0$

$$|\lambda_1^S| < |\lambda_{1\pm}^{AS}| \tag{33}$$

• $T \to 0$

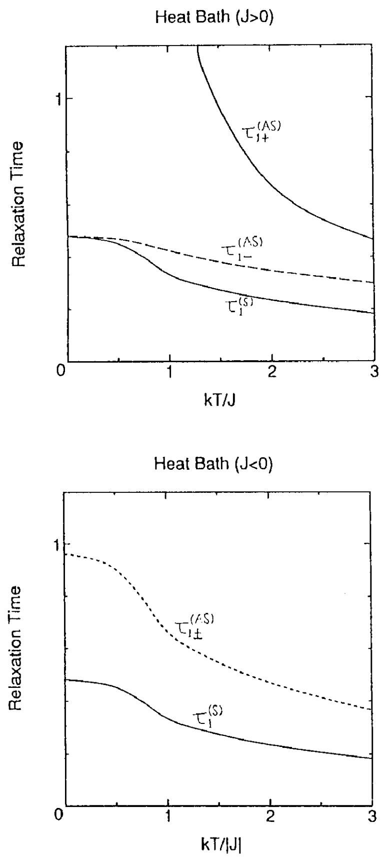

Figure 2: Relaxation times of the dynamics by heat bath method. Continuous, dashed and dotted lines denote the modes which is non-oscillating, oscillating with a period 2 and oscillating with a variable period respectively.

$\circ\ J > 0$

$$\lambda_1^S \to 1/2,\ \lambda_{1+}^{AS} \to 1,\ \lambda_{1-}^{AS} \to -1/2 \quad (34)$$

$\circ\ J < 0$

$$\lambda_1^S \to -1/2,\ \lambda_{1\pm}^{AS} \to \frac{-1 \pm \sqrt{7}i}{4}$$
$$(|\lambda_{1\pm}^{AS}| \to 1/\sqrt{2}) \quad (35)$$

$\lambda_{1\pm}^{AS}$ take complex values for the case $J < 0$ and λ_{1-}^{AS} is a complex conjugate of λ_{1+}^{AS}. Its corresponding mode oscillates with a variable period. λ_{1-}^{AS} takes a negative value for the case $J > 0$. The osccilations reflect the time lag of feed back in a physical sense. The behavior of relaxation times is shown in fig.2.

The characters of dynamics of heat bath method are summerized followingly.

- There are four modes whose eigen-value $\lambda = 0$. (Total number of modes is eight.) Each elementary process P_i has two eigen-values $\lambda = 1$ (stationary mode) and $\lambda = 0$. It implies that each elementary process P_i relaxes the local spin state to equibrium one perfectly after once operation P_i. The local zero modes of P_i survive in the zero modes of the total process P.

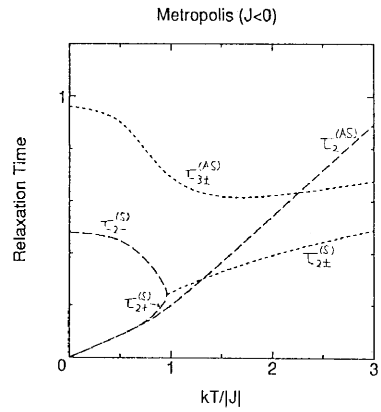

Figure 3: Relaxation times of the dynamics by Metropolis method. Notations are in common with those in fig.2.

- The mode with the longest relaxation time diverges at $T \to 0$ for the case $J > 0$ and no divergent modes exist for the case $J < 0$. The reason is in common with that for dynamics on continuous time. It is interesting that the symmetry (antisymmetric under the spin inversion) and the degeneracy of the modes with the longest relaxation time are common to two dynamics both for the cases $J > 0$ and $J < 0$.

(c) Metropolis method

In the present part, the dynamics P (eqn. 23) constructed from the elementary processes (22) is analyzed. Only the antiferromagnetic case $J < 0$ is discussed here because there is no unified form of the elementary process (22) available both for the cases $J > 0$ and $J < 0$. The operator P has a spin inversion symmetry and no translational symmetries as in the case of heat bath method. There is one stationary mode (canonical distribution) whose eigen-value $\lambda_0 = 1$. The relevant properties of other eigen-modes are listed followingly. (Notations are in common with those in the part (b).)

$$\lambda_1^S = \lambda_1^{AS} = 0 \quad (36)$$

$$\lambda_{3\pm}^{AS} = \begin{cases} \text{real negative value} & (\alpha \leq 1/8) \\ \text{complex conjugate pair} & (\alpha > 1/8) \end{cases} \quad (37)$$

$$\alpha = e^{-2\beta|J|}$$

$\bullet T \to 0 \quad (\alpha \to 0)$

$$\lambda_{2+}^S \sim -\alpha,\ \lambda_{2-}^S \sim -1/2 + \alpha$$

$$\lambda_2^{AS} \sim -\alpha,\ \lambda_{3\pm}^{AS} \to \frac{-1 \pm \sqrt{7}i}{4}$$

$$(|\lambda_{3\pm}^{AS}| \to 1/\sqrt{2}) \quad (38)$$

The behavior of relaxation times is shown in fig.3. Obtained results are summerized followingly.

- There are two zero modes ($\lambda = 0$) which have exactly same elements as those for heat bath method.
- There are no divergent modes at $T \to 0$.

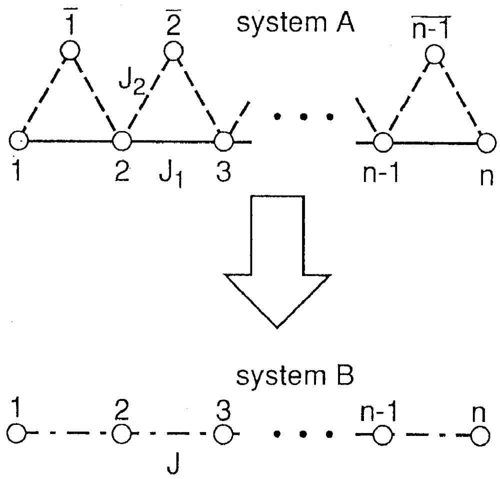

Figure 4: The Ising model on the $\Delta-$ chain and the equivalent Ising chain.

- All relaxation modes have oscillation. Oscillation of the modes whose eigenvalue $\tau^S_{2\pm}$ translate from the constant period 2 to continuously variable period at $\alpha = 1/8$.

6. Percolation properties of thermodynamically equivalent systems

The Ising percolation models have more information than the original Ising models as discussed in section 3. In the present section, it is shown that the connection proberbility between two distant spins in the Ising percolation model takes distinct values even for thermodynamically equivalent Ising systems. It implies that the connection proberbility is not characterized by the equibrium properties of the Ising model. Taking the Ising model (A) on the Δ-chain (fig.4) as a sample, the connection proberbility is analyzed. The Hamiltonian is given as

$$\mathcal{H}_{\mathcal{A}} = -J_1 \sum_i \sigma_i \sigma_{i+1} - J_2 \sum_i (\sigma_i \sigma_{\bar{i}} + \sigma_{i+1} \sigma_{\bar{i}}). \quad (39)$$

By summing up the Boltzmann weight over the states for $\sigma_{\bar{i}}'$s, the Ising chain (B) with a renormalized coupling constant J is obtained. Two spin systems (A) and (B) are thermodynamically equivalent and the following correlation equality holds.

$$\langle \sigma_1 \sigma_n \rangle_A = \langle \sigma_1 \sigma_n \rangle_B \quad (40)$$

The system (A) is frustrated for the case $J_1 < 0$. Two kinds of percolation proberbility P_+ and P_- (eqn. 10) of frustrated systems have non zero value from the general results in section 3. Then using the relation (9), the following relations are obtained.

$$\begin{aligned} P^A = P^A_+ + P^A_- > |P^A_+ - P^A_-| &= |\langle \sigma_1 \sigma_n \rangle_A| \quad (41) \\ &= |\langle \sigma_1 \sigma_n \rangle_B| = P^B \end{aligned}$$

P^X is a total probability to percolate in the system X. The inequality (42) implies that the quantity P^X takes distinct value for two thermodynamically equivalent systems. To take a subtrace is interpreted physically as the operation to relax the substate much more rapidly than the rest. The fact that the procedure for subtrace lead to erroneous results suggests the quantity P^X has some informations for the dynamical properties.

7. Summary and future problems

The properties of three kinds of dynamics have been analyzed on the simplest system only with three spins in the present article. Two kinds of dynamics on discrete time have the properties similar to those in the case on continuous time concerning the mode with the longest relaxation time. The process by heat bath method have more zero modes than that by Metropolis method. The fact does not mean that dynamics by heat bath method converges more rapidly than that by Metropolis method because relaxation depends on the slowest mode. No prominent differences are found in the two dynamics. In fact, each dynamics can be deformed to another one continuously via the proper intermediate dynamics.

Relevant properties of dynamics of frustrated systems seem to be missed in observing only the relaxation of a small number of quantities with specific symmetries because such systems have no easy axis. Damage spreading is a method extracting the dynamical properties of topologically complex modes by comparing evolution from two initial conditions. The picture is interpreted as observing a dynamics on two replicas with dynamical interlayer interactions. It is a future problem to study dynamics on two replicas and make clear the relation between the property "damage spreading" and the percolation property.

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