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Localization and Quantum Hall Effect in Two-Dimensional Systems Under Strong Magnetic Fields*

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Synopsis

Experimental researches of quantum transport properties of semiconductor two-dimensional electron systems in Si-MOSFETs and GaAs/AlGaAs heterostructures in high magnetic fields up to 27 T and at low temperatures down to 20 mK are performed. Analysis of the Hall conductivity of Si-MOSFETs based on a mobility edge model shows that the temperature dependence of the mobility edge can not be explained by existing theory of localization. The fractional quantum Hall effect is observed at the filling factor of 1/7 in heterostructures. Sample size dependence and magnetic field dependence of the breakdown of the integral quantum Hall effect in heterostructures reveal that the Hall current is carried not by the edge states but by the extended states in the localization in the bulk of the two-dimensional systems.

I. Introduction

In two-dimensional(2D) electron systems of n-channel MOS inversion layers on Si (001) surfaces or GaAs inversion layers in GaAs/AlGaAs heterostructure interfaces in sufficiently strong magnetic fields and at sufficiently low temperatures, it is possible to realize an extreme-quantum-limit condition ($k_B T<\Gamma<\hbar\omega_c$), Γ is the broadening of a Landau level and ω_c is the cyclotron frequency.

In such a system , there exist gap regions in the density of states between the boundaries of each Landau level. When random potentials are incorporated in the system, localized states are expected to exist near lower and higher edges of each Landau level. If the range of the random potential δ is much longer than the radius of Landau orbit $\ell_N=\ell_0(2N+1)^{1/2}$, N is the Landau index, it is easy to see that centers of Landau orbits near the bottoms and the tops of the random potentials move along closed trajectories lying in equipotential lines. When the range of the random potentials is short ($\delta < \ell_N$), they also produce localized states near the lower and higher edges. Extended states which exist near the center of each Landau level are expected to show a different behavior from those in absence of magnetic fields.[1)]

A classical expression of the Hall conductivity in a 2D system is given by

$$\sigma_{xy} = - N_s e/B + \sigma_{xx}/\omega_c \tau \qquad (1)$$

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where N_s is the total electron concentration, σ_{xx} the diagonal conductivity and τ electron elastic scattering time. It is easy to see that when an integral number, i, of Landau levels are completely full and the higher Landau level is empty as $N_s = ieB/h$, we have $\sigma_{xx} = 0$ because τ becomes infinite in the extreme-quantum-limit condition. Then we have $\sigma_{xy} = -ie^2/h$. Ando et al.[2] derived theoretically that the relation $\sigma_{xy} = -ie^2/h$ holds when σ_{xx} is zero due to the localization even if $N_s \neq ieB/h$. This is the integral quantum Hall effect(IQHE). IQHE is also expressed by using resistivity tensors as $\rho_{xy} = h/ie^2$ when $\rho_{xx} = 0$.

If the strength of the random potentials is weaker than the mutual Coulomb interactions of electrons, a strong magnetic field enhances correlated states of electrons. Such correlated states show the fractional quantum Hall effect(FQHE); i.e., $\rho_{xy} = -e^2/\nu h$, ν being p/q where p is an integer and q is an odd integer, near the fractional filling of Landau levels $\nu = N_s h/eB$. Recently the FQHE with even denominater has been reported. The correlated electron state which exhibits the fractional quantum Hall effect is believed to be a quantum liquid state. When the filling factor ν decreases, a transition into the Wigner solid state or the charge density wave state is expected to occur.

The present paper reviews experimental researches carried out by using facilities of high magnetic fields and low temperatures in IMR. In the next section the localization in Landau levels in Si-MOSFETs will be described. The FQHE at low electron concentration will be described in the third section, and the breakdown of IQHE will be described in the final section.

II. Localization in Landau Levels of Si-MOSFETs

The localization plays the most important role in IQHE.[3] Our method of analysis of the localization is as follows.

Kawaji et al.[4] first tried to analyze temperature dependence of the Hall conductivity σ_{xy} by an effective mobility edge model. They assumed first that the second term is much smaller than the first term in eq.(1). Their second assumption is that the Hall conductivity in a Landau level with the filling factor ν is given by

$$\sigma_{xy}(\nu,T) = -(e^2/h)n_M(\nu,T)/N_M \tag{2}$$

where N_M is the total number of delocalized states and n_M the number of electrons in the delocalized states.

In addition to the two assumptions above, if one assumes that the density of states and the mobility edge E_c are symmetric with respect to the center of the Landau level and that the broadening Γ does not depend on ν, it is easy to analyze temperature dependence of E_c/Γ .

The temperature dependence of the mobility edge arises from the temperature dependence of the localization length ξ due to the inelastic scattering. At zero temperature, the localization length diverges at the center of the Landau level. At a finite temperature T , however, inelastic scattering cut off the maximum localization length and delocalize the electrons with localization length longer than the inelastic scattering length. This produces localized states at around the center of the Landau level with a finite energy width. Then we can define a mobility edge where the localization length is equal to the inelastic scattering length, L_ε , which is given by $L_\varepsilon \sim (\tau_\varepsilon/\tau)^{1/2}\ell_0$, where τ_ε is the inelastic scattering time and τ is the elastic scattering time.

Theoretical studies by many authors[5-12] have shown that the energy dependence of the localization length, $\xi(E)$, shows a critical behavior of

$$\xi(E) \propto |E|^{-s} \tag{3}$$

where the energy is measured from the center of the Landau level.

The temperature dependence of the inelastic scattering time so far obtained in 2D systems in the absence of magnetic field has shown the relation $\tau_\varepsilon \propto T^{-p}$. Although the temperature dependence of τ_ε in strong magnetic fields is not yet understood, if we use this relation, the mobility edge is expected to show the temperature dependence like

$$E_c/\Gamma \propto T^{\kappa} \tag{4}$$

where $\kappa = p/2s$.

In the mobility edge model, E_c/Γ is related to the Hall conductivity like,

$$E_c/\Gamma \propto (-\,d\sigma_{xy}/dN_s)_{\max}^{-1}, \tag{5}$$

under the condition $k_BT << E_c$. Then we can determine the exponent κ from the measurements of temperature dependence of $-\,d\sigma_{xy}/dN_s$ or $-\,d\sigma_{xy}/dB$. Thus we can estimate the critical exponent s provided that the the exponent p of the inelastic scattering time is given.

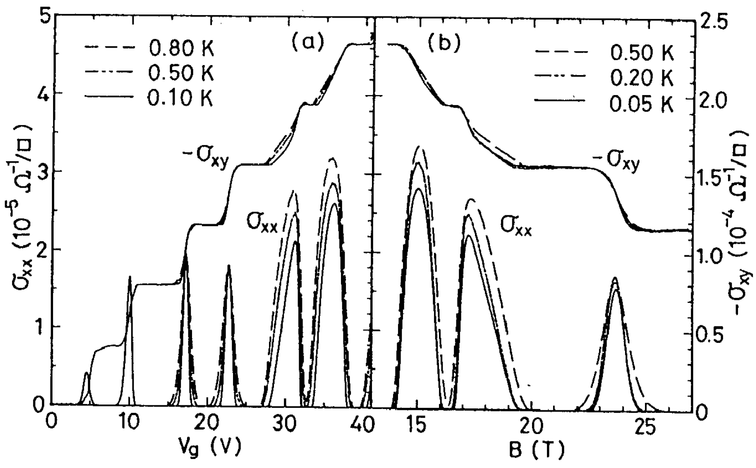

Fig.1. Diagonal conductivity σ_{xx} and Hall conductivity $-\sigma_{xy}$ as a function of gate voltage, V_G , at 25 T (a), and as afunction of magnetic fiueld, B, at V_G =21.4 V, at three temperatures, respectively.[13]

The Hall conductivity and the diagonal conductivity of Si-MOSFETs were measured in magnetic fields up to 27 T and at tempertures down to 50 mK.[13] Figures 1(a) and 1(b) show typical traces of V_G dependence in a magnetic field of 25 T and B dependence at V_G = 21.4 V of σ_{xx} and σ_{xy} at three temperatures. Figure 1(a) shows six σ_{xx} peaks associated with the (0↑+), (0↑-), (0↓+), (0↓-), (1↑+), (1↑-), (1↓+), (1↓ -) Landau level from left to right and σ_{xy} plateaus between them. Temperature dependences of E_c/Γ in the (0↓-) and the (1↑+) Landau level are described as $E_c/\Gamma \propto T^{\kappa}$ at temperatures between 0.2 and 0.8 K; i.e., κ=0.29±0.10 for the (0↓-) level and κ=0.16±0.20 for the (1↑-) level. Values of exponent κ's are close to the values obtained in the measurements in the magnetic field of 15 T.[14]

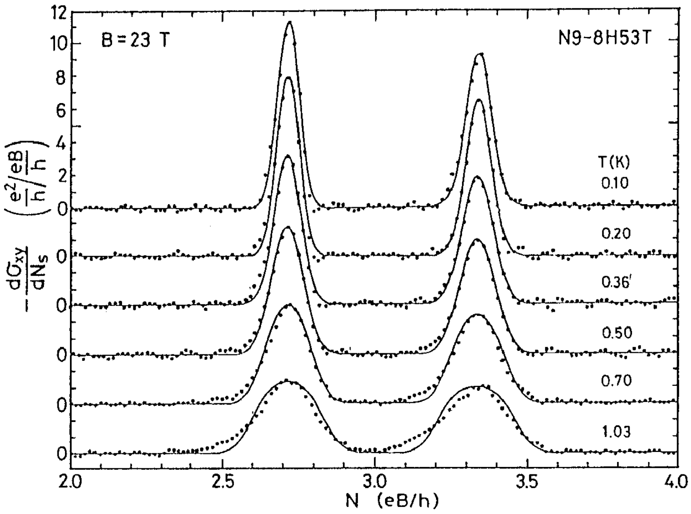

Fig.2. Temperature dependence of - $d\sigma_{xy}/dN_s$ versus N_s of (0↓+) and (0↓-) level of low mobility sample(N9-8H53) measured in a magnetic field of 23 T.[15]

Measurements of Si-MOSFETs were further carried out at temperatures between 50 mK and 1.45 K and in magnetic fields between 14 and 23 T using samples having two grades of electron mobility: μ(peak at 1.5 K) = 1.4 $m^2/V \cdot s$ (92-17H53T) and 0.38 $m^2/V \cdot s$ (N9-8H53T).[15] As an example, Fig. 2 shows temperature dependence of - $d\sigma_{xy}/dN_s$ versus N_s of (0↓+) and (0↓-) level of low mobility sample(N9-8H53) measured in a magnetic field of 23 T. In Fig. 2, solid lines are results of a model calculation using Γ = 3.59 meV (41.6 K) for (0↓+) level and Γ = 3.57 meV (41.4 K) for (0↓-) level for the Gaussian like density of state function which takes into account a smearing of the mobility edge. Figure 3 shows temperature dependences of E_c/Γ of (0↓-) and (0↓+) Landau level measured for two samples. As the temperature decreases, E_c/Γ decreases like T^{κ} with κ =1.1±0.2 near 1 K and then saturates below 300 mK for both high and low mobility samples. We carried out the source-drain voltage dependence of the temperature dependence, and we conclude that the saturation observed at low temperature is not the Joule heating effect.

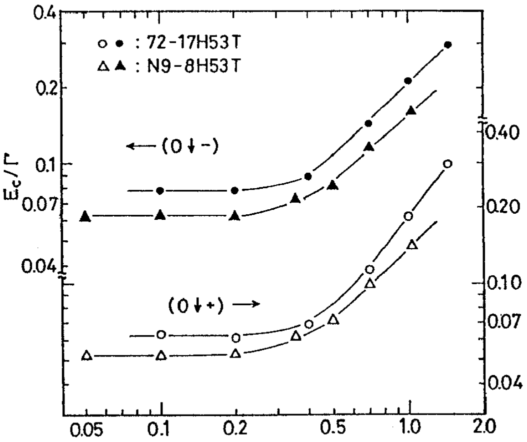

Fig.3. Temperature dependence of E_c/Γ of (0↓-) and (0↓+) level measured for two samples.[15]

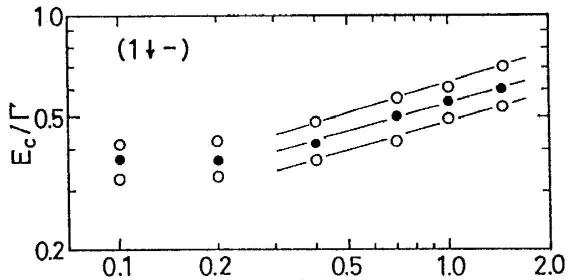

Fig. 4. Temperature dependence of E_c/Γ of (1↓-) level.[15]

It should be noted here that the temperature dependence of $(-\,d\sigma_{xy}/dN_s)_{\max}^{-1}$ at high temperatures of the present experiments give slightly weak temperature dependence like T^{κ} with $\kappa = 0.79 \sim 0.95$. In these temperature range, the condition E_c equals to about $4k_BT$ for eq.(5) is not fulfilled.

Temperature dependence of E_c/Γ of (1↓-) level shows that the exponent κ lies between 0.28 and 0.3 at high temperatures. The temperature dependence saturates below 1.2 K.

The saturation of the temperature dependence of E_c/Γ observed at low temperatures suggests the existence of the finite width of the delocalized states arising from the potential fluctuation in a macroscopic scale.

The present experiments show that the temperature exponent κ of the mobility edge E_c depends on the Landau quantum number. According to the theoretical results by Aoki and Ando[6], the exponent s in the critical localization for $N = 0$ and $N = 1$ Landau level is 2 and 4. When we use these values for s and common values for p, the expected ratio $\kappa(N = 0)/\kappa(N = 1) = 2$. However, the ratio $\kappa(N = 0)/\kappa(N = 1) \approx 3.7$ obtained in our experiments is about 2 times as large as the theoretical prediction. Theoretical studies on the inelastic scattering in strong magnetic fields are necessary to understand these results.

III. Fractional Quantum Hall Effect at $\nu = 1/7$ of GaAs/AlGaAs Heterostructure

Until 1988, the lowest filling where FQHE was observed was $\nu = 1/5$. We carried out magneto-transport experiments on a high mobility GaAs/AlGaAs heterostructure in magnetic fields up to 27 T and at temperatures down to 20 mK and observed a structure in ρ_{xy} which shows the existence of FQHE at $\nu = 1/7$.[16]

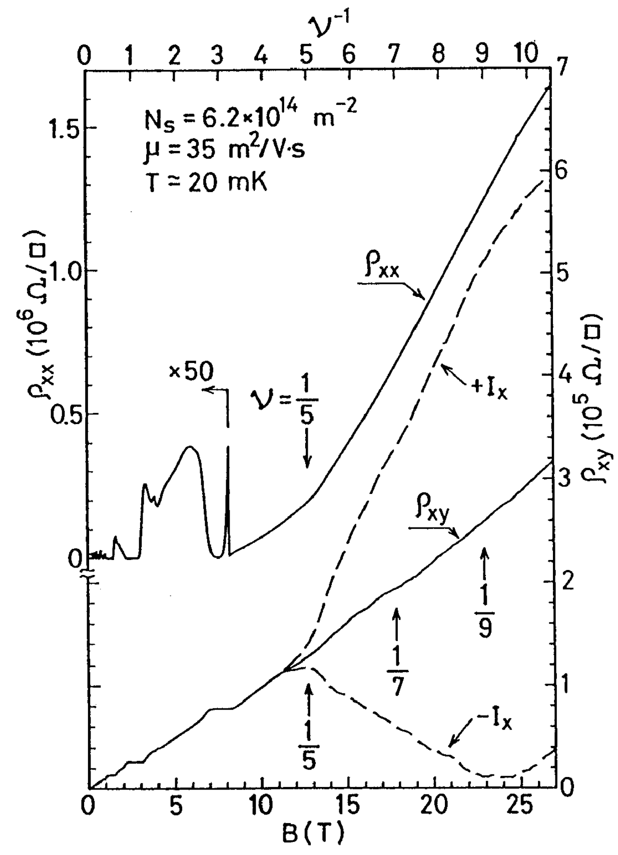

Fig. 5. Magnetic field dependence of diagonal resistivity ρ_{xx} and Hall resistivity at zero gate bias. Broken lines show the anomalous Hall voltages at positive and negative polarities of the sample current.[16]

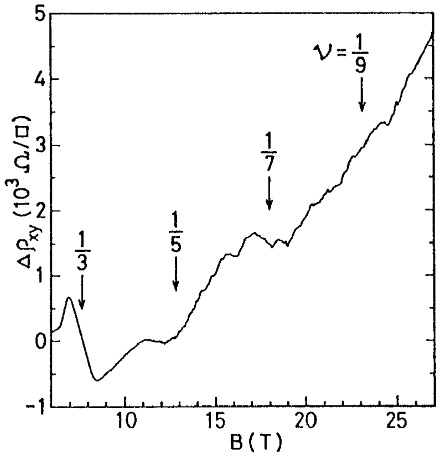

Fig. 6. Deviation of ρ_{xy} from the classical Hall resistivity $B/N_s e$, $\Delta\rho_{xy}$, versus magnetic field B of data in Fig. 5.[16]

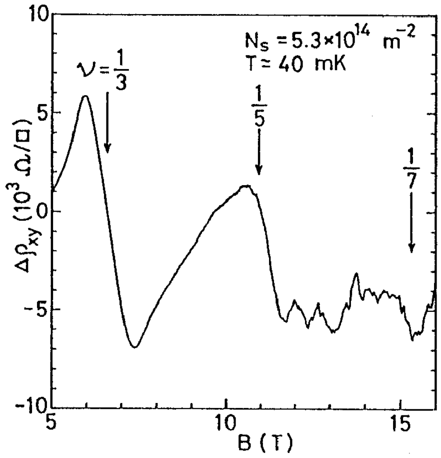

Fig. 7. Deviation of ρ_{xy} from the classical Hall resistivity $B/N_s e$, $\Delta\rho_{xy}$, versus magnetic field B at reduced electron concentration.[16]

The sample used had an electron concentration N_s of 6.2×10^{14} m^{-2} and electron mobility μ of 35 m^2/V·s at zero gate bias. The electron concentration was decreased to 5.3×10^{14} m^{-2} and was increased to 8.7×10^{14} m^{-2} when a bias voltage of -30 V and +82 V was applied to the backside gate, respectively. The electron mobility at these electron concentrations were 24 m^2/V·s and 60 m^2/V·s, respectively.

Figure 1 shows the magnetic field dependence of the diagonal resistivity ρ_{xx} and the Hall resistivity ρ_{xy} at a temperature T ≅ 20 mK with zero gate bias. A shallow minimum in ρ_{xx} is observed at $\nu=1/5$. However, ρ_{xx} increased rapidly and no appreciable structure is observed at $\nu<1/5$. On the other hand, weak structures are observed in ρ_{xy} at $\nu<1/5$ as well as at $\nu=1/5$. In order to observe the structures in ρ_{xy} clearly, deviation of ρ_{xy} from the classical Hall resistivity line $B/N_s e$ against the magnetic field B is plotted in Fig.2. A structure corresponding to $\nu=1/7$ is observed at $B\sim$ 18 T as well as the structure of $\nu=1/5$ at $B\sim$ 13 T.

When the electron concentration was reduced to 5.3 x 10^{14} m^{-2} by applying a bias voltage of -30 V to the backside gate, a structure in ρ_{xy} at $\nu=1/7$ was also observed at about 15.5 T at a temperature T≅ 40 mK. This is shown in Fig. 7 as the deviation of ρ_{xy} from the classical Hall resistivity line $B/N_s e$ against the magnetic field.

These results shown in Fugs. 6 and 7 demonstrate that a quantum liquid state exist at $\nu=1/7$.

IV. Breakdown of the Quantum Hall Effect in GaAs/AlGaAs Heterostructures

Most important feature of IQHE is that the quantized Hall resistance is independent of structure of samples, measurement current or magnetic field and so on. The IQHE is explained by a picture of localization of electrons in Landau levels where extended as well as localized states exist in each Landau level since the first theoretical work by Ando et al..[2] Recently, a different explanation of the IQHE based on an edge channel picture has been proposed.[17] Several experimental works appear to support the edge channel picture.[18,19,20]

Essential points in both pictures are following: In the localization picture, the quantized Hall current depends on the Hall electric field. On the other hand, the quantized Hall current depends not on the Hall electric field but on the Hall voltage in the edge channel picture.

Quite recently, Kawaji, Hirakawa and Nagata[21] observed that critical current of breakdown of the IQHE is proportional to the width of samples. This result clearly favours the localization picture in the IQHE. However, the mechanism of the breakdown is not yet clear. We carried out experiments on magnetic field dependence of the critical breakdown current which is proportional to the sample width.[22]

The samples used have three different widths: $w=10$, 20 and 35 μm. We note here that the width w is the width of the mask pattern in photolithography. The magnetic field dependence of the breakdown current in these samples were measured in magnetic firlds up to 22 T at temperatures down to 0.5 K. The width in the magnetic field ΔB for the quantized plateau region with $i = 1$, 2 and 4 in $R_H(i) = h/ie^2$ was obtained from the diagonal resistance $R_{xx}(i)$ vs magnetic field B trace. The critical current I_{cr} was determined from the current at which the plateau disappeared.

Sample width dependences of the critical current measured in three samples are shown in Fig. 8. Figure 8 shows that the critical currents in each magnetic field for $i = 1, 2$ or 4 against the sample width lie on a straight line. Extrapolations of three lines do not pass through the origin. The three lines intersect the abcissa approximately at 6 μm. Actual width of samples is reduced from the size of the photolithographic mask pattern by chemical etching. Effective width of the samples in which electrons flow is reduced further by formation of depletion layers on both edges of the samples. Therefore, we take the effective width of the sample $w' = w - 6$ μm. Secondly, Fig. 8 shows that the critical current is not in regular order against the magnetic field in each sample.

If the critical current does not depend on the quantum number of the Hall plateau, i, as reported by Stormer et al.[23], the electron drift velocity determined by the magnetic field and the average critical electric field does not depend on the quantum number i. The constant electron drift velocity at the breakdown favoiur a mechanism of breakdown where phonon emission like Cherenkov radiation causes the breakdown. Our result does not support this mechanism.

We calculate the critical field, F_{cr}(th), based on the quasi-elastic inter-Landau level scattering proposed by Eaves and Sheard[24] and compared with experimental results shown as F_{cr}'(exp) $/F_{cr}$(th) in Fig. 9. Here, the experimental breakdown field is calculated

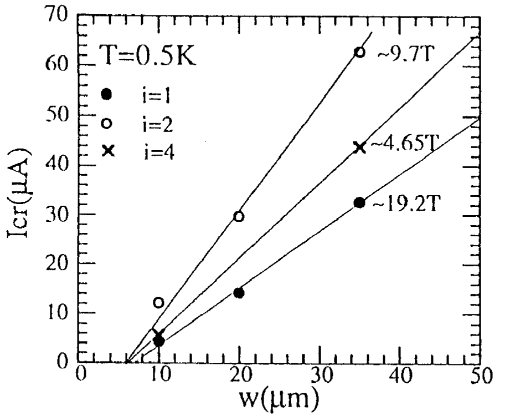

Fig. 8. Sample width dependence of breakdown critical current in three magnetic fields.[22]

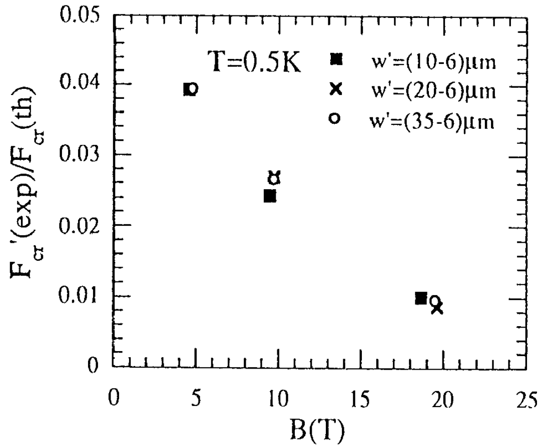

Fig.9. Magnetic field dependence of critical electric field normalized by theoretical breakdown field.[22]

using the effective width of the samples w'. Figure 9 shows that $F_{cr}'(\exp)/F_{cr}(\text{th})$ in different samples are in agreement with each other and they are in order against the magnetic field. The result that the experimental critical field is much smaller than calculated one can be explained by enhancement of the electic field in extended states in localization. Another possible explanation is that the breakdown occurs near the sample edges where the field is stronger than the average field as observed by Fontein et al..[25)] If the latter is the case, the ratio of the average field to the field near the edge depends strongly on the sample width. However, the experimental results do not depend on the sample width as shown in Fig. 9.

In conclusion magnetic field dependence of the critical breakdown current density can be explained by the bulk Hall current picture where the enhancement of the field in extended states in localization is stronger in higher magnetic field.

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