On the Relationship between Bond's Work Index and Mechanical Properties of Brittle Materials

著者	Yashima Saburo, Awano Osamu, Saito Fumio
journal or publication title	Science reports of the Research Institutes, Tohoku University. Ser. A, Physics, chemistry and metallurgy
volume	30
page range	138-152
year	1981
URL	http://hdl.handle.net/10097/28208

On the Relationship between Bond's Work Index and Mechanical Properties of Brittle Materials.*

Saburo Yashima, Osamu Awano** and Fumio Saito
The Research Institute of Mineral Dressing and Metallurgy
(Received May 11, 1981)

Synopsis

By a dimensional analysis method, we have obtained the following relationship between the Bond's work index and the mechanical properties of brittle materials.

$$Wi \cdot \rho = 0.623\ Y_1^{0.35}\ Sc^{0.15}\ St^{0.50}\ (1-\nu_1^{2})^{0.20}\ Rc^{-0.09}\ Rt^{-0.48}$$

Where Wi [Kg-cm/kg] is the work index, ρ [kg/cm^3] is the density, Sc [Kg/cm^2] is the compressive strength, St [Kg/cm^2] is the tensile strength, Y_1 [Kg/cm^2] is the Young's modulus, ν_1 [−] is the Poisson's ratio, Rc [−] is the ratio of specific surface area of fractured product to the rod specimen and Rt [−] is the ratio of specific surface area to the spherical specimen, both under slow rate of compressive loading.

Dividing Wi calculated from the above equation by 4.05×10^4 Kg-cm/kg, we can obtain Wi having the unit of [kWh/ton]. The above relationship is applicable to materials having the Moh's hardness H=2.0 - 6.5.

I. Introduction

Indeces with which the crushing resistance or the grindability in a wide meaning is expressed include the Rittinger number, crushing resistance, Hardgrove index and Bond's work index. One of them, the Bond's work index which was reported in the third theory of comminution by F.C.Bond[1] has been regarded to be significant in practical

---

* The 349th report of the Research Institute of Mineral Dressing and Metallurgy. Reported in Japanese in Kagaku Kogaku, 34 (1970),1199.
** Takasago Thermal Engineering Company Ltd.

problems because it means a relative crushing resistance, it can be used as an indication of power estimation of crushing machines and be useful for the process control in crushing operation.[2,3]

The Bond's work index is regarded as one of the crushing resistance as mentioned above, it can therefore be presumed that the work index is correlated to the mechanical properties of brittle materials. Then, the satisfactory result was obtained from the dimensional analysis in the relationship between the Bond's work index and the mechanical properties of brittle materials. Besides, the contents of the Bond's work index were investigated from the other facet by T.Tanaka.[4] If the results obtained from various facets are summarized, it can be thought that the whole aspect on the work index will be made apparent.

II. Mechanical properties concerned the fracture of solids

The mechanical properties affecting the fracture of brittle solid particles were investigated prior to the measurements of the work index, the mechanical properties of brittle solids and the dimentional analysis.

II-1. Young's modulus and Poisson's ratio

The Young's modulus is a measure of structure insensitive properties and it makes a contribution to the elastic strain energy in fracture phenomena. Since part of the strain energy stored in a specimen up to a fracture is converted into the new surface energy of fractured product, the Young's modulus also affects substantially the size of fractured product. Furthermore, it is associated with the rate of crack propagation and the propagation of the impact wave.

The Poisson's ratio shown in Table 1 tended to decrease with the increase of the Young's modulus, but the correlativity of both properties was not necessarily constant.[a] Also, the Poisson's ratio was not related to the compressive strength, but it was related theoretically to both the elastic strain energy and the Young's modulus as given in Eqs.(9) and (11).

II-2. Tensile strength and compressive strength

---

a) The relationship between the Young's modulus Y_1, the shearing modulus G and the Poisson's ratio ν_1 is given by Eq.(a).

$$Y_1 = 2G(1+\nu_1) \qquad \text{(a)}$$

Therefore, if G is constant, the Young's modulus is proportional to the Poisson's ratio. But, because the shearing modulus varies with the variation of the Young's modulus, the Young's modulus may not always be proportional to the Poisson's ratio.

One of the characteristics of brittle solids is the fact that they have a strong resistant against the compressive force, on the contrary they have a weak resistant against the tensile force. The tensile strength is a measure of structure sensitive properties and it makes a remarkable contribution to fracture phenomena. The compressive strength is a measure of structure sensitive properties, but brittle solids are not fractured only by the compressive stress itself, they are fractured by the tensile stress generated at the direction of right angle to the acting direction of compressive load, the shear stress generated by the combined force of tension and compression. The mechanism of compressive fracture is very complicated and it is difficult to practice any theoretical analysis, therefore, the compressive strength is defined by the compressive stress acting on the specimen up to a fracture point.

II-3. Loading rate and status of fracture

The Young's modulus and strength of specimen was in a tendency to increase with the increasing of strain rate,[5)] that is, loading rate. The function formula in both of the relationships was different according to material of samples and it was necessary to solve these many unknowns that are remained at the present stage. In this study, the relationship between the Bond's work index and the mechanical properties of brittle materials under slow rate of loading, a most fundamental condition, was investigated. The status of fracture was varied according to material of samples so that it might be based on the combined force of material. Also, the status of fracture was changed by differentiating the loading rate. However, in this study, these relationships were expressed as the ratio of specific surface area of fractured product to specimen used in the measurements of the compressive strength and the tensile strength as described hereinafter. The difinition and measurement method of the ratio of specific surface area will be described later.

III. Measurement of the work index and the mechanical properties

Several different kinds of materials considered to be brittle were chosen for this experiment. These were seven kinds of brittle solids, that is, quartz glass, borosilicate glass, quartz, feldspar, limestone, marble and gypsum. Two kinds of glassy specimens were sufficiently annealed. The makers of glassy specimens, the places of production and the purities of natural minerals were the same as that in the previous paper.[6)]

The work index was measured by setting the size apertures of test sieve for size classification to 208 μm (65 mesh in Tyler standard sieves) by the standard ball mill method proposed by F.C.Bond.[7]

The compressive strength, the Young's modulus and the Poisson's ratio were measured by applying the Y.Shimomura's method.[8] That is, these properties measured by using a rod specimen of 2.0 cm in diameter having ratio of diameter "d" to length "l" of d/l=1/2 giving the most stable measurements. The Young's modulus and the Poisson's ratio of the specimens, which were measured by using a cross wire strain gage mounted on the central portion of the loading direction to the specimen, were read from the load-deformation curves obtained by a load cell and a dynamic strain meter. The compressive tester used for the experiment was a universal testing machine having 30 tons capacity.

The compressive strength Sc is given by

$$Sc = P/A \qquad (1)$$

Where P is the fracture load and A is the cross-sectional area of specimen at a right angle to the loading direction. The Young's modulus was expressed as the secant modulus of elasticity by applying the method used in the field of rock mechanics and measured from the load-deformation curve in the compression test.[b] The Poisson's ratio was obtained from the results of longitudinal and lateral strains recorded in a similar manner as above. The sphere compressive strength (tensile strength) St measured by using a sphere specimen having diameter of 2.0 cm as illustrated in Fig.1 is obtained from the following equations proposed by Y.Hiramatsu et al.[9][c]

$$St = 0.7\ P/\pi r^2 \qquad (2)$$

Where r is one half of the distance between two loading points. The brittleness index Br of samples obtained from Eqs.(1) and (2) is given by Eq.(3).

$$Br = Sc/St \qquad (3)$$

Number of specimens were 40 for the rod specimen and 20 for the spherical specimen.[d] A pair of loading platens mounted on the aforementioned testing machine was made of tungsten carbide, and its surface roughness is 1/100 mm. The Young's modulus and the Poisson's ratio of the tungsten carbide discs were $6.05\text{x}10^6$ Kg/cm^2, 0.21, respectively. The density of samples was calculated from masses and volumes of several specimens shaped to a fairly high precision by hand. The size analysis of fractured products of rod and sphere specimens in compressive fracture test was also carried out carefully. The method

---

b) The secant modulus of elasticity was measured from the stress correspond to the origin and 40 % strength on the stress-strain coordinate. Similarly, the Poisson's ratio was obtained from the two strains at the same stress as mentioned above.

of preparation, precision of finishing, methods of storage of specimens, experimental apparatus, methods of size analysis and its expression were omitted in this paper since these have been reported in the previous paper.[6] The loading rate for the spherical specimen was set at 3 ton/min for the glassy materials and 0.3 ton/min for the minerals, whereas, the loading rate for the rod specimen were 15 ton/min for the glassy materials and 0.75 to 7.5 ton/min for the minerals, so as to hold the same time as the time up to a fracture of the spherical specimen.[e] As the same time, it was necessary to obtain the specific surface area of fractured products in the measurement of the aforementioned Sc and St. From this purpose, a differential transformers were always attached to record the load-deformation curve of specimen with a X-Y recorder beside the measurement of the strain obtained by the strain gage mounted on the rod specimen. Immediately after the specimen is fractured, the load was released rapidly to protect the specimen from overgrinding. The experimental results are tabulated in Table 1. The ratios of specific surface areas of fractured products

---

c) This method was proposed by Y.Hiramatsu et al.[9] as the tensile strength measurement method of brittle materials such as rocks that it can be applied in the case of compressive fracture of irregular particles compressed in the contact state of two points. At the same time, they showed that the strength obtained by this method was in a good agreement with the result of the Brazilian test. In this method, St in the previous paper[6] was named as the sphere compressive strength because the compressive force was generated in part of the sphere specimen, but St can be named as the tensile strength.

d) As the size distribution of fractured products of rod specimen was different from that of rod specimen mounting the strain gages, a 20 extra number of rod specimens without the strain gage was prepared in order to investigate the size distribution.

e) In the crushing test of the rod specimens, the surface roughness of the two surfaces of rod specimens was finished within $\pm 20\mu m$. The upper and lower surfaces of rod specimen was polished so that the upper surface is parallel to the lower surface. And a suspension of MoS_2 was spread on the upper and lower surfaces of a rod specimen in order to minimize the friction between the two platens. By this procedure, it is often possible to prevent the fracturing of ends of both surfaces during the course of loading process.

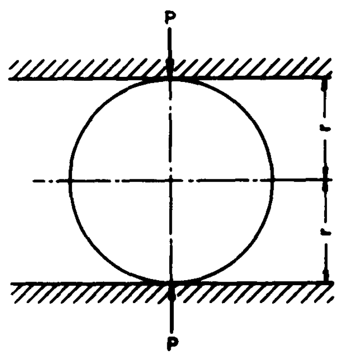

Fig.1 Compression of sphere

to rod and sphere specimens, Rc and Rt respectively, are presented in Table 1. These are required values for the subsequent dimentional analysis.[f)] The specific surface areas of original specimens were calculated from the geometrical sizes and the masses of specimens. The specific surface areas of fractured products having regular shapes such as in the marble rod specimens, the sphere specimens made of limestone, marble and gypsum were calculated geometrically from the sizes of a fragments and the masses of specimens. On the other hand, the specific surface areas of fractured products having irregular shapes and fine sizes with a wide size distribution as determined by a test sieving were obtained from the usual method, and calculated by assuming the specific surface shape factor of fractured fragments is to be 6.0.[g)]

Table 1 Properties of samples

Kinds of samples	H [–]	ρ [g/cm³]	Sc [Kg/cm²]	St [Kg/cm²]	Y_1 [Kg/cm²]	ν_1 [–]	Br [–]	Wi [kWh/ton]	Rc [–]	Rt [–]
Quartz glass	6.5	2.20	4300	240	7.50×10^5	0.16	17.9	14.8	31.8	5.62
Borosilicate glass	6.5	2.33	2100	460	6.24×10^5	0.21	4.57	15.2	13.4	6.71
Quartz	6.5	2.62	1020	115	8.89×10^5	0.16	8.87	13.3	9.13	1.65
Feldspar	6.0	2.55	1400	85.8	5.99×10^5	0.26	16.3	12.4	7.20	1.50
Limestone	4.0	2.70	630	44.1	6.94×10^5	0.32	14.3	9.4	3.37	1.50
Marble	3.0	2.70	560	31.7	5.45×10^5	0.30	17.7	6.7	1.25	1.50
Gypsum	2.0	2.30	350	28.3	3.86×10^5	0.32	12.4	6.3	2.06	1.50

f) The value of the Young's modulus and the Poisson's ratio of quartz glass, limestone and gypsum in Table 1 were partly different from those reported in the previous paper.[6)] This difference may be due to the difference in periods of purchase of sampling.

g) It is recognized that the shape factor varys with the size of fractured fragments. However, in this study, the shape factor 6.0 is selected for convenience in the calculation.

IV. The dimensional analysis concerned the work index

The dimensional analysis in relation to the work index based on Π theory is advanced as below. As the foregoing factors such as the tensile strength St, the compressive strength Sc, the Young's modulus Y_1, the Poisson's ratio ν_1, the strain rate ζ, the density ρ and the size distribution function of fractured product Φ_D are interrelated to the work index, the work index Wi is given by the following equation.[h)]

$$Wi = f(St, Sc, Y_1, \nu_1, \zeta, \rho, \Phi_D) \qquad (4)$$

It seems that St, Sc, Y_1, ν_1, and Φ_D respectively are the function of the strain rate correspond to the loading rate in the compression test. If the experimental values of strength and other measurements of every kind of samples were measured under the constant strain rate and the slow rate of loading, the influence of the strain rate or the loading rate to the other kinds of mechanical properties of samples was almost negligible. The term of Poisson's ratio ν_1 is omitted from Eq.(4) tentatively. The term of ν_1 will be reconsidered in some detail at the time when the theoretical relationship concerning the elastic strain energy is introduced.[i)]

Consequently, Eq.(4) can be given by Eq.(4')

$$Wi = f(St, Sc, Y_1, \rho, \Phi_D) \qquad (4')$$

The dimensional analysis will be advanced by introducing K for Eq.(4') under the difinition of Eq.(4'').

$$K = Wi^a\ St^b\ Sc^c\ Y_1{}^d\ \rho^e\ \Phi_D{}^f \qquad (4'')$$

By summarizing the relationships among each of the indexes after consolidating the dimensions of each of the terms, we obtain Eq.(5).

$$\left.\begin{aligned} 2a-b-c-d-3e &= 0 \\ b+c+d+e &= 0 \\ a+b+e+d &= 0 \end{aligned}\right\} \qquad (5)$$

Consequently we obtain Eq.(6)

$$a = e = -(b+c+d) \qquad (6)$$

Accordingly, the following equation is derived from Eq.(4'').

---

h) The symbol Φ_d means $\Phi_D = f(Rc, Rt)$ of which details are given in Eq. (14), also it means the status of fracture inherent to the sample, and Φ_D at the present stage is expressed as the size distribution function in general term.

i) Strictly speaking, if the shear strength τ and shearing modulus G are introduced into Eq.(4), the result of a better precision will be obtainable. However, in this study, τ and G were omitted, and only the elastic constant was considered, a consideration concerning ν_1 was introduced in Eq.(11) as will be shown later.

$$K = Wi^{-(b+c+d)}\, St^{b}\, Sc^{c}\, Y_1^{d}\, \rho^{-(b+c+d)}\, \Phi_D^{f} = \left(\frac{St}{Wi\cdot\rho}\right)^{b} \left(\frac{Sc}{Wi\cdot\rho}\right)^{c} \left(\frac{Y_1}{Wi\cdot\rho}\right)^{d} \Phi_D^{f} \qquad (7)$$

The meaning of each terms in Eq.(7) must be investigated in order to determine the four kinds of indexes. The terms of the strength including St and Sc of all the nondimensional terms are related to the elastic strain energy stored in specimen until the specimen is fractured as well as the term of the Young's modulus Y_1. In the compressive fracture test of rod specimen, the elastic strain energy per unit volume of specimen is given theoretically by the following equations.

$$Ec = Sc^2/2Y_1 \qquad (8)$$

$$Ec \propto Sc^2/Y_1 \qquad (8')$$

On the other hand, the theoretical equation of the elastic strain energy per unit volume of sphere specimen Et as applied to the compression of sphere set between two parallel platens, as shown in Fig. 1, reported by S.P.Timoshenko et al.[10] based on the Hertz's theory is given by the following equation.

$$Et = 1.93\ \{(1-\nu_1^2)/Y_1+(1-\nu_2^2)/Y_2\}^{2/3}\ St^{5/3} \qquad (9)$$

Suffixes 1 and 2 in Eq.(9) designate the sample and platens, respectively. As the value of $[(1-\nu_2{}^2)/Y_2]$ is negligible as compared to the value of $[(1-\nu_1{}^2)/Y_1]$, then Eq.(9) is reduced to Eq.(9').

$$Et \propto [(1-\nu_1{}^2)/Y_1]^{2/3}\ St^{5/3} \qquad (9')$$

From Eqs.(8') and (9'), Eq.(7) is expressed by Eq.(10).

$$K = \left(\frac{Y_1}{Wi\cdot\rho}\right)^{\alpha} \left(\frac{Sc}{Wi\cdot\rho}\right)^{\beta} \left(\frac{St}{Wi\cdot\rho}\right)^{\gamma} (1-\nu_1{}^2)^{\delta}\ \Phi_D^{\varepsilon} = Y^{*\alpha}\ Sc^{*\beta}\ St^{*\gamma}\ (1-\nu_1{}^2)^{\delta}\ \Phi_D^{\varepsilon} \qquad (10)$$

where $Y^{*}=(Y_1/Wi\cdot\rho)$; $Sc^{*}=(Sc/Wi\cdot\rho)$; $St^{*}=(St/Wi\cdot\rho)$

It is considered that the terms of Young's modulus Y^*, compressive strength Sc^* and tensile strength St^* substituted for the nondimensional terms are the values to evaluated the grindability (in this study, the condition of the work index measurement). In other words, as the Young's modulus Y_1 is a structure sensitive property correlated to the ideal strength reflecting the strength of bondage of sample, whereas, (Wi·ρ) means the energy required to fracture a specimen, it is thought that the value of Y^* represented by the ratio of Y_1 and (Wi·ρ) is constant regardless of the kinds of samples, provided that there is no structural defects in the sample. However, the available brittle materials has a lot of faults such as pre-existent cracks, therefore, it is considered that the larger the value of Y^* having a wide scatter reflecting the characteristics of structure of sample the more heterogeneity in the structure of sample.

It was thought that the distribution function Φ_D had to be introduced in this analysis because the status of fracture observed on the fractured fragments of sphere specimens obtained under the slow rate of compressive loading was characterized by kinds of samples, as described in the previous paper.[6)] This status is classified into three groups by kinds of samples as given below.

In the first group, with regard to the specimens of limestone, marble and gypsum, specimens are fractured into two semispheres at the great circle cross-section passing through the upper and lower loading points. In the second group, with regard to the specimens of quartz and feldspar, large fractured fragments of 3 or 4 pieces and a small number of fine sized fragments are produced. In the third group, with regard to the specimens of quartz glass and borosilicate glass, the core of sphere, where the axis of the two loading points passes, are fractured into fine particles and surrounding portions to the core are disintegrated into several crescent-shaped fragments.

The samples of the first group are polycrystal, the ratios of the elastic strain energy to the Young's modulus have a constant value approximately as recognized in the same kinds of samples. In other words, it means that the maximum strains until the specimens are fractured are nearly equal, it can therefore be presumed that the mechanisms of energy consumption are approximately equal among various kinds of samples. It is considered that the specimens are fractured into large-sized fragments because the structure is of polycrystal including many faults, low level strength and low level elastic strain energy.

The samples of the second group consist relatively of large crystal grains. It is considered that the fracture may be due to the properties of samples itself, a small number of fine sized fragments are produced in a liberation process of the elastic strain energy which is larger than that of the first group.

The samples of the third group establishing glassy and noncrystalline structure have a high compressive strength because imperfections or faults in the samples is relatively few. It is thought that a lot of fine particles of fractured fragments are produced in a liberation process of the elastic strain energy.

On the other hand, the status of fracture of the rod specimens is classified approximately into three groups by kinds of samples in a similar manner as above. This status of fracture may be due to the Young's modulus and the Poisson's ratio. The rod specimen such as marble and gypsum having a small value of the Young's modulus in relative term and a large value of the Poisson's ratio are fractured into two pieces along with a diagonal line linking up with one end of upper plane to the other end of lower plane of rod specimen. The rod specimen such as limestone having large values in both the Young's modulus and the Poisson's ratio are fractured into many narrow-paper-tablet-shaped fragments along in parallel with the direction of loading. The fractured products of quartz and feldspar rod specimens become an irregular shape because of being cracked not only in parallel but also in right angle to the loading direction. The rod specimens such as quartz glass and borosilicate glass are, on the whole, fractured into very fine particles. It is considered that accumulation of the elastic strain energy stored in the specimen is related in these status of fracture.[j] The ratios of specific surface area of fractured product to specimen Rc and Rt tabulated in Table 1 are used as the distribution function including the characteristic status of fracture.

By considering Eqs.(8′) and (9′), the elastic strain energy transformed non-dimensional terms is expressed by Eq.(11)

$$\left.\begin{aligned} A &= Sc^{*2}/Y^{*} \\ B &= [(1-\nu_1^{2})/Y^{*}]^{2/3}\, St^{*5/3} \end{aligned}\right\} \quad (11)$$

By using these equations, the function representing a characteristic of the structure of sample, Y^{*} and the ratios of specific surface area of fractured product to specimen, Rc and Rt, Eq.(12) is obtained from Eq.(10).

$$K = (A^{g}\, B^{h}\, Y^{*k})/(Rc^{i}\, Rt^{j}) \qquad (12)$$

j) With respect to rubbers, plastics, metal materials, as a general rule, it is not always said that the finer fractured fragments were produced with the increase of the elastic strain energy. However, in the brittle materials used in this experiment, it was found that the elastic strain energy was significantly concerned the fracture phenomena.

The reasons for such transformation as given above are as follows. The elastic strain energy stored in a specimen until it is fractured is related to the ratios of specific surface area Rc and Rt, when these indexes affecting adequately on each of the terms of Eq.(12) as factors are given, it is thought that the ratio of the terms of energy A and B, and the terms of the ratio of specific surface area Rc and Rt converges to a constant value. Furthermore, when there is a wide spread in this ratio, the cause can be attributed to the difference in the structure of sample, consequently, that is because that an adequate constant value K is obtained by giving an adequate index to Y^* representing the characteristic of a brittle materials. These indexes are determined so as to hold the value of K constant regardless of the kinds of samples by a trial-and-error method. In the determination of these indexes, it is convenient to imagine that Rc and Rt correspond to A and B, respectively. And as can be seen from the relationships between the strengths and work indexes of quartz glass, borosilicate glass, and between the strengths and the work indexes of quartz, feldspar, it can be considered that the value of index "h" is larger than that of index "g" because the fracture mechanism near a compressive fracture of sphere specimen occupies to a great extent, compared to a compression of rod specimen in a rotating cylindrical ball mill. Therefore, in order to determine the value of index "g" as a criterion of the index "h" of B in Eq.(12), the scatter of the constant value of K obtained by substituting arbitrary values in indexes g, i, j, k was investigated. Eventually, it is found that the constant value of K at $g=h/4$ gives a better convergency. Subsequently, in order to determine the indexes of i, j, k, the indexes of i, j, k so as to let the constant value of K is minimized are obtained by a trial-and-error method. Further, the scatter of the constant values of K is investigated by fixing two indexes and varying one index of these three indexes. The relationship between index "k" and coefficient of variation C for $i=0.3$, $j=1.6$ is shown in Fig.2. From Fig.2, it is found that the minimum value of C lies in the range of "k" are 2.05 to 2.25. The relationship between index "j" and coefficient of variation C by using index "k" as a parameter and 0.3 as a fixed value "i" is shown in Fig.3. From the observation of the curves shown in Fig.3, it is thought that C varying continuously with "j" has a minimum value at near the region from 2.0 to 2.2 of "j". The value of "j" giving minimum values in these curves as a parameter "k" decreases with increasing of the value of "k". The value of C in Fig.3 has a minimum value, 0.191 at $j=1.6$ in the curve for $k=2.1$. The similar tendency is observed for the different value of "i", as shown in Fig.4

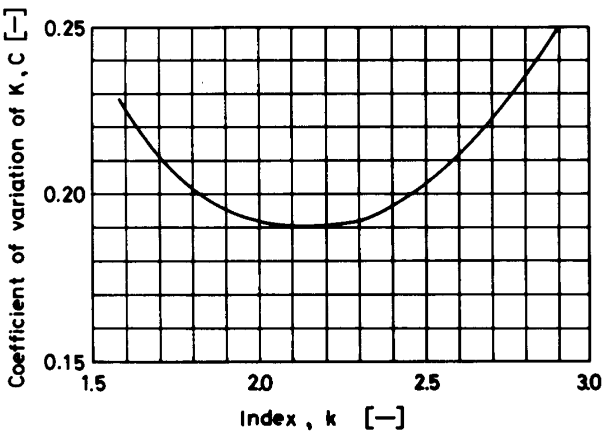

Fig.2 Relationship between index k and coefficient of variation (i=0.3, j=1.6)

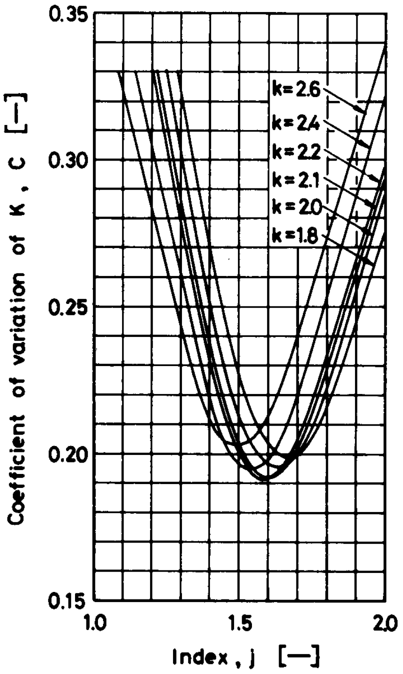

Fig.3 Relationship between index j and coefficient of variation - 1 (i=0.3)

for i=0.4 and Fig.5 for i=0.2. The values of "j" and "k" so as to let the coefficient of variation C minimize are j=1.5, k=2.0 in Fig.4 and j=1.7, k=2.2 in Fig.5. And the minimum value of the coefficient of variation obtained from the abovecited values of "j" and "k" in Fig.4 is smaller than that in Fig.5. From the above experimental results, it may be confirmed that value of the coefficient of variation has the minimum value all at i=0.3, j=1.6 and k=2.1.

By placing these experimental results in Eqs.(10) and (12), we obtain Eq.(13).

$$\left.\begin{aligned} 4.66 &= (Y^{*2.1}\ A^{1/4}\ B)/(Rc^{0.3}\ Rt^{1.6}) \\ &= \frac{Y^{*2.1}\ (Sc^{*2}/Y^{*})^{1/4}\ [\{1-\nu_1^2)/Y^{*}\}^{2/3}\ St^{*5/3}]}{Rc^{0.3}\ Rt^{1.6}} \\ &= Y^{*1.18}\ Sc^{*0.5}\ St^{*1.67}\ (1-\nu_1^2)^{0.67}\ Rc^{-0.3}\ Rt^{-1.6} \\ &= \frac{\left(\frac{Y_1}{Wi\cdot\rho}\right)^{1.18} \left(\frac{Sc}{Wi\cdot\rho}\right)^{0.5} \left(\frac{St}{Wi\cdot\rho}\right)^{1.67} (1-\nu_1^2)^{0.67}}{Rc^{0.3}\ Rt^{1.6}} \end{aligned}\right\} \quad (13)$$

where $\alpha=1.18$; $\beta=0.50$; $\gamma=1.67$; $\delta=0.67$

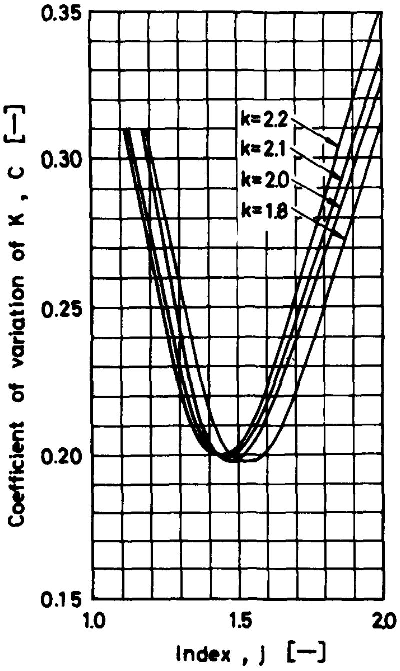

Fig.4 Relationship between index j and coefficient of variation - 2 (i=0.4)

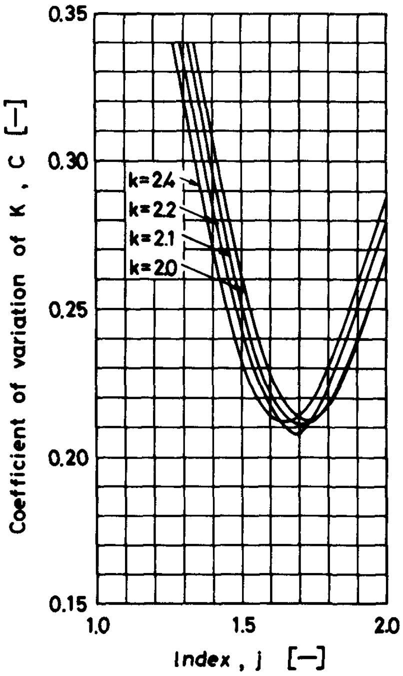

Fig.5 Relationship between index j and coefficient of variation - 3 (i=0.2)

From Eq.(13), the distribution function shown by Eq.(10) is determined as follows.

$$\Phi_D = Rc^{-0.3}\ Rt^{-1.6} \qquad (14)$$

Eq.(13) can be rewritten as follows.

$$Wi\cdot\rho = 0.632\ Y_1^{0.35}\ Sc^{0.15}\ St^{0.5}\ (1-\nu_1^2)^{0.2}\ Rc^{-0.09}\ Rt^{-0.48} \qquad (15)$$

Using the relation given in Eq.(3), we obtain the following equation.

$$(\frac{Wi\cdot\rho}{St}) = 0.632\ (\frac{Y_1}{St})^{0.35}\ Br^{0.15}\ (1-\nu_1^2)^{0.2}\ Rc^{-0.09}\ Rt^{-0.48} \qquad (16)$$

The values of $(1-\nu_1^2)^{0.2}$ obtained in every kinds of samples are approximately constant, and as the average of these values is 0.986 (within the precision of ± 0.008), as a practical equations we obtain Eqs.(15'), (16').

$$Wi\cdot\rho = 0.623\ Y_1^{0.35}\ Sc^{0.15}\ St^{0.5}\ Rc^{-0.09}\ Rt^{-0.48} \qquad (15')$$

$$(\frac{Wi\cdot\rho}{St}) = 0.623\ (\frac{Y_1}{St})^{0.35}\ Br^{0.15}\ Rc^{-0.09}\ Rt^{-0.48} \qquad (16')$$

Since the unit of Wi calculated from Eqs.(15), (16), (15') and (16') is [Kg-cm/kg], dividing Wi by 4.05×10^4 Kg-cm/kg (=1 kWh/ton) we

Table 2 Comparison of the Bond's work indexes

Kinds of samples	Tested by the standard ball mill method Wi [kWh/ton]	Calculated from Eq.(15) using the values of Table 1 Wi [kWh/ton]	Calculated from Eq.(15') using the values of Table 1 Wi [kWh/ton]
Quartz glass	14.8	14.0	13.8
Borosilicate glass	15.2	15.2	15.0
Quartz	13.3	14.0	13.8
Feldspar	12.4	12.0	11.8
Limestone	9.4	8.1	8.0
Marble	6.7	6.8	6.7
Gypsum	6.3	5.9	5.8

can obtain Wi having the unit of [kWh/ton]. The calculated values of the work index obtained from the mechanical properties of samples as shown in Table 1 are in a fairly good agreement with the experimented values obtained by the standard ball mill method, as tabulated in Table 2.

V. Consideration

It seems that the Bond's work index is an inherent value to a material, although T.Ishihara[11] reported that the value of the work index increased rapidly with the decrease of the classification size in the measurements of the work index, as also recognized by F.C. Bond.[12] Bond explained that this phenomenon was attributable to deviation from the homogeneous broken material. He proposed that the work index obtained from the classification size of 100μm in the measurements of the work index, so called constant work index, was employed as the reference. This tendency shows that the Bond's theory approximates more and more clossly to the Rittinger's theory. From analogy with the experimental results obtained by T.Ishihara, the classification size in this experiment was set at 208μm (65mesh) as the region neglecting the influence of the classification size. In this study, the effects of impact among balls and tippings were not considered. It was stated in the previous paper[6] that the Weibull's relation[13] is applicable to the brittle materials. The introduction of this relation into experiments is expected in the subsequent studies.

VI. Conclusions

Eqs.(15), (16), (15'), (16') were obtained as a result of considerating the relationships between the work index and the mechanical properties of brittle materials by the dimensional analysis. These relationships can be applied to such samples as shown in Table 1, and the work index measured by the standard ball mill method was obtained by setting the classification size of fractured products at 208μm (65 mesh). The compressive strength Sc, the Young's modulus Y_1 and the Poisson's ratio ν_1 were measured by using the rod specimen having diameter of 2.0 cm and length of 4.0 cm, and the tensile strength St was measured by using the sphere specimen having diameter of 2.0 cm under slow rate of loading, respectively. These values tabulated in Table 1 are the representative values measured by using each of 20 specimens. Rc and Rt are the ratios of specific surface area of fractured product to each of the specimens used in the measurements of strengths as described previously. In the case of talc having the Moh's hardness H=1.0, application of this experimental result is impracticable because of the large plastic deformation.

References

(1) F. C. Bond, Trans., A.I.M.E., 193 (1952), 484.

(2) H. Oya, J. Min. Inst. Japan, 80 (1964), 180.

(3) N. Takahashi, J. Min. Inst. Japan, 80(1964), 176.

(4) T. Tanaka, "Funtai purosesu nyumon" (An Introduction to Powder Processing Methods), (1968), 127.

(5) R. Kobayashi, J. Min. Inst. Japan, 81 (1964), 595.

(6) S. Yashima, S. Morohashi, O. Awano and Y. Kanda, Kagaku Kogaku, 34 (1970), 210.

(7) F. C. Bond, British Chem. Eng., 6 (1961), 378.

(8) Y. Shimomura and A. Takata, J. Min. Inst. Japan, 77 (1961), 377.

(9) Y. Hiramatsu, Y. Oka and H. Kiyama, J. Min. Inst. Japan, 81 (1965), 1024.

(10) S. Timoshenko and J. N. Goodier, Theory of Elasticity, (1951), 371.

(11) T. Ishihara, J. Min. Inst. Japan, 81 (1965), 1064.

(12) F. C. Bond, Eng. & Min. J., 164 (1963), 108.

(13) W. Weibull, J. Appl. Phys., 18 (1951), 293.