A Method of Measuring Ion Beam Profiles

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Synopsis

A method of measuring beam profiles is described. The curves to be obtained in measuring beam intensities of given profiles with a large detector hole are first looked for by calculating the volume common between a cylinder and a cylinder, a cone, or a truncated cone. The original beam profile is given by comparing the experimental curve with calculated curves. Some results of numerical calculation were obtained with a computor. As an example, a profile of K^+-ion beams was determined by this method.

I. Introduction

The elastic scattering of high speed particles (ions or neutrals) in a gas has long been used as the direct experimental method of determining the intermolecular potential between the particle and the gas molecule at small internuclear separations. Much important information was acquired by this method of investigation. There is, however, a case which shows discrepancy between experimental results and theoretical calculation. Jordan and Amdur(1) have shown in their work on He-He interaction that a cause of the discrepancy exists in taking little account of the intensity distribution across the beam.

Thus, the knowledge of intensity distribution across the beam is essential to an analysis of the results of an elastic scattering experiment. This work was begun in relation to an experiment on elastic scattering of high speed K^+-ions with Ar-atoms,(2) and this article describes a method of measuring an ion beam profile with a large detector hole. The results of the calculation given below will be available also for a neutral beam profile, for which a small detector hole was used previously.(1)

II. Principle

The profile of an ion beam can be measured in the way that follows.(2) As shown in Fig. 1, displacement X of the ion of energy E is given by the following formula, when the ion passes through the electric field formed by a pair of parallel electrodes:

(1) J. E. Jordan and I. Amdur, J. Chem. Phys., **46**, 165 (1967).
(2) I. Amdur, H. Inouye, A. N. van der Steege, and A. J. H. Boerboom, to be published.

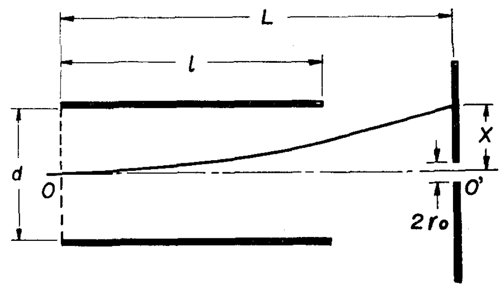


Fig. 1. Scheme of the experimental device for measuring ion beam profiles.

$$X = \frac{1}{2}\frac{P}{E}\frac{l}{d}(L - l/2), \tag{1}$$

where P is voltage applied on the electrodes, d the distance between the electrodes, l the length of the electric field, and L the distance from the point of incidence to the detector plane. Since d, l, and L are constants specific to the apparatus, the displacement X is proportional to the applied voltage P when the energy E is fixed. When the radius r_0 of the detector hole is sufficiently small, the beam profile can be obtained by plotting the ion currents passing through the detector hole against P, that is, X. When r_0 is large, an intensity distribution across the beams is averaged over the area of the detector hole. The measured curve is quite different from the original profile. The beam profile will be distorted to some extent by the deflection, but the distortion is negligibly small if the displacement is small as in the apparatus described in reference 2 (the maximum displacement is about 0.4 mm).

While the measurement with a small detector hole directly gives a beam profile, it requires a detection device of high sensitivity and complex structure. Therefore the authors employed another method: ——

The detector hole behind the collision chamber in the total scattering experiment is used also as the detector hole for beam profile measurements, though it is rather large. The curves to be obtained in measuring some given profiles with a large detector hole are first looked for by calculation. Then the real profile is deduced from the calculated curve which fits the measured curve. Details of the calculation are described below.

III. Calculation

1. *Rectangular Profile*

When an intensity distribution across the beam is of axial symmetry and uniform along the radial direction, then it is represented by a cylinder AA' with height H_0 corresponding to the uniform intensity (Fig. 2). When the beam axis, that is, the axis AA' of the cylinder is displaced by X from the axis of the apparatus (OO' in Fig. 1), the ion current passing through the detector hole is thought to be proportional to the volume common between the cylinder AA' with

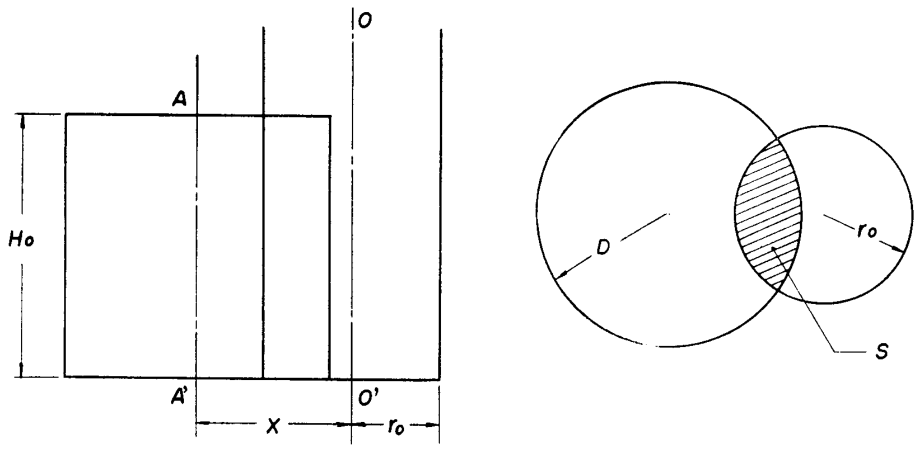


Fig. 2. Representation of a rectangular beam profile displaced from the detector axis.

radius D and the cylinder OO′ with radius r_0. Since this common volume is proportional to the cross section S parallel to the detector plane, a curve given by plotting S against the interaxial distance X is an intensity curve that is obtained when a beam with a rectangular profile is measured behind the detector hole of radius r_0. S as a function of X is given by the following expression:

$$S=\frac{\pi r_0^2}{2}+\frac{\pi D^2}{2}+D^2\sin^{-1}\frac{r_0^2-D^2-X^2}{2DX}-r_0^2\sin^{-1}\frac{r_0^2-D^2+X^2}{2r_0X}$$
$$-\frac{1}{2}\sqrt{4D^2X^2-(r_0^2-D^2-X^2)^2}\,. \tag{2}$$

Relative values of S for some values of D/r_0 are given in Fig. 3 and Table I. The ordinate in Fig. 3 denotes V/V_0. V_0 is the common volume at $X=0$, and

Table I. V/V_0 for beams of rectangular profiles

X \ D	1.2	1.6	2.0
0	100	100	100
0.2	100	100	100
0.4	90.66	100	100
0.6	78.07	100	100
0.8	65.36	92.40	100
1.0	52.91	80.91	100
1.2	40.99	68.36	93.13
1.4	29.84	55.68	82.08
1.6	19.69	43.30	69.90
1.8	10.89	31.59	57.19
2.0	3.91	20.88	44.65
2.2	0	11.56	32.69
2.4	0	4.15	21.65
2.6	0	0	11.98
2.8	0	0	4.28
3.0	0	0	0

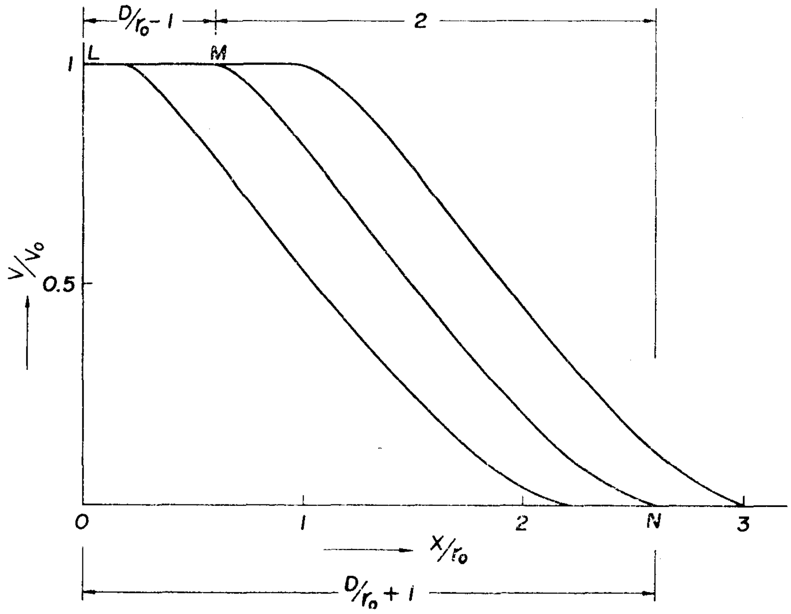


Fig. 3. V/V_0-curves for beams of rectangular profiles.

$$V/V_0 = H_0 S/H_0 S_0 = S/S_0 .$$

Here $S_0 = \pi r_0$ for $r_0 < D$;

$= \pi D$ for $r_0 \geqq D$.

V/V_0 is independent of absolute values of ion beam intensity and gives only the fraction of ion currents passing through the detector hole. The portion MN of the curve shows nearly the same behavior independent of the value of D in every curve, as seen in Fig. 3. When a measured curve fits a curve in Fig. 3, the original beam profile is rectangular. The radius D is obtained from

$$\overline{\mathrm{ON}} = D/r_0 + 1 ,$$

or

$$\overline{\mathrm{LM}} = D/r_0 - 1 .$$

2. *Triangular Profile*

When a beam profile is of axial symmetry and of triangular shape, the intensity distribution can be represented by a cone of base diameter $2D$ and height H_0 corresponding to the maximum intensity. Ion currents measured behind the detector hole of radius r_0 is represented by the volume common between the cone and the cylinder of radius r_0. Calculation of the common volume is performed in four different ways according to the cases described below:

(i) When $X \geqq r_0$, $X - r_0 < D$, and $X + r_0 > D$ (Fig. 4), the common volume is given by the following expression:

$$V = \int_0^h S\mathrm{d}h = \frac{H_0}{D} \int_{X-r_0}^{D} S\mathrm{d}R . \tag{3}$$

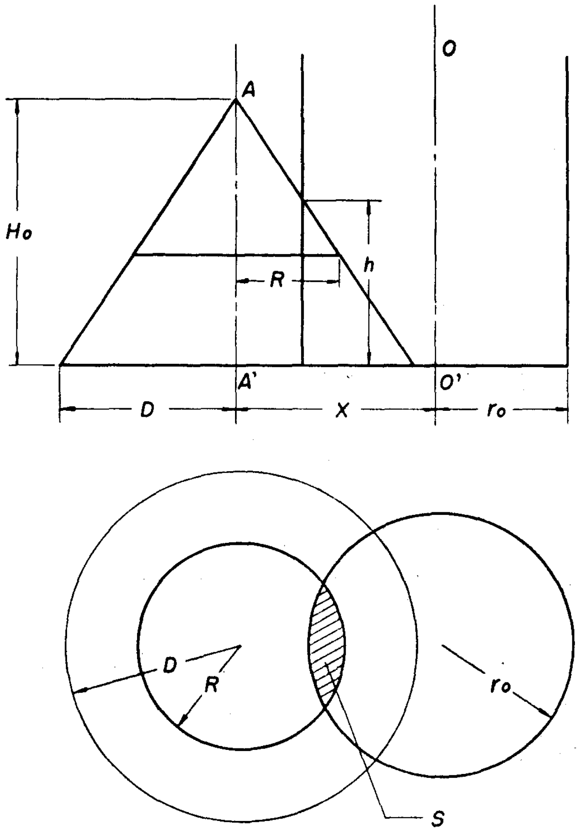


Fig. 4. Representation of a triangular beam profile displaced from the detector hole axis.

Here S is expressed by the formula that D in Eq. (2) is replaced by R, radius of cross section of the cone parallel to the base plane at an arbitrary height. Integration gives the following formula:

$$V=\frac{H_0}{D}\left\{\frac{\pi}{2}\left(r_0^2 D+\frac{1}{3}D^3\right)-r_0^2 D\sin^{-1}\frac{r_0^2+X^2-D^2}{2r_0X}+\frac{D^3}{3}\right.$$

$$\times\sin^{-1}\frac{r_0^2-X^2-D^2}{2XD}-\left[a_0+a_2+\frac{1}{3}a_4\left(1+\frac{1}{k^2}\right)\right]$$

$$\times\int_{z_0}^{1}\frac{dz}{\sqrt{(1-z^2)(1-k^2z^2)}}+\left[a_2+\frac{2}{3}a_4\left(1+\frac{1}{k^2}\right)\right]$$

$$\left.\times\int_{z_0}^{1}\sqrt{\frac{1-k^2z^2}{1-z^2}}\,dz+\frac{1}{3}a_4 z_0\sqrt{(1-z_0)(1-k^2z_0^2)}\right\}, \tag{4}$$

where

$$a_0=-\frac{1}{2}(X-r_0)(X^2-r_0^2)+\frac{4}{3}(X^2+2r_0^2)(X+r_0)-\frac{5}{6}(X+r_0)^3,$$

$$a_2=-\frac{4}{3}(X^2+2r_0^2)(X+r_0)+\frac{5}{3}(X+r_0)^3,$$

$$a_4 = -\frac{10}{3} X r_0 (X + r_0)\,,$$

$$k^2 = \frac{4 r_0 X}{(X + r_0)^2}\,,$$

and

$$z_0 = \sqrt{\frac{(X + r_0)^2 - D^2}{4 r_0 X}}\,.$$

(ii) For $D \leqq 2r_0$, $r_0 > X > 0$, and $X + r_0 > D$, the volume is given by

$$V = \frac{H_0}{D}\left[\int_{X-r_0}^{D} S dR + \frac{\pi}{3}(r_0 - X)^3\right]. \tag{5}$$

Integration gives a formula quite similar to Eq. (4).

(iii) For $r_0 > X > O$, and $X + r_0 < D$, the volume is given by

$$V = \frac{H_0}{D}\left[\int_{X-r_0}^{r_0} S dR + \frac{\pi}{3}(r_0 - X)^3 + \pi r_0^2 (D - r_0 - X)\right]. \tag{6}$$

The first term is given by an expression similar to Eq. (4), except that the upper limit of integration is changed.

(iv) For $D > 2r_0$, $X \geqq r_0$, and $X + r_0 < D$, the volume is given by

$$V = \frac{H_0}{D}\left[\int_{X-r_0}^{X+r_0} S dR + \pi r_0^2 (D - r_0 - X)\right]. \tag{7}$$

Integration is performed in a similar way to that in section (iii).

In every case calculation of the common volume comes to that of Eq. (4) or similar formulas. Since direct numerical calculation of Eq. (4), etc., took much time and labor, approximate calculation was made with a computor by replacing the expression on the right side of Eq. (3) with Simpson's formula.

$$V = \frac{H_0}{D}\,\frac{[D - X + r_0]}{3 \times 2n} \sum_{i=1}^{n} (S_{2i-2} + 4\,S_{2i-1} + S_{2i})$$

$$= \frac{H_0}{D}\,\frac{r_0}{30000} \sum_{i=1}^{n} (S_{2i-2} + 4 S_{2i-1} + S_{2i})\,,$$

Here $2n$ was substituted by $\dfrac{\mathrm{D} - \mathrm{X} + r_0}{r_0/10000} = 10000 \times \dfrac{\mathrm{D} - \mathrm{X} + r_0}{r_0}$ for the convenience of computation.

The quantities r_0 and H_0 are assumed in the computation to be one since X and D are measured with r_0 as unit and the ordinate V/V_0 is independent of H_0. V_0 is the volume at $X=0$, given by $(H_0/D) \times \pi r_0^2 [D - (2/3) r_0]$. The results of computation are listed in the second column of Table II ($B=0$).

3. *Trapezoid Profile*

When a beam profile is of axial symmetry and of trapezoid shape, the intensity distribution is represented by a truncated cone of height H_0 corresponding to the maximum intensity. Ion currents measured behind the detector hole of radius r_0 is given by the volume common between the truncated cone and the cylinder of radius r_0.

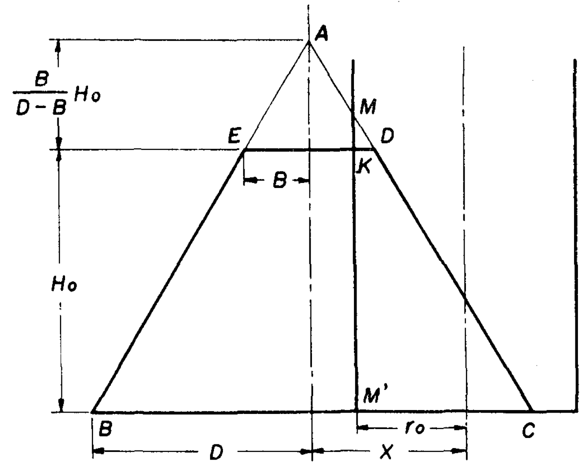


Fig. 5. Representation of a trapezoid beam profile displaced from the detector hole axis.

If a truncated cone whose base is $2D$ and whose plateau is $2B$ ($B \leqq D$) is considered as the difference of two cones, ABC and ADE (Fig. 5), the results of calculation on a cone in section 2 can be used for this case. For $X \geqq r_0$, $X - r_0 < B$, and $X + r_0 \geqq D$ (Fig.5), for instance, the generating line MM' of the cylinder crosses the line segment AD. The volume is given by the difference between the volume of the solid MM'C and that of the solid MKD. The former volume is given by Eq. (8) and the latter by the expression in which B is used in place of D as the upper limit of integration in Eq. (8).

$$V_{\mathrm{MM'K}} = \frac{H_0}{D-B} \int_{X-r_0}^{D} S\mathrm{d}R. \tag{8}$$

$$V = \frac{H_0}{D-B} \left[\int_{X-r_0}^{D} S\mathrm{d}R - \int_{X-r_0}^{B} S\mathrm{d}R \right]. \tag{9}$$

Similar calculation is available for the other cases.

Numerical calculation was performed for

$D/r_0 = 1.0,\ 1.2,\ 1.4,\ 1.6,\ 1.8,$ and 2.0;

$B/r_0 = 0,\ 0.2,\ 0.4,\ \cdots,$ and $(D-0.2)/r_0\ (\leqq 1.4)$;

and $X/r_0 = 0,\ 0.2,\ 0.4,\ \cdots,$ and $(D/r_0) + 0.8$

Table II. V/V_0 for beams of triangular and trapezoid profiles.

(1) $D=1.0$

X \ B	0	0.2	0.4	0.6	0.8
0	100.00	100.00	100.00	100.00	100.00
0.2	97.04	97.02	96.84	96.22	93.93
0.4	88.76	88.68	88.00	85.67	80.61
0.6	75.95	75.76	74.31	70.87	66.68
0.8	59.75	59.43	57.87	55.65	53.07
1.0	41.88	41.83	41.55	40.98	40.14
1.2	25.45	25.65	26.52	27.43	28.10
1.4	12.95	13.05	13.83	15.56	17.32
1.6	4.86	4.90	5.20	6.20	8.27
1.8	0.89	0.89	0.95	1.13	1.81

(2) $D=1.2$

X \ B	0	0.2	0.4	0.6	0.8	1.0
0	100.00	100.00	100.00	100.00	100.00	100.00
0.2	96.26	96.25	96.11	95.68	94.50	94.96
0.4	86.57	86.50	86.00	84.47	81.87	82.90
0.6	73.83	73.70	72.74	70.63	68.68	70.30
0.8	59.08	58.87	57.91	56.65	55.77	57.95
1.0	43.39	43.37	43.26	43.12	43.33	45.92
1.2	28.88	29.03	29.68	30.54	31.69	34.61
1.4	17.16	17.24	17.87	19.29	21.10	24.17
1.6	8.62	8.66	8.98	9.96	11.91	14.87
1.8	3.21	3.23	3.34	3.71	4.72	7.09
2.0	0.58	0.58	0.60	0.67	0.85	1.55

(3) $D=1.4$

X \ B	0	0.2	0.4	0.6	0.8	1.0	1.2
0	100.00	100.00	100.00	100.00	100.00	100.00	100.00
0.2	97.28	97.27	97.20	96.98	96.45	97.48	100.00
0.4	89.17	89.14	88.85	88.00	86.94	89.52	96.13
0.6	76.92	76.83	76.23	75.02	74.53	77.72	85.15
0.8	62.76	62.63	62.03	61.47	61.75	65.28	72.60
1.0	47.93	47.93	47.97	48.23	49.26	59.97	60.02
1.2	34.03	34.15	34.76	35.75	37.40	41.18	47.75
1.4	22.29	22.37	22.96	24.34	26.39	30.07	35.97
1.6	13.12	13.17	13.51	14.55	16.61	20.00	25.12
1.8	6.55	6.58	6.75	7.27	8.54	11.28	15.47
2.0	2.43	2.44	2.50	2.69	3.17	4.45	7.36
2.2	0.44	0.44	0.45	0.49	0.57	0.80	1.61

with a computor by replacing the integrals with Simpson's formula. The results of the computation are listed in Table II and some of them ($D/r_0=1.2$, 1.6, and 2.0) are shown in Figs. 6(a)-(c).

IV. Comparison of a Measured Curve with Calculated Curves

As seen in Figs. 6(a) - (c), every curve is essentially in contact with the abscissa at the point of $(D/r_0)+0.8$, although V/V_0 becomes zero at $X=D+r_0$ in

(4) $D=1.6$

X \ B	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
0	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
0.2	97.86	97.86	97.81	97.68	97.38	98.32	100.00	100.00
0.4	91.49	91.47	91.30	90.78	90.36	93.01	98.07	100.00
0.6	81.16	81.11	80.72	80.06	80.35	84.06	90.94	96.73
0.8	67.84	67.74	67.38	67.27	68.22	72.33	79.52	86.43
1.0	53.54	53.56	53.70	54.24	55.78	60.00	67.07	74.11
1.2	39.89	40.01	40.60	41.70	43.69	47.92	54.57	61.39
1.4	28.00	28.08	28.65	30.05	32.30	36.36	42.45	48.94
1.6	18.23	18.28	18.66	19,75	21.95	25.65	31.05	36.97
1.8	10.68	10.71	10.93	11.58	13.08	16.13	20.65	25.83
2.0	5.32	5.33	5.44	5.76	6.51	8.27	11.63	15.91
2.2	1.97	1.97	2.01	2.13	2.41	3.06	4.59	7.57
2.4	0.35	0.35	0.36	0.38	0.43	0.55	0.83	1.65

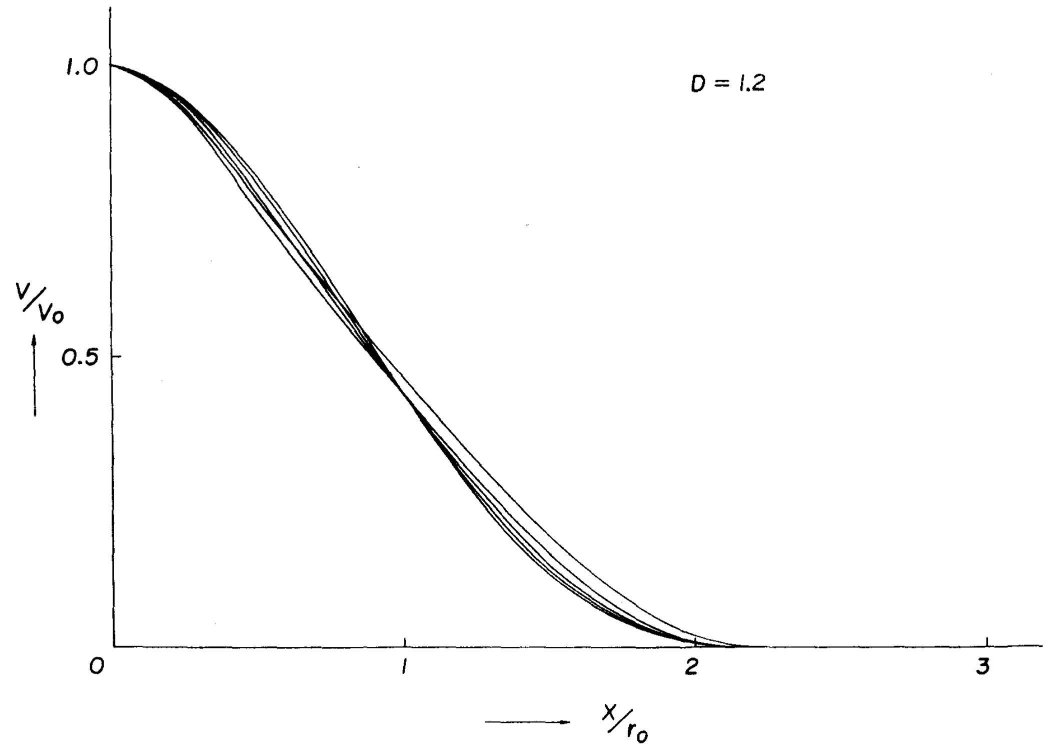
(5) $D=1.8$

X \ B	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
0	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
0.2	98.24	98.24	98.21	98.12	97.93	98.74	100.00	100.00
0.4	93.00	92.98	92.86	92.56	92.37	94.76	98.71	100.00
0.6	84.49	84.45	84.19	83.82	84.44	88.04	93.96	98.37
0.8	72.95	72.88	72.68	72.83	74.16	78.45	85.28	91.62
1.0	59.46	59.49	59.70	60.43	62.28	66.79	73.74	80.60
1.2	46.10	46.21	46.80	47.99	50.21	54.71	61.41	68.24
1.4	34.08	34.16	34.73	36.16	38.57	42.89	49.12	55.70
1.6	23.81	23.87	24.27	25.43	27.75	31.71	37.32	43.42
1.8	15.46	15.49	15.75	16.51	18.20	21.53	26.35	31.78
2.0	9.04	9.06	9.21	9.65	10.64	12.81	16.56	21.16
2.2	4.49	4.50	4.58	4.80	5.29	6.36	8.48	11.92
2.4	1.66	1.66	1.69	1.77	1.95	2.35	3.13	4.70
2.6	0.30	0.30	0.30	0.32	0.35	0.42	0.56	0.84

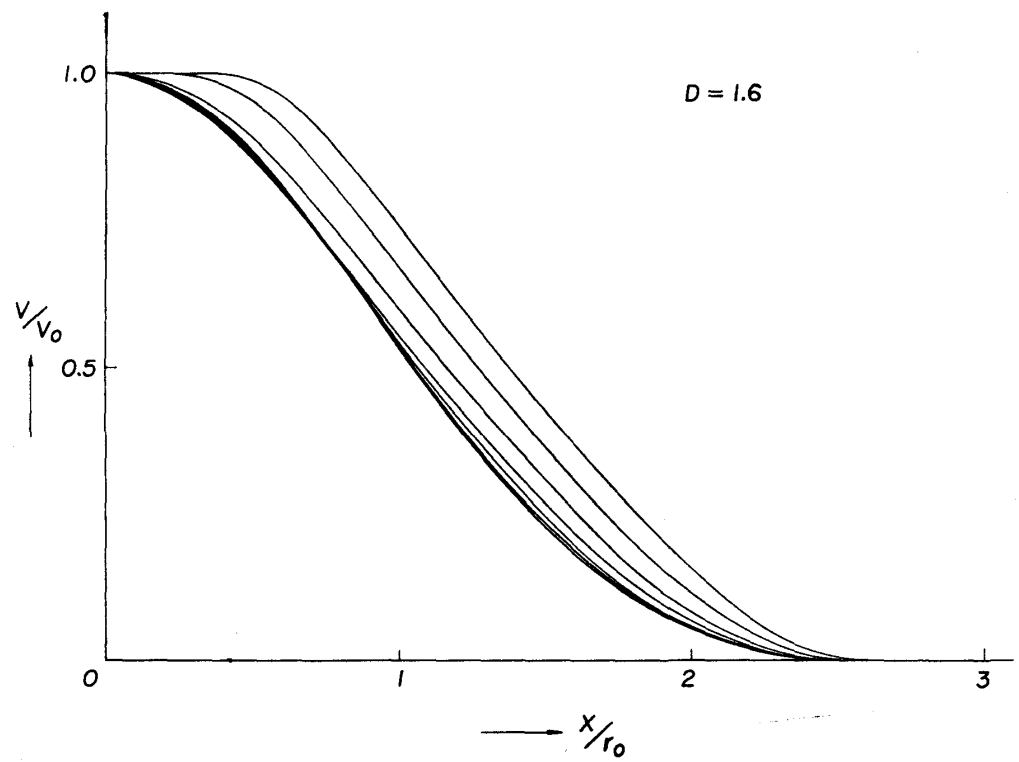
(6) $D=2.0$

X \ B	0	0.2	0.4	0.6	0.8	1.0	1.2	1.4
0	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
0.2	98.50	98.50	98.48	98.42	98.28	98.99	100.00	100.00
0.4	94.05	94.03	93.95	93.71	93.68	95.81	99.03	100.00
0.6	86.81	86.79	86.60	86.38	87.11	90.44	95.47	98.91
0.8	77.01	76.96	76.84	77.14	78.61	82.76	88.96	94.41
1.0	65.10	65.14	65.40	66.24	68.27	72.84	79.57	86.09
1.2	52.33	52.43	53.02	54.27	56.65	61.29	67.95	74.70
1.4	40.34	40.42	41.00	42.45	44.97	49.47	55.80	62.40
1.6	29.73	29.79	30.21	31.42	33.85	38.01	43.80	50.02
1.8	20.73	20.77	21.06	21.91	23.77	27.34	32.41	38.05
2.0	13.44	13.46	13.65	14.20	15.41	17.91	22.01	26.89
2.2	7.85	7.86	7.98	8.30	9.00	10.46	13.08	16.90
2.4	3.90	3.90	3.96	4.12	4.47	5.19	6.49	8.66
2.6	1.44	1.44	1.46	1.52	1.65	1.92	2.40	3.20
2.8	0.26	0.26	0.26	0.27	0.30	0.34	0.43	0.57

principle. One can obtain the value of D from the X-value of the point where the measured curve comes into contact with the abscissa. If the measured curve fits a certain one with the same value of D, the value of B employed for calculation of



(a)



(b)

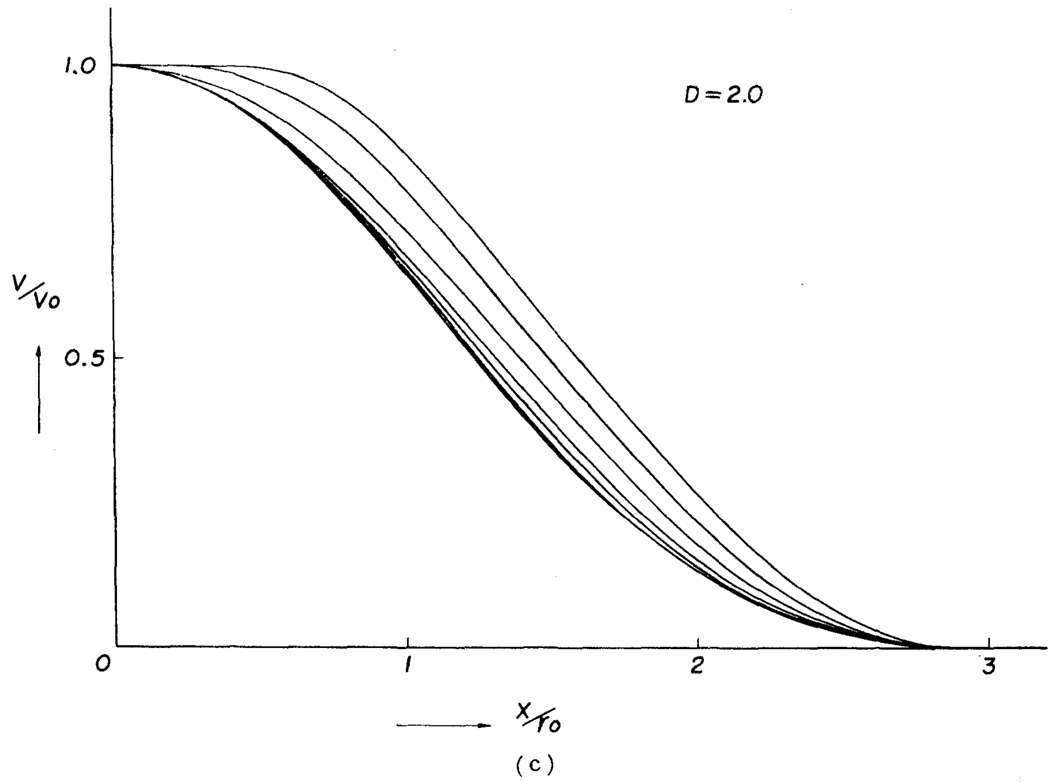


Fig. 6. Calculated curves for (a) $D=1.2$, (b) 1.6, and (c) 2.0 (measured with r_0 as unit).

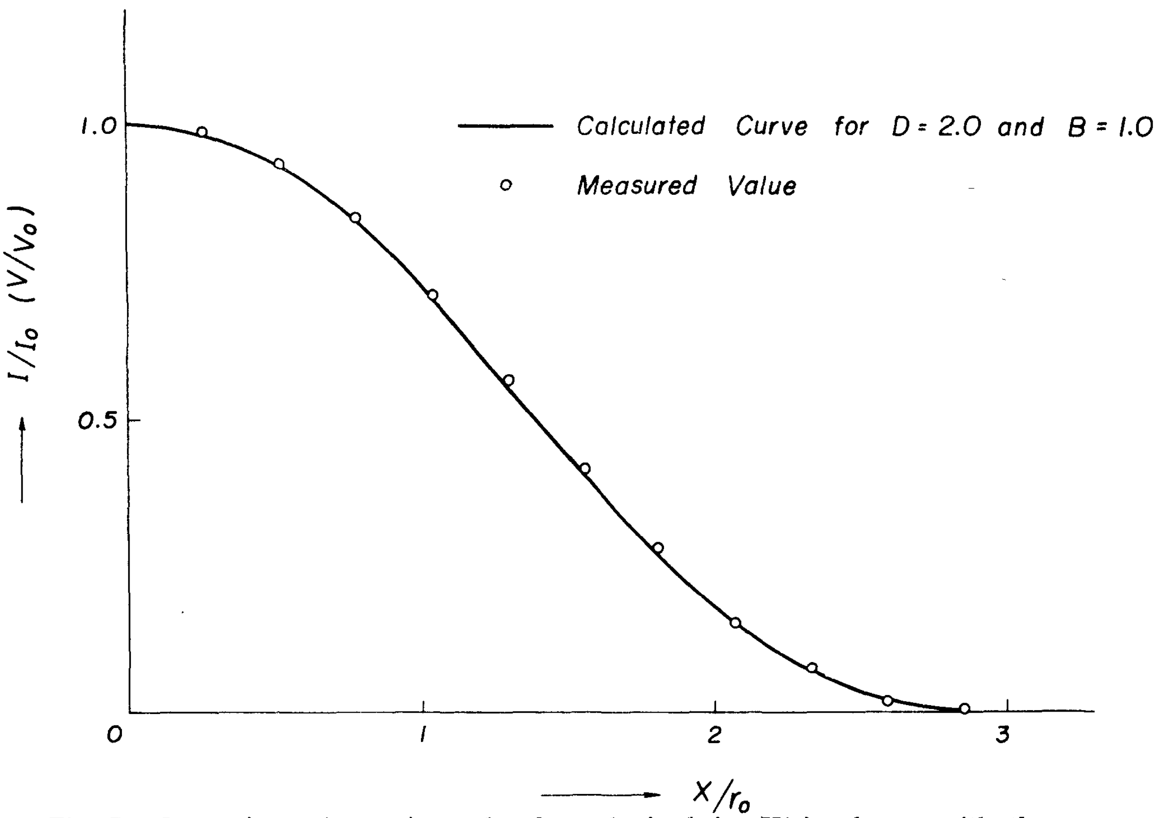


Fig. 7. Comparison of experimental values obtained for K^+-ion beams with the curve calculated for $D=2.0$ and $B=1.0$ (measured with r_0 as unit).

the latter is true also for the original beam profile. A result of a comparison of experimental values* with a calculated curve is shown in Fig. 7. As seen in the figure, those values are in good agreement with the curve calculated for $D/r_0=2.0$ and $B/r_0=1.0$. This fact means that the original beams have the trapeoizd profile with base $4r_0$ and plateau $2r_0$. ($r_0=0.125$ mm in the apparatus used in the experiment.)

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* The data were obtained for K^+-ion beams in FOM-Institute for Atomic and Molecular Physics, Amsterdam (see reference 2).