Fringe Disorder in Three-Beam Interference

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Fringe Disorder in Three–Beam Interference

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Synopsis

The intensity distribution curves in three-beam interference between three rectangular slits in a beam of quasi-monochromatic light from an incoherent slit source are investigated by the photoelectric method with Ebert optical system. Examples and discussions are given to illustrate the relation between the widths of primary slit source and the patterns. We must take note of that no fringe disorder is in the patterns with some special primary source widths and that the fringe disorder, if it exists, always comes up either at the parts of intensity maxima or at the parts of intensity minima of the patterns.

I. Introduction

The degree of coherence between three slits in a beam of quasi-monochromatic light from an incoherent slit source was studied by the photoelectric method with Ebert optical system and the three-beam interference was explained as the combination of the two-beam interference(1). Since we discussed mainly the degree of coherence there, we did not take note of the fringe patterns in detail, except the situation whether the intensity at the center of each pattern has a relative

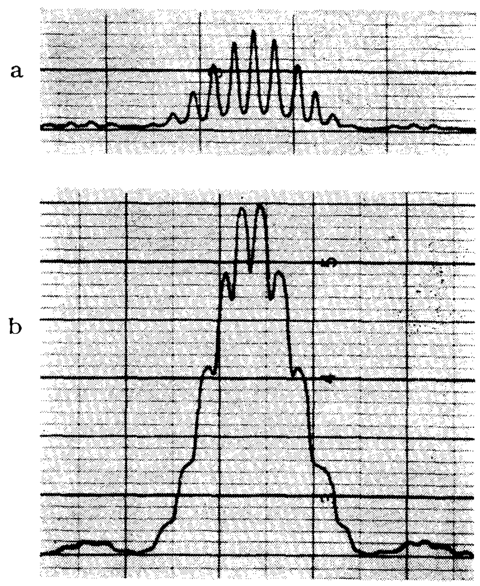


Fig. 1. Observed intensity distribution curves by three-beam interference. Results are based on $\lambda=0.589\,\mu$, $2w=2\,\text{mm}$, separation between double slits is 10 mm each, $2w'=90\,\mu\mp\varepsilon$.

(1) S. Nawata, Sci. Rep. RITU, A **16** (1964), 54.

maximum or minimum. The detailed observation of fringe patterns in the three-beam interference gives us something like the fringe disorders depending upon the primary slit widths and they exist either in the neighbourhoods of intensity maxima or of intensity minima. For instance, in Fig. 1 we refer the observed patterns shown in the previous paper. We will see the parts of intensity maxima look sharp, while those of intensity minima do round a little in (a), and this situation is just opposite in (b), as well as the phase change occurs at the center. We never observe such phenomena – we refer to them for fringe disorders here, though they are the intensity distribution themselves — under any primary slit widths in two-beam interference. The purpose of this paper is to study the relation between the fringe disorders and the primary slit widths.

II. Experimental apparatus and results

Since the apparatus shown in Fig. 2 is the same as that in previous paper, we will omit details here. The primary slit source S_1 irradiated by the sodium yellow line effectively acts as an incoherent slit source. The intensity distribution is observed through the exit slit S_2 by the photoelectric method.

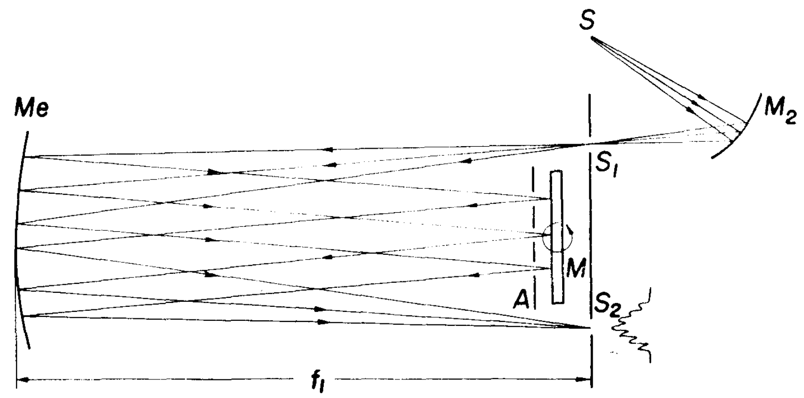
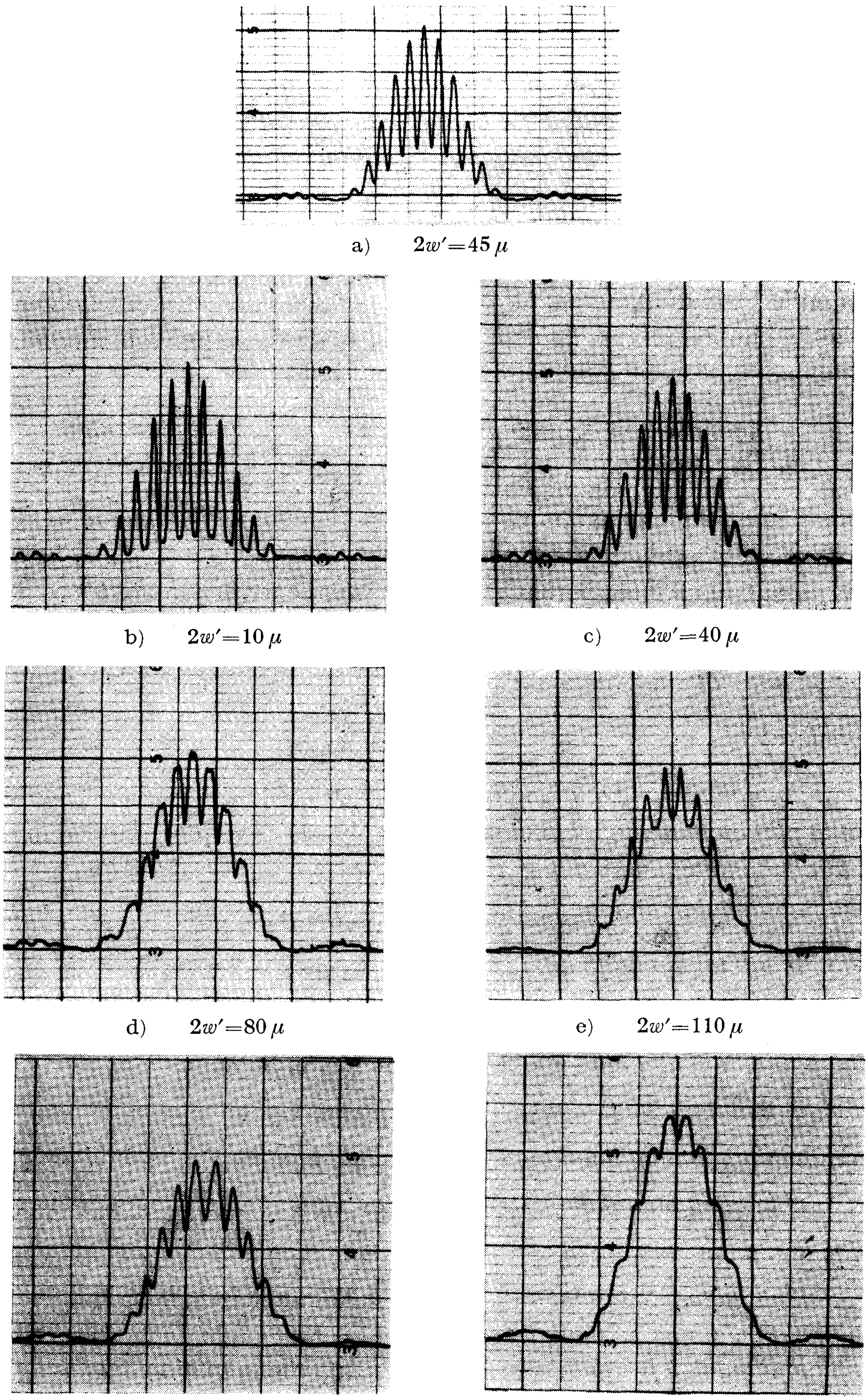


Fig. 2. Schematic diagram of experimental arrangement.
M=flat mirror, Me=Ebert mirror (1510 mm focal length, 200 mm diam), A=diffraction mask with three rectangular slits, s=quasi-monochromatic light source, s_1, s_2 entrance and exit slits.

In Fig. 3 are shown the observed distribution curves with various primary slit widths in three-beam interference, except (a) in two-beam interference. In case of (a), the mask used has two rectangular slits whose widths $2w$ are 2 mm each and their separation $2h$ is 10 mm. We only show the observed curve with the primary slit width $2w'=45\mu$, because as mentioned above we never have fringe disorders under any primary slit widths in two-beam interference. In cases of (b)~(g) in three-beam interference, the mask with three rectangular slits whose widths $2w$ are 2mm each and their symmetrical separations are 10mm each was used.

In (b) with $2w'=10\mu$ and (e) with 110μ, the intensity maxima look sharp, while the intensity minima do round a little. On the contrary, the situations in (d)



a) $2w'=45\,\mu$

b) $2w'=10\,\mu$ c) $2w'=40\,\mu$

d) $2w'=80\,\mu$ e) $2w'=110\,\mu$

f) $2w'=135\,\mu$ g) $2w'=170\,\mu$

Fig. 3. Observed intensity distribution curves by three-beam interference, except (a) by two-beam interference. Results are based on $\lambda=0.589\,\mu$, $2w=2$ mm, separation between double slits is 10 mm each.

with $2w'=80\mu$ and (g) with 170μ are just opposite. In (c) with $2w'=40\mu$ and (f) with 135μ, we have fringe disorders neither at intensity maxima nor at intensity minima, whose situation is the same as (a) in two-beam interference. We may have now to conclude that fringe disorders exist either at the parts of intensity maxima or of intensity minima if they exist, and there is no primary slit width range with fringe disorders in both.

III. Discussion

In order to study the situation at intensity maxima and at intensity minima in three-beam interference, we use the following intensity expression calculated by the combination of intensities in two-beam interference:

$$I=3\left(\frac{\sin n\pi}{n\pi}\right)^2\left(1+\frac{4}{3}\gamma_{0,1}\cos\frac{n\pi h}{w}+\frac{2}{3}\gamma_{+1,-1}\cos\frac{2n\pi h}{w}\right), \tag{1}$$

in which $n=2w\phi/\lambda$, $\gamma_{0,1}=\sin(m\pi/2)/(m\pi/2)$, $\gamma_{+1,-1}=\sin m\pi/m\pi$ and $m=4w'h/\lambda f_1$. This expression can induce from theorem, too.[(2)]

By using $x=(n\pi h)/w$ for convenience, (1) may be written in the form

$$I=3\left\{\frac{\sin(wx/h)}{(wx/h)}\right\}^2\left(1+\frac{4}{3}\gamma_{0,1}\cos x+\frac{2}{3}\gamma_{+1,-1}\cos 2x\right). \tag{2}$$

If we denote by

$$f(x)=1+(4/3)\gamma_{0,1}\cos x+(2/3)\gamma_{+1,-1}\cos 2x, \tag{3}$$

the positions with extreme value of $f(x)$ approximately correspond with them of (2). From (3),

$$f'(x)=-(4/3)\sin x(\gamma_{0,1}+2\gamma_{+1,-1}\cos x), \tag{4}$$

$$f''(x)=-(4/3)(4\gamma_{+1,-1}\cos^2 x+\gamma_{0,1}\cos x-2\gamma_{+1,-1}). \tag{5}$$

Under $f'(x)=0$, we have

$$x=n_0\pi \quad \text{or} \quad x=2n_0\pi\pm\cos^{-1}(-\gamma_{0,1}/2\gamma_{+1,-1}),$$

where $n_0=0, \pm 1, \pm 2, \ldots$ and $|-\gamma_{0,1}/2\gamma_{+1,-1}|\leqq 1$.

1. In case of $\gamma_{0,1}>0$ and if $x=2m_0\pi$ ($2m_0$ is even), from (5),

$$f''(2m_0\pi)=-(8/3)\gamma_{0,1}\{\cos(m\pi/2)+1/2\},$$

so we have

$$f''(2m_0\pi)<0 \text{ (maximum) when } -1/2<\cos(m\pi/2)\ [\leqq 1],$$

and

$$f''(2m_0\pi)>0 \text{ (minimum) when } [-1\leqq]\cos(m\pi/2)<-1/2.$$

Next if

$$x=(2m_0+1)\pi \quad \text{where} \quad (2m_0+1) \text{ is odd},$$

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(2) A. Lohmann, Optica Acta, **9** (1962), 1.

we have

$$f''\{(2m_0+1)\pi\}<0 \text{ (maximum) when } 1/2<\cos(m\pi/2)[\leqq 1],$$

and

$$f''\{(2m_0+1)\pi\}>0 \text{ (minimum) when } [-1\leqq]\cos(m\pi/2)<1/2.$$

2. In a similar way, in case of $\gamma_{0,1}<0$ and if $x=2m_0\pi$, we have

$$f''(2m_0\pi)>0 \text{ (minimum) when } -1/2<\cos(m\pi/2)[\leqq 1],$$

and

$$f''(2m_0\pi)<0 \text{ (maximum) when } [-1\leqq]\cos(m\pi/2)<-1/2.$$

If $x=(2m_0+1)\pi$,

$$f''\{(2m_0+1)\pi\}>0 \text{ (minimum) when } 1/2<\cos(m\pi/2)[\leqq 1],$$

and

$$f''\{(2m_0+1)\pi\}<0 \text{ (maximum) when } [-1\leqq]\cos(m\pi/2)<1/2.$$

We will discuss next the case of

$$x=2n_0\pi\pm\cos^{-1}(-\gamma_{0,1}/2\gamma_{+1,-1}) \tag{6}$$

Similarly we have,

$$f''\left\{2n_0\pi\pm\cos^{-1}\left(-\frac{\gamma_{0,1}}{2\gamma_{+1,-1}}\right)\right\}=\frac{8\gamma_{0,1}}{3\cos(m\pi/2)}\cdot\left(\cos\frac{m\pi}{2}-\frac{1}{2}\right)\left(\cos\frac{m\pi}{2}+\frac{1}{2}\right).$$

Under the range of $|-\gamma_{0,1}/2\gamma_{+1,-1}|\leqq 1$ where (6) is the root of $f'(x)=0$,

3. when $\gamma_{0,1}>0$, we have

$$f''(\alpha)<0 \text{ (maximum) when } [-1\leqq]\cos(m\pi/2)<-1/2,$$

and

$$f''(\alpha)>0 \text{ (minimum) when } 1/2<\cos(m\pi/2)[\leqq 1],$$

with $\alpha=2n_0\pi\pm\cos^{-1}(-\gamma_{0,1}/2\gamma_{+1,-1})$.

4. Similarly when $\gamma_{0,1}<0$, we have

$$f''(\alpha)>0 \text{ (minimum) when } [-1\leqq]\cos(m\pi/2)<-1/2,$$

$$f''(\alpha)<0 \text{ (maximum) when } 1/2<\cos(m\pi/2)[\leqq 1].$$

We collect together the calculated results into Table 1. $\gamma_{0,1}$ in the first file means the complex degree of coherence in two-beam interference and it depends upon the primary slit width $2w'$ as shown in Fig. 4. Under our experimental conditions we actually have $\gamma_{0,1}>0$ when $2w'=0\sim 90\mu$, $180\mu\sim 270\mu$, ... and $\gamma_{0,1}<0$ when $2w'=90\mu\sim 180\mu$, $270\mu\sim 360\mu$, ... The positions with the principal extreme value in the second file reverse by $\gamma_{0,1}\gtrless 0$, it means the positions with the principal maximum (minimum) when $\gamma_{0,1}>0$ change into the positions with the principal minimum (maximum) when $\gamma_{0,1}<0$.

In three-beam interference, we sometimes have the third terms in the second file, which situate with symmetry at principal extreme values and they do give fringe disorders never existing in two-beam interference.

The third file shows the conditions for maximum or minimum and the fourth

Table 1. Calculated conditions between fringe disorder and width of primary slit source.

Sign of $\gamma_{0,1}$	Positions with extreme value	Conditions for max. or min.	Max or min	Ranges of $2W'$ with max and min including conditions of $\gamma_{0,1} \gtrless 0$.	Calculated ranges of $2W'$ based on $\lambda = 0.59\,\mu$, $f_1 = 1510$ mm and $h = 10$ mm (μ unit)
$\gamma_{0,1} > 0$	(1) $x = 2m_0\pi$ (top)	$-\frac{1}{2} < \cos\frac{m\pi}{2} (\leq 1)$	max	$0 \leq 2W' < \frac{1}{3}P$ and $P + n_0P < 2W' < \frac{4}{3}P + n_0P$	$0 \leq 2W' < 60$ and $180 + n_0 \times 180 < 2W' < 240 + n_0 \times 180$
		$(-1 \leq) \cos\frac{m\pi}{2} < -\frac{1}{2}$	min	$\frac{1}{3}P + n_0P < 2W' < \frac{1}{2}P + n_0P$	$60 + n_0 \times 180 < 2W' < 90 + n_0 \times 180$
	(2) $x = (2m_0+1)\pi$ (bottom)	$(-1 \leq) \cos\frac{m\pi}{2} < -\frac{1}{2}$	min	$\frac{1}{6}P + n_0P < 2W' < \frac{1}{2}P + n_0P$	$30 + n_0 \times 180 < 2W' < 90 + n_0 \times 180$
		$\frac{1}{2} < \cos\frac{m\pi}{2} (\leq 1)$	max	$0 \leq 2W' < \frac{1}{6}P$ and $P + n_0P < 2W' < \frac{7}{6}P + n_0P$	$0 \leq 2W' < 30$ and $180 + n_0 \times 180 < 2W' < 210 + n_0 \times 180$
	(3) $x = 2m_0\pi \pm \cos^{-1}\left(-\frac{\gamma_{0,1}}{2\gamma_{+1,-1}}\right)$	$(-1 \leq) \cos\frac{m\pi}{2} < -\frac{1}{2}$	max	$\frac{1}{3}P + n_0P < 2W' < \frac{1}{2}P + n_0P$	$60 + n_0 \times 180 < 2W' < 90 + n_0 \times 180$
		$\frac{1}{2} < \cos\frac{m\pi}{2} (\leq 1)$	min	$0 \leq 2W' < \frac{1}{6}P$ and $P + n_0P < 2W' < \frac{7}{6}P + n_0P$	$0 \leq 2W' < 30$ and $180 + n_0 \times 180 < 2W' < 210 + n_0 \times 180$
$\gamma_{0,1} < 0$	(1) $x = (2m_0+1)\pi$ (top)	$(-1 \leq) \cos\frac{m\pi}{2} < -\frac{1}{2}$	max	$\frac{1}{2}P + n_0P < 2W' < \frac{5}{6}P + n_0P$	$90 + n_0 \times 180 < 2W' < 150 + n_0 \times 180$
		$\frac{1}{2} < \cos\frac{m\pi}{2} (\leq 1)$	min	$\frac{5}{6}P + n_0P < 2W' < P + n_0P$	$150 + n_0 \times 180 < 2W' < 180 + n_0 \times 180$
	(2) $x = 2m_0\pi$ (bottom)	$-\frac{1}{2} < \cos\frac{m\pi}{2} (\leq 1)$	min	$\frac{2}{3}P + n_0P < 2W' < P + n_0P$	$120 + n_0 \times 180 < 2W' < 180 + n_0 \times 180$
		$(-1 \leq) \cos\frac{m\pi}{2} < -\frac{1}{2}$	max	$\frac{1}{2}P + n_0P < 2W' < \frac{2}{3}P + n_0P$	$90 + n_0 \times 180 < 2W' < 120 + n_0 \times 180$
	(3) $x = 2m_0\pi \pm \cos^{-1}\left(-\frac{\gamma_{0,1}}{2\gamma_{+1,-1}}\right)$	$\frac{1}{2} < \cos\frac{m\pi}{2} (\leq 1)$	max	$\frac{5}{6}P + n_0P < 2W' < P + n_0P$	$150 + n_0 \times 180 < 2W' < 180 + n_0 \times 180$
		$(-1 \leq) \cos\frac{m\pi}{2} < -\frac{1}{2}$	min	$\frac{1}{2}P + n_0P < 2W' < \frac{2}{3}P + n_0P$	$90 + n_0 \times 180 < 2W' < 120 + n_0 \times 180$
	$m_0 = 0, \pm 1, \pm 2, \cdots$, $0 \leq \cos^{-1}\left(-\frac{\gamma_{0,1}}{2\gamma_{+1,-1}}\right) \leq \pi$			$P = 2\lambda f_1/h$, $n_0 = 0, 1, 2, \cdots$	

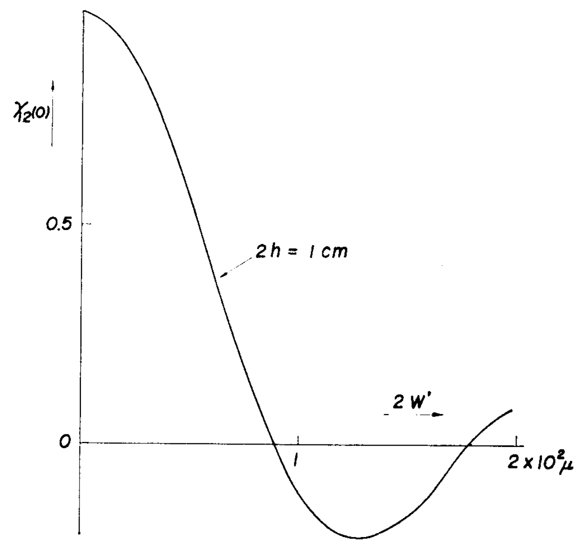

Fig. 4. Calculated degree of coherence as a function of the source slit width $2w'$. Result is based on $f = 1510$ mm, $\lambda = 0.589\,\mu$.

Table 2. Relation between the width of primary slit source and the pattern.

$\gamma_{0,1} > 0$ (0–90) | $\gamma_{0,1} < 0$ (90–180) | $\gamma_{0,1} > 0$ (180–270)

2W′(μ)	0–30	30–60	60–90	90–120	120–150	150–180	180–210	210–240	240–270
Top	max	max	min	max	max	min	max	max	min
Bottom	max	min	min	max	min	min	max	min	min
Two extreme values symmetrically situated on both sides of top or bottom	min	no existence	max	min	no existence	max	min	no existence	max

2W′(μ)	0–30	30–60	60–90	90–120	120–150	150–180	180–210	210–240	240–270
Neighbourhood of top	o	o	x	o	o	x	o	o	x
Neighbourhood of bottom	x	o	o	x	o	o	x	o	o

10μ 35μ 40μ 80μ 110μ 135μ 165μ 170μ

o--- without disorders
x--- with disorders

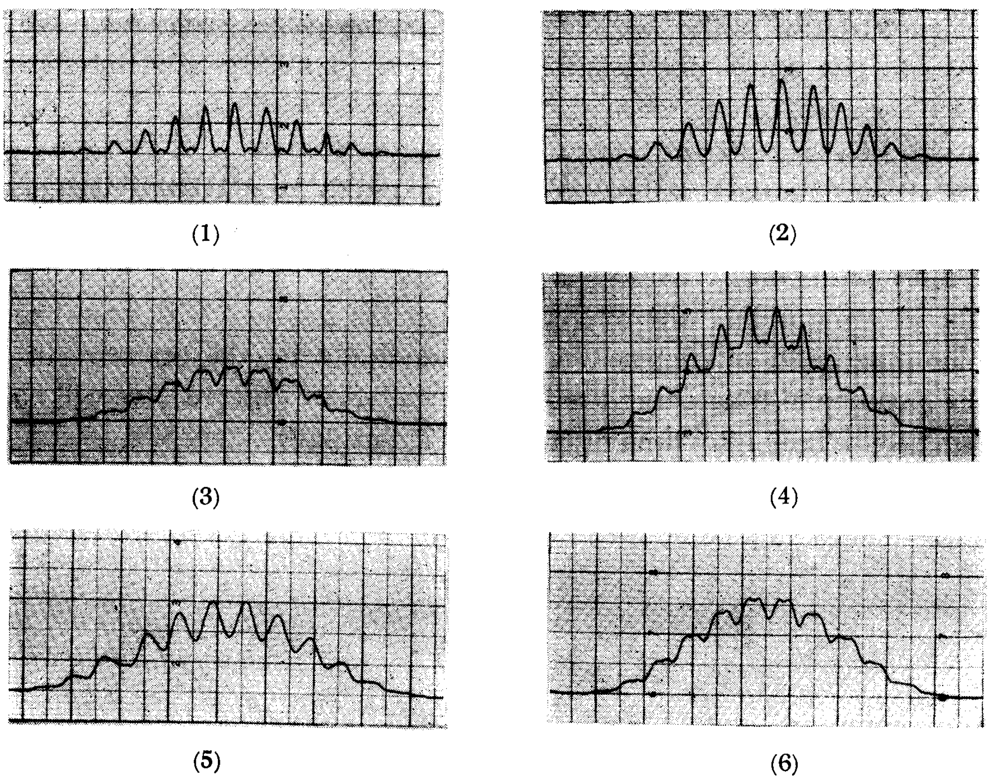

Fig. 5. Observed intensity distribution curves. Same as Fig. 3, except $2w' = 10\,\mu$, $35\,\mu$, $80\,\mu$, $110\,\mu$, $135\,\mu$ and $165\,\mu$ in the order.

file shows the situations which depend upon the range of $2w'$ including conditions of $\gamma_{0,1} \gtrsim 0$ shown in the fifth file. In the sixth file the ranges of $2w'$ under our experimental condition are shown.

Table 2 concretely shows calculated results in Table 1. No fringe disorders exist in the neighbourhoods of top with maximum, while fringe disorders exist there with minimum. Similarly, no fringe disorders exist in the neighbourhoods of bottom with minimum, while fringe disorders exist there with maximum. We show symbolically the situations in the neighbourhoods of top and bottom in Table 2 bellow.

It will be concluded that fringe disorders depend upon the primary slit width and they exist either in the neighbourhoods of principal intensity maxima or of principal minima and we have no primary slit width range with fringe disorders in both. Fig. 5 shows observed patterns with higher resolving power than those shown in Fig. 3 and arrows in Table 2 bottom show the primary slit widths in Fig. 3 and Fig. 5. It will be seen that the situations of pattern have a good agreement with theoretical results.

IV. Conclusion

The observation of intensity distribution curves in three-beam interference gives us the following conclusions, which are explained theoretically:

(1) The intensity distribution curve in three-beam interference is, in general, different from that in two-beam interference, that is, the proper extreme values exist symmetrically in the neighbourhoods of principal extreme value and they look like to give fringe disorders which never exist in two-beam interference.

(2) There is a certain range of primary slit width without any fringe disorders, whose situation is similar to that in two-beam interference.

(3) Fringe disorders, if they exist, are either in the neighbourhoods of principal intensity maxima or of principal intensity minima and we have no primary slit width range with fringe disorders in both.

Though we can deal with analytically that the positions with proper extreme value in three-beam interference change periodically by the primary slit width and the quantity of fringe disorders is very small as we observed, we will not take them up here. They have been discussed by T. Suzuki[(3)].

(3) T. Suzuki, Bull. RISM Tohoku Univ., **13** (1964), 49.