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著者	TANAKA Kunihide, TANUMA Seiichi, FUKUROI Tadao
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Transverse Galvanomagnetic Effect of Bismuth Single Crystal in a Strong Magnetic Field*

Kunihide TANAKA, Seiichi TANUMA and Tadao FUKUROI

The Research Institute for Iron, Steel and Other Metals

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Synopsis

Transverse galvanomagnetic effects of bismuth single crystal are measured in a strong magnetic field up to about 100 kilo Oersted at 4.2, 3.0 and 1.8°K. And the twelve components of the galvanomagnetic tensor are obtained with respect to the magnetic field dependence. Furthermore the behaviors of the galvanomagnetic tensor components near the quantum limit of the magnetic quantization are studied, expecting that they can lend themselves to analyse the energy band structure.

In a strong magnetic field, the amplitudes of the oscillatory part of the galvanomagnetic tensors are nearly temperature independent, and the behaviors of Hall effect appear to be different from the expected one from the classical theory of the two bands model.

I. Introduction

In the magnetic field bismuth crystal shows the particular oscillatory behaviors with the change of the field strength at low temperatures, not only in the magnetic susceptibility, but also in the electrical conductivity, the Hall effect and the thermal conductivity and so on. In 1930 Schubnikov and de Haas(1) found that at the temperature of liquid helium the electrical resistance of a bismuth single crystal changed oscillatory its magnitude with the magnetic field strength. Since then many investigators(2)~(7) had measured the oscillatory galvanomagnetic effects of various metals in the magnetic field. Theories of the galvanomagnetic effect in a strong magnetic field(8)~(14) have shown that the observed oscillatory behaviors in various cases arise from the result of magnetic quantization of the electron orbits in the conduction band much as in the de Haas-van Alphen effect in the magnetic susceptibility.

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This experiment concerns with the magnetic field dependence of oscillatory and non-oscillatory parts of the galvanomagnetic tensor and the behaviors of magnetoresistance and Hall effect in a very strong magnetic field near the quantum limit. For that purpose, the magnetic field is demanded to be as larger as possible from the following requirements.

$$\omega\tau \gg 1 \quad \text{and} \quad \beta H = \hbar\omega \gg kT. \tag{1}$$

where $\omega = \frac{eH}{m^* c}$, $\beta = \frac{e\hbar}{m^* c}$ and τ is the time of relaxation. Owing to these conditions, it is desirable that the specimen should be the purest one in order that the relaxation time is sufficiently long, and the effective mass of carriers is smal so that the cyclotron frequency ω and the effective Bohr magneton β are to be large enough. Furthermore, very low temperature experiment is necessary to fulfil the requisitions. Moreover, the Fermi energy ζ of the substance is to be adequately small that the quantum limit condition $\zeta \lesssim \hbar\omega$ may be easily realized. And the pure bismuth single crystal is considered to be one of the substances which satisfy the above-mentioned requirements.

II. Experimental procedure

1. Apparatus

For the purpose of getting a magnetic field intensity higher than 100 KOe, a solenoid, which was made by winding a copper wire of S.W.G. #30, is immersed in liquid helium and about 150A at the peak value during a half cycle of commercial alternating current is flowed through it as referred to below. The dimensions and constants of the solenoids used are listed in Table 1.

Table 1. Geometrical shape and size of the solenoids used to get a strong magnetic field.

Solenoid No.	Inner diameter (cm)	External diameter (cm)	Length of solenoid (cm)	Number of turns	Coil constant k
1	0.78	1.68	2.02	1,200	0.856
2	0.8	1.52	2.20	1,225	0.878
3	0.8	1.82	2.02	1,228	0.833

We used the discharge of a kind of ignitron called "Sendaitron" as the source of pulsive currents, which was connected with the recorder to register the magnitude of the currents through the solenoid. The ignitor of Sendaitron, which consists of a thin walled glass bulb containing silicon-carbide powder (referred in Figs. 1, 2) is immersed in the mercury pond of the tube. Hence the tube is ignited by the discharge of cold emission towards mercury through the thin glass wall by applying a pulsive voltage on the ignitor. By applying a pulsive voltage on the ignitor, a half wave current discharge during 10 msec or less flows through the circuit, i.e., a direct current of a half phase sine wave flows through the solenoid from 200 volts main line of 50 c/sec which gives rise to an intense magnetic field proportional to the current. And we recorded the change of the current with

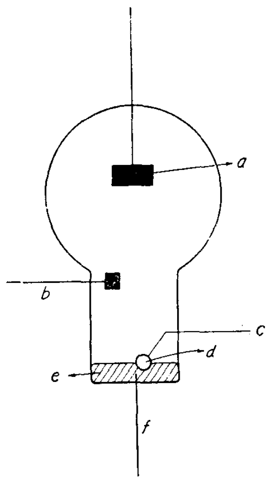

Fig. 1. Structure of Sendaitron.
a : anode, *b* : sub-cathode, *c* : ignitor, *d* : silicon-carbonide powder in thin walled glass bulb, *e* : mercury pond, *f* : cathode.

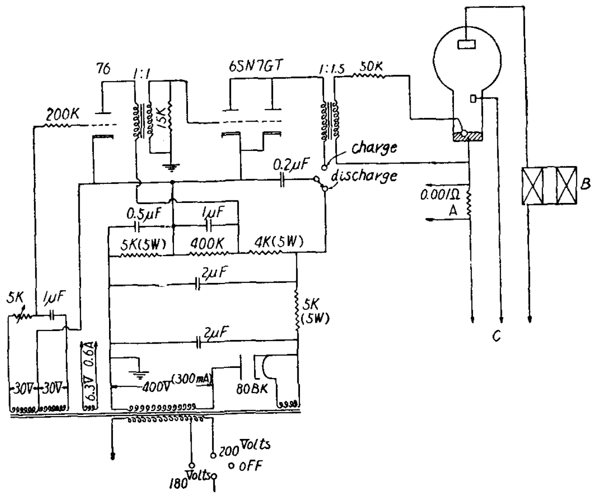

Fig. 2. Igniting circuit of Sendaitron rectifier tube.
A : standard resistance.
B : solenoid in Dewar's vessel.
C : three phase current of 200V.

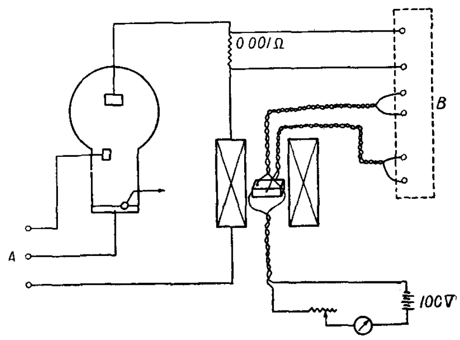

Fig. 3. The circuit of measuring galvanomagnetic effects in a strong magnetic field.
A : AC source of 200V.
B : electromagnetic oscillograph.

either an electromagnetic oscillograph or a dual beam synchroscope along with the changes of the Hall voltage or the potential drop of the magnetoresistance induced by the magnetic field.

The circuit is illustrated in Fig. 3; the mixing of the parasitic voltage developed by the induction effect is avoided by twisting the lead-wires. We estimate the strength of the magnetic field from the trace of the coil current on the oscillogram and plot the relations between the magnetic field strength and the magnitude of the phenomena corresponding to the Hall voltage or the magnetoresistance potential drop synchronous to the magnetic field, and by taking a mean of measured values corresponding for both polarities of magnetic field, we can obviate any parasitic components which may contaminate into the Hall effect and the magnetoresistance. The strength of magnetic field is estimated by the intensity of current. The formula used in this calculation is the one by Wall[(15)], that is as follows:

$$H=k\frac{4\pi}{10}wi, \tag{2}$$

where i the current in ampere, w the number of turns per unit length of the solenoid, and k the correction coefficient defined by dimension of the solenoid that is given by

$$k=\frac{2b}{4d}\ln\left\{\frac{(a+d)+\sqrt{(a+d)^2+b^2}}{(a-d)+\sqrt{(a-d)^2+b^2}}\right\} \tag{3}$$

(15) T. F. Wall, J. Inst. Elect. Engrs., **64** (1926), 745.

in which a, $2d$ and $2b$ denotes the mean radius, the mean radial depth, and the length of solenoid, respectively.

In this experiment, the solenoid is immersed directly into liquid helium, so that the electrical resistance of the solenoid becomes as small as its residual resistance of copper wire, hence its impedance can be regarded to be nearly equal to its inductance and also the Joule's heat will be dissipated very quickly. Thus we can easily get a strong magnetic field as high as about 100 KOe.

2. Specimen

In order to get the specimen as pure as possible, pure bismuth ingot, which was made from Tadanac bismuth having the nominal purity of 99.9999 per cent, was further refined by zone melting repeated twenty-four times (Bi I). The specimens which have definite orientations, were cut out from a large single crystal grown by the Bridgman method. And also for the sake of comparison, an impure single crystal was grown from the raw material of a nominal purity of 99.99 per cent as obtained from Nihon-kogyo Co. (Bi II). The specimens were cut out first to a dimension of $0.3 \times 0.3 \times 1.5\,\mathrm{cm}^3$ and polished up with emery-paper and then etched with conc. HNO_3 in order to remove the treated layer on the surface.

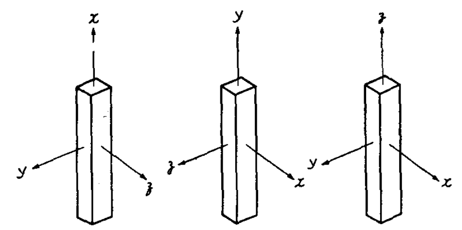

Fig. 4. Crystallographic orientations of specimens.

A rhombohedral crystal, such as bismuth, has three principal crystallographic orientations, i.e. those of parallel to the trigonal, binary and bisectrix axes. The said orientations were determined by the light figure method and three specimens such as shown in Fig. 4 were prepared, whose sizes are $0.18 \times 0.2 \times 0.7\,\mathrm{cm}^3$ respectively. Here we designate the binary, bisectrix and trigonal axes as x, y and z axes respectively.

In this experiment, the following six cases for the orientation with regard to the sample current (J) and the magnetic field (H) are considered.

$$\begin{cases} H /\!/ z \\ J /\!/ x, \end{cases} \begin{cases} H /\!/ z \\ J /\!/ y, \end{cases} \begin{cases} H /\!/ y \\ J /\!/ z, \end{cases} \begin{cases} H /\!/ y \\ J /\!/ x, \end{cases}$$

$$\begin{cases} H /\!/ x \\ J /\!/ y, \end{cases} \begin{cases} H /\!/ x \\ J /\!/ z. \end{cases}$$

And every possible care was taken for setting the specimens at the same position in the center of the solenoid, by mounting the specimen on a holder made of acrylic acid resin as shown in Fig. 5.

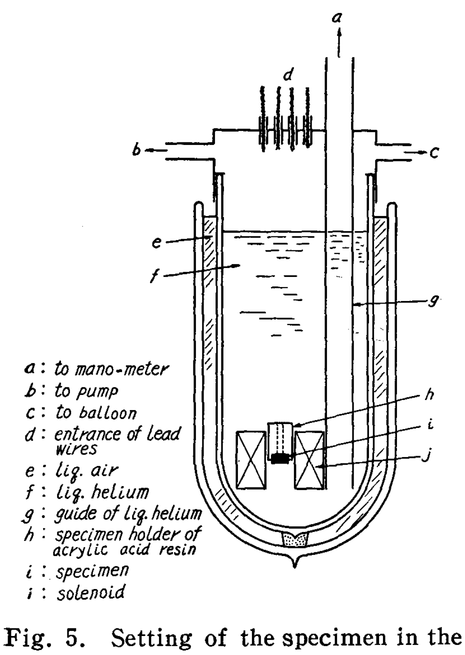

Fig. 5. Setting of the specimen in the solenoid.

III. Experimental results

In general, the following relationship exists between the electrical field and the electrical current density.

$$E_i=\sum_k \rho^s_{ik}(H)J_k+\sum_k \rho^a_{ik}(H)J_k,\ \rho^{s,a}_{ik}(H)=\rho^{s,a}_{ki}(-H) \tag{4}$$

where ρ^s_{ik} and ρ^a_{ik} are the symmetric and antisymmetric components of galvanomagnetic tensor. By this experiment, we can obtain the following components of galvanomagnetic tensor of bismuth.

$$\rho_{xx}(y),\ \rho_{xx}(z),\ \rho_{yy}(z),\ \rho_{yy}(x),\ \rho_{zz}(y),\ \rho_{zz}(x),$$
$$\rho_{xy}(z),\ \rho_{yx}(z),\ \rho_{zx}(y),\ \rho_{xz}(y),\ \rho_{yz}(x),\ \rho_{zy}(x).$$

Each component of upper row means the transverse magnetoresistance and of lower row means the Hall effect.

In Fig. 6, if $J\parallel x$, $H\parallel z$ and the potential difference between A and B be V_R. We get

$$\rho_{xx}(z)=\overline{V}_R\cdot\frac{1}{J}\cdot\frac{a\cdot b}{l} \tag{5}$$

in which l is a distance between A and B and $\overline{V}_R=\frac{1}{2}(V_R(H)+V_R(-H))$. If the Hall voltage between C and D is given by $\overline{V}_H$, we get

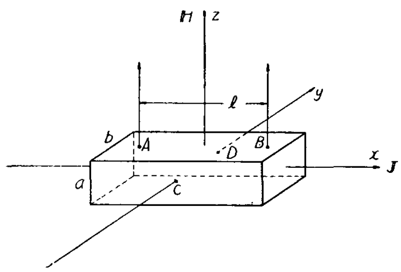

Fig. 6. Setting of probes on the specimen.

$$\rho^a_{yx}(z)=\overline{V}_H\cdot\frac{a}{J} \tag{6}$$

where $\overline{V}_H=\frac{1}{2}(V_H(H)-V(-H))$ in order to separate the magnetoresistive components in the transverse voltage from the Hall components. Furthermore the Hall coefficient R is obtained by

$$R=\rho^a_{yz}(z)/H \tag{7}$$

In this experiment, we did not measure the longitudinal galvanomagnetic tensor components, $\rho_{xx}(x)$, $\rho_{yy}(y)$, $\rho_{zz}(z)$. Notwithstanding the measurements were made at 4.2°, 3°, 1.8°K, the temperature dependence of the phenomena in such a high magnetic field could not be observed. So we present here only the case at 1.8°K, of which the original records are shown in Fig. 7 (a) and (b), the former is the record taken by an electromagnetic oscillograph and the latter is that by a dual synchroscope.

The residual resistance of the pure crystals (Bi I) at 4.2°K are respectively,

$$\rho_{xx}(0)=1.21\times10^{-6}\Omega\,\text{cm},\qquad \rho_{yy}(0)=1.39\times10^{-6}\Omega\,\text{cm},$$
$$\rho_{zz}(0)=1.03\times10^{-6}\Omega\,\text{cm},$$

and the ratios of the residual resistance to the resistance at room temperature are respectively,

$$\rho_{xx}(0)_{4.2^\circ\text{K}}/\rho_{xx}(0)_{r.t.}=9.54\times10^{-3},\ \rho_{yy}(0)_{4.2^\circ\text{K}}/\rho_{yy}(0)_{r.t.}=1.37\times10^{-2},$$
$$\rho_{zz}(0)_{4.2^\circ\text{K}}/\rho_{zz}(0)_{r.t.}=6.13\times10^{-3}.$$

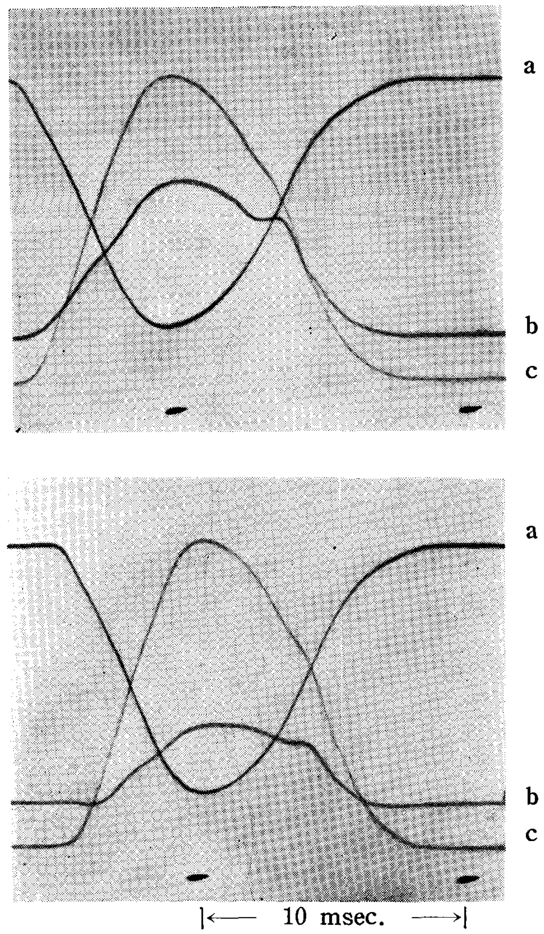

Fig. 7 (a). The original traces of the phenomena in the magnetic field of opposite polarities by the electromagnetic oscillograph (J//bis. H//trig.) a : current through the solenoid. b : Hall effect. c : magnetoresistance effect.

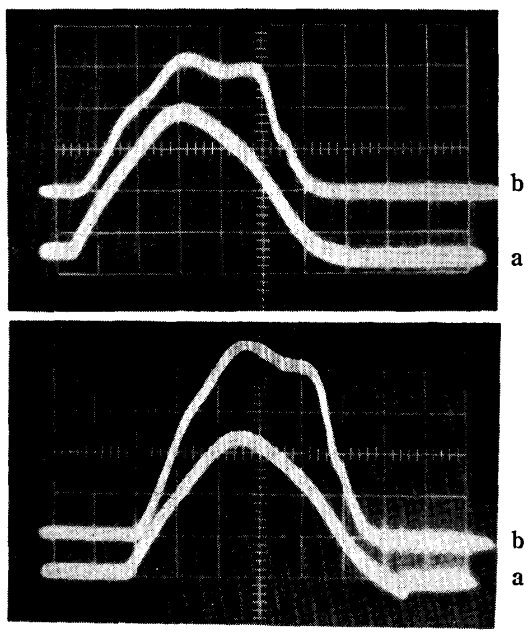

Fig. 7 (b). The original traces of the change of Hall effect and magnetic field by the dual beam synchroscope (J//bis. H//trig.)
a : current through the solenoid. b : Hall effect.

The magnetoresistances of the same crystals in about 80KOe at 1.8°K are

$$\Delta\rho_{xx}(y)/\rho_{xx}(0)=6.8\times10^5,\qquad \Delta\rho_{yy}(z)/\rho_{yy}(0)=2.9\times10^5,$$
$$\Delta\rho_{zz}(x)/\rho_{zz}(0)=2.11\times10^5.$$

While in the case of impure samples, residual resistance at 4.2°K and the ratios of residual resistance to the resistance at room temperature are as follows.

$$\rho_{xx}(0)_{4.2^\circ K}=3.96\times10^{-5}\Omega\ \mathrm{cm},\qquad \rho_{yy}(0)_{4.2^\circ K}=2.17\times10^{-5}\Omega\ \mathrm{cm},$$
$$\rho_{zz}(0)_{4.2^\circ K}=1.86\times10^{-5}\Omega\ \mathrm{cm},$$

and

$$\rho_{xx}(0)_{4.2^\circ K}/\rho_{xx}(0)_{290.5^\circ K}=3.07\times10^{-1},\qquad \rho_{yy}(0)_{4.2^\circ K}/\rho_{yy}(0)_{290.5^\circ K}=1.74\times10^{-1},$$
$$\rho_{zz}(0)_{4.2^\circ K}/\rho_{zz}(0)_{290.5^\circ K}=9.12\times10^{-2}.$$

The magnitude and the magnetic field dependence of the magnetroresistance are as shown in Fig. 8 (a) and (b) that were obtained from the traces on the three elements electromagnetic oscillograph of Yokogawa Electric Co.

The monotonic parts of the magnetoresistance of the purer bismuth (Bi I) vary sharply with the magnetic field at low temperatures. On comparing Fig. 8 (a) of Bi I and (b) of Bi II, it is found that the magnetoresistance of Bi I is about one hundred times larger than that of Bi II and the magnetic field dependencies of both specimens are clearly different. In the case of Bi II, the magnetoresistances increase proportional to the square root of H in the whole range from 10 to 100 KOe, while in the case of Bi I they vary as shown in Table 2. In an intermediate magnetic field, the dependence on H changes continuously. From the above table, it is known that the magnetic field dependence of magnetoresistance get less noticeable as the magnetic field increases; it is, however, not yet evident from this data alone whether or not the exponent of H decreases below 0.5 and even tends to zero, i.e. saturates perfectly, in a stronger magnetic field.

The results of the measurements of Hall effects on an electromagnetic oscillograph are displayed in Fig. 9. From these results, the behavior of Hall effect seems to be similar to the case of the magnetoresistance. And the values of Hall field of the pure bismuth (Bi I) are almost 10 times larger than the impure bismuth (Bi II), and these amounts seem to approach the constant values as

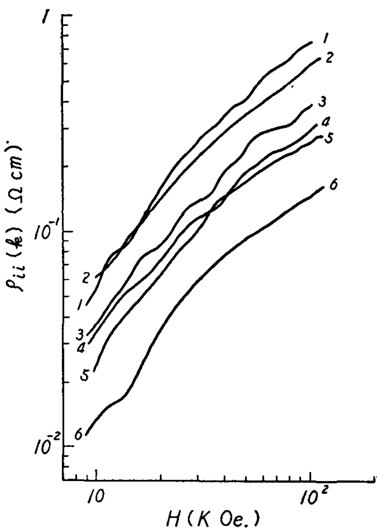

Fig. 8 (a). The magnetic field dependence of transverse magnetoresistance of Bi I at 1.8°K.
1: $\rho_{zz}(x)$, 2: $\rho_{zz}(x)$, 3: $\rho_{xx}(z)$, 4: $\rho_{yy}(z)$, 5: $\rho_{xx}(y)$, 6: $\rho_{yy}(x)$.

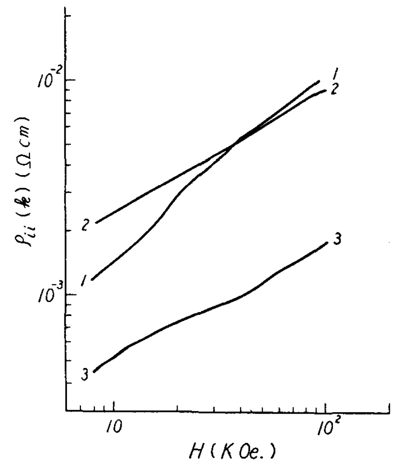

Fig. 8 (b). The magnetic field dependence of transverse magnetoresistance of Bi II at 1.8°K.
1: $\rho_{xx}(y)$, 2: $\rho_{zz}(x)$, 3: $\rho_{xx}(z)$.

Table 2. Dependence of the magnetoresistance on the magnetic field of Bi I. $\rho_{ii}(k) \propto H^a$.

$\rho_{ii}(k)$ \ H	10~20KOe	25~45KOe	50~60KOe
$\rho_{xx}(y)$	$H^{1.3}$	$H^{0.9}$	$H^{0.58}$
$\rho_{xx}(z)$	$H^{1.39}$	$H^{0.81}$	$H^{0.36}$
$\rho_{yy}(z)$	$H^{1.4}$	$H^{0.83}$	$H^{0.46}$
$\rho_{yy}(x)$	$H^{1.58}$	$H^{0.81}$	$H^{0.68}$
$\rho_{zz}(x)$	$H^{1.15}$	$H^{0.97}$	$H^{0.71}$
$\rho_{zz}(y)$	$H^{1.48}$	$H^{0.94}$	$H^{0.68}$

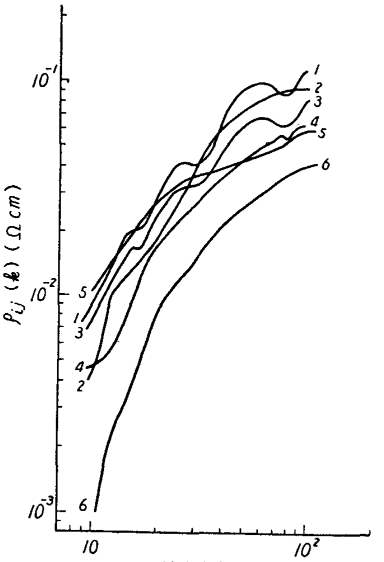

Fig. 9 (a). The magnetic field dependence of Hall field of Bi I at 1.8°K.
1 : $\rho_{yx}(z)$, 2 : $\rho_{zx}(y)$, 3 : $\rho_{xy}(z)$,
4 : $\rho_{yz}(x)$, 5 : $\rho_{xz}(y)$, 6 : $\rho_{zy}(x)$.

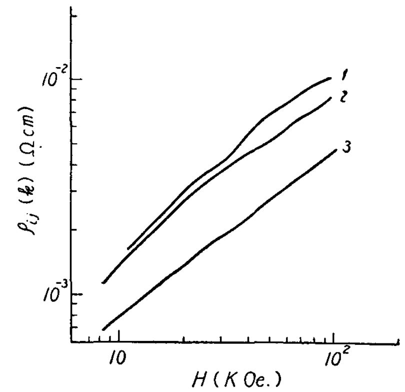

Fig. 9 (b). The magnetic field dependence of Hall field of Bi II at 1.8°K.
1 : $\rho_{zx}(y)$, 2 : $\rho_{yz}(x)$, 3 : $\rho_{yx}(z)$.

Table 3. The dependence of Hall field on the magnetic field. $\rho^a_{ij}(k) \propto H^a$.

$\rho^a_{ij}(k)$ \ H	10~20KOe	25~40KOe	50~80KOe
$\rho_{xz}(y)$	$H^{1.4}$	$H^{0.5}$	$H^{0.4}$
$\rho_{zx}(y)$	$H^{1.9}$	$H^{1.9}$	$H^{0.3}$
$\rho_{yz}(x)$	$H^{2.7}$	$H^{1.3}$	$H^{0.5}$
$\rho_{zy}(x)$	$H^{1.8}$	$H^{1.0}$	$H^{0.6}$
$\rho_{xy}(z)$	$H^{1.7}$	$H^{0.9}$	$H^{0.4}$
$\rho_{yx}(z)$	$H^{2.1}$	$H^{1.0}$	$H^{0.3}$

the magnetic field increases. The magnetic field dependence of $\rho^a_{ij}(k)$ of Bi I are as given in Table 3. According to the symmetric relation of Hall effect, the conditions $|\rho^a_{xy}(z)| = |\rho^a_{yx}(z)|$, $|\rho^a_{zy}(x)| = |\rho^a_{yz}(x)|$, $|\rho^a_{zx}(y)| = |\rho^a_{xz}(y)|$ must be satisfied.

While these requisitions are not completely satisfied in the data of Fig. 9(a). These disagreements are likely due to the missetting of the specimens in the solenoid. On the other hand, $\rho_{ij}^{a}(k)$ of Bi II increases proportional to $H^{0.7}$ through the whole range of magnetic field from 10 to 100KOe as shown in Fig. 9 (b).

The magnetoresistance and Hall effect of the specimens cut from the same single crystal of Bi I was measured in a low magnetic field below 10 KOe by Mase and Tanuma[16]. By their data as shown in Figs. 10, 11, the magnetoresistance and Hall field vary in proportion to $H^{1.7}$ and $H^{2.1\sim2.4}$ respectively. On comparing

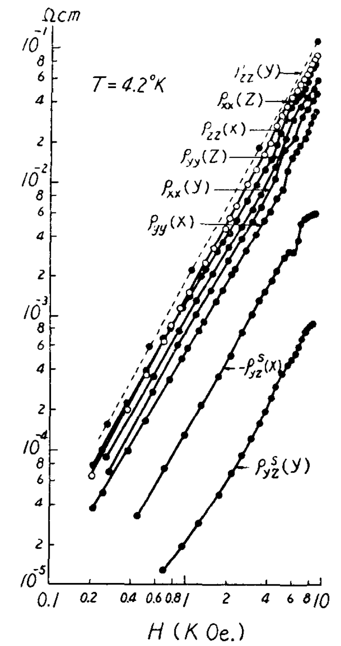

Fig. 10. Symmetric tensor components *vs.* *H* curves.

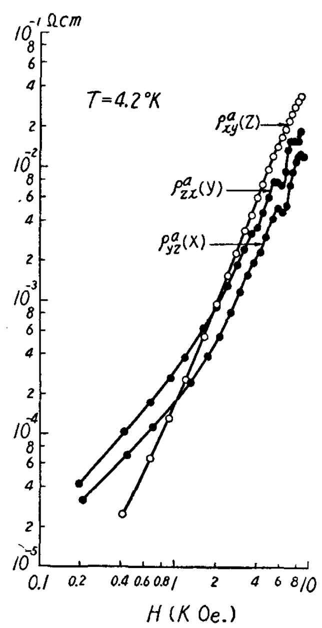

Fig. 11. Antisymmetric tensor components *vs.* *H* curves.

$\rho_{ii}(k)$ and $\rho_{ij}(k)$ in the present experiment with their data the values at 10KOe in this experiment almost coincide with the values at 7KOe in their case. And both curves can be connected smoothly, if the intensity of magnetic field effective in the solenoid is reduced by 30 per cent admitting the overestimation of such an extent. In this experiment such an overestimation may be probable since we have regarded that the magnetic field in the central space of solenoid is homogeneous where the specimen is placed. Then, Hall effects of pure samples (Bi I) are proportional to H^2 in the weak field and tend to saturate in the higher field.

In Fig. 12 (a) are shown the Hall coefficients $\rho_{ij}^{a}(k)/H$ of Bi I, which increase with field up to 20 KOe and become constant values from 20 to 40 KOe except for a certain undulations due to de Haas-van Alphen effect, then decrease in the range beyond 40 KOe. And in the case of Bi II, Hall coefficients show rather a tendency of decrease with field than remaining constant values in the whole range of field from 10 to 100 KOe as in Fig. 12 (b).

(16) S. Mase and S. Tanuma, Sci. Rep. RITU, **A12** (1960), 35.

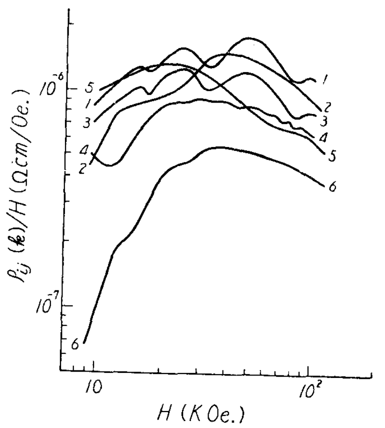

Fig. 12 (a). Hall coefficints of Bi I *vs.* *H* curves at 1.8°K.

1 : $\rho_{yx}(z)/H$, 1 : $\rho_{zx}(y)/H$, 3 : $\rho_{xy}(z)/H$, 4 : $\rho_{yz}(x)/H$, 5 : $\rho_{xz}(y)/H$, 6 : $\rho_{zy}(x)/H$.

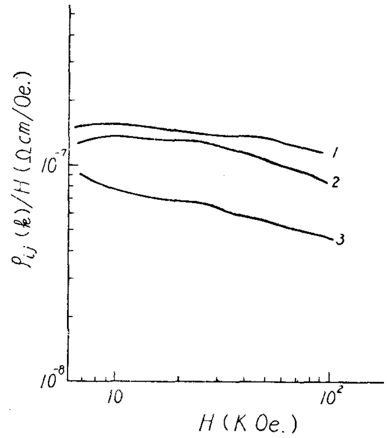

Fig. 12 (b). Hall coefficients of Bi II *vs.* *H* curves at 1.8°K.

1 : $\rho_{zx}(y)/H$, 2 : $\rho_{yz}(x)/H$, 3 : $\rho_{yx}(z)/H$.

The above data in this experiment were obtained by an electromagnetic oscillograph and the tendency that the Hall coefficient decreases in a strong field appeared to us rather incomprehensible and we feared that such a result might be caused from a low impedance of measuring apparatus. In our case, the electrical resistance of the coil of galvanometer in the electromagnetic oscillograph is 2 ohm, while electrical resistance of the specimen is about 1×10^{-4} ohm at $H=0$ and about 1×10^{-2} ohm at $H=100$ KOe, so that the change of input impedance due to magnetic field is too large in comparison with the internal impedance of the measuring apparatus.

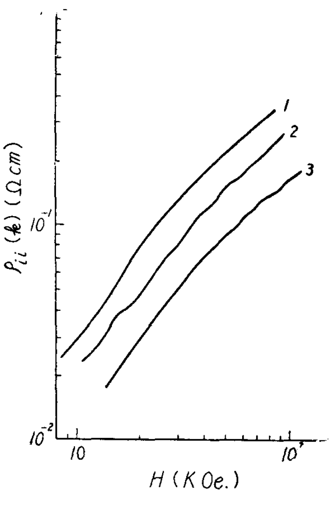

Fig. 13. The magnetic field dependence of magnetoresistance of Bi I at 1.8°K measured by a dual synchroscope.

1 : $\rho_{zz}(y)$, 2 : $\rho_{yy}(z)$, 3 : $\rho_{xx}(y)$.

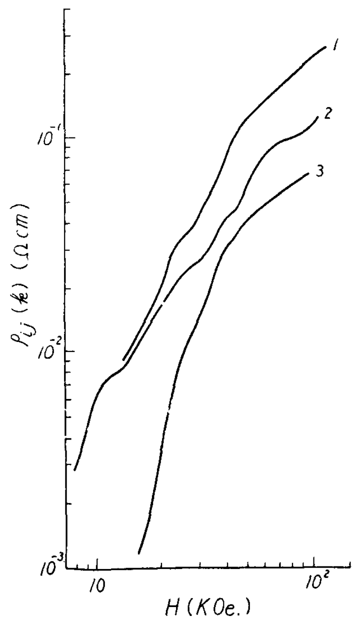

Fig. 14. The magnetic field dependence of Hall field of Bi I at 1.8°K measured by a dual synchroscope.

1 : $\rho_{zx}(y)$, 2 : $\rho_{xy}(z)$, 3 : $\rho_{xz}(y)$.

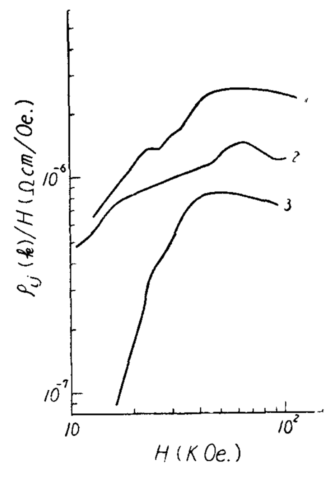

Fig. 15. Hall coefficient of Bi I *vs.* *H* curves at 1.8°K measured by a dual synchroscope.

1 : $\rho_{zx}(y)/H$, 2 : $\rho_{xy}(z)/H$, 3 : $\rho_{xz}(y)/H$.

So we repeated the measurements of $\rho_{xx}(y)$, $\rho_{zz}(y)$, $\rho_{yy}(z)$, $\rho_{zx}(y)$, $\rho_{xy}(z)$ and $\rho_{xz}(y)$ by use of a dual synchroscope of Iwasaki Inc. Type No. 5101. Since the input impedance of this apparatus can be selected appropriately below 1 megohm, there may leave no longer ambiguity in the result as in the case of the electromagnetic oscillograph. The results in this case are given in Figs. 13, 14, 15, which reveal, nevertheless, the similar tendency of galvanomagnetic tensor components with magnetic field as in the case of the electromagnetic oscillograph. Therefore we have ruled out the previous deduction that the Hall coefficient apparently decreases as the magnetic field is increased owing to the use of a low impedance apparatus.

IV. Discussion

The theories of galvanomagnetic effects of metals in a strong magnetic field have been worked out by several authors.(10),(11),(12),(13),(14) The theories, however, have not yet been completed to such a degree as to explain quantitatively the observed results for each metal. But we shall try to discuss the results of this experiment of bismuth by the knowledge of existing theories. Since Landau(8) had pointed out that kinetic energy of electrons in a magnetic field is quantized in a plane perpendicular to the direction of the field, Davydov and Pomeranchuck(10) obtained, making use of two bands model for bismuth, the expressions of the magnetoresistance in a strong magnetic field. The mechanism of scattering employed by them was the interaction of carriers with localized impurities and they calculated the transverse current on the basis of classical conception. Their expressions show an unlimited increase in the magnetoresistance with magnetic field but do not agree with the experimental results. Recently Argyres(13) worked out quantum mechanically the expressions of galvanomagnetic properties of an isotropic metal based upon one electron model in the temperature range in which the phonon scattering predominates. And his expressions show that the Hall coefficient is a linear function of the magnetic field and the resistance increases proportional to H^2 up to a sufficiently high magnetic field. Though his energy level scheme is too simple to deal with such a highly anisotropic substance as bismuth, certain qualitative comparison may be made concerning the behavior of the Hall effect and the magnetoresistance.

Here we try to proceed with the argument on the basis of Zilberman's expression(12) of galvanomagnetic effect for an isotropic two bands model of a metal in a strong magnetic field at low temperatures ($\omega\tau \gg 1$).

By assuming that exp $(2\pi^2 kT/\beta H) \gg 1$, $kT \ll \beta H$, $\beta H < E_0$ and $E_0 \gg kT$, the transverse magnetoresistance is given as follows,

$$\rho_H = \frac{CH^2(f_1+f_2)}{ec^2H^2(N_1-N_2)^2+cC^2(f_1+f_2)^2} \tag{8}$$

where

$$f_1 = \frac{8}{3}E_0^2 m_1^2 \left\{1+\frac{9}{40}\left(\frac{\beta_1 H}{E_0}\right)-\left(\frac{5\pi^2\sqrt{2}\,kT}{(\beta_1 H E_0)^{1/2}}\right)\exp\left(-\frac{2\pi^2 kT}{\beta_1 H}\right)\cos\left(\frac{2\pi E_0}{\beta_1 H}-\frac{\pi}{4}\right)\cdots\cdots\right\}$$

and f_2 is obtained from f_1 by replacing m_1 by m_2 and E_0 by $A_0 - E_0$, the subscripts

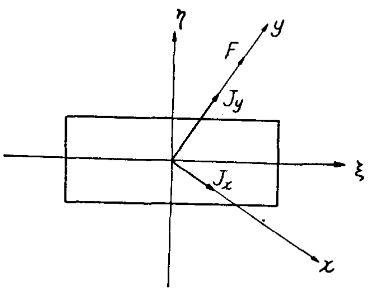

Fig. 16. The components of the electrical current and field in the specimen.

1 and 2 refer to the conduction band and the valence band respectively.

After the method by Borovik[17], Zilberman's expression of the Hall effect is described in terms of the ratio of the Hall field F_η and the electric field F_ξ parallel to the current flow, (Fig. 16)

$$\frac{F_\eta}{F_\xi}=\frac{J_x}{J_y}=\frac{-H(N_1-N_2)}{C(f_1+f_2)}=R\sigma H \tag{9}$$

Then the Hall coefficient would be

$$R=\frac{-(N_1-N_2)H^2}{ec^2H^2(N_1-N_2)+cC^2(f_1+f_2)^2} \tag{10}$$

where N_1 is the number of electrons per unit volume and N_2 is the number of holes.

Following these expressions, if $N_1 \fallingdotseq N_2$, the magnetoresistance increases up to the saturation and Hall field is proportional to H, and if $N_1=N_2$, the magnetoresistance increases infinitely in proportion to H^2 and the Hall field is expected to be very small or practically zero. Notwithstanding we must be cautious to apply these expressions to an anisotropic substance such as bismuth, we shall try to compare the general trend of the observed behavior of the galvanomagnetic tensor components with the above theory.

Following these expressions, under the reasonable assumption that N_1 is very nearly equal to N_2, still they are slightly different in our specimen Bi I, the results of magnetoresistance in the present experiment show a tendency of gradual saturation as expected from the theory; but the behaviors of the Hall effect and the Hall coefficient in the range of a strong field cannot be perfectly explained, because the Hall field increases with a power of H less than unity and the Hall coefficient decreases with the increase of H in a strong field.

Using the results of the conduction theory, we can correlate the elements of the conductivity tensor σ as follows.

$$\left.\begin{aligned}J_x&=\sigma_1E_x-\sigma_2E_y\\J_y&=\sigma_2E_x+\sigma_2E_y\end{aligned}\right\} \tag{11}$$

In terms of σ_1 and σ_2 the Hall coefficient R is given by

$$R=-\frac{1}{H}\frac{\sigma_2}{\sigma_1{}^2+\sigma_2{}^2} \tag{12}$$

For instance, if we take the case in that the magnetic field and the electric current are parallel to z and y axes respectively and the measurement of the Hall voltage is made in the direction parallel to x axis, the tensor component of the Hall coefficient can be expressed by following,

$$R_{xy}(z)=\rho^n_{xy}(z)/H=-\frac{\sigma^n_{xy}(z)}{H(\sigma_{xx}(z)^2+\sigma^n_{xy}(z)^2)}, \tag{13}$$

where

$$\sigma_{xx}(z)=\sum_l\frac{n_le\mu_l}{1+\omega_l{}^2\tau_l{}^2}$$

(17) E. S. Borovik, J. Exptl. Theoret. Phys. U.S.S.R., **30** (1956), 262.

$$\sigma^{a}_{xy}(z)=\sum_{l}\frac{n_l ec}{H}\cdot\frac{\omega_l^2\tau_l^2}{1+\omega_l^2\tau_l^2}$$

and n_l is the number of electrons or holes in l th band and μ_l is the mobility tensor in this band. τ_l and ω_l are the relaxation time and the cyclotron frequency in l th band, respectively.

When the effective masses of electrons and holes are small and the magnetic field is weak, i.e. $\omega_l\tau_l \ll 1$, the expression of Eq. (13) is considered to become roughly

$$R_{xy}(z)\approx-\frac{\sum_l \frac{n_l ec}{H^2}\cdot\omega_l^2\tau_l^2}{\sum_l n_l^2e^2\mu_l^2+\sum_l\frac{n_l^2e^2c^2}{H^2}\omega_l^4\tau_l^4} \tag{14}$$

From this equation, $R_{xy}(z)$ is a constant value at $H\approx 0$, and decreases inversely proportional to H^2. On the other hand, as the magnetic field is increased, the condition $\omega_l\tau_l \ll 1$ must be replaced by $\omega\tau \gtrsim 1$. In the case that $\omega_l\tau_l \gg 1$, the Hall coefficient becomes as follows from Eq. (13),

$$R_{xy}(z)\approx\frac{\Delta n\, H^2}{A\nu_s^2+B(\Delta n)^2H^2}, \tag{15}$$

which is derived by Mase and Tanuma[16], where ν_s is the number of scattering centers and Δn is the difference of the concentrations of electrons and holes. According to Eq. (15), when $\Delta n \neq 0$, $R_{xy}(z)$ increases proportional to H^2 at first and then becomes saturated to a constant value in a strong magnetic field. The present experiment was carried out up to the region of magnetic field satisfying $\omega_l\tau_l \gg 1$ for the light mass carriers.

If there is another hole having heavy mass for which the condition $\omega_h\tau_h \ll 1$ is met in the magnetic field of which intensity corresponds to $\omega_l\tau_l \gg 1$ for light mass carriers, this heavy mass hole contributes markedly to the Hall coefficient in such an intense magnetic field under consideration here. That is to say, $R_{xy}(z)$ must be expressed in a complicated combination of Eq. (15) for light mass carriers and Eq. (14) for heavy mass holes and it causes to decrease with increasing magnetic field.

The presence both of electrons having very light mass and of holes having light mass are found in bismuth by the studies of the cyclotron resonance and the galvanomagnetic phenomena[16], but the presence of a heavy hole has not yet been observed in any experiment, and we are not at all certain whether the assumption of the heavy hole is right or not in bismuth. The observation of the Hall effect in an intense magnetic field as high as 1000 KOe would be of use to examine this.

The oscillatory phenomena in bismuth single crystal have been investigated by several researchers,[1]~[7],[18],[19] and it is well known that the oscillation arises from the quantization of electron orbit in a magnetic field. The several results of the period are listed in Table 4.[20]

(18) D. Shoenberg, Phil. Trans. Roy. Soc., **A245** (1952), 1.
(19) M. C. Steele and J. Babiskin, Phys. Rev., **98** (1955), 359.
(20) After Seitz and Turnbull, *Solid State Physics*, Vol. 9, p. 282.

Table 4. Period of oscillations $[\Delta \frac{1}{H}]$ in bismuth crystal (in units of 10^{-5} Oersted^{-1}).

Properties referred / Orientation	χ Shoenberg[18]	R_H Gerritsen and de Haas[2]	$G, \kappa, \rho,$ Steele and Babiskin[19]	R_H Reynolds et al.[4]	R_H and ρ_t Connel and Marcus[6]	R_H Overton and Berlincourt[5]	ρ_t Alers and Webber[3]	ρ_t Babiskin[7]
H//trig.	1.4	—	—	1.5	1.6	1.57	—	1.57
H//bin.	7.4	~7	7.1	—	7.5	—	—	7.1
//	0.25	—	—	—	—	—	—	0.3
H//bis.	4.3	~5	4.1	—	4.2	—	4.0	—
//	8.5	—	8.2	—	8.8	—	7.9	—

We could observe clearly the period of the oscillation with magnetic field parallel to the trigonal axis, while in the case that the field is parallel to the binary axis, the period is hardly recognized, inasmuch as the period is very short and the amplitude is very small. The periods, which are analysed from this data, are 1.65×10^{-5} Oersted^{-1} for the case of H//trig. and 0.2×10^{-5} Oersted^{-1} for the case of H//bin., but in the case of H//bis. the oscillation could not be observed. At any rate, the oscillations disappear beyond the quantum limit. The intensity of magnetic field which induce the quantum limit is calculated from the period of the oscillation, i.e.

$$\zeta=\beta H, \quad \text{hence} \quad H=\frac{1}{\left(\frac{\beta}{\zeta}\right)}=\frac{1}{\left(\Delta\frac{1}{H}\right)}. \tag{16}$$

So by making use of the values of the period in Table 4, we can derive the quantum limit with respect to the crystallographic orientations of bismuth as in Table 5.

Table 5. Quantum limit of bismuth. (in units of 10^5 Oersted)

Properties referred / Orientation	χ Shoenberg	R_H Gerritsen and de Haas	R_H and ρ_t Connel and Marcus
H//trig.	0.714	—	0.625
H//bin.	0.135	0.143	0.133
//	4.0	—	—
H//bis.	0.232	0.2	0.238
//	0.118	—	0.114

From this results, except for one case ($H=4.0\times10^5$Oe) of H//bin. and the case ($H=0.714\times10^5$Oe) of H//trig., it is said that our measurement was carried out beyond the quantum limit.

The oscillation was found in a proper orientation of the pure specimen (Bi I), while it did not appear in the case of impure sample (Bi II). In the former case, the temperature dependence of the amplitude and the monotonic part of the oscillation in the range between 4.2° and 1.8°K was not apparently recognized. This remarkable feature can be understood by the following consideration, that the amplitude of the oscillation is proportional to $\frac{kT}{2\beta_0^* H}\left(\frac{\beta_0{}^* H}{\zeta}\right)^{3/2}\left[\sinh\frac{\pi^2 kT}{\beta_0{}^* H}\right]^{-1}$, where $\beta_0{}^*$ is the effective Bohr magneton and ζ is the Fermi energy, and in such a strong magnetic field as $\frac{\pi^2 kT}{\beta_0{}^* H} \ll 1$, $\frac{kT}{2\beta_0{}^* H}\left(\frac{\beta_0{}^* H}{\zeta}\right)^{3/2}\left[\sinh\frac{\pi^2 kT}{\beta_0{}^* H}\right]^{-1}$ becomes approximately equal to $\frac{1}{4\pi^2}\left(\frac{\beta_0{}^* H}{\zeta}\right)^{3/2}$, then the amplitude becomes independent of temperature. The oscillatory behavior in a strong magnetic field is different from the one in a weak field, i.e., the mass tensor does not change in such a strong magnetic field.

A possibility of an existence of a heavy hole which satisfies the condition $\omega_h \tau_h \ll 1$ even in a strong magnetic field is introduced. In order to make sure this problem, we are setting about the measurement in a magnetic field of some hundreds kilo-Oersted.

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