On the Demagnetizing Factor of Cylindrical Rods

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On the Demagnetizing Factor of Cylindrical Rods*

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Synopsis

The ballistic demagnetizing factor of cylindrical rods of ferromagnetic substances, whose dimension ratio is smaller than 10, was experimentally determined by a new method using the demagnetizing curve of New K.S. Magnet alloy having a high coercive force. The relationship between the demagnetizing factor, N, and the dimension ratio, m, was expressed as follows:

$$N=5\,5(m-0.54)^{-1.4}, \qquad m<10.$$

I. Introduction

As well known the demagnetizing factor N is dependent on the form of specimen. When the form of specimen is a rotational ellipsoid, the surface distribution of the intensity of magnetization is uniform; hence, the demagnetizing factor can be calculated and expressed by elliptic function. When the form is arbitrary, the effective magnetic field and the intensity of magnetization are partially different in the specimen; hence, it is difficult to calculate the demagnetizing factor. In this case, however, if the effective magnetic field and the intensity of magnetization are measured for the same portion of the specimen, the usual relation between these quantities will be allowable. Therefore, in general, the demagnetizing factor is given as the function of the form and the portion of specimen to be measured, and of the intensity of magnetization at that portion there of. As the demagnetizing factor of rods the approximate values have been obtained experimentally.

To study the properties of a magnet, cylindrical rods having the dimension ratio (length/diameter), m, of 10~20 are generally used, and the hysteresis is measured. As the demagnetizing factor the value by Shuddemagen(1) is used in the case of ballistic galvanometer method, or the value by Mann(2) in the case of magnetometer method.

Recently, the magnet alloy having so high coercive force was produced that the small sized magnets even of dimension ratio less than 1 became to be practically used. Generally, in designing the form of a magnet, the apparent residual magnetic induction appearing on the both poles of the magnet in its own form

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* The 979th report of the Research Institute for Iron, Steel and Other Metals. Published in Japanese in Nippon Kinzoku Gakkai-Si (J. Japan Inst. Metals), **21** (1957), 294.

(1) C.L.B. Shuddemagen, Proc. Amer. Acad., **43** (1907), 185.

(2) R. Mann, Dissert, Berlin, (1895).

must be used, but not the residual magnetic induction in the case of the effective field being zero. In Shuddemagen's table of demagnetizing factor the values are shown only down to $m=10$ and therefore, the demagnetizing factor in the case of small dimension ratio $m<10$ must be obtained from the relation between the demagnetizing factor and the dimension ratio drawn in logarithmic scale by extrapolation, which, however, leads to the illogicality giving rise to $N>4\pi$ at last. Of course, the demagnetizing factor will easily be obtained if the effective field is directly measured, but it is very difficult to measure this field in the case of the very small dimension ratio. M. M. Tschetwerikowa(3) has obtained the demagnetizing factor in the case $0.3\leqq m\leqq 25$ from the magnetizing curve of Cr-magnet steel. In practice, a magnet is used on the demagnetizing curve; hence, if the demagnetizing factor is obtained directly from this curve, it is very convenient for the design of a magnet. Consequently, in the present study a new method was suggested of obtaining the demagnetizing factor directly from the demagnetizing curve, and the corresponding demagnetizing factors.

II. The apparent residual magnetic induction

The residual magnetic induction, B_r, of magnet material is the magnetic induction when the effective field $H=0$. However, when a magnet is of an arbitrary form, it becomes $H\neq 0$ even in the case of the external field $H_e=0$, according to the demagnetizing field produced by its own magnetic poles. In the case of usual magnet, the residual magnetic induction is that when $H_e=0$, but not when $H=0$. This residual magnetic induction is called the apparent residual magnetic induction, B_d, which is distinguished from ordinary residual magnetic induction when $H=0$. Since $H_e=H$ in the case of a ring form specimen, it becomes $B_d=B_r$. In order to obtain the apparent magnetic induction, it is necessary to obtain the demagnetizing curve of the magnet.

Generally, the effective magnetic field, H, and the magnetic induction B, are expressed by the following formulas:

$$H=H_e-NI \tag{1}$$

$$B=4\pi I+H. \tag{2}$$

In the present case, by substituting the value of the intensity of magnetization, I, obtained by putting $H_e=0$ in the formula (1) into the formula (2), the apparent residual induction, B_d, was obtained as follows,

$$B_d=\left(1-\frac{4\pi}{N}\right)H_d, \tag{3}$$

where H_d is the effective field when $H_e=0$, and it corresponds to the demagnetizing field of the specimen. If the suitable point for $B/H=1-4\pi/N_m$ is obtained on the hysteresis curve in the second quadrant, the demagnetizing curve, (B, H) at this point will correspond to (B_d, H_d) of the manget with the demagnetizing

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(3) M. M. Tschetwerikowa, Russ. Z. tech. Phys., 3(1933), 1071.

factor, N_m. Consequently, the demagnetizing factor could be experimentally obtained from these relationships even in the case of low residual magnetizing induction, that is, of small dimension ratio.

III. The method of measurement

First, by using a specimen having $m>10$, the demagnetizing factor was obtained from Shuddemagen's table, the hysteresis was measured by the ballistic galvanometer method, and the demagnetizing curve was drawn as shown in Fig. 1. The coordinate refers to the magnetic induction, B, and the abscissa refers to the effective field, H, which is negative. Next, a point P on the demagnetizing curve and the origin O were joined by a straight line, and the angle between the line PO and H axis is called θ. By using the formula (3), the demagnetizing factor, N_d, of a magnet having the apparent residual magnetic induction, B_d, is obtained as follows:

$$N_d=4\pi/(1-\tan\theta). \qquad (4)$$

Then, N_d corresponding to B_d of any point on the demagnetizing curve in Fig. 1 is obtained by the above-mentioned procedure and from the formula (4). The relation between B_d and N_d is shown in Fig. 2, and this curve will be called the demagnetizing factor curve. In Fig. 2, when $N_d=0$, $B_d=B_r$, and when $B_d=0$, $N_d=4\pi$.

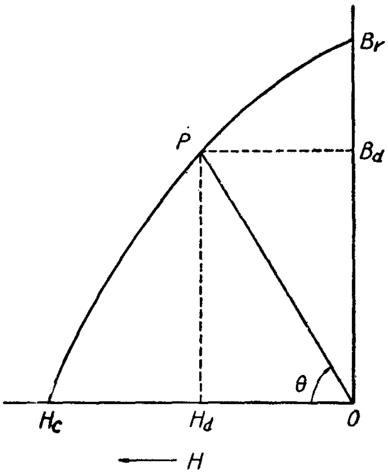

Fig. 1. Demagnetizing curve of magnet.

Next, the apparent residual magnetic induction was measured by the ballistic galvanometer method for the magnet having $m<10$, the property of which is the same as those in Fig. 1. When the deflection of galvanometer is denoted by φ, the apparent residual magnetic induction B_d is given by the following formula:

$$B_d=\frac{4\pi-N}{A-CN}\cdot K\varphi \qquad (5)$$

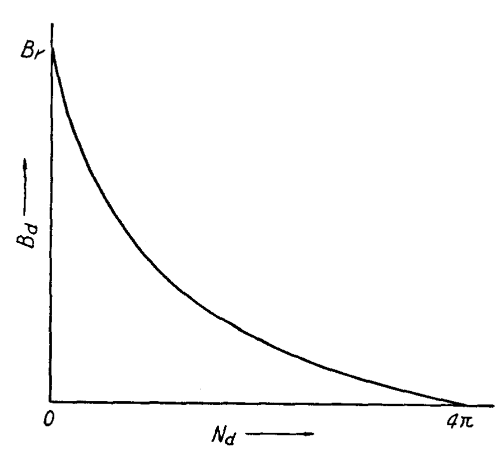

Fig. 2. Apparent residual induction B_d vs. demagnetizing factor N curve obtained from demagnetizing curve.

where A and C are the constants determined by the sectional area of the specimen, the area of the search coil and its turn numbers, and K is the constant of the ballistic galvanometer. As shown in the formula (5), so far as the demagnetizing factor, N, is unknown, the magnetic induction cannot be obtained. When the dimension ratio is determined, the deflection of galvanometer is obtained as a constant, so that B_d is able to be plotted as the function of N as seen in Fig. 3, by using the formula (5). This curve will be called the residual induction factor curve, and it is a rectangular hyperbola having the asymptote given by $B_d = \frac{K}{C}\varphi$ and $N = A/C$. A/C becomes to be equal to 4π, when the area of the search coil, S', becomes to be equal to the sectional area of the specimen, S. In practice, $S' > S$, and so, $A/C < 4\pi$. But in practice $0 < N \leqq 4\pi$ and $B_d \geqq 0$; hence, in the formula (5) $B_d < 0$ in the region $A/C < N < 4\pi$ and it was not allowable. Therefore, in the formula (5) only the region $N < A/C$ was effective. The demagnetizing factor curve in Fig. 2 and the residual induction factor curve in Fig. 3 can be put one upon another, supposing that the specimens for both of them possess the same property. Further, it seems that the closed point of them gives the apparent residual magnetic induction and the demagnetizing factor of the specimen having the dimension ratio which is marked by φ. For specimens with the same property but of various dimension ratios, the residual magnetic induction factor curves similar to that shown in Fig. 3 will be obtained, from which the relation between the dimension ratio and the demagnetizing factor is obtained by the same procedure as the above.

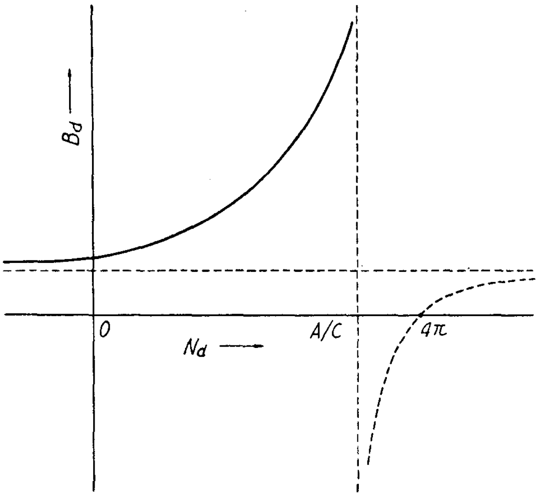

Fig. 3. Relation between the induction B_d and demagnetizing factor N, obtained from ballistic method measurement.

IV. The experimental results and discussion

New K.S. magnet alloy(4), which does not contain cobalt and has possibly large coercive force, was used as the specimen, because the larger the coercive force, the higher the sensitivity of measurement is. First, the alloy was cast into a rod, 1 cm in diameter, and treated to become a magnet. From the rod, 6 specimens having the dimension ratio, m, shown in Table 1 were prepared by the routin work in Suita Plant, Sumitomo Metal Industries, Ltd., and therefore, it may be said that all specimens possess the same property.

Table 1. Dimension ratio m and obtaind demagnetizing factor N of the specimens used.

Specimen No.	1	2	3	4	5	6
m	14.1	6.7	3.0	2.0	1.1	0.5
N	(0.112)	0.38	0.90	1.47	2.83	5.04

Specimen No. 1 has m of 1.41, and N of 0.12 is obtained from Shuddemagen's

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(4) K. Honda, H. Masumoto and Y. Shirakawa, Sci. Rep. Tôhoku Univ., **23** (1934), 365.

table by interpolation. The demagnetizing curve was obtained by the ballistic galvanometer method and is shown in the left side of Fig. 4. As seen from the figure this magnet possesses the properties of the residual magnetic induction, $B_r=6000$ G, and the coercive force, $H_c=500$ Oe.

Next, the demagnetizing factor curve was calculated by the formula (4) for a point on the demagnetizing curve as shown in the right side of Fig. 4. The magnetic induction when $N=0$ is the so-called residual magnetic induction, being a characteristic property of this specimen, which is able to be obtained when $m=\infty$, that is, in the case of a specimen of ring form. Further, as seen from the formula (3) the demagnetizing factor when $B=0$ is equal to 4π, which corresponds to the value in the direction perpendicular to the oblate plane of an oblate rotational ellipsoid.

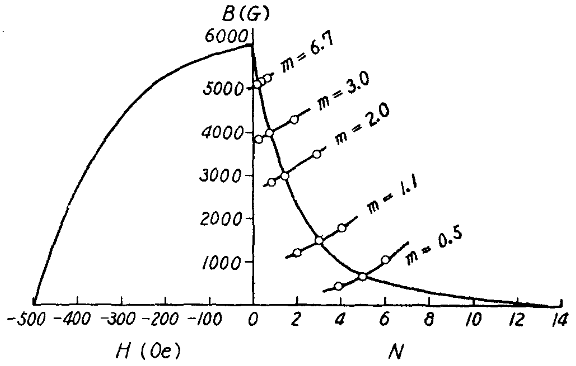

Fig. 4. Demagnetizing curve, and relation between the induction B and the demagnetizing factor N for New K.S. Magnet I.

Then, by using specimen No. 2 having m of 6.7, the deflection, φ, of the ballistic galvanometer showing the apparent residual magnetic induction was measured. By substituting this value into the formula (5), the residual induction factor curve was drawn on the demagnetizing factor curve in Fig. 4, by using the approximate values of the demagnetizing factor estimated for this specimen or thereabout. The values of B_d of 5120 G and N of 0.38 were obtained at the closed point of the above two curves. These values are the apparent residual induction and the demagnetizing factor for the specimen No. 2. A couple of values, $m=6.7$ and $N=0.38$, were obtained by this measurement.

As for specimen No. 3 and the rest, the same measurements as the above were carried out and their demagnetizing factors, N, were obtained as shown in Table 1. These values are not highly accurate, but it seems that these are sufficient for designing magnets having small dimention ratio. In the present experiment, even the specimen having m down to 0.5 was measured and in every case $N \ll 4\pi$, which is far smaller than that when $A/C=7.45$ in the formula (5).

From Table 1, the demagnetizing factor vs. the dimension ratio curve is obtained as shown in Fig. 5 by the mark ○ which is substantially expressed by the following formula:

$$N=5.5(m+0.54)^{-1.4} \qquad (6)$$

This curve in the region $10>m>0.5$ is shown in Fig. 5. The dotted line in the region $0.5>m\geqq 0.1$ is calculated by the formula (6). Moreover, the values by Shuddemagen are shown by the dotted line in the region $m>10$ in the figure.

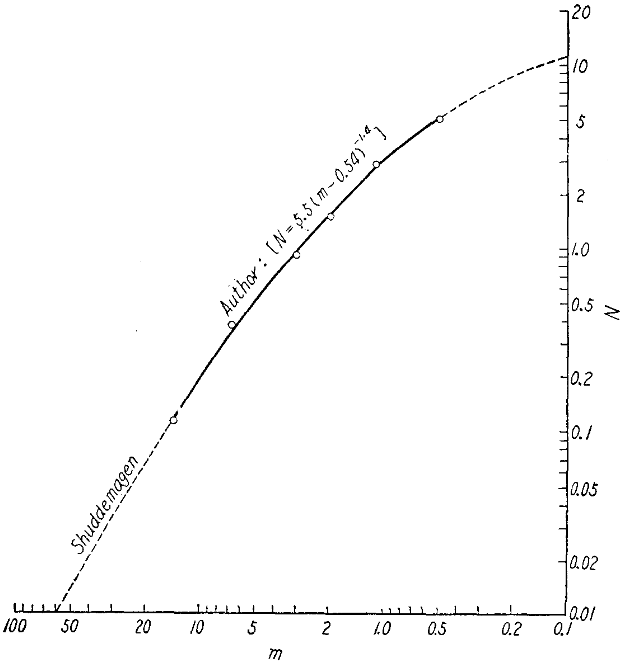

Fig. 5. Demagnetizing factor N vs. dimension ratio m curve.

By using the formula (6), the demagnetizing factors in various cases in which $m<10$ were obtained as shown in Table 2. In the table also the values by Tschetwerikowa are partially shown, which were obtained from the magnetizing curve for Cr magent steel by the ballistic galvanometer method. It is seen from the table that the values obtained from the demagnetizing curve in the present experiment nearly coincide with the results by Tschetwerikowa. In addition, taking the susceptibilities, χ, into consideration, Warmuth⁽⁵⁾ theoretically obtained the demagnetizing factor of a rod specimen for the ballistic galvanometer method from the demagnetizing factor of rotational ellipsoid. Plotting the values of N obtained in the present experiments on the demagnetizing factor vs. the dimension ratio curves which were drawn by Warmuth's values keeping χ constant, it appears that the present values in the cases $m>5$ and $m<1$ approach to Warmuth's values when $\chi=\infty$ and 0, respectively.

Table 2. Dimension ratio m and demagnetizing factor N of the magnets in the case $m<10$, according to the experimental fomula (5) and obtaining by M.M. Tschetwerikowa.

m	0.1	0.2	0.5	1.0	2.0	5.0	10
N	10.3	8.4	5.2	3.0	1.5	0.50	0.20
Tschetwerikowa	—	—	—	2.90	1.65	0.51	0.19

Summary

A new method for the determination of the demagnetizing factor of the cylindrical specimen having small dimension ratio from the demagnetizing curve was discussed theoretically. The experiment was carried out by using the cylindrical specimens of New K.S. magnet alloy I, and the relationship between the

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(5) K. Warmuth, Arch. Electrotechnik, 33 (1939), 747.

demagnetizing factor, N, and the dimension ratio, m, seems to be expressed approximately by the following equation:

$$N=5.5(m+0.54)^{-1.4}, \qquad m<10$$

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