Magnetostriction Constants of Face-Centered Cubic Nickel-Copper and Nickel-Cobalt Alloys

著者	YAMAMOTO Mikio, NAKAMICHI Takuro
journal or publication title	Science reports of the Research Institutes, Tohoku University. Ser. A, Physics, chemistry and metallurgy
volume	11
page range	168-182
year	1959
URL	http://hdl.handle.net/10097/26913

Magnetostriction Constants of Face-Centered Cubic Nickel-Copper and Nickel-Cobalt Alloys*

Mikio YAMAMOTO and Takurô NAKAMICHI

The Research Institute for Iron, Steel and Other Metals

(Received February 12, 1959)

Synopsis

The magnetostriction constants, λ_{100} and λ_{111}, of face-centered cubic Ni-Cu and Ni-Co alloys have been determined at room temperature, using single-crystal disc specimens and the strain gauge technique. In Ni-Cu alloys, both constants are negative irrespective of the composition and their absolute values decrease monotonously with increasing copper content. In Ni-Co alloys, the concentration dependence is quite different for λ_{100} and λ_{111}; λ_{111} is negative irrespective of the composition, showing a flat minimum centered at about 30% Co, while λ_{100} increases almost linearly with increasing cobalt content, passing through zero at about 20% Co, and eventually reaches to a large value of 116×10^{-6} at 55% Co.

It has been found that the experimental results on the concentration dependence of the magnetostriction constants of face-centered cubic Ni-Co alloys agree quite well with the Vonsovsky-Nêel formula for binary cubic solid solutions consisting of two types of magnetic atoms A and B, which is of the form

$$\lambda = C(N_A{}^2E_{AA}+2N_AN_BE_{AB}+N_B{}^2E_{BB}),$$

where λ is λ_{100} or λ_{111}, C the constant, N_A and N_B the concentrations of the A and B atoms, respectively, and E_{AA}, E_{AB} and E_{BB} the energies of magnetic interaction between A and A atoms, between A and B atoms, and between B and B atoms, respectively. For binary cubic solid solutions consisting of magnetic and non-magnetic atoms such as Ni-Cu alloys, it is shown theoretically and experimentally that the following relations hold approximately among the magnetostriction constants, saturation magnetization, I_s, and concentration of non-magnetic atoms, N:

$$\lambda_{100} \propto \lambda_{111} \propto I_s{}^2 \propto (1-N/N_c)^2,$$

where N_c is the concentration where the spontaneous magnetization disappears.

I. Introduction

The theory of magnetostriction has already found a probable origin of magnetostriction, but it encounters the difficulty of mathematical formulation. Accordingly, problems such as the temperature and concentration dependence of the magnetostriction constants can only be discussed qualitatively(1). While, on the experimental side, not so much have yet been made for solving these problems; for basically important nickel-cobalt alloys, we have only data for the concentration dependence of the magnetostriction constants which the senior of the present authors and Miyasawa(2) obtained from the analysis of the magnetostriction curves

---

* The 934th report of the Research Institute for Iron, Steel and Other Metals. A short note of the experimental part of this work was published previously in J. Phys. Soc. Japan, **2** (1958), 228.

(1) Cf., for example, E. W. Lee, Rep. Progr. Phys., **18** (1955), 184.

(2) M. Yamamoto and R. Miyasawa, Nippon Kinzoku Gakkai-shi, **B15** (1951), 509; Sci. Rep. RITU, **A5** (1953), 113.

of polycrystalline specimens, and even simple nickel-copper alloys have not yet been studied. So the present authors have determined, using the resistance strain gauge technique(3) and single crystal disc specimens, the concentration dependence of the magnetostriction constants of face-centered cubic nickel-cobalt and nickel-copper alloys at ordinary temperatures and made some theoretical considerations concerning the data obtained. This work constitutes the first program of our researches of the magnetostriction constants of nickel alloys.

II. principle of the measurement, single crystal specimens, and measuring equipments

1. Principle of the measurement

The saturation magnetostriction of single crystals of cubic ferromagnetic materials is described accurately by an expression in 5 constants h_1, h_2, h_3, h_4 and h_5. But, among these constants, the latter 3 constants are in general small by an order of magnitude as compared with the former 2 constants and may be neglected usually. The 2-constant expression is

$$\lambda = h_1(\alpha_1^2\beta_1^2+\alpha_2^2\beta_2^2+\alpha_3^2\beta_3^2-1/3)+2h_2(\alpha_1\alpha_2\beta_1\beta_2+\alpha_2\alpha_3\beta_2\beta_3+\alpha_3\alpha_1\beta_3\beta_1). \tag{1}$$

Here λ is the fractional change in length measured in the direction with the direction cosines β_1, β_2 and β_3 with respect to the crystal axes when the crystal is saturated in the direction with direction cosines α_1, α_2 and α_3, and $h_1=(3/2)\lambda_{100}$, and $h_2=(3/2)\lambda_{111}$, where λ_{100} and λ_{111} are the saturation magnetostrictions in the [100] and [111] directions, respectively.

In the present measurements, changes in length originated from the magnetostriction are measured by the strain gauge method(3). The gauge is cemented along a definite position on the plate surface of a disc crystal and the disc is magnetized in a number of directions in the plate surface. Thus, the β's of Eq.(1) are fixed and the α's varied. As the plate surface of a crystal disc, we take a (110) or (100) plane, the gauge positions being the [001] and [$1\bar{1}1$] directions for a (110) disc crystal and the [001] and [$0\bar{1}1$] directions for a (100) disc crystal. For both disc crystals, magnetic field of some 4,000 oersteds, sufficient to saturate them, is applied to the [001] direction and at various angles, φ, to this direction up to $\varphi=\pm 90°$ in steps of 10°. The observed strain values of the gauge are designated λ_φ. In order to eliminate the effect of initiated domain distribution, the data are given in terms of $e_\varphi=\lambda_\varphi-\lambda_0$, where λ_0 is the strain of the gauge for $\varphi=0°$ or the [001] direction. According to Eq.(1), the relationships between e_φ and φ are as follows: In the (100) disc crystal, for the gauge position parallel to the [001] direction

$$e_\varphi=(3/4)\lambda_{100}(\cos 2\varphi-1), \tag{2}$$

and for the gauge position parallel to the [$0\bar{1}1$] direction

$$e_\varphi=(3/4)\lambda_{111}\sin 2\varphi, \tag{3}$$

---

(3) J.E. Goldman, Phys. Rev., **72** (1947), 529.

and in the (110) disc crystal, for the gauge position parallel to the [001] direction

$$e_\varphi=(3/4)\lambda_{100}(\cos 2\varphi-1), \tag{4}$$

and for the gauge position parallel to the $[1\bar{1}1]$ direction

$$\begin{aligned} e_\varphi &= (1/4)\lambda_{111}(1-\cos 2\varphi+2\sqrt{2}\sin 2\varphi) \\ &= (3/4)\lambda_{111}\{(1/3)+\cos 2(\varphi-54°44')\}. \end{aligned} \tag{5}$$

Thus, the magnetostriction constants λ_{100} or λ_{111} can be determined from the amplitudes of the observed $(e_\varphi-\varphi)$ curves.

2. *Preparation of disc crystal specimens*

Block single crystals of nickel-copper and face-centered cubic nickel-cobalt alloys were grown in a vacuum by the Bridgman method. The mixture of about 50 gr in a desired ratio of electrolytic nickel (99.92 % pure) and electrolytic copper (99.96 % pure) or electrolytic cobalt (99.86 % pure) was melted in an alumina tube, 12 mm in inner diameter, hung in a vacuum Tammann (carbon tube) furnace(4) and then the tube containing the melt was lowered at the rate of 1~4 cm/hr (the lowering rate of the tube was varied according to the composition of alloys), thus the melt being solidified from its bottom. The state of crystallization of the ingot was examined by repeated etching with saturated aqueous solutions of ferric chloride plus a small quantity of concentrated nitric acid.

The single crystal ingots thus made were orientated with the light figure method(5) after etching with boiling saturated aqueous solution of ferric chloride (plus a very small quantity of concentrated nitric acid for high-cobalt face-centered cubic nickel-cobalt alloys). Single crystal plates, 1~2 mm thick, with the plate surface parallel to a {100} or {110} plane were cut out from the ingots by jewler's saw, and then the plates were shaped into circular discs by being polished on emery papers. The discs were electropolished in the 9 : 1 mixture of phosphoric acid and chromic acid in order to remove the surface layer with a large internal stress caused by the mechanical polishing, and then annealed at 1000°C for an hour in a vacuum in order to remove the stress in its interior as well as to homogenize the composition. The disc face was made to be within 2 degrees to the {100} or {110} plane.

Disc crystals of nickel-copper alloys are about 15 mm in diameter and 1.5 mm in thickness, and those of nickel-cobalt alloys are about 10 mm in diameter and 1.0 mm in thickness. The chemically-analysed compositions and crystallographic indices of the plate suface of the disc crystals are given in Table 1.

---

(4) See H. Takaki, S. Nakamura, Y. Nakamura, T. Hayashi, K. Furukawa and M. Aso, J. Phys. Soc. Japan, **9** (1954), 204.

(5) As for the light figures of nickel-copper alloy crystals, see M. Yamamoto and J. Watanabe, Nippon Kinzoku Gakkai-shi, **18** (1954), 595; Sci. Rep. RITU, **8** (1956), 125. Light figures of nickel-cobalt alloy crystals will be described by them in the near future.

Table 1. Composition and Miller indices of the plate surfaces of the specimen crystals and the measured values of λ_{100} and λ_{111}.

(a) Ni-Cu alloys

Specimen mark	Composition (at.%-Cu)	Plate surface	$\lambda_{100}\times 10^6$	$\lambda_{111}\times 10^6$
Pure Ni	0	(100)	−54.1	−22.4
Ni-Cu, A	7.0	(100)	−48.1	−18.4
Ni-Cu, B	16.4	(110)	−29.3	−10.0
Ni-Cu, C	25.8	(110)	−13.1	− 4.0

(b) Ni-Co alloys

Specimen mark	Composition (at.%-Co)	Plate surface	$\lambda_{100}\times 10^6$	$\lambda_{111}\times 10^6$
Ni-Co, A	10.5	(110)	−28.1	−31.9
Ni-Co, B	21.4	(110)	6.5	−34.3
Ni-Co, C	33.1	(110)	41.9	−35.7
Ni-Co, D	45.4	(110)	80.0	−32.8
Ni-Co, E	55.2	(110)	116.3	−30.6

3. *Measuring equipments*

Measuring equipments are an apparatus for supporting and rotating a crystal disc in magnetic field produced by an electromagnet and an equipment for detecting the magnetostrictive strain of the crystal disc. The apparatus for supporting and rotating a disc specimen is as shown in Fig. 1. It is a rotable quartz rod provided with a specimen support made of copper attached to its lower end and with a circular board of the scale, expressing the rotation angle of the specimen, attached to its upper end. The disc crystal is pasted on the lower surface of the specimen support, and the resistance gauge is cemented within a few tenth of a degree to the required crystallographic direction on the lower surface of the crystal disc by alligning the directions of the gauge wires with a scratch drawn parallel to the required direction on the crystal surface. Lead wires from the gauge are taken out from the outlet at the top of the apparatus so that their twist may be as little as possible when the specimen is rotated. The specimen together with the quartz rod can be kept in a vacuum.

The strain-detecting device is composed of a Wheatstone bridge, one arm of which is the resistance gauge cemented on the disc crystal, connected with a high-sensitivity galvanometer (Fig. 2). Other similar gauges forming the remaining arms of the bridge are cemented on the surface of a rectangular brass rod which is placed in a Dewer vessel. Variable resistances of the order of 100 kΩ are connected parallel to the gauge cemented on the specimen and to a neighboring gauge, respectively, in order to obtain the balance of the bridge as well as to

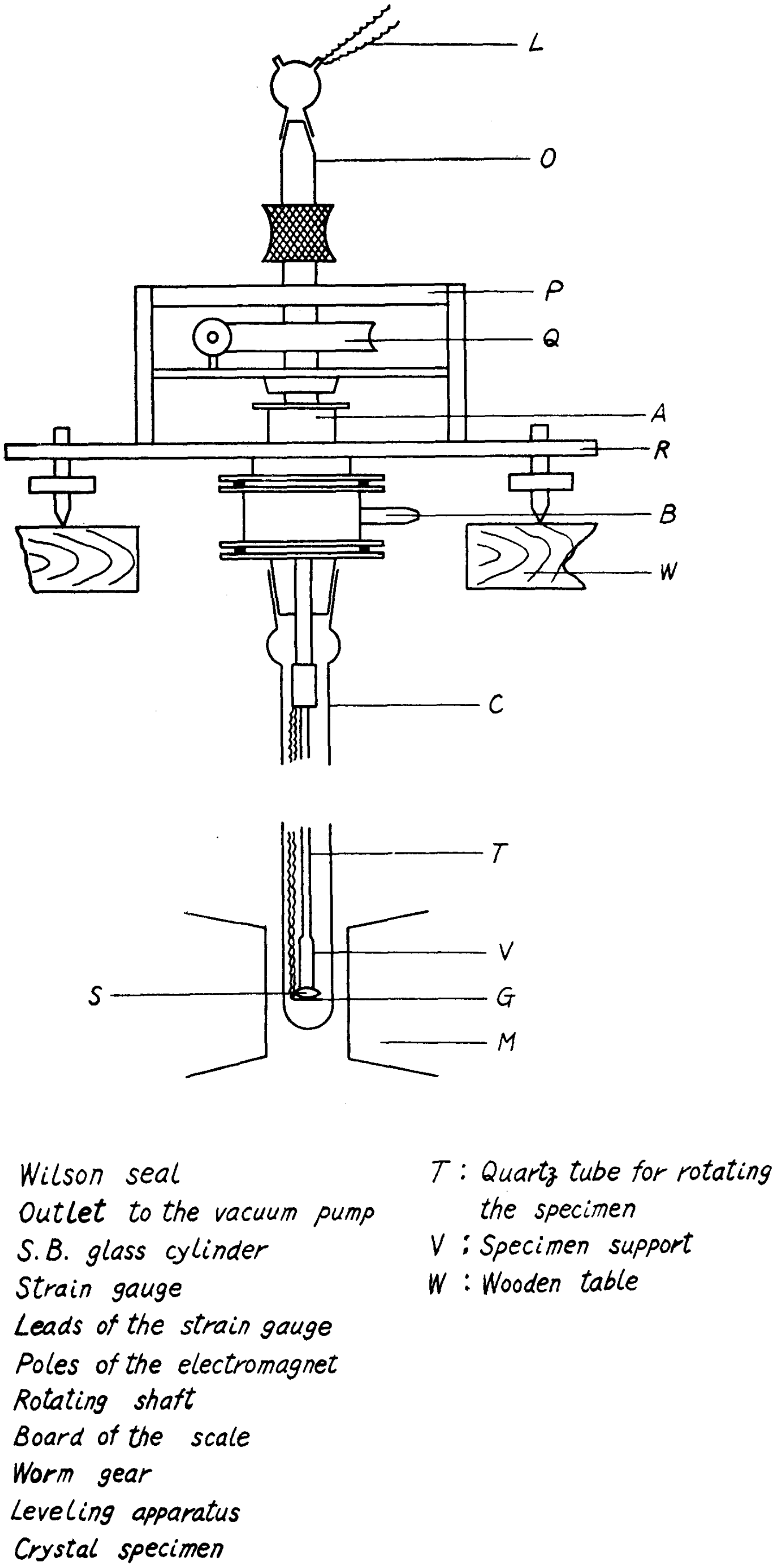


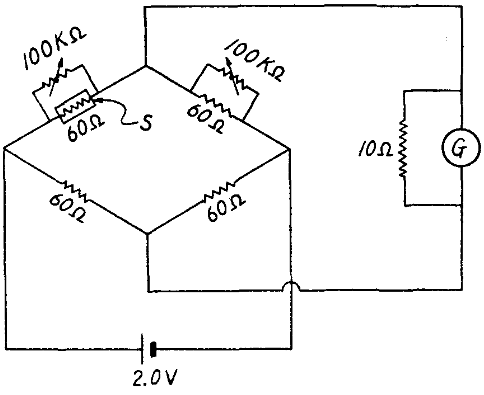
Fig. 1. Apparatus for supporting and rotating the specimen.

measure the sensitivity of the strain-detecting device. The sensitivity τ of the strain-detecting apparatus is given by

$$\tau = R \cdot \Delta r / \kappa r_0^2 \cdot \Delta\theta, \qquad (6)$$

where R is the resistance of the gauge, κ the gauge factor, r_0 the resistance of the rheostat connected parallel to the gauge cemented on the disc crystal (cf. Fig. 2), Δr the variation of r_0, and $\Delta\theta$ the deflection of the galvanometer corresponding to Δr. The magnetostriction λ and the corresponding deflection θ of the galvanometer are connected through τ as

$$\lambda = \tau\theta, \qquad (7)$$



S : Specimen,
G : Galvanometer

Fig. 2. Connection diagram of the Wheatstone bridge employed for detecting the magnetostrictive strain of the specimen.

A 2-volt battery is connected to the bridge. The sensitivity of the galvanometer is about 10^{-9} A/mm and that of the strain-detecting device is 0.53×10^{-6} per mm of the scale. The gauge factor and resistance of the gauges used are 2.0 and 60 (or 120) Ω, respectively.

The Weiss type electromagnet used produces magnetic field of about 3,200 Oe for the pole-distance of 5.4 cm and the current of 4 A, which is sufficient to saturate the disc crystals studied here.

4. *Experimental error*

The error of the magnetostriction constants determined in such a way may mainly be due to the shift of the null point of the galvanometer during the measurements which is caused by the thermoelectric effect in the gauge wires and the lead wires and by the thermal expansion of the specimen, other errors due to the magnetoresistance effect of the gauge, involved in the determination of crystallographic orientations, etc. being masked by the first one. The total error may amount to within 1×10^{-6}.

III. Results of the measurements

1. *Nickel-copper alloys*

The relations between the observed magnetostriction, e_φ, and the direction, φ, of magnetization are shown in Fig. 3; (a) and (b) are those for the determination of λ_{100} and of λ_{111}, respectively. As seen from the figure, the maximum and minimum points of the observed curves are shifted a little from the expected directions, owing to the difficulty of setting the direction of magnetic field accurately parallel to the [001] direction in the disc surface of the specimen. The theoretical curves (Eqs. (2)~(5)) corrected for this deviation of the field direction from the [001] direction, agree very well with the observed points, so that the two-constant expression (1) used here is sufficient to describe the magnetostriction of nickel-copper alloys. This is also the case for face-centered nickel-cobalt alloys.

The measured data for the magnetostriction constants are given in Table 1. For pure nickel, $\lambda_{100}=-54.1\times10^{-6}$ and $\lambda_{111}=-22.4\times10^{-6}$, which are in good accordance with the previous data $^{(6)\sim(9)}$, as seen from Table 2. The concentration

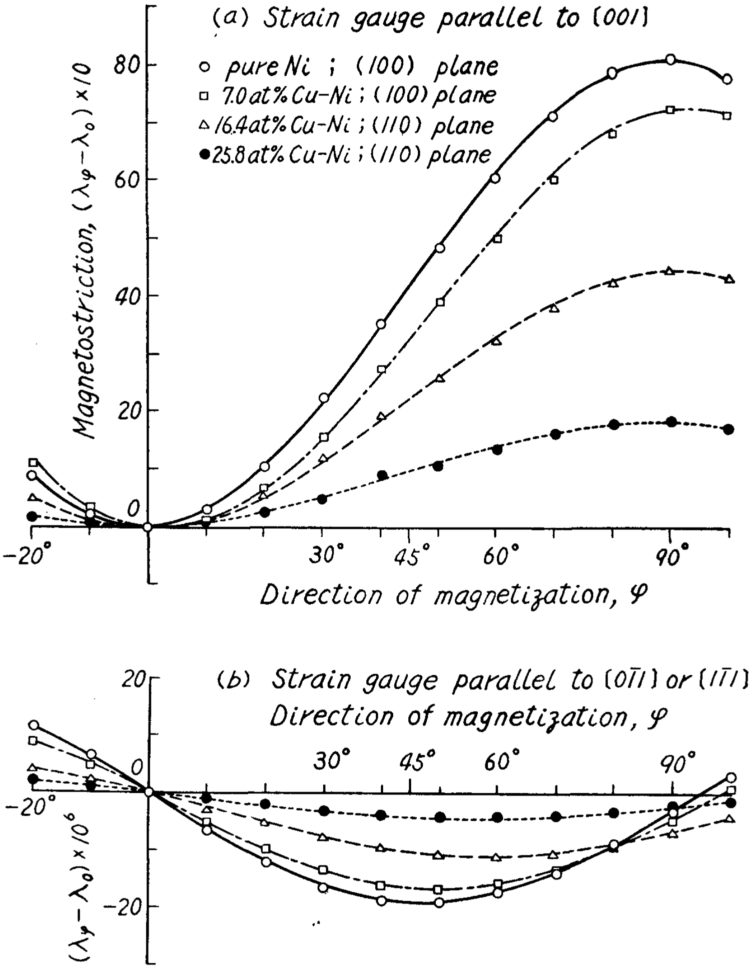


Fig. 3. Magnetostriction, relative to that for the [001] direction, $\lambda_\varphi-\lambda_0$, as dependent on the direction of magnetization, φ, in the (100) or (110) disc plane in Ni-Cu single crystals.

Table 2. Magnetostriction constants of nickel.

Worker	$\lambda_{100}\times10^6$	$\lambda_{111}\times10^6$
Present authors	−54.1	−22.4
Masiyama(6)	−49	−30
Bozorth and Hamming(7)	−50.9±6	−23.5±1.8
Corner and Hunt(8)	−51	−19.6
Chikazumi and Wakiyama(9)	−55.8	−29.5

---

(6) Y. Masiyama, Sci. Rep. Tôhoku Univ., **17** (1928), 945.
(7) R. M. Bozorth and R. W. Hamming, Phys. Rev., **89** (1953), 865.
(8) W. D. Corner and G. H. Hunt, Proc. Phys. Soc., **A68** (1955), 133.
(9) S. Chikazumi and T. Wakiyama, Read at the sectional meeting of Phys. Soc. Japan,(1957).

dependence of the magnetostriction constants are shown in Fig. 4, which shows that both constants remains negative for any copper content and their absolute values decrease monotonously with increasing copper content, eventually vanishing at 35 at.-% copper, as may be expected from the magnetostriction data for polycrystalline specimens of the alloys(10). In Fig. 5, the values of the saturation magnetostriction for pseudo-isotropic polycrystal, $\bar{\lambda}_S$, calculated from our single crystal data according to the formula derived under the assumption of uniform stress over all crystallites, namely.

$$\bar{\lambda}_S=(1/5)(2\lambda_{100}+3\lambda_{111}), \qquad (8)$$

are compared with the measured data by Went(1) and by Shirakawa and Numakura(10). The calculated data are somewhat higher in magnitude than the measured ones. This discrepancy may possibly be due to the fact that polycrystalline rod specimens used by Went and by Shirakawa and Numakura had the domain distribution predominant along the rod axis at unmagnetized state.

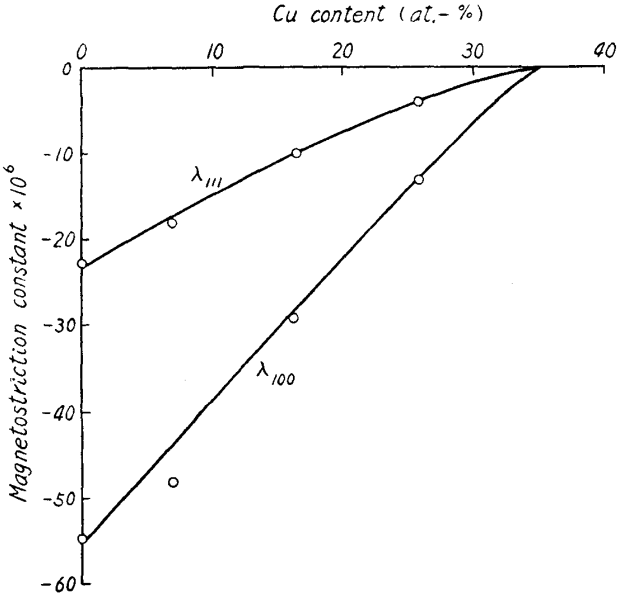


Fig. 4. Concentration desendence of the magnetostriction constants of Ni-Cu alloys.

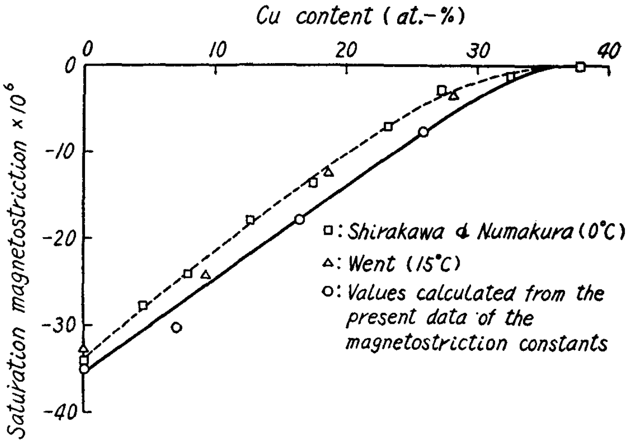


Fig. 5. Measured and calculated values of the saturation magnetostriction for polycrystalline Ni-Cu alloys.

2. *Face-centered cubic nickel-cobalt alloys*

In face-centered cubic nickel-cobalt alloys, the concentration dependencies are quite different for λ_{100} and λ_{111}, as seen from Fig. 6. λ_{111} is negative irrespective of the composition, showing a flat minimum centered at about 30 at.-% cobalt. While, λ_{100} increases almost linearly with increasing cobalt content and, after passing through zero at about 20 at.-% cobalt, reaches to a large value of 116×10^{-6} at 55 at.-% cobalt. The extrapolation of this relation to 68 at.-% cobalt, namely, the

---

(10) Y. Shirakawa and K. Numakura, Kinzoku Gakkai-shi, **18** (1954), 731 (in Japanese).

(11) J.J. Went, Physica, **17** (1951), 99.

boundary between the γ (f.c.c.) phase and the $\gamma+\varepsilon$ (h.c.p.) phase yields the value of about 160×10^{-6}, which is comparable with the maximum value of the magnetostriction constant hitherto found: $\lambda_{100}=153\times10^{-6}$ for 30% Co-Fe alloy$^{(12)}$.

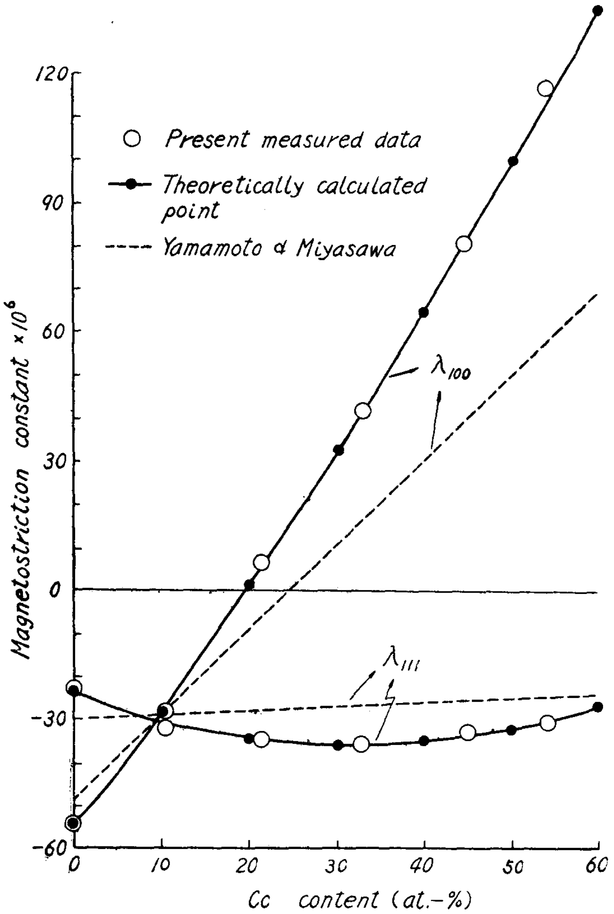


Fig. 6. Concentration dependence of the magnetostriction constants of face-centered cubic Ni-Co alloys.

In Fig. 6, the results obtained by one of the present authors (Yamamoto) and Miyasawa$^{(2)}$ from the analysis of the magnetostriction vs. field curves of polycrystalline nickel-cobalt alloy specimens are also given. As seen from the figure, their results agree qualitatively well with, but are genearlly lower in magnitude than, the single crystal data obtained here. This quantitative difference, which is larger for higher cobalt content alloys, may be due mainly to the lack of saturation in the magnetostriction vs. field curves of the polycrystalline specimens measured by them$^{(2)}$.

The values of the saturation magnetostriction for pseudo-isotropic polycrystals, $\bar{\lambda}_S$, calculated from the magnetostriction constants obtained by us according to Eq. (8) are compared with the measured data by Masiyama$^{(13)}$, by Went,$^{(11)}$ and by Yamamoto and Miyasawa$^{(2)}$ in Fig. 7. The calculated data are in best agreement with the measured ones by Went. Yamamoto and Miyasawa's and Masiyama's

---

(12) K. Azumi, Unyu Giken Hôkoku, **4** (1954), 1 (in Japanese).
(13) Y. Masiyama, Sci. Rep. Tôhoku Univ., **22** (1933), 338.

measurements were made in magnetic fields less than about 1,000 Oe which is far lower than the maximum measuring field of 7,000 Oe in Went's measurements. Thus the difference between the calculated data and the measured data by Masiyama and by Yamamoto and Miyasawa may also be due to the lack of the magnetic saturation in their measurements.

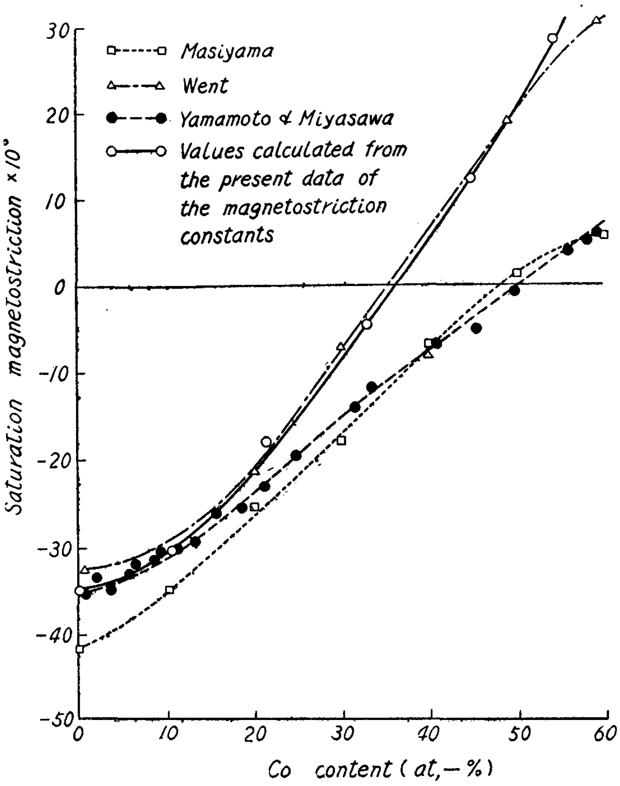


Fig. 7. Measured and calculated values of the saturation magnetostriction for polycrystalline face-centered cubic Ni-Co alloys.

IV. Comparison with theories

It is now accepted that the greater part of the magnetostriction has its origin in the spin-orbit interaction within the atoms. Van Vleck(14) showed that spin-orbit interaction simulated dipolar coupling (or quadrupole coupling if j=1) with a constant (in place of $\mu_i\mu_j/r_{ij}^3$, where μ_i and μ_j are the magnetic moments of the ith and jth atoms and r_{ij} is the distance between the two atoms) which has its origin in the spin-orbit coupling and is between ten and one hundred times larger than $\mu_i\mu_j/r_{ij}^3$.

Vonsovsky(15) treated the subject basing on a Heisenberg model, and obtained the following expressions for the magnetostriction constants:-

$$\left.\begin{aligned} \lambda_{100} &= -\frac{2L}{3}\,\frac{I_s^2/I_{s0}^2}{C_{11}-C_{12}}\,\{S-S'[\frac{1}{A}(1-e^{-A/kT})-\frac{1}{B}(1-e^{-B/kT})]\} \\ \text{and}\qquad \lambda_{111} &= -\frac{L}{3}\,\frac{I_s^2/I_{s0}^2}{C_{44}}\,\{P-P'[\frac{1}{A}(1-e^{-A/kT})-\frac{1}{B}(1-e^{-B/kT})]\}, \end{aligned}\right\} \quad (9)$$

---

(14) J. H. Van Vleck, Phys. Rev., **52** (1937), 1178.
(15) S. V. Vonsovsky, J. Phys. USSR, **3** (1940), 181.

where L is Avogadro's number, I_s and I_{s0} are the values of saturation magnetization at the absolute temperature T and the absolute zero, C_{11}, C_{12} and C_{44} are the elastic constants of cubic crystal, and S, S', P, P', A, and B are constants. The first term in the bracket represents the spin-spin part of the magnetostriction, which varies with Temperature as $(I_s/I_{s0})^2$, and the second term represents the perturbing influence of the spin-orbit interaction.

A different approach was employed by Katayama(16) and Fletcher(17), who used a collective electron approximation with tight-binding approximation wave functions and spin-orbit coupling introduced as a perturbation. Fletcher derived the following expression for the magnetostriction constants of face-centered cubic crystal at 0°K:

$$\left.\begin{aligned} \lambda_{100} &= -(4/3)\frac{nA^2}{C_{11}-C_{12}}\ \Sigma f(\delta, \vec{k}), \\ \lambda_{111} &= -(2/9)\frac{nA^2}{C_{44}}\ \Sigma g(\delta, \vec{k}), \end{aligned}\right\} \tag{10}$$

and $$\delta = n_\beta J(I_s/I_{s0}),$$

where n is the number of atoms in unit volume, A is the spin-orbit interaction parameter, f and g are the complex functions of the wave vector $\vec{k}$ in the energy band and δ, which vanish as δ vanishes, Σ the sum taken over all of the occupied states of the band in the unstrained crystal, n_β the Bohr magneton, and J the exchange integral.

On the other hand, Néel(18) put formally the magnetic interaction energy between nearest neighbour atoms, w, as

$$w = \left(-\frac{3\mu_i\mu_j}{r_{ij}^3} + l + m\delta r_{ij}\right)\left\{\cos^2(\vec{\mu_i}, \vec{r_0}) - \frac{1}{3}\right\},$$

where $$r_{ij} = r_0 + \delta r_{ij},$$

and r_0 is the nearest neighbor distance, and, using the value of the change in this energy resulted from deformation averaged over all of the possible nearest neighbor sites, obtained the following expressions for the magnetostriction constants:

$$\left.\begin{aligned} \lambda_{100} &= -\frac{nL}{32V(C_{11}-C_{12})}(6l + mr_0) \\ \text{and}\qquad \lambda_{111} &= -\frac{nL}{9VC_{44}}(2l + mr_0), \end{aligned}\right\} \tag{11}$$

where V is the atomic volume.

Now, as to the concentration dependence of the magnetostriction constants in binary solid solutions, two cases must be distinguished: solid solutions consisting of two types of magnetic atoms and those consisting of magnetic and nonmagnetic atoms. For the case of binary solid solution consisting of two types of magnetic

---

(16) T. Katayama, Sci. Rep. RITU, **3** (1951), 341.

(17) G. C. Fletcher, Proc. Phys. Soc., **A68** (1955), 1066.

(18) L. Néel, J. physique, **15** (1954), 225.

atoms A and B, Vonsovsky$^{(15)}$ shows that the concentration dependence of the magnetostriction constants should be of the form

$$\lambda = C(N_A{}^2E_{AA} + 2N_AN_BE_{AB} + N_B{}^2E_{BB}), \tag{12}$$

wher N_A and N_B are the concentrations of atoms A and B, respectively, and E_{AA}, E_{BB} and E_{AB} are the energies of interaction between like and unlike atoms. This is originated from the fact that all of the parameters of the spin-spin and spin-orbit interactions (S, S', P, and P' in Eq. (9)) have such a concentration depndence as

$$S = n(N_A{}^2S_{AA} + 2N_AN_BS_{AB} + N_B{}^2S_{BB}).$$

It is quite natural that Néel has also derived Eq. (12), since the pair interaction parameters l and m in Néel's formulae (11) have the nature similar to S, S', P and P' in Vosnovsky's ones (9).

The analysis in terms of the Vonsovsky-Néel formulae (12) of our experimental results on the magnetostriction constants in face-centered cubic nickel-cobalt alloys yield for the pair interaction parameters the following values:-

$$\left.\begin{array}{ll} Ll_{NiNi} = 1.03 \times 10^8 \text{ ergs}, & Lm_{NiNi}r_0 = 3.37 \times 10^8 \text{ ergs}, \\ Ll_{NiCo} = -1.78 \times 10^8 \text{ ergs}, & Lm_{NiCo}r_0 = 17.9 \times 10^8 \text{ ergs}, \\ Ll_{CoCo} = -11.9 \times 10^8 \text{ ergs}, & Lm_{CoCo}r_0 = 19.5 \times 10^8 \text{ ergs}. \end{array}\right\} \tag{13}$$

The values of λ_{100} and λ_{111} calculated using these values of pair interaction parameters are plotted as crosses in Fig. 6, which indicates that the Vonsovsky-Néel formula (12) holds quite well for face-centered cubic nikel-cobalt alloys.

It is needless to say that the Vonsovsky-Néel formula (12) can not be applied to binary solid solutions consisting magnetic and non-magnetic atoms, though Chikazumi and Wakiyama$^{(9),(19)}$ have made such an application to nickel alloys containing vanadium and chrome. In fact, the following unreasonable values of the pair interaction parameters are deduced from our experimental data for nickel-copper alloys:-

$$\left.\begin{array}{ll} Ll_{NiCu} = 1.3 \times 10^8 \text{ ergs}, & Lm_{NiCu}r_0 = -10.3 \times 10^8 \text{ ergs}, \\ Ll_{CuCu} = 12.0 \times 10^8 \text{ ergs}, & Lm_{CuCu}r_0 = -28.0 \times 10^8 \text{ ergs}, \end{array}\right\} \tag{14}$$

which indicate that the Cu-Cu interaction is very larger than the Ni-Ni one.

In binary solid solution consisting of magnetic atoms and non-magnetic atoms, both of the spin-spin and spin-orbit interaction parameters can be considered approximately as

$$S \propto I_{S_0}{}^2, \tag{15}$$

so that the concentration dependence at 0°K of the magnetostriction constants may also be approximately of the form

$$\lambda_0 \propto I_{S_0}{}^2. \tag{16}$$

---

(19) S. Chikazumi and T. Wakiyama, Read at the annual meeting of the Phys. Soc. Japan, (1957).

On the other hand, the temperature dependence of the magnetostriction constants of a solid solution may be given approximately as

$$\lambda = \lambda_0 (I_S/I_{S_0})^2. \tag{17}$$

Accordingly, we have approximately

$$\lambda \propto I_S^2. \tag{18}$$

Now, it has been established that, in binary nickel alloys containing non-magnetic atoms, the concentration dependence of the saturation magnetization can be expressed approximately

$$I_S = (I_S)_{Ni}(1 - N/N_C)^2, \tag{19}$$

where $(I_S)_{Ni}$ is the value of I_S for nickel, N the concentration of non-magnetic atoms, and N_C the value of N where I_S vanishes. Thus we have approximately

$$\lambda_{100} \propto \lambda_{111} \propto I_S^2 \propto (1 - N/N_C)^2. \tag{20}$$

We compare the above results of considerations with our experimental data on nickel-copper alloys. The measured values of λ_{100} and λ_{111} given in Table 1 indicate that an approximate proportionality between them is valid, although the proportionality constant rises somewhat with increasing copper content. In Fig. 8, λ_{100} and λ_{111} are plotted against I_S^2, taking the values of I_S obtained previously by one of the present authors (Yamamoto)[(20)]. Fig. 8 shows that the proportionality of λ_{100} and of λ_{111} with I_S^2 holds approximately. Taking N_C as 42 at.-%-Cu, by extrapolating the linear portion of the I_S vs. N curve[(20)], λ_{100} and λ_{111} are plotted against $(1-N/N_C)^2$ in Fig. 9, which indicates that the proportionality of λ_{111} with $(1-N/N_C)^2$ is good and the proportionality of λ_{100} with $(1-N/N_C)^2$ holds very approximately. Thus, it may be concluded that the relations (20) holds approximately in nickel-copper alloys.

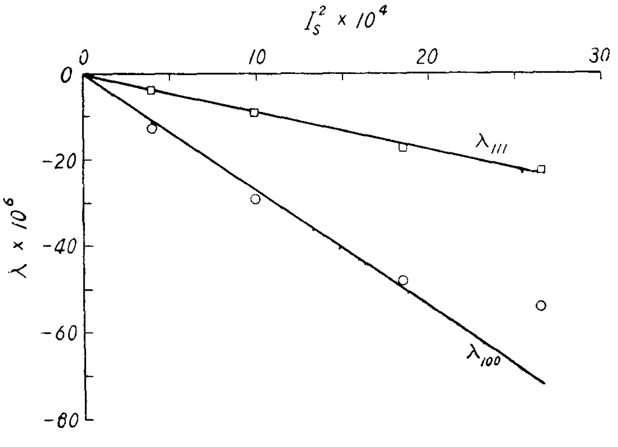


Fig. 8. Approximate quadratic dependence of the magnetostriction constants, λ_{100} and λ_{111} upon the spontaneous magnetization, I_s, in Ni-Cu alloys.

Summary

The magnetostriction constants, λ_{100} and λ_{111}, of nickel-copper and face-centered cubic nickel-cobalt alloys at room temperature have been determined, using single-crystal disc specimens and the strain gauge technique. The specimens used are

---

(20) M. Yamamoto, Nippon Kinzoku Gakkai-shi, **6** (1942), 249; Sci. Rep. RITU, **6** (1954), 446.

discs with (110) or (100) plate surfaces, cut out from single crystal ingots prepared by the Bridgman method in a vacuum Tammann furnace. Nickel-copper crystal discs containing 0, 7.0, 16.4 and 25.8 at.-% Cu are 15 mm in diameter and 1.5 mm in thickness, and nickel-cobalt crystal discs containing 10.5, 21.4, 33.1, 45.4 and 55.2 at-% Co are 10 mm in diameter and 1.0 mm in thickness. The out-put unbalanced voltage of a Wheatstone bridge formed by a strain gauge cemented along a certain crystallographic direction on the plate surface of the specimen together with other three similar gauges, which was proportional to the strain of the specimen, was detected directly by a galvanometer. The strain sensitivity was 0.53×10^{-6} per mm of the scale. The specimen was placed in magnetic field of some 4000 Oe, strong enough to saturate it, as produced from a Weiss-type electromagnet, and its magnetostrictive strain along the fixed direction was measured as a function of the direction of the magnetic field.

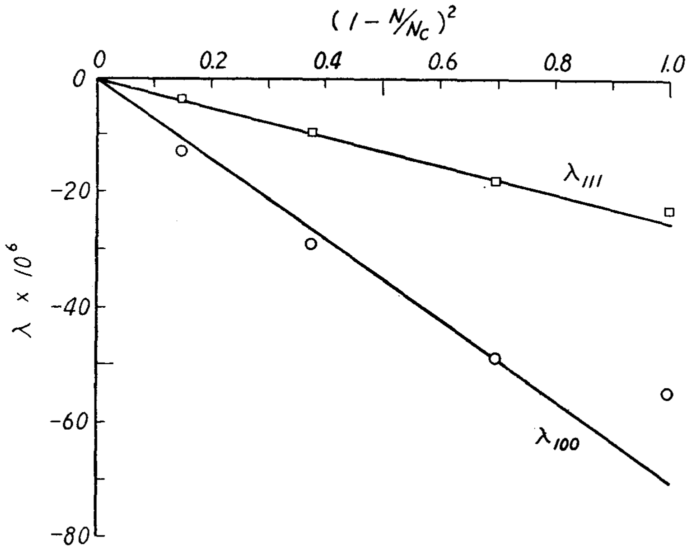


Fig. 9. Approximate quadratic dependence of the magnetostriction constants, λ_{100} and λ_{111}, on the concentration, N, in Ni-Cu alloys.

Values of λ_{100} and λ_{111} for nickel are -54.1 and -22.4×10^{-6}, respectively, which are in good agreement with available data. In nickel-copper alloys. both constants are negative irrespective of the composition and their absolute values decrease monotonously with increasing copper content, vanishing at some 35 at.-% Cu. In face-centered cubic nickel-cobalt alloys, the concentration dependencies are quite different for λ_{100} and λ_{111}; λ_{111} is negative irrespective of the composition, showing a flat minimum centered at about 30 at-% Co, while λ_{100} increases almost linearly with increasing cobalt content and, passes through zero at about 20 at-% Co, reaching to the large value of 116×10^{-6} at 55 at-% Co. The present data agree qualitatively well, but quantitatively not so well, particularly for cobalt-rich alloys, with the data obtained previously by one of the authors (Yamamoto) and Miyasawa from the analysis of the magnetostriction vs. magnetic field curves of polycrystalline specimens. This discrepancy may be due mainly to the lack of saturation of the magnetostriction vs. field curves measured by them.

It has been found that our experimental results on the concentration dependence of the magnetostriction constants of face-centered nickel-cobalt alloys agree quite well with the Vonsovsky-Néel formula for binary cubic solid solutions consisting of two type of magnetic atomes A and B, which is of the form

$$\lambda = C(N_A{}^2E_{AA} + 2N_AN_BE_{AB} + N_B{}^2E_{BB}),$$

where λ is λ_{100} or λ_{111}, C the constant, N_A and N_B the concentrations of the A and B atoms, respectively, and E_{AA}, E_{AB} and E_{BB}, the energies of magnetic interaction between A and A atoms, between A and B atoms, and between B and B atoms, respectively. For binary cubic solid solutions consisting of magnetic and non-magnetic atoms such as nickel-copper alloys, it has been shown theoretically and experimentally that the following relations hold approximately among the magnetostriction constants, λ_{100} and λ_{111}, saturation magnetization, I_S, and concentration of non-magnetic atoms, N:

$$\lambda_{100} \propto \lambda_{111} \propto I_S{}^2 \propto (1-N/N_c)^2,$$

where N_C is the concentration where the spontaneous magnetization disappears.

In closing, the authors express their hearty thanks to Dr. S. Taniguchi who gave valuable advices and to Messrs. J. Watanabé and K. Hoshi who helped them in preparing single crystal specimens and in determining the orientations of the crystal specimens. It is to be noted that a part of the fund required for the present work was supplied from the Grant in Aid for Scientific Researches of the Ministry of Education.