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Magnetic Flux Increase in Non-Circular Tin Rods at the Superconducting Transition*

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Synopsis

In the previous reports, it has been shown that the current minimum I_0 required for the occurrence of magnetic flux increase is represented in the $(I\text{-}H\text{-}T)$ space by the simultaneous equations $I_0 = \xi\gamma d(T_c - T)$ and $H_0 = \xi(T_c - T) - I_g/\gamma d$. Here I_g, T_c and ξ are the characteristic constants of the superconductor measured in amp, °K and Oe/deg respectively. d is the diameter of the specimen in mm and γ seems to have a nearly constant value of 0.25 measured in amp/mm Oe, irrespective of the superconductor. It has been shown also that the formula $I_0 = I_g + \gamma dH$ proposed for the minimum current requirement up to that time can be derived from the above simultaneous equations after eliminating T and that these simultaneous equations are understood in a good approximation as those for the intersection of the plane $I = I_g + \gamma dH$ with the transition surface in the $(I\text{-}H\text{-}T)$ space. In the present paper the experimental results on the magnetic flux increase in non-circular tin rods at the superconducting transition are reported. Under the expectation that the mysterious constant I_g should be the same in non-circular cylindrical specimens, if the paramagnetic effect should be observed in them and the current minimum should be required also for the appearance of the paramagnetic effect in them, as in the circular cylindrical specimens, measurements were done with cylindrical tin specimens of non-circular cross-sections of several types. In the specimens in which we expected the magnetic flux increase according to the proposed idea of the paramagnetic effect, we observed the magnetic flux increase and obtained the formula $I_0 = I_g + \beta H_0$ for the current minimum required for the occurrence of the paramagnetic effect. Although β differs from specimen to specimen according to the shape of the specimen cross-section, the magic constant I_g proves to be the same as in the case of circular rod, irrespective of the shape of the specimen cross-section, i.e. I_g is 1.2 amp for tin.

I. Introduction

A superconductor through which an externally supplied electric current flows in the presence of an external magnetic field shows a remarkable quasi-paramagnetism preceding the change to perfect diamagnetism, which is well-known now as the so-called paramagnetic effect in superconductors.(1) In the previous papers(2) we reported the experimental results on the paramagnetic effect in tin. In these reports(2) we confirmed that the paramagnetic effect was not an apparent but an intrinsic one observed, irrespective of the measuring procedure, and claimed that the current minimum required for the appearance of the paramagnetic effect should be determined in the $(I\text{-}H\text{-}T)$ space, instead of merely on the $(I\text{-}H)$ plane

* The 915th report of the Research Institute for Iron, Steel and Other Metals.

(1) A historical bibliography is found in references (2) and (5).

(2) Y. Shibuya and S. Tanuma, Phys. Rev. **98** (1955), 938; Sci. Rep. RITU, **A7** (1955), 549.

as proposed up to that time, where I, H, and T were the current supplied, the external magnetic field and the temperature respectively. The current minimum was represented in the $(I\text{-}H\text{-}T)$ space by the simultaneous equations $I_0=\xi\gamma d(T_c-T)$ and $H_0=\xi(T_c-T)-I_g/\gamma d$, where ξ, T_c and I_g were characteristic constants of the superconductor and had values 1.1×10^2 Oe/deg, 3.732°K and 1.2 amp respectively for the case of tin: γ was 0.23 amp/mm Oe. H_0, the external magnetic field maximum beyond which we cannot observe the magnetic flux increase at a given temperature was measured in oersted and d, the specimen diameter in mm. It has been shown that the formula proposed by Steiner and extended by W. Meissner et al. i. e. $I_0=I_g+\gamma dH$[3][4] could be derived from the above simultaneous equations after eliminating T and H in this formula should be understood as H_0. In the subsequent paper[5] which dealt with circular indium rods, it has been shown that the current minimum necessary for the occurrence of the magnetic flux increase in indium is represented in the $(I\text{-}H\text{-}T)$ space by the same simultaneous equations as in the case of tin, and that these equations are understood in a good approximation as those for the intersection of the plane $I=I_g+\gamma dH$ with the transition surface in the $(I\text{-}H\text{-}T)$ space, $H_c^2=H^2+(4I/d)^2$.

In the formula $I_0=I_g+\gamma dH$ for the circular cylindrical specimens, the characteristic quantity which specifies the cross-section of the specimen appears only as d, the specimen-diameter in the second term to the right side. As I_g, the so-called mysterious constant in the paramagnetic effect, which appears in the first term is independent of the characteristic which specifies the specimen cross-section, we are inclined to suppose that it should be the same in a cylindrical superconductors of non-circular cross-section, if the paramagnetic effect should be observed and the current minimum should be required for the occurrence of the paramagnetic effect. To obtain the analytical expression for the magnetic field distribution around a cylindrical specimen of non-circular cross-section through which an externally supplied current flows in the presence of an external magnetic field is, however, difficult except the special cases e. g. the case of an ellipsoidal cylindrical specimen. Therefore, at first sight, it seems meaningless to perform an experiment on the paramagnetic effect in non-circular cylindrical superconductors, because the experimental result cannot be compared easily with the theory. Nevertheless it may be some aids to the clarification of the origin of the mysterious constant I_g to ascertain whether non-circular rods, if the paramagnetic effect occurs in them and the current minimum is required for its occurrence, give the same I_g as the circular rods. This is the reason why we undertook an experiment with non-circular cylindrical superconductors. We observed the paramagnetic effect in a rectangular, a semi-circular, and a regular triangular cylindrical tin specimen.

The critical line representing the current minimum in the $(I\text{-}H\text{-}T)$ space was determined after the same procedure as in the case of circular rods. It was repre-

(3) K. Steiner, Z. Naturforsch. **4a** (1949), 271.
(4) Meissner, Schmeissner and Meissner, Z. Phys. **130** (1951), 529.
(5) The preceding paper in this series.

sented by the simultaneous equations:

$$I_0 = \xi\beta(T_c - T)\,,$$
$$H_0 = \xi(T_c - T) - I_g/\beta\,.$$

From these two equations the *I-H* relation i. e. $I_0 = I_g + \beta H_0$ could be derived as in the former cases. Although β was different from specimen to specimen according to the shape of their cross-sections, I_g proved to be the same, irrespective of the shape of cross-section i. e. I_g was 1.2 amp for tin. The result that the paramagnetic effect can be observed in these specimens can be understood at least qualitatively in terms of the idea(6) on which the theory of H. Meissner based(7).

Although it seems not adequate to deal here with the experiment with a hollow circular cylinder, a description of additional measurements performed with it is given. The result was in agreement with that reported already by W. Meissner et al.(6)

Furthermore we performed an experiment with non-circular tin rods having re-entrant cross-sections without observing the paramagnetic effect in them. We retraced also the experiment of W. Meissner and R. Doll(8) with a tin specimen of sandwich-type consisted of two semi-circular rods which were insulated from each other by a mica foil and in accordance with their result we observed no paramagnetic effect in it. These results can be also understood qualitatively by the idea that the helical path for the current cannot be completed in them.

II. Experimental details

As the experimental apparatus and procedure were almost the same as appeared in the previous report(5), we do not touch much upon them here. Most of the magnetic measurements were performed by the so-to-speak dynamic method and some by the so-to-speak static method. The former method consisted mainly in holding T constant throughout, taking H as a parameter and I as a variable which was changed in small steps: at each step the ballistic deflection of the galvanometer connected to search coils was measured when H was reversed. A search coil was fixed around the centre of the specimen, and a compensating coil connected in opposition to the search coil was fixed around a copper lead at a position sufficiently apart from the specimen in the uniform field. The difference in the magnetic flux through two coils induced a current in the ballistic-type galvanometer circuit, when the magnetic field was reversed. In some dynamic measurement we used the dynamic method of holding T constant throughout, taking I as a parameter, and reversing H which was taken as a variable and changed in small steps. In the static method we observed the ballistic deflection of the galvanometer when a search coil was dropped in a uniform external magnetic field from a position far apart from the specimen to a position around the centre of the

(6) Meissner. Schmeissner and Meissner. Z. Phys. **132** (1952), 529.
(7) H. Meissner, Phys. Rev. **97** (1955), 1627; **101** (1956), 31.
(8) W. Meissner and R. Doll, Z. Phys. **140** (1955), 340.

specimen under the equilibrium conditions with constant I, H and T. The difference of the magnetic flux at the initial and the final position of the search coil induced a current in the galvanometer circuit when the search coil was dropped.

All the tin specimens except the rectangular cylindrical one were prepared from Johnson-Matthey spectroscopically standardised tin. The rectangular cylindrical tin specimen was prepared from Kahlbaum pure tin. The specimen was machined to the size of 5 mm in breadth, 1 mm in thickness and 80 mm in length. A tin rod of 6 mm in diameter was cut longitudinally right in two with a cutter of 0.5 mm in thickness, one of which was used as the semi-circular cylindrical specimen whose length was 80 mm. Therefore, strictly speaking, the specimen was not semi-circular. The tin specimen of sandwich-type was constructed from a pair of semi-circular rods thus prepared and insulated from each other by a mica foil. The regular triangular cylindrical tin specimen was prepared by machining a tin rod to the size of 5 mm in an edge of the regular triangle and 70 mm in length. The hollow cylindrical specimen was machined so as to have the outer diameter of 5 mm, the inner diameter of 2 mm, and the length of 70 mm.

In three kinds of non-circular tin rods we could not observe the paramagnetic effect. One was prepared in such a way that a circular rod of 5 mm in diameter and 70 mm in length was cut away to the cylinder axis longitudinally with a cutter having the angle of 30° [see Fig. 1 (a)]. The other was prepared from a hollow cylinder of 5 mm in outer diameter, 2 mm in inner diameter and 70 mm in length by cutting it longitudinally and perpendicularly to its surface at one side of the wall with a cutter of 1 mm thick. Its cross-section is shown in Fig. 1 (b). The third was the specimen of sandwich-type afore-mentioned.

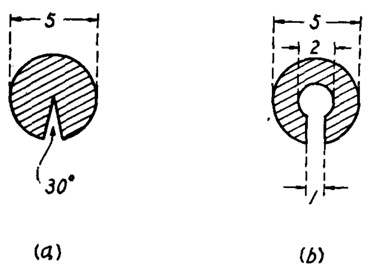

Fig. 1. Cross-sections of tin rods showing no paramagnetic effect. The length is in the unit of mm.

The annealing procedures of all the specimens were quite the same i. e. the specimens were sealed in evacuated glass tubes separately and were annealed at 190°C for two hours.

All these specimens were soldered with Wood's metal at their upper ends to copper rods of the same cross-section and length as theirs respectively.

III. Experimental results

1. Non-circular rods showing the paramagnetic effect

(a) Rectangular rod

A typical result obtained by the dynamic method is shown in Fig. 2, in which the ballistic deflection of the galvanometer was plotted as a function of current I for a given field H at 3.691°K. According to the similar procedure to that adopted in the previous reports the apparent permeability μ was defined. μ^* which corresponded to the maximum of quasi-paramagnetism for constant values of magnetic field and temperature was plotted as a function of the current I^*, where μ^* occurred, in Fig. 3 (a). It can be seen that in this case also there exists the current minimum I_0 for the occurrence of the paramagnetic effect. In the same way the external magnetic field maximum H_0 at that temperature was determined (Fig. 3 (b)). I_0 and H_0 thus determined at several temperatures gave the I_0–T, the H_0–T, and the I_0–H_0 relation shown in Fig. 4 (a), (b) and (c) respectively. These three relations are represented respectively by three equations as follows:

$$I_0 = \xi\beta(T_c - T)\,, \qquad (1)$$

$$H_0 = \xi(T_c - T) - I_g/\beta\,, \qquad (2)$$

$$I_0 = I_g + \beta H_0\,. \qquad (3)$$

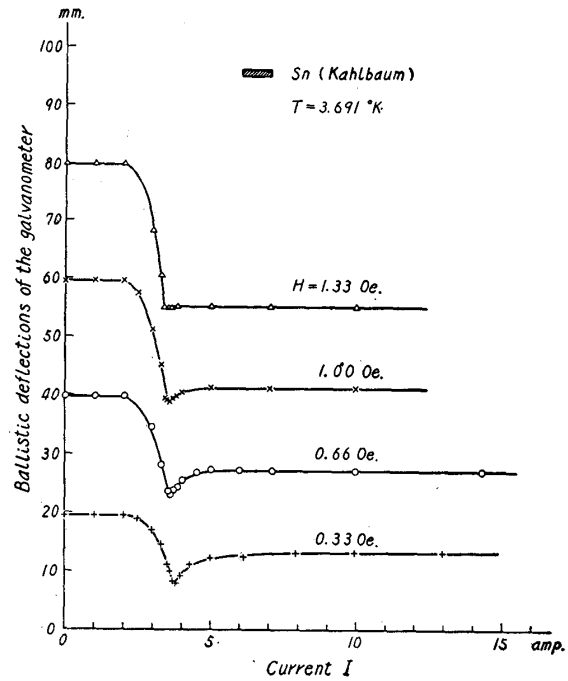

Fig. 2. Ballistic deflections of the galvanometer for the rectangular tin rod as a function of current I for the specified values of magnetic field H, when H was reversed.

The critical line (the line for the current minimum I_0) is represented in the (I-H-T) space by the simultaneous eqations of arbitrary two equations of the above three. These equations are quite similar to those obtained in the case of circular rods except γd replaced by β. The mysterious constant I_g exists also in the present case and its numerical value is quite the same as in circular rods, i. e. $I_g = 1.2$ amp. Other constants are $T_c = 3.742$°K, $\xi = 62.0$ Oe/deg and $\beta = 1.61$ amp/Oe. Similarly to the case of circular rods the critical line terminates at the point (I_g, T_g) on the (I-T)

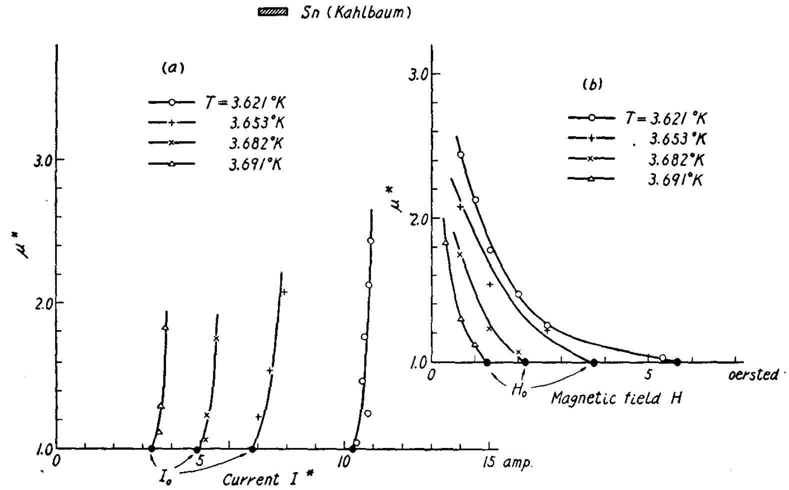

Fig. 3. (a) μ^* for the rectangular tin rod as a function of current I^* at the specified temperatures. (b) μ^* for the same as a function of external magnetic field H at the specified temperatures.

plane, where T_g, which does not appear explicitly in above equations, is equal to $T_c - I_g/\xi\beta$ and its value is 3.712°K in this case.

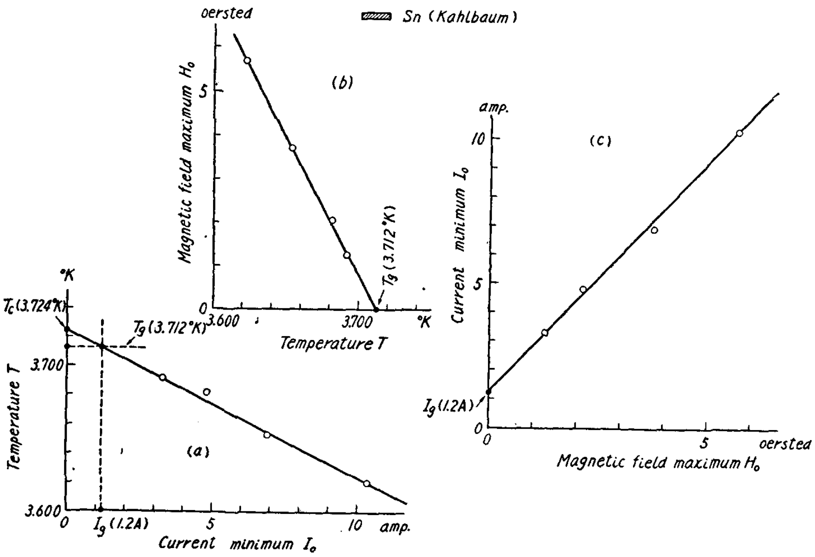

Fig. 4. (a) I_0-T relation, (b) H_0-T relation, and (c) I_0-H_0 relation for the rectangular tin rod.

(b) Semi-circular rod

One example of the ballistic deflections of the galvanometer plotted against I is shown in Fig. 5 which was obtained for specified values of H at 3.672°K. It seems at first sight curious that whereas the ballistic deflections caused by reversing H in the superconducting state should be proportional to H, the figure does not show such a proportionality. This is due to the procedure that we changed the shunt-resistance of the galvanometer with the magnetic field in order to plot the ballistic deflections for the magnetic field 0.66 Oe to 3.32 Oe in a limited space. This procedure does not, however, affect the determination of the apparent permeability μ. Determining I_0 and H_0 for several temperatures, we obtained the I_0-T, the H_0-T, and the I_0-H_0 relation shown in Fig. 6 (a), (b) and (c) respectively. These relations are represented also by above three equations. In this case also the mysterious constant I_g exists and its value is again

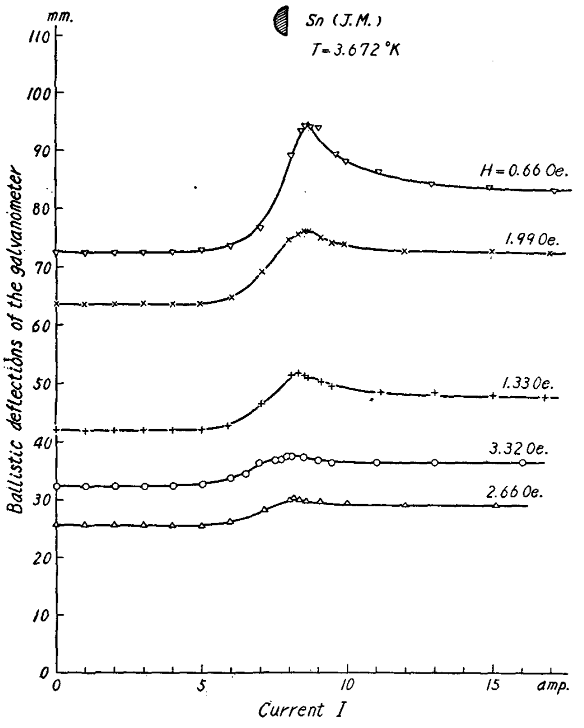

Fig. 5. Ballistic deflections of the galvanometer for the semi-circular tin rod as a function of current I for the specified values of magnetic field H, when H was reversed.

1.2 amp. Other constants appearing in the equation are $T_c = 3.732°K$, $\xi = 85.5$ Oe/deg and $\beta = 1.44$ amp/Oe. T_g is 3.722°K.

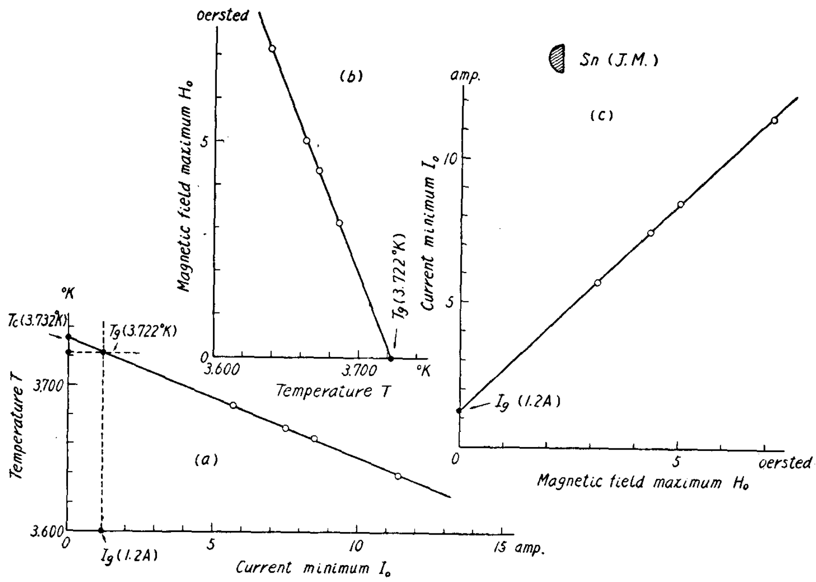

Fig. 6. (a) I_0-T relation, (b) H_0-T relation and (c) I_0-H_0 relation for the semi-circular tin rod.

(c) Regular triangular rod

The ballistic deflection curves obtained at 3.694°K by the dynamic method are shown in Fig. 7. The circumstances are quite similar to those in former two cases. The (I-T), the (H-T) and the (I-H) projection of the critical line in the (I-H-T) space are shown in Fig. 8 (a), (b) and (c) respectively. These projections are represented also well by the above three equations (1), (2) and (3). We obtained I_g=1.2 amp in this case also. The values of T_c, ξ and β are 3.734°K, 81.9 Oe/deg and 1.08 amp/Oe respectively. T_g is 3.720°K

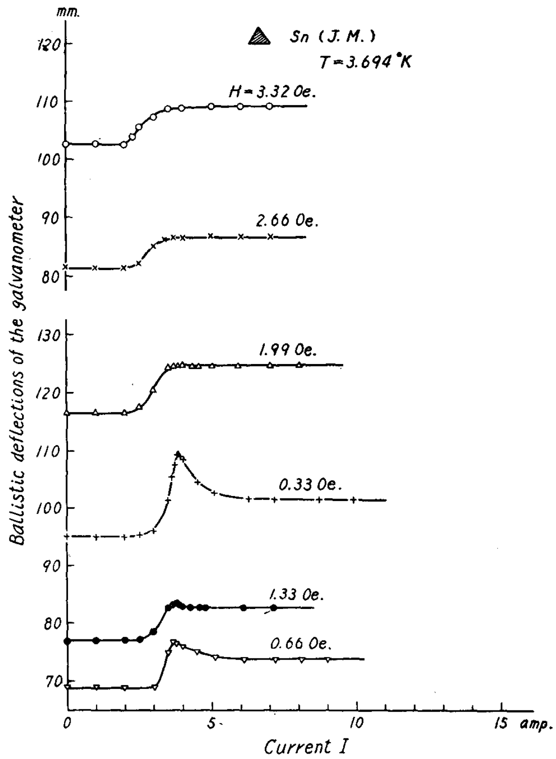

Fig. 7. Ballistic deflections of the galvanometer for the regular triangular tin rod as a functions of current I for the specified values of magnetic field H, when H was reversed.

2. Hollow circular rod

We observed the quasi-paramagnetism in this specimen both with a search coil surrounding the specimen and with a search coil placed into the hole. One of the results obtained by the dynamic method at 3.662°K is shown in Fig. 9. It can be seen that the flux increase

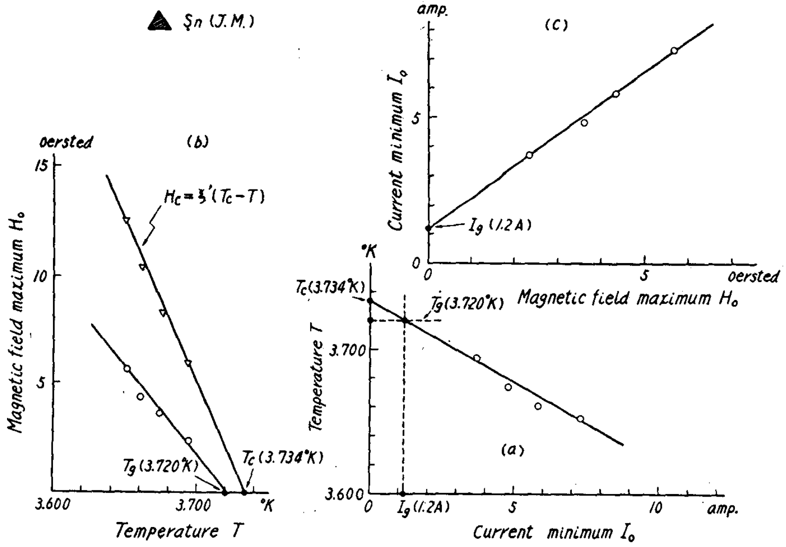

Fig. 8. (a) I_0-T relation, (b) H_0-T relation and (c) I_0-H_0 relation for the regular triangular tin rod. The critical field H_c is represented by $H_c = \xi'(T_c-T)$, where $\xi' = 150.5$ Oe/deg and $T_c = 3.734$°K.

occurred not only in the body of the specimen but also in the hole.

The result obtained by the static method is shown in Fig. 10 which ensures that the paramagnetic effect is not a transient, apparent phenomenon accompanying the change in time of any one of I, H or T. The critical line for the current minimum required for the appearance of the paramagnetic effect was determined from the results obtained with the outer search coil by the dynamic method. It is represented in the (I-H-T) space by the same equations as in circular tin or indium rods, which need not be repeated here. We note here only that I_g is also 1.2 amp in this case. We did not ascertain the current minimum requirement with the inner search coil placed in hole. We observed a hysteresis in the magnetization curve with $I=0$ in this specimen which is a characteristic of hollow cylinders. The hysteresis is shown in Fig. 11 obtained by the static method at 3.667°K.

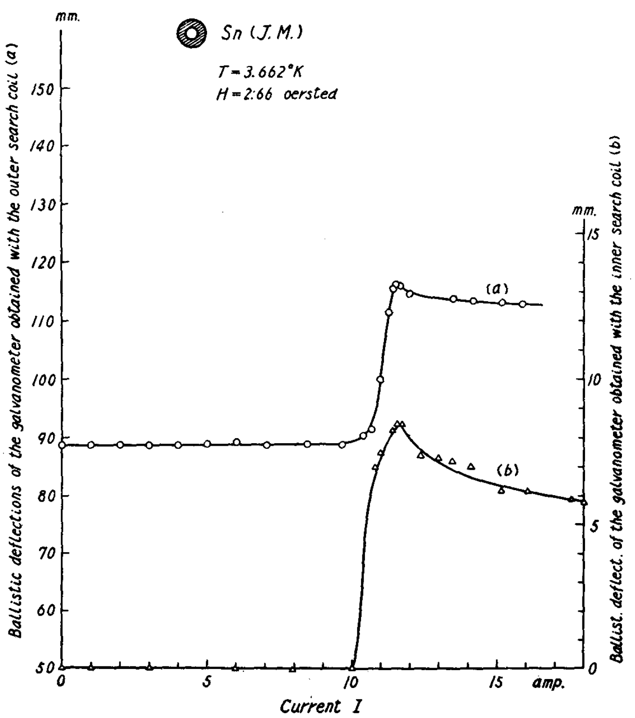

Fig. 9. (a) Ballistic deflections of the galvanometer for the hollow tin rod obtained with the outer search coil as a function of current I for $H=2.66$ Oe and $T=3.662$°K, when H was reversed. (b) The same obtained with the inner search coil.

The magnetic field was increased from 0 to 13.30 Oe by small steps, and vice versa; at each step a moving search coil connected to the galvanometer was dropped and the ballistic deflection was measured.

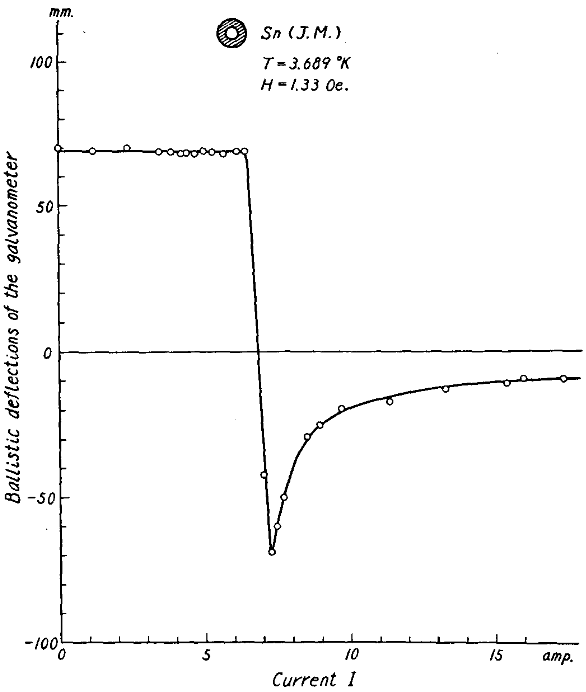

Fig. 10. Ballistic deflections of the galvanometer for the hollow tin rod as a function of current I for $H=1.33$ Oe and $T=3.689°K$ obtained by the static method.

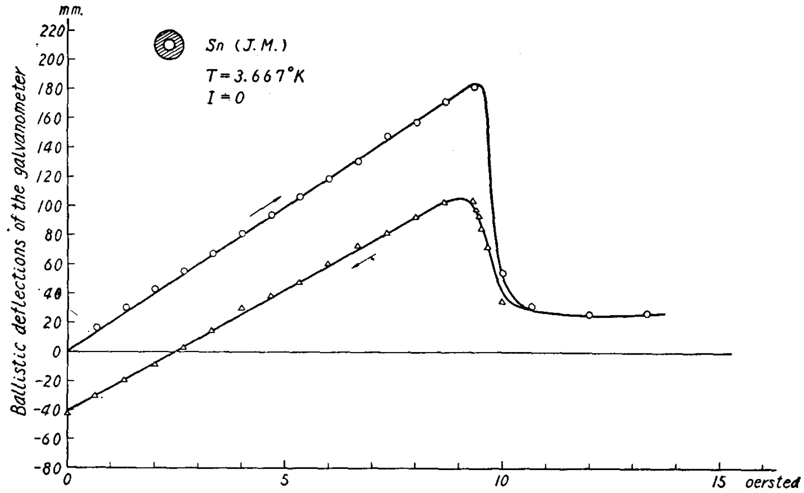

Fig. 11. Hysteresis in the magnetization curve for the hollow tin rod obtained by the static method.

3. Non-circular rods showing no paramagnetic effect

(a) Rod having a longitudinal, wedge-shaped slot (Fig. 1 (a))

It may be justified both from Fig. 12 obtained by the dynamic method and from Fig. 13 by the static method to conclude that the quasi-paramagnetism cannot be observed in this specimen. This specimen showed a hysteresis in the magnetization curve with $I = 0$, as shown in Fig. 14 which was obtained by the static method, in spite of the fact that the specimen was a singly-connected body. In Fig. 14 the ballistic deflections were plotted as a function of the magnetic field which varied from zero to 18.61 Oe by small steps without returning back and vice versa.

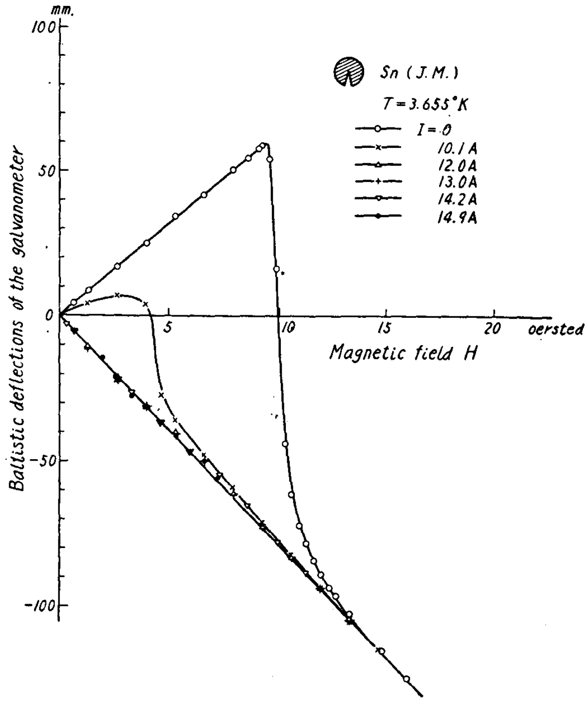

Fig. 12. Ballistic deflections of the galvanometer for the tin rod having a wedge-shaped longitudinal slot as a function of magnetic field H for the specified values of current I at 3.655°K, when H was reversed.

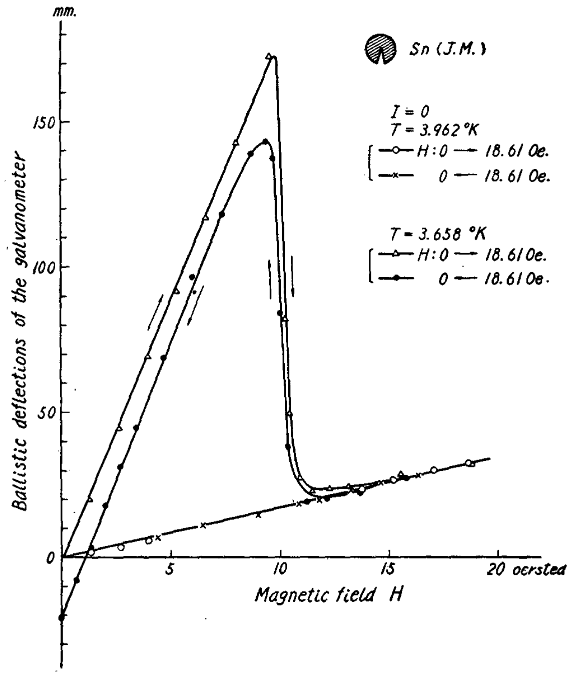

Fig. 14. Hysteresis in the magnetization curve for the tin rod having a wedge-shaped longitudinal slot obtained by the static method.

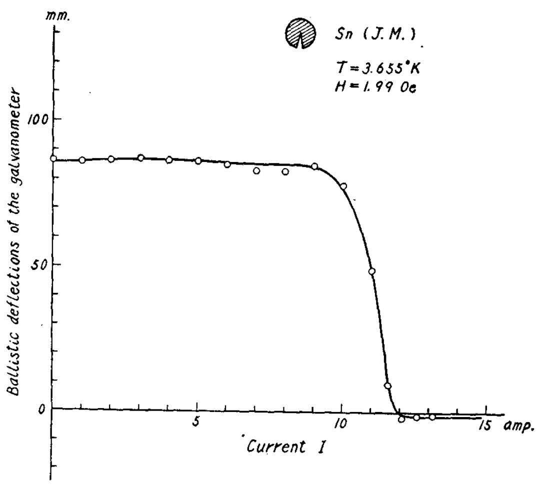

Fig. 13. Ballistic deflections of the galvanometer for the tin rod having a wedge-shaped longitudinal slot as a function of current I for H=1.99 Oe and T=3.655°K obtained by the static method.

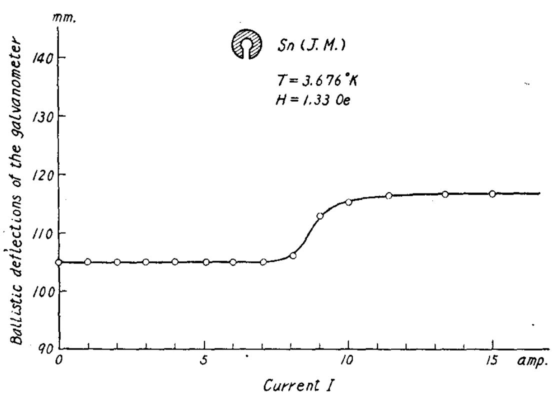

Fig. 15. Ballistic deflections for the hollow tin rod having a longitudinal slot as a function of current I for H=1.33 Oe and T=3.676°K, when H was reversed.

(b) Hollow rod having a longitudinal slot at one side of its thickness (see its cross-section in Fig. 1 (b))

The ballistic deflection vs. current curve at 3.676°K for a fixed value of H which was reversed is shown in Fig. 15. We could not observe the quasi-paramagnetism in this specimen.

(c) Sandwich-type rod

The results of dynamic measurement and of static measurement are shown in Figs. 16 and 17 respectively. In this case also we found no paramagnetic effect.

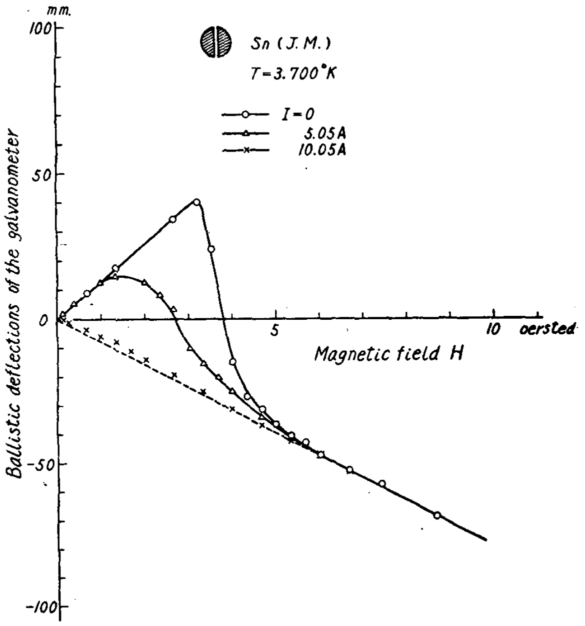

Fig. 16. Ballistic deflections for the tin rod of sandwich-type as a function of magnetic field H for the specified values of current I at 3.700°K, when H was reversed.

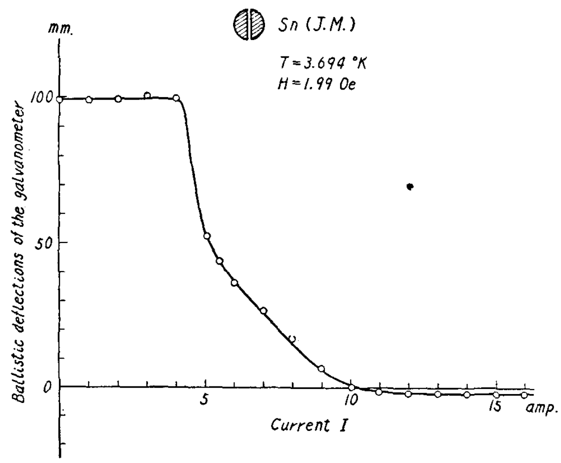

Fig. 17. Ballistic deflections for the tin rod of sandwich-type as a function of current I for H=1.99 Oe and T=3.694°K obtained by the static method.

IV. Conclusions

The results obtained with non-circular tin rods showing the paramagnetic effect as seen in III. 1. are tabulated in Table 1. As the experimental results could not be compared with the theory in contradiction to the case of circular rods, because the analytical expression for the magnetic field distributions around such specimens could not be easily obtained, the critical field H_c was determined only for the triangular rod, which is represented by $H_c=\xi'(T_c-T)$ where $\xi'=150.5$ Oe/deg and $T_c=3.734$°K.

The shapes of specimen cross-section dealt with in this paper are not, of course, geometrically accurate ones, because their edges were more or less rounded off. The fact that the paramagnetic effect can be observed in such specimens can be explained qualitatively with the idea[6] on which the theory of H. Meissner[7] bases. In such specimens also the resultant magnetic field of the external field and the field produced by the specimen current shows a helical path, irregular as it is, and the current flowing so as to connect the oblong superconducting grains which remain in the intermediate state along the resultant field produces a longitudinal flux greater than that due to the external field.

Table 1. Numerical values of constants which appear in the formulas for the current minimum I_0 for non-circular tin rods showing the paramagnetic effect.

$$I_0 = \xi\beta(T_c - T)\,,$$
$$H_0 = \xi(T_c - T) - I_g/\beta\,,$$
$$I_0 = I_g + \beta H_0\,.$$

	Sn (Kahlbaum)	Sn (J. M.)	Sn (J. M.)
I_g (amp)	1.2	1.2	1.2
ξ (Oe/deg)	62.0	85.5	81.9
β (amp/Oe)	1.61	1.44	1.08
T_c (°K)	3.724	3.732	3.734
[T_g (°K)]	3.712	3.722	3.720

On the other hand in such specimens showing no paramagnetic effect as dealt with in III. 3. the paramagnetic effect may not be observed owing to the possibility that the helical path for the current can not be completed.

β corresponds to γd in the formula obtained for circular rods, where γ having the dimension amp/mm Oe is more or less constant i. e. nearly 0.25 in both cases of tin and indium and d is the specimen-diameter in mm. β seems to depend on the cross-section of the specimen. It is not, however, clear from the present investigation in what way β depends on the cross-section of the specimen. Similar arguments can be said in regard to ξ. The difference of T_c between the rectangular and the semi-circular or the triangular rod should be ascribed to the difference of their sources.

The mysterious constant I_g which appeared in the formulas for the current minimum I_0 in the $(I\text{-}H\text{-}T)$ space for circular rods was 1.2 amp for tin. The fact that I_g exists also in non-circular rods and its value is also 1.2 amp, irrespective of the cross-section of the specimen, may throw a light on the explanation for I_g.

The discrepancies between our experiment reported in the previous paper[(2)] which dealt with circular tin rods and the theory of H. Meissner[(7)] could not be removed completely by our recent results on circular indium rods.[(5)] However, the facts that our recent results showed a qualitative agreement with the theory and the paramagnetic effect observed in some specimens of non-circular rods can be explained qualitatively with the idea due to W. Meissner et al.[(6)] seem to substantiate to some extent the theory of H. Meissner on the paramagnetic effect.[(7)] Nevertheless the explanation for the origin of the mysterious constant I_g seems to be sought in a new direction.

Acknowledgement

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