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Magnetic Flux Increase and Resistance of Circular Indium Rods at the Superconducting Transition*

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Synopsis

The so-called paramagnetic effect in superconductors has been investigated in indium. The current minimum I_0 required for the occurrence of the magnetic flux increase has been represented by the same functions of the temperature T and the external magnetic field maximum H_0, beyond which we cannot observe the quasi-paramagnetism at that temperature, as in the case of tin i. e. $I_0=\xi\gamma d(T_c-T)$ and $H_0=\xi(T_c-T)-I_g/\gamma d$. Here I_g, ξ and T_c are characteristic constants of the superconductor and have values 0.6 amp, 94.4 Oe/deg and 3.422°K respectively for the case of indium: γ is 0.27 amp/mm Oe. The external magnetic field maximum H_0 and the specimen diameter d are measured in Oe and in mm respectively. It has been shown also that these formulas are understood in a good approximation as those for the intersection of the plane $I=I_g+\gamma dH$ with the transition surface in the $(I\text{-}H\text{-}T)$ space, and that the resultant magnetic field H_t (the external magnetic field H plus the magnetic field H_I due to the current) at the specimen surface for the maximum of quasi-paramagnetism at constant magnetic field and temperature is, however, less than H_c. In parallel with the measurement of the magnetic flux, plotted as a function of the current at constant temperature and external magnetic field, a measurement of the resistance of the same specimen has been performed. The result of the resistance measurement is in good accordance with an expression derived in a modified form from the theory proposed by H. Meissner.

The results obtained in the present investigation are not in a complete agreement with H. Meissner's theory. A possible explanation for this discrepancy is briefly touched.

I. Introduction

Shortly after H. K. Onnes succeeded for the first time in the liquefaction of helium gas, he discovered superconductivity of mercury the resistance of which he measured at the liquid helium temperatures. Onnes came to the conclusion that the resistance of mercury vanished below 4.15°K, by measuring the potential drop between two points of specimen and the current flowing through it. Afterwards W. Meissner and R. Ochsenfeld discovered that superconductivity was accompanied by zero induction in the magnetic field below the critical one H_c appropriate to the temperature. Two important features of the superconductor are characterized by the disappearance of its electrical resistance and its magnetic induction below the critical temperature T_c. The independent variables of the state which may determine whether a metal is in the superconducting state, if it becomes one at all, are the temperature T and the external magnetic field H. We need only consider the superconducting transition on the $(H\text{-}T)$ plane. However, if a superconductor through which an externally supplied current flows

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is placed in an external magnetic field, we must consider also the current I as a variable, because the current through the specimen produces its own magnetic field H_I. When a cylindrical superconductor through which a current flows in the presence of an external longitudinal magnetic field, it is reasonable to consider the superconducting transition in the $(I\text{-}H\text{-}T)$ space instead of merely on the $(H\text{-}T)$ plane.

Experiment on the magnetic flux in a superconductor through which a current flowed in the presence of an external magnetic field was performed by K. Steiner and H. Schoeneck,[1] who observed first the increase of magnetic induction in the specimen immediately before the superconducting transition. It is quite natural to expect that a superconductor through which an externally supplied current flows in the presence of an external magnetic field, may become normalconducting in the external magnetic field smaller than H_c, because the effective magnetic field which destroyes superconductivity may be the resultant of the external magnetic field and the magnetic field due to the current. It was, however, quite unexpected that a superconductor showed a remarkable quasi-paramagnetism[2] preceding the change to diamagnetism. Indeed Steiner and Schoeneck observed the magnetic flux larger than that due to the external magnetic field with a cylindrical superconductor through which a current larger than a critical one flowed in the presence of an external longitudinal magnetic field. Steiner reported afterwards[3] that there was a relation between the current minimum I_0 and the external magnetic field H for the occurrence of the magnetic flux increase, and that he observed the same effect also with a hollow cylindrical superconductor through which a current larger than a critical one flowed in the presence of an external, circular, magnetic field perpendicular to the specimen-axis. These phenomena are well-known now as the paramagnetic effect in superconductors.

It was, however, conceived once that the paramagnetic effect might have possibly been only an apparent one, because in earlier works the observation of quasi-paramagnetism was coupled with a simultaneous variation of one such as H or T of the variables of the state. Meissner et al.[4],[5] emphasized, however, that the magnetic flux increase was not an apparent but an intrinsic phenomenon, from their measurement by the fluxmetric recording method in which the temperature was changed very slowly at fixed values of current and external magnetic field, because they confirmed that the magnetic flux increase could be maintained permanently if current, magnetic field and temperature were kept constant. Furthermore they clarified that the coefficient of the magnetic field which appeared in the

(1) K. Steiner and H. Schoeneck, Phys. Z. **44** (1943), 346.

(2) For instance, tin is a paramagnetic metal from room temperature down to very low temperatures. Its magnetic susceptibility is, however, of the small order of 10^{-8} and hence this paramagnetism can be discarded here. The quasi-paramagnetism discussed in the present paper is of the same order as the absolute value of the perfect diamagnetism, the volume susceptibility of which is $-1/4\pi$.

(3) K. Steiner, Z. Natforsch. **4a** (1949), 271.

(4) Meissner, Schmeissner and Meissner, Z. Phys. **130** (1951), 521, 529.

(5) Meissner, Schmeissner and Meissner, Z. Phys. **132** (1952), 529.

formula for the minimum current requirement proposed by Steiner was proportional to the specimen-diameter. They performed measurements on hollow cylindrical specimens,(5) besides on cylindrical one, with the result that they could observe also the magnetic flux increase with both a search coil to measure the magnetic flux inside the hole and another one to measure the total magnetic flux through the specimen, and concluded that the magnetic flux increase was not due to the volume magnetization but to the circular component of current in the specimen. In order to verify this conclusion they performed experiments on both a cylinder and a hollow cylinder, splitted so as to hinder the circular current, without observing the magnetic flux increase.

In the previous papers(6) we reported the experimental results of the magnetic flux increase in tin. Employing the so-to-speak static method of the measurement i.e. the method of measuring the ballistic deflection of the galvanometer when a search coil was dropped in a uniform external magnetic field from a position far apart from the specimen to a position around the centre of the specimen under the equilibrium conditions with constant I, H and T, we confirmed that the paramagnetic effect was not an apparent but an intrinsic one observed irrespective of the measuring procedure. This result was, in conjunction with both the confirmation by Meissner et al. who employed the fluxmetric recording method, and that by T. S. Teasdale and H. E. Rorschach(7) who employed the static method similar to ours, enough to disprove the question that the paramagnetic effect might be an apparent one probably due to the dynamical method of the measurement. Employing the dynamical method of the measurement i.e. the method of measuring the ballistic deflection of the galvanometer connected to search coils when the external magnetic field was reversed at constant values of current and temperature, we determined the current minimum necessary for the occurrence of the magnetic flux increase and claimed that it should be determined in the $(I\text{-}H\text{-}T)$ space, instead of merely on the $(I\text{-}H)$ plane as formerly proposed. The current minimum I_0 was represented in the $(I\text{-}H\text{-}T)$ space by the simultaneous equations $I_0 = \xi\gamma d(T_c - T)$ and $H_0 = \xi(T_c - T) - I_g/\gamma d$, where ξ, T_c and I_g were characteristic constants of the superconductor and had values 1.1×10^2 Oe/deg, 3.732°K and 1.2 amp respectively for the case of tin: γ was 0.23 amp/mm Oe. H_0, the external magnetic field maximum beyond which we could not observe the magnetic flux increase at a given temperature was measured in oersted and d, the specmen-diameter in mm. Showing that the formula for the minimum current requirement proposed by Steiner and extended by W. Meissner et al, i.e. $I_0 = I_g + \gamma dH$ could be derived from the above simultaneous equations after eliminating T, we concluded that the formula $I_0 = I_g + \gamma dH$ was the one for the orthogonal projection on the $(I\text{-}H)$ plane of the critical line (I_0-line) in the $(I\text{-}H\text{-}T)$ space and that H in this formula should be understood as H_0.

(6) Y. Shibuya and S. Tanuma, Phys. Rev. **98** (1955), 938: Sci. Rept. RITU. **A7** (1955), 549. These papers are referred to in the text as I and II.
(7) T. S. Teasdale and H. E. Rorschach, Phys. Rev. **90** (1953), 709.

H. Meissner formulated a theory of the paramagnetic effect[8] at about the same time of the publication of the paper I, basing upon the idea[5] that in the intermediate state the specimen current following a helical path so as to connect the oblong superconducting grains oriented along the resultant magnetic field H_t (the vector sum of external field H and current field H_I) produced a longitudinal flux greater than that due to the external field. According to H. Meissner, the maximum of quasi-paramagnetism, e. g. at constant external magnetic field and temperature, occurs when the resultant field H_t is equal to the critical field H_c at that temperature. Our previous result (the paper II) did not, however, agree with this prediction. Afterwards H. Meissner gave a detailed analysis on our experimental result (the paper II) and claimed that our result substantiated rather than invalidated his theory.[9]

In order to confirm this point we performed measurements on the paramagnetic effect in solid circular indium rods. Further the resistance measurement was thought quite important for the understanding of the nature of the paramagnetic effect. Nevertheless it seemed that the simultaneous measurement of the resistance and the magnetic flux was hitherto very scanty except those of W. Meissner and R. Doll[10] and of A. Sellmaier.[11] No quantitative analysis of the experimental result was, however, given in these investigations. Basing upon H. Meissner's theory, we have derived an expression for the resistance as a function of current at constant magntic field and temperature. In order to compare this result with the experiment we performed the resistance measurement in parallel with the magnetic flux measurement.

Special care was taken in the determination of the temperature within the specimen throughout the present investigation.

The experimental results obtained have shown (i) that the maximum of quasi-paramagnetism at constant field and temperature occurs when the resultant field $H_t[=(H^2+H_i^2)^{\frac{1}{2}}]$ is less than H_c at that temperature in disagreement with the theory proposed by H. Meissner, (ii) that H_t for the maximum of quasi-paramagnetism at that temperature approaches, however, H_c with the decrease of current and H_t at (I_0, H_0) coincides nearly with H_c in accordance with the theory proposed by H. Meissner, (iii) that the resistance as a function of current under the same condition agrees qualitatively with the theoretical prediction, and (iv) that the formulas for the current minimum I_0 in the $(I\text{-}H\text{-}T)$ space coincide in a good approximation with those for the intersection of the plane $I=I_g+\gamma dH$ with transition surface $H_c^2=H^2+(4I/d)^2$.

In course of the present investigation it came to our notice that J. C. Thompson who obtained the same expression as ours for the resistance as a function of current at constant field and temperature, performed also the resistance measurement

(8) H. Meissner, Phys. Rev. **97** (1955), 1627; **101** (1956), 31.
(9) H. Meissner, Phys. Rev. **103** (1956), 39.
(10) W. Meissner and R. Doll, Z. Physik **140** (1955), 340.
(11) A. Sellmaier, Z. Physik **141** (1955), 550.

with the result showing a fair agreement with the theory.[12] He obtained, moreover, an analytical expression for the apparent permeability as a function of current under the same condition as in the resistance measurement, which was also in a fair agreement with his experimental result. Our magnetic measurement showed also a good agreement with the expression for the apparent permeability derived by J. C. Thompson.

II. Experimental details

The experimental apparatus used in the present investigation is shown schematically in Fig. 1. and can be seen in Plate 1. A circular indium rod S and circular copper rods L_1 and L_2 of about the same diameter as that of S, soldered together with Wood's metal at the upper and the lower end of S, were placed in a uniform external longitudinal magnetic field, and an externally supplied electric current flowed down or up through them.

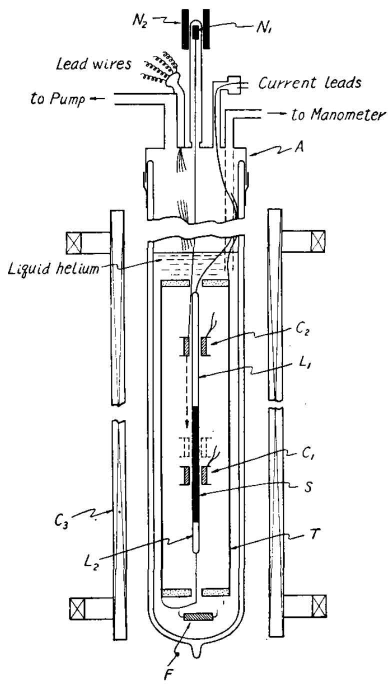

Fig. 1. Schematic diagram of the experimental apparatus. The outer Dewar vessel for containing liquid nitrogen and two pairs of large Helmholtz coils of square-type for nullifying the earth magnetic field are not shown.

The magnetic flux in the specimen was measured with the use of a ballistic-type galvanometer having a period of 13 sec. as in the case of ferromagnetic substances. We employed mainly the so-to-speak dynamic method[15] of holding T constant throughout, taking H as a parameter and I as a variable which was changed in small steps: at each step the galvanometer deflection was measured when H was reversed. A search coil C_1 was fixed around the center of the specimen, and a compensating coil C_2 connected in opposition to C_1 was fixed in a unform magnetic field around a copper lead at a position sufficiently far from the specimen. The difference in the magnetic flux through two coils C_1 and C_2 induced a current in the ballistic-type galvanometer circuit, when the external magnetic field H was reversed. Two coils C_1 and C_2 have about the same construction. They were wound with 3,000 turns of BS# 40 P. V. F. wire

(12) J. C. Thompson, Journ. Phys. Chem. Solid **1** (1956), 61. L. Rinderer who investigated only the resistance of cylindrical superconductors in the magnetic field obtained independently also the same expression for the resistance as a function of current as J. C. Thompson and we did [Helvetica Physica Acta **29** (1956), 339].

(13) We are indebted to H. Meissner who recommended us this dynamic procedure in place of the dynamic one, previously adopted, of holding T constant throughout, taking I as a parameter, and reversing H which was taken as a variable and changed in small steps. The latter procedure might yield some inaccuracy in the measurement in small magnetic field.

on bakelite forms. As it was already ascertained by the static measurement in the previous paper that the paramagnetic effect was not a transient, apparent phenomenon accompanying the change in time of any one of I, H or T but an intrinsic one observed, irrespective of the measuring procedure, we do not touch further this problem than mention that we confirmed the intrinsic nature of the paramagnetic effect also in indium by the static method of dropping the search C_2, which was used as a compensating coil in the dynamic measurement, in the uniform field from its position around the copper cylinder to a position around the center of the indium specimen and observing the ballistic deflection of the galvanometer connected only to C_2 in this case. Use was made of two N. K. S. magnets N_1 and N_2 and Nylon thread shown in Fig. 1 for the dropping and the pulling up of C_2.

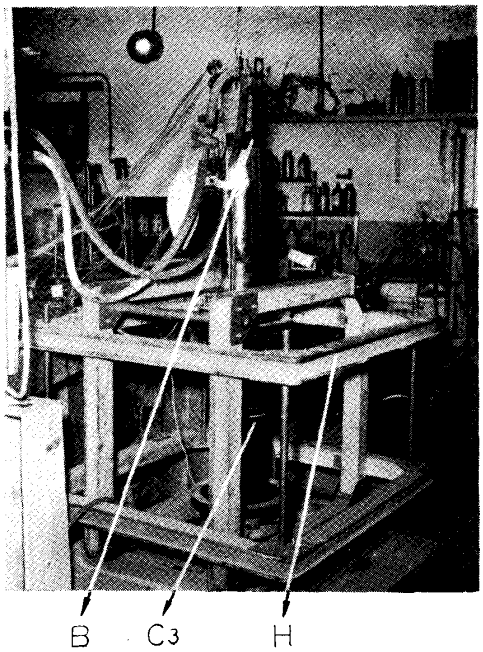

Plate 1. Liquid helium Dewar B, the P. V. F. wire-wound solenoid system C_3, and two pairs of large Helmholtz coils of square-type are shown.

The thin-walled, slotted copper tube T surrounding concentrically the specimen was used as the return current lead in order to nullify the magnetic field around the specimen due to the return current. The P. V. F. wire-wound solenoid system C_3 produced a uniform longitudinal magnetic field homogeneous to within 0.07 per cent in the volume of 2 cm in diameter and 20 cm in length in the central portion of the solenoid system. The earth magntic field was cancelled with two pairs of large Helmholtz coils of square-type located perpendicularly to each other (one horizontally, the other vertically) which are not shown in Fig. 1 for spatial reasons but can be seen in Plate 1. A small heater F was placed in the bottom of the helium Dewar for the purpose of making uniform the temperature-gradient in the liquid helium column. The manostat previously reported[6] was used to fix the helium vapour pressure to within 0.2 mm Hg. The helium vapour pressure was measured with a mercury manometer. The temperature within the specimen was determined together by vapour pressure thermometry using the 1948 Mond Laboratory Tables and by taking the liquid helium column head into consideration. The determination of the liquid helium column head was perfomed as follows. Observing the liquid helium levels at the beginning and at the end of the measurement at a given temperature, we determined the mean liquid helium level during the measurement at that temperature and defined the mean liquid helium column head by the distance of the mean liquid helium level from the specimen-centre. The lowering of the liquid level during one run of measurements at a given temperature was, generally speaking, considerable and sometimes amounted to about 10 cm, when the level was high i. e. it lay near the liquid helium Dewar cap A in Fig. 1, but it became 3~4 cm when the level lay far below the cap A. The helium vapour pressure above the liquid helium and the liquid helium column head thus determined served to determine the true temperature within the specimen.

The indium specimens were prepared from Johnson-Matthey spectroscopically standardised indium (>99.99 % I_n). They were cast in vacuo in glass tubes. After they were removed from glass tubes they were drawn to the required diameter. After cutting the specimen to about 80 mm long, the specimen was sealed in a evacuated glass tube and annealed at 100°C for several hours. Although the melting point of indium is relatively low (about 156°C) and the room temperature annealing would be sufficient for indium, the annealing process above mentioned was performed by way of precaution. The specimen No. 1 was 2.38 mm in diameter and about 80 mm in length, and No. 2 was 1.63 mm in diameter and about 75 mm in length. Both specimens showed the same critical temperature 3.422°K.

The resistance measurement was performed only with the specimen No. 2 by the use of a Diesselhorst compensation apparatus which ensured to measure such a small potential difference as 10^{-8}V. The galvanometer used for the resistance measurement had a voltage sensitivity of 3×10^{-8}V. Two BS # 38 enamelled wires soldered with Wood's metal to the specimen No. 2 near its both ends were used as the potential probes. The potentiometer circuit for measuring resistance is shown in Fig. 2.

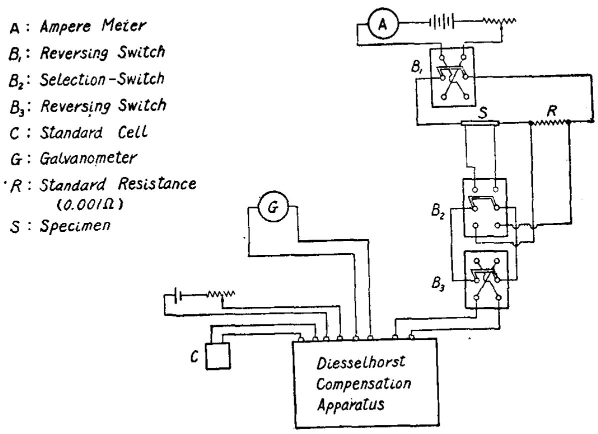

Fig. 2. Potentiometer circuit for measuring resistance.

The parallel measurement of the magnetic flux and the resistance was carried out in such a way as follows. After measuring the potential drop between two potential probes for a certain value of I at constant values of H and T, we observed the ballistic deflection of the galvanometer caused by the reversal of H. Then I was somewhat increased while H and T were kept constant, and the same procedure was repeated. This procedure started with $I = 0$ and ended with such a large current as 15 amp. The current I was increased by about 1 amp except near the current I^* where the maximum of quasi-paramagnetism occurred. The current increase was made in smaller steps near I^*.

The method employed for finding the critical field H_c consisted in raising the field steadily and watching for signs from the search coil that magnetic flux was entering the specimen as the superconducting phase collapsed. This method was sometimes supplemented by the method of determining H_c from the magnetization curve of the specimen obtained by reversing H. We observed supercooling of some degree in both specimens.

III. Experimental results

1. The determination, in the $(I\text{-}H\text{-}T)$ space, of the current minimum I_0 required for the occurrence of the paramagnetic effect.

In the magnetic flux measurement for this purpose we employed the dynamic method described above. Typical examples of results obtained with the specimens Nos. 1 and 2 are shown in Figs. 3 and 4 respectively. a/b shown in the inset of Fig. 3 or Fig. 4 gives the apparent permeability μ, which is a function of I only, provided H and T are constant. μ^* which designated the maximum of μ for fixed

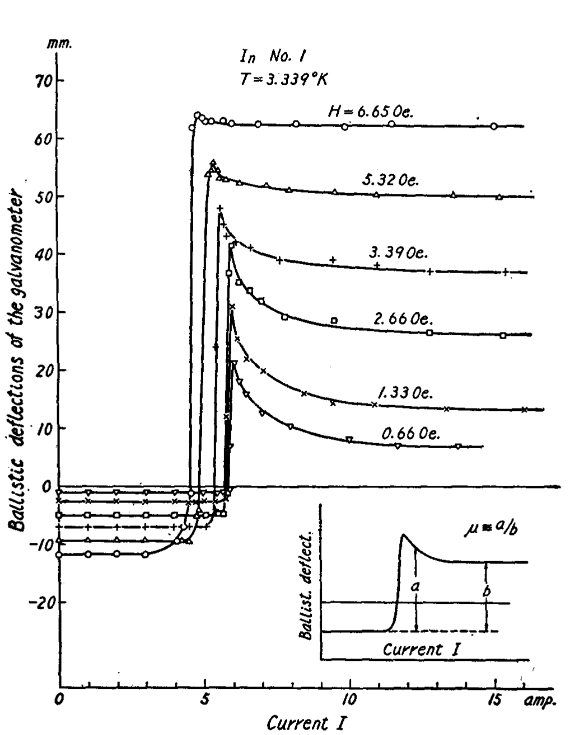

Fig. 3. Ballistic deflections of the galvanometer for No. 1 specimen as a function of current I for the specified values of magnetic field H, when H was reversed. a/b shown in the inset defines the apparent permeability μ.

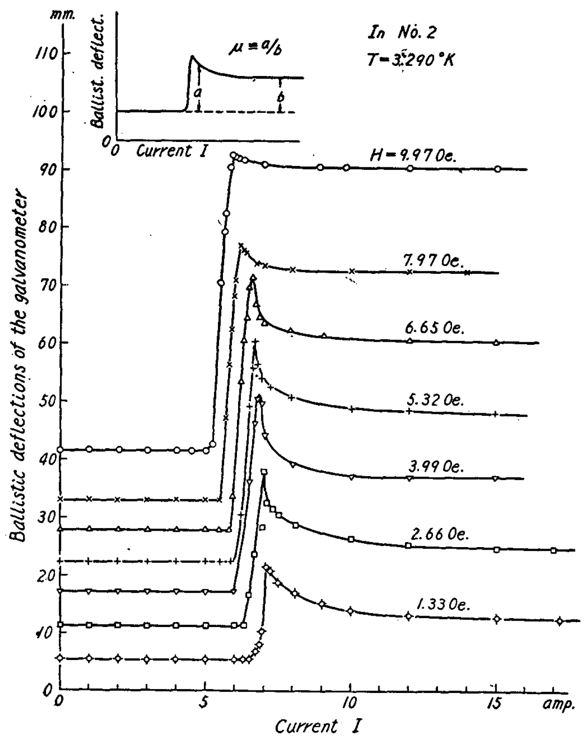

Fig. 4. Ballistic deflections of the galvanometer for No. 2 specimen as a function of current I for specified values of magnetic field H, when H was reversed. a/b shown in the inset gives the apparent permeability μ.

values of H and T was plotted against I^* where μ^* occurred, and the extrapolation to the abscissa where $\mu = 1$ defined the current minimum I_0 at that temperature. I_0 changed with temperature and there was a one-to-one correspondence between I_0 and T. Figs. 5 and 6 show these extrapolations at the specified temperatures for the specimens Nos. 1 and 2 respectively. From Figs. 5 and 6 we obtained the I_0-T relations for two specimens. In a similar way we plotted μ^* against H and obtained H_0, the magnetic field maximum beyond which we could not observe the quasi-paramagnetism at a given temperature, by the extrapolation of μ^* to the abscissa (Figs. 7 and 8). There was also a one-to-one correspondence between H_0

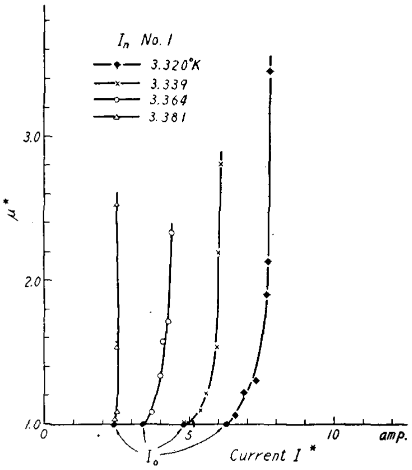

Fig. 5. μ^* for No. 1 specimen as a function of current I^* at the specified temperatures.

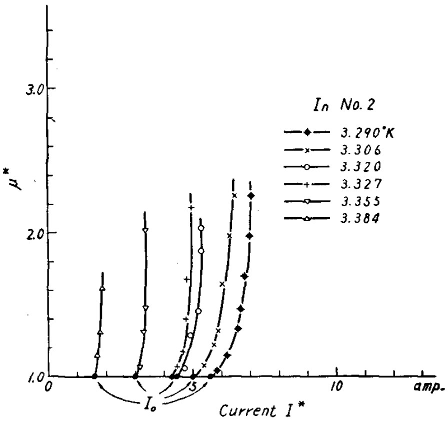

Fig. 6. μ^* for No. 2 specimen as a function of current I^* at the specified temperatures.

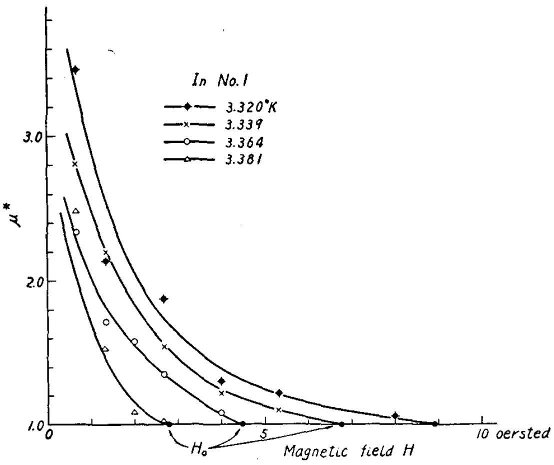

Fig. 7. μ^* for No. 1 specimen as a function of external magnetic field H at the specified temperatures.

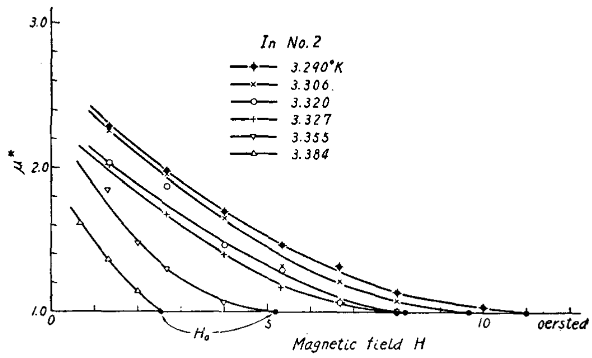

Fig. 8. μ^* for No. 2 specimen as a function of external magnetic field H at the specified temperatures.

and T. Thus we obtained the H_0-T relations. Further we obtained the I_0-H_0 relations for two specimens from Figs. 5, 6, 7 and 8. The I_0-T, the H_0-T, and the I_0-H_0 relation thus obtained for two specimens are shown as straight lines at least in the measured region in Figs. 9, 10 and 11 respectively. Just as in the case of tin, it was ascertained that the formula for the current minimum required for the appearance of the paramagnetic effect was represented graphically by a straight line (the critical line) in the $(I\text{-}H\text{-}T)$ space, the orthogonal projections on the $(I\text{-}T)$, the $(H\text{-}T)$ and the $(I\text{-}H)$ plane of which are the straight lines shown in Figs. 9, 10 and 11 respectively.

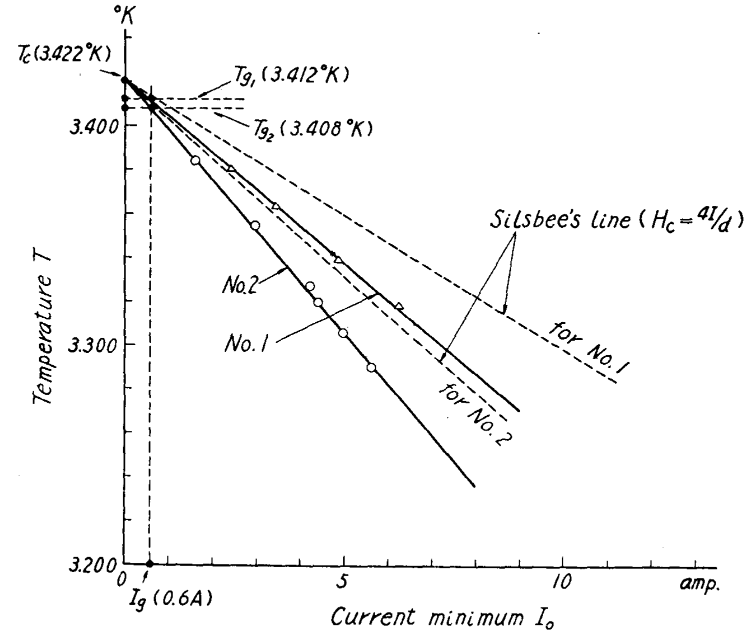

Fig 9. $I_c\text{-}T$ relations for the specimens Nos. 1 and 2.

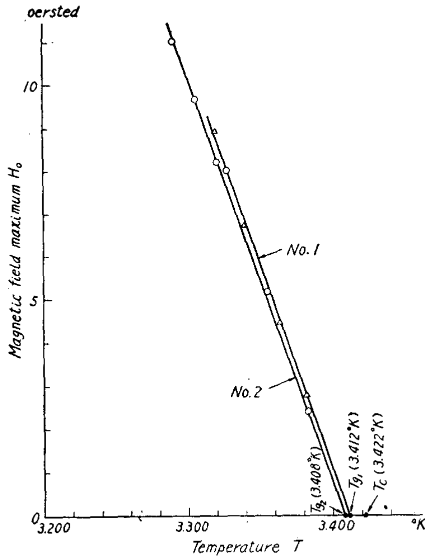

Fig. 10. $H_c\text{-}T$ relations for two specimens.

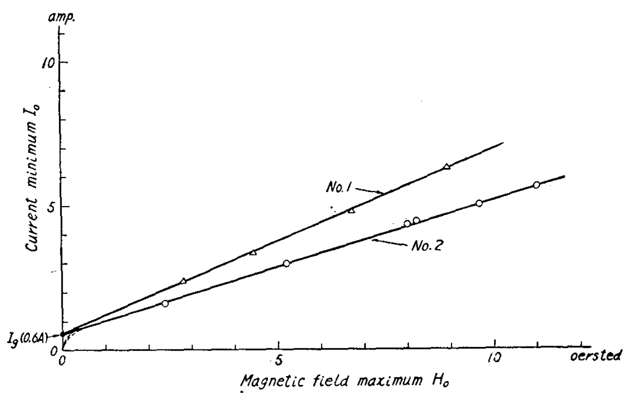

Fig. 11. $I_0\text{-}H_0$ relations for two specimens.

Both the $(I\text{-}T)$ projections having inclinations, proportional to the specimen-diameter d, to the T-axis point to the transition temperature T_c. The $(H\text{-}T)$ projections are approximately parallel to each other and $T_c\text{-}T_g$ is inversely proportional to d, where T_g is the intersection of the projection with the T-axis

(Fig. 12). The $(I\text{-}H)$ projections have inclinations, proportional to d, to the H-axis but intersect with the I-axis at a definite value of current I_g, irrespective of the specimen-diameter. Thus the critical lines terminate at the point (I_g, T_g) on the $(I\text{-}T)$ plane.

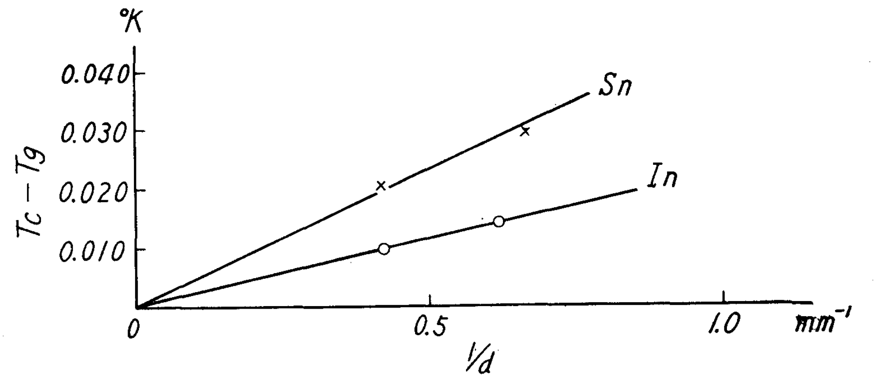

Fig. 12. Proportionality of $T_c - T_g$ to the reciprocal of specimen diameter, $1/d$. The straight line for tin was drawn by using data obtained previously (the paper II).

In a similar way to the case of tin we obtained as the formulas for the $(I\text{-}T)$, the $(H\text{-}T)$ and the $(I\text{-}H)$ projection which satified those relations described above the following three equations (1), (2) and (3) respectively.

$$I_0 = \xi\gamma d(T_c - T)\,, \tag{1}$$

$$H_0 = \xi(T_c - T) - I_g/\gamma d\,, \tag{2}$$

$$I_0 = I_g + \gamma d H_0\,. \tag{3}$$

Here ξ, T_c and I_g are the characteristic constants of the superconductor. The value of γ which was believed formerly to be also a characteristic constant of the superconductor was found to scatter widely according to the recent measurement[14].

Table 1. Numerical values of constants which appear in the formulas for the current minimum I_0 for indium rods.

$$I_0 = \xi\gamma d(T_c - T),$$
$$H_0 = \xi(T_c - T) - I_g/\gamma d\,,$$
$$I_0 = I_g + \gamma d H_0\,.$$

	In No. 1 (2.38 mm ϕ, 80 mm l)	In No. 2 (1.63 mm ϕ, 75 mm l)	Mean value
I_g (amp)	0.6	0.6	0.6
ξ (Oe/deg)	95.5	93.2	94.4
γ (amp/mm Oe)	0.26	0.28	0.27
T_c (°K)	3.422	3.422	3.422
[T_g (°K)]	3.412	3.408	

(14) J. C. Thompson, Phys. Rev. **102** (1956), 1004.

This may be mainly due to the fact that μ^* in Figs. 7 or 8 is a slowly varying function of *H*. It cannot be decided at the present stage of investigation whether γ is a characteristic constant of the superconductor, a constant originated merely from the circular nature of the specimen cross-section or a quantity depending upon the condition of the specimen surface. The numerical values of ξ, T_c, I_g and γ for two specimens are tabulated in Table 1. Averaging the values for two specimens, we decided that $\xi = 94.4$ Oe/deg, $T_c = 3.422°$K, $I_g = 0.6$ amp and $\gamma = 0.27$ amp/mm Oe for indium. Of three equations (1), (2) and (3) only arbitrary two equations are independent and the critical line in the $(I\text{-}H\text{-}T)$ space is represented by the simultaneous equations of the two. Fig. 13 shows schematically the critical lines for two specimens in the $(I\text{-}H\text{-}T)$ space. The quasi-paramagnetism can be observed only in the region of large I and smaller H at each temperature than those given by the critical line. Eq. (3) coincides formally with formula $I_0 = I_g + \gamma dH$ obtained by Meissner et al. T_g which does not appear explicitly in the above equations is given by $T_c - I_g/\xi\gamma d$.

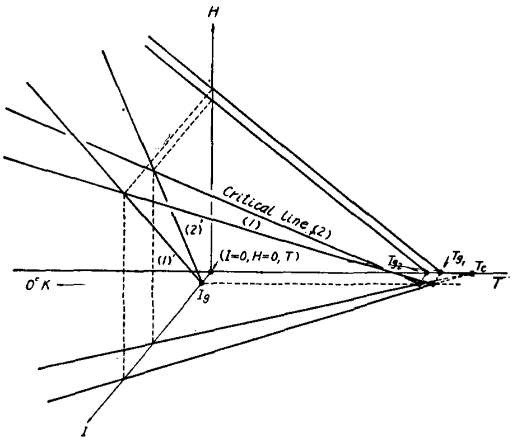

Fig. 13. Critical lines (lines of current minimum) for two specimens in the $(I\text{-}H\text{-}T)$ space. The $(I\text{-}H)$ projections (1)' and (2)' of the critical lines are represented well by Eq. (3) in the text which is analogous to the equation obtained by Meissner et al. i.e. $I_0 = I_g + \gamma dH$.

It should be emphasized that such a somewhat systematic structure in the apparent permeability curve as reported in the previous report (Fig. 16 in the paper II.) could not be observed throughout this investigation. The existence of such a structure in the μ-curve as found previously seems now to us to be apparent. Furthermore we could not obtain in this investigation such closed contours as found in the previous report (Fig. 17 in the paper II). These results formerly obtained might be spurious and be attributed to the experimental procedure formerly adopted.[13]

Although the extension of the I_0–T relation to the absolute zero of temperature would not be permissible, it seems, in a similar way to the case of tin, from this extension that there is a lower limit of specimen–diameter, d_0 for the appearance of the paramagnetic effect. This limit can be deduced from the equation $I_g = \xi\gamma d_0 T_c$. For the case of indium, d_0 becomes 6.9×10^{-3} mm.

2. The verification of the theory which predicts that the apparent permeability maximum μ^* at a given temperature occurs at the point where the resultant magnetic field $H_t[=(H^2+H_I^2)^{\frac{1}{2}}]$ equals the critical field H_c at that temperature.

As the previous report II suffered criticism in regard to this problem[9], we tried to confirm the prediction in this investigation. In Fig. 14 the resultant fields

$H_t[=(H^2+H_i^{*2})^{\frac{1}{2}}=(H^2+(4I^*/d)^2)^{\frac{1}{2}}]$ at the surface of the specimen No. 2 for several temperatures, obtained at the points where μ^* occurred were compared with the critical field H_c. As can be seen in Fig. 14, most of measured points lie below the threshold curve $H_c=136\ (T_c-T)$, i. e.

$$H_t=[H^2+(4I^*/d)^2]^{\frac{1}{2}}<H_c(T)\,.$$

Denoting provisionally H_t for the maximum of quasi-paramagnetism at a fixed temperature by $H_c^*(I^*,\ T)$ (the effective critical field),

$$H_c^*\ (I^*,\ T)<H_c\ (T).$$

However, we found the general trend that H_c^* at a fixed temperature approached H_c with the decrease of current I^*, and H_c^* at the point $(I_0,\ H_0)$ lies nearly on the H_c-curve. This situation can be seen more clearly in Fig. 15, where the

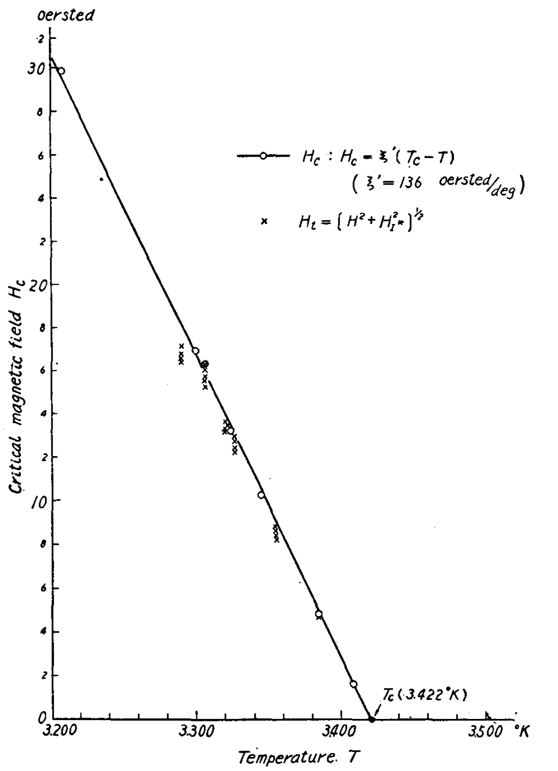

Fig. 14. Comparison of the resultant magnetic field H_t at the specimen surface (the vector sum of external field H and current field H_i^*) with the critical magnetic field H_c.

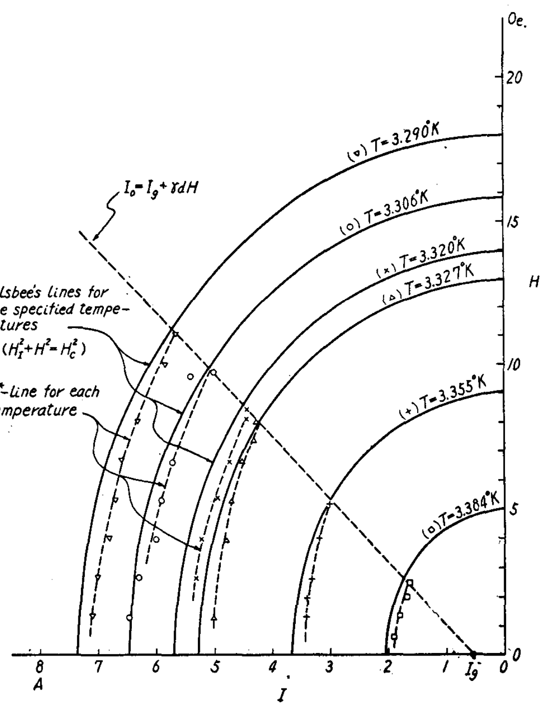

Fig. 15. Orthogonal projections, on the $(I\text{-}H)$ plane, of the intersections (the Silsbee's lines), for the specified temperatures, of the planes $T=$const. with the transition surface $[H^2+(4I/d)^2]^{\frac{1}{2}}=H_c(T)$, and the μ^*-lines for the corresponding temperatures.

orthogonal projections on the $(I\text{-}H)$ plane, of the intersections (the Silsbee's lines), for several temperatures, of the planes $T=$ const. with the transition surface $[H^2+(4I/d)^2]^{\frac{1}{2}}=H_c(T)$ are shown together with the μ^*-lines for the corresponding temperatures. It can be said that H_c^* at the point $(I_0,\ H_0)$ coincides nearly with H_c in accordance with the theory proposed by H. Meissner. This point will be touched again in the following.

3. Comparison of the resistance measurement at the superconducting transition with the theoretical prediction.

The ratio of electric resistance at 4.2°K of the specimen No. 2 to that at 290°K was 1.7×10^{-4}. The result of resistance measurement for $H = 2.66$ Oe at 3.301°K are shown in Fig. 16 (lower figure) where the resistance is plotted as the relative resistance R/R_n against the current I. R_n, the resistance of the specimen in the normal state at that temperature which was measured in an external magnetic field strong enough to destroy superconductivity, was 2.1×10^{-7} ohms in this case. The solid curve represents the relative resistance as a function of the current at constant magnetic field and temperature, which was derived from a calculation based upon H. Meissner's theory (Appendix);

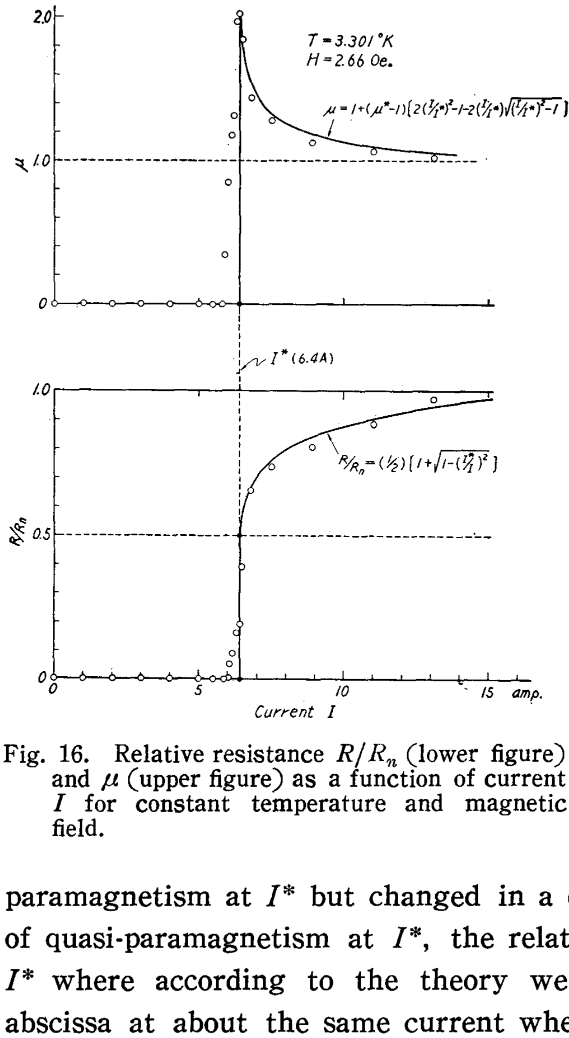

Fig. 16. Relative resistance R/R_n (lower figure) and μ (upper figure) as a function of current I for constant temperature and magnetic field.

$$R/R_n = (1/2)\left[1+\sqrt{1-(I^*/I)^2}\right],$$

where I^* represents the current where the maximum of quasi-paramagnetism occurs. This expression is quite the same as that obtained by F. London[15] for the case of zero magnetic field. In accordance with the result that the perfect diamagnetism did not change abruptly to the maximum of quasi-paramagnetism at I^* but changed in a certain range of current to the maximum of quasi-paramagnetism at I^*, the relative resistance did not appear abruptly at I^* where according to the theory we should expect it to be 1/2, but left the abscissa at about the same current where the perfect diamagnetism began to be destroyed (see the upper figure in Fig. 16). It can be seen, however, that the agreement between theory and experiment is fairly good.

As described in the introduction, J. C. Thompson who obtained the same expression as ours for the electric resistance as a function of current and found also a fair agreement between theory and experiment, obtained furthermore an expression for μ at constant values of H and T as a function of I i. e.

$$\mu = 1+(\mu^*-1)\left[2(I/I^*)^2-1-2(I/I^*)\sqrt{(I/I^*)^2-1}\right],$$

and found that the experiment showed a good agreement with the theory in this case also[12]. Our results of magnetic measurement performed in parallel with the

(15) F. London, Superfluids (New York, John Wiley and Sons), Vol. **1** (1950), 120.

resistance measurement above described is shown also in Fig. 16 (upper figure) in which μ was plotted against I. The solid curve represents the expression obtained by J. C. Thompson. The experiment agrees also fairy with the theory.

IV. Discussion and conclusion

Although the same formulas for the current minimum required for the appearance of the paramagnetic effect as obtained in the previous reports which dealt with tin specimens have been obtained in the present investigation with indium specimens, it has become adequate to consider from the present investigation that some of conclusions in the previous report (the paper II) was apparent. We have not been able to draw out from the present investigation some of the previous conclusions (i) that the closed contour lines for μ could be obtained on the (I-H) planes at relatively higher temperatures near T_g, and (ii) that a somewhat systematic structure in the μ-curve as a function of H could be obtained also at relatively higher temperatures near T_g.

As once thought of by us[6], suggested also by J. C. Thompson[16] and pointed out by H. Meissner[9], the critical line (I_0-line) determined in the (I-H-T) space can be understood as the intersection of the plane $I=I_g+\gamma dH$ with the transition surface $H_c^2=H^2+(4I/d)^2$. We obtain the following equations for the intersection.

$$I_0 = \frac{I_g+\gamma\sqrt{(1+16\gamma^2)d^2H_c^2-16I_g^2}}{(1+16\gamma^2)},$$

$$H_0 = \frac{-16\gamma(I_g/d)+\sqrt{(1+16\gamma^2)H_c^2-16(I_g/d)^2}}{(1+16\gamma^2)}.$$

Although these equations are not such linear equations as Eqs. (1) and (2) but quadratic ones, they can be approximated by the following linear equations respectively, provided $H_c \gg 4I_g/d$.

$$I_0 = \frac{\xi'\gamma d(T_c-T)}{(1+16\gamma^2)^{\frac{1}{2}}}+\frac{I_g}{(1+16\gamma^2)},$$

$$H_0 = \frac{\xi'(T_c-T)}{(1+16\gamma^2)^{\frac{1}{2}}}-\frac{16\gamma I_g}{(1+16\gamma^2)d},$$

since $H_c=\xi'(T_c-T)$. ξ should be understood as $\xi'/(1+16\gamma^2)^{\frac{1}{2}}$, which is 95.5 Oe/deg for $\xi'=136$ Oe/deg and $\gamma\sim 0.25$ amp/mm Oe as compared with $\xi=95.4$ Oe/deg (see Table 1). Then we have

$$I_0 = \xi\gamma d(T_c-T)+I_g/2, \quad (4)$$

$$H_0 = \xi(T_c-T)-I_g/(\gamma d)\times 0.5. \quad (5)$$

Eq. (4) differs from Eq. (1) by the second constant term, which is only 0.3 amp in the present case. Eq. (5) differs from Eq. (2) by a factor 1/2 in the second term. When allowance has been made for the difficulty of the quantitative

(16) Private communication.

measurement inherent in such a investigation as on the paramagnetic effect, it should be approved that the agreements between Eqs. (1) and (4) and between Eqs. (2) and (5) are good.

The result that the perfect diamagnetism and the zero resistance at constant values of H and T began to be destroyed at about the same current slightly smaller than I^* cannot be explained by the theory. This unexplained result may be due to other causes e. g. the end effect of the specimen.

The result that the paramagnetic effect vanishes abruptly at I_g seems quite curious. What comes across the mind in this regard is that the line $I_0=I_g+\gamma dH_0$, though obtained as a straight line in the measured range, may not be a straight line near T_c but the one, shown in trial with a dotted line in Fig. 10, so curved as to lead the paramagnetic effect to appear at T_c and to be observed continuously from T_c down to lower temperatures. Such a speculation, however, would diminish in some measure the mystery of I_g. At the present stage of investigation we have no theoretical reason why the I_0-H_0 line should become thus curved. Alternatively the mysterious constant I_g may be connected with the boundary effect between normal and superconducting regions.(17) If this is the case, the fact that the paramagnetic effect vanishes abruptly at I_g may not be so curious as it seems. It may be, however, meaningless to continue such a speculation further without establishing a concrete theory.

As seen in Fig. 16, the specimen shows a small resistance of $R=(1/2)R_n$ at the point where μ^* occurs. If the heating due to the current causes the temperature of the specimen to be slightly higher than that of the liquid bath, we shall understand in some degree the discrepancy between H_c and H_c^*. If this is the case, the theory of the paramagnetic effect proposed by H. Meissner will be a fairly satisfactory one, except for its drawback that it cannot explain the mysterious constant I_g. Although we cannot give an estimate of the heating of the specimen due to the current, it seems, however, unlikely that such small resistance of the specimen as 1×10^{-7} ohm, through which the current of 5~10 amp flows, causes a noticeable change in the temperature of the specimen. Another possible explanation for the discrepancy between H_c and H_c^* is that it may be due to the surface irregularity of the specimen or other secondary effect.(18) Experimental results show that at larger current the discrepancy is remarkable, it become smaller with the decrease of current and finally H_c^* coincides nearly with H_c at the point (I_0, H_0). If the discrepancy should be due to the surface irregularity, it must explain these experimental results. What may be thought of by one as a secondary effect is that the discrepancy may be due to the finite length of the specimen. Even if we take the demagnetization-coefficients of the specimens into consideration, we cannot, however, explain the discrepancy, because the demagnetization-coefficients of two specimens are at most of the order of 1 per cent. In case that a current is flowing through the specimen an unknown circumstance may occur at

(17) J. Bardeen, Theory of Superconductivity, p. 340 [Handbuch der Physik, Vol. **15** (1956)].
(18) C. J. Gorter, Private discussion: see also C. J. Gorter, Physica **23** (1957), 45.

the junctions of the superconductor and the normalconducting leads. At the present stage of investigation, however, we can say nothing further. The discrepancy between H_c and H_c^* is left to be subjected to further investigation. At any rate an essentially new concept seems necessary for the explanation of the mysterious constant I_g.

I_g which appears in Eq. (3) as a material constant seems independent of the cross-section of the rod, because the quantity which specifies the characteristics of the cross-section of the rod appears only as d, the diameter in second term. If the paramagntic effect should be observed in non-circular rods and the current minimum also required for its appearance, we should obtain the same I_g in these rods as in the circular ones. In this expection we performed an experiment on the paramagnetic effect in non-circular tin rods. As expected, we observed the paramagnetic effect in them and obtained the same I_g as in circular rods. The result will be reported in the following paper, Part II of this series.

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Appendix

As described in the text, the apparent permeability maximum occurs for given external magnetic field H and temperature T at I^* where the resultant magnetic field H_t at the surface of the specimen is less than the critical field H_c:
$H_t=[H^2+(I^*/\pi d)^2]^{\frac{1}{2}}=H_c^*(I^*,T)<H_c(T)$,
where we used the rationalized mks-unit system in order to reconcile the unit system with that which was adopted by H. Meissner. As the current is increased above I^*, the apparent permeability does not drop at once from its maximum value to the normal state value but decreases

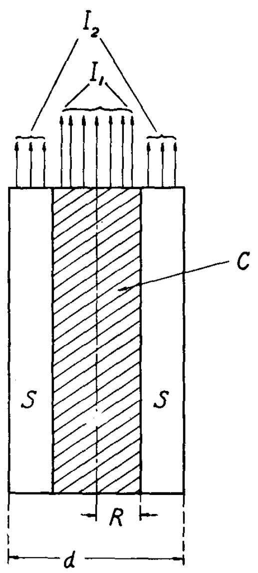

Fig. A1. Cylindrical specimen in the stage $I>I^*$ for constant temperature and magnetic field, where I^* is the current at which the paramagnetic maximum occurs. C is the intermediate core of radius R through which the current I_1 flows and S is the normal conducting sheath through which the current I_2 flows.

slowly toward it.

It seems reasonable to assume following F. London and H. Meissner, that the specimen in this stage consists of an intermediate core C of radius R, through which the current I_1 flows, surrounded by a normal conducting sheath S through which the current I_2 flows (see Fig. A1). The radius of the intermediate core R is determined by the condition that the resultant field at R is equal to H_c^*:

$$I^*/\pi d = I_1/2\pi R. \qquad (1)$$

Regarding our H_c^* as H_c in H. Meissner's theory, from his theory [Equation (17)][3], we have

$$R = I^*/\pi d\sigma_n E, \qquad (2)$$

where E is the electric field and σ_n is the normal state conductivity.

Combining (1) and (2), we get

$$I_1 = 2I^*/\pi d^2\sigma_n E.$$

The current I_2, which passes through the normal conducting sheath S, is given by

$$I_2 = \int_R^{d/2} \sigma_n E 2\pi r dr = \sigma_n E\pi(d/2)^2 - I^{*2}/\pi d^2\sigma_n E.$$

The sum of the current I_1 and I_2 must be equal to the total current I.

Then

$$I = I_1 + I_2 = \sigma_n E\pi(d/2)^2 + I^{*2}/\pi d^2\sigma_n E = E/\Omega_n + \Omega_n I^{*2}/4E,$$

where $\Omega_n = [\pi(d/2)^2\sigma_n]^{-1}$ is the normal resistance per unit length R_n/l, R_n and l being the normal resistance and the length of the specimen respectively.

Solving the above equation with respect to E, we get

$$E = (1/2)\, I\Omega_n [1+\sqrt{1-(I^*/I)^2}] \qquad \text{or}$$

$$\Omega = (\Omega_n/2)[1+\sqrt{1-(I^*/I)^2}], \quad \text{where } \Omega = E/I.$$

Then we have for the resistance $R = \Omega l$

$$R = (R_n/2)[1+\sqrt{1-(I^*/I)^2}].$$