

A Method of Accurate Determination of Absolute Values of X-Ray Levels in Copper. (II, 6)

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A Method of Accurate Determination of Absolute Values of X-Ray Levels in Copper. (II, 6)*

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Synopsis

From the analysis of the absorption spectrum of copper thin film observed by A. Smakula, the absolute value of the energy level E_2 , was determined. Then, the absolute values of the others, K , L_1 , L_{II} , L_{III} , M_I , M_{II} , M_{III} , M_{IV} , M_V , E_1 , E_3 and E_4 , were determined by combinations. Here, E_1 , E_2 , E_3 and E_4 are the energy levels associated with the valence electrons.

I. Introduction

H. R. Robinson and the co-workers⁽¹⁾ have made very extensive investigations on magnetic spectroscopy, and determined absolute values of X-ray levels of many elements including copper. The experimental method was as follows: the substance to be studied was excited by X-rays of known wave-lengths and the expelled electrons were focussed by a magnetic field perpendicular to the plane of the electron path. Then, from the field intensity, H and the radius of curvature of the electron path, r , the electron velocity, v was determined by the equation

$$\frac{mv}{e} = rH,$$

where m =mass of the electron and e =electron charge in emu. By putting $\beta = \frac{v}{c}$, in which c =light velocity, the following equation was deduced,

$$\beta^2 = \frac{\left(\frac{mv}{e}\right)^2 \left(\frac{e}{m_0}\right)^2}{c^2 + \left(\frac{mv}{e}\right)^2 \left(\frac{e}{m_0}\right)^2},$$

where m_0 is the rest mass of the electron, and then

$$\beta^2 = \frac{(rH)^2 \left(\frac{e}{m_0}\right)^2}{c^2 + (rH)^2 \left(\frac{e}{m_0}\right)^2}.$$

Hence, the energy of the electron, W , could be computed by the equation

* The 786th report of the Research Institute for Iron, Steel and Other Metals.

(1) H.R. Robinson and W.F. Rawlinson, *Phil. Mag.*, **28** (1914), 277; H.R. Robinson, *Proc. Roy. Soc.*, A **104** (1923), 455; H.R. Robinson, *Phil. Mag.*, **50** (1925), 241; H.R. Robinson and A.M. Cassie, *Proc. Roy. Soc.*, **113** (1926-27), 282; R.H. Robinson and C.L. Young, *Proc. Roy. Soc.*, A **128** (1930), 92; H.R. Robinson, J.P. Andrews and E.J. Irons, *Proc. Roy. Soc.*, A **143** (1933-34), 48; H. R. Robinson, *Phil. Mag.*, **18** (1934), 1086; H. R. Robinson, *Proc. Phys. Soc.*, **46** (1934), 693.

$$W = m_0 c^2 \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right).$$

Accordingly, for the expelled electron

$$\nu^*/R = \frac{m_0 c}{R h} \left(\frac{1}{\sqrt{1-\beta^2}} - 1 \right).$$

Here the mark * was used to discriminate it from ν/R of the primary X-ray. When the electron in question had been produced by a single ionization of an electron shell, the absolute value of the resulting level must be equal to the difference between ν/R and ν^*/R .

For convenience of the following treatment, some results on copper obtained by H. Robinson⁽²⁾ are reproduced in Table 1, in which the primary X-ray was $K\alpha_1$ of Cu, $\nu/R=592.8$ and $R=109737$.

Table 1. Determinations of absolute values of X-ray levels in copper.

rH	ν^*/R	$592.8-\nu^*/R$	Explanation	Combined ⁽³⁾ transitions	Absolute value of K
280.6	510.8	82.0	L_I	$\{ K\beta_1, K - M_{II, III} = 655.9$ $L\beta_{3,4}, L_I - M_{II, III} = 75.3$	662.6
284.0	523.1	69.7	L_{III}	$K\alpha_1, K - L_{III} = 592.8$	662.5
300.8	586.3	6.5	$M_{II, III}$	$K\beta_1, K - M_{II, III} = 655.9$	662.4

In this table, the first column shows the values of rH , on which he remarked that the accuracy would be within the error of 0.1 per cent. From these values, ν^*/R was deduced by the equations shown above, in which the values, $c = 3 \times 10^{10}$ cm/sec, $e/m_0 = 1.7686 \times 10^7$ emu/gr, $e = \frac{4.774 \times 10^{-10}}{c}$ emu and $h = 6.545 \times 10^{-27}$ erg sec were used. In the third column, the differences between the value of ν/R of the primary X-ray, Cu $K\alpha_1$, and ν^*/R are shown. By comparing these values with those for absorption edges, the explanations as shown in the fourth column were given. Further, for convenience of a later reference, the absolute values of K -level were computed by the present writer. The transitions utilized in the above computation are shown in the fifth column and the results obtained are shown in the sixth column. These values are in good agreement with one another, and nearly equal to the value determined by the absorption edge, 661.59⁽³⁾. From these facts, the absolute values of the X-ray levels obtained above seem to be plausible.

But, as pointed out by A. E. Sandström⁽⁴⁾, in the computation of ν^*/R the errors were introduced from the fundamental constants, which might be larger than those generally supposed. Hence, it will not be useless to recalculate the values of ν^*/R from the experimental data of rH by using the values of the fundamental constants accepted now to be accurate enough for the present purpose. So, adopting the values⁽⁵⁾, $c = 2.99796 \times 10^{10}$ cm/sec, $e/m_0 = 1.759_0$ emu/g, $e = 1.6019 \times 10^{-20}$ emu, $h = 6.623 \times 10^{-27}$ erg sec

(2) H. Robinson, Proc. Roy. Soc., A 104 (1923), 455.

(3) M. Siegbahn, *Spectroscopie d. Röntgenstrahlen*, 2te Aufl. (1931).

(4) A. E. Sandström, Phil. Mag., 22 (1936), 171.

(5) Landolt-Börstein, *Tabelle*, 1 Band, 1 Teil (1950), 30.

and $R=1.097375 \times 10^5$ and using the relation $1000 X U = 1002.02 \times 10^{-8}$ cm, ν^*/R was re-computed in the crystal scale as shown in the second column of Table 2, the first column being the same as that in Table 1. The values in other columns were obtained in the same way as in Table 1.

Table 2. Recalculations of absolute values of X-ray levels in copper.

rH	ν^*/R	$592.7 - \nu^*/R$	Interpretation	Absolute value of K
280.6	506.6	86.1	L_I	666.7
284.0	518.9	73.8	L_{III}	666.6
300.8	581.6	11.1	$M_{II, III}$	667.1

From this table, it will evidently be seen that the absolute value of K is too large, the excess being estimated to be about 5 Ryd. This must be resulted from the incorrectness of the interpretation shown in the fourth column, that is, the ionization was not single, but double or more. Further, from the numerical value of the above excess, it is highly probable that the second ionization took place in $M_{II, III}$ -shell.

In the first case in which $rH=280.6$, L_I -shell must first be ionized by the incidence of the primary ray, and then by the impingement of the ejected electron, $M_{II, III}$ -shell, must secondly be ionized, the two ionizations being supposed to be occurred in the same atom. By the second ionization, the impinged electron would lose the energy, $M'_{II, III} + \epsilon$, where $M'_{II, III}$ is the ionization energy of the $M_{II, III}$ -shell, of which the inner shell, L_I , had already been ionized, and ϵ is the kinetic energy of the electron detached from the $M_{II, III}$ -shell. Then, from Table 2,

$$86.1 = L_I + (M'_{II, III} + \epsilon)$$

It may be assumed that $(M'_{II, III} + \epsilon)$ is approximately equal to $M_{II, III}$. Then,

$$L_I + M_{II, III} = 86.1$$

On the other hand⁽³⁾,

$$L\beta_{3,4}, L_I - M_{II, III} = 75.3$$

Accordingly,

$$L_I = 80.7, \quad M_{II, III} = 5.4$$

Inserting the above value of $M_{II, III}$ into the equation

$$K\beta_1, K - M_{II, III} = 655.9^{(3)},$$

we have

$$K = 661.3$$

For the second case in which $rH = 284.0$, it may be assumed

$$L_{III} + M_{II, III} = 73.8$$

similarly to the first case. Inserting the above value of $M_{II, III}$ into this equation, we obtain

$$L_{III} = 68.4$$

Accordingly, from the equation

$$K\alpha_1, K - L_{III} = 592.7$$

it gives

$$K = 661.1$$

In the third case in which $rH = 300.8$, it may be assumed,

$$2M_{II,III} = 11.1$$

Accordingly,

$$M_{II,III} = 5.6$$

Then, combining it with

$$K\beta_1, K - M_{II,III} = 655.9^{(3)}$$

we have

$$K = 661.5$$

The above treatments and the results obtained are summarised in Table 3.

Table 3. Summarizing table showing the recalculations of the absolute values of Cu X-ray levels in copper.

rH	ν/R	$592.7 - \nu/R$	Interpretation	Combined transition ⁽³⁾	Absolute values of levels			
					$M_{II,III}$	L_{III}	L_I	K
280.6	506.6	86.1	$L_I + (M'_{II,III} + \epsilon)$ or $L_I + M_{II,III}$	$L\beta_{3,4}, L_I - M_{II,III} = 75.3$ $K\beta_1, K - M_{II,III} = 655.9$	5.4		80.7	661.3
284.0	518.9	73.8	$L_{III} + (M''_{II,III} + \epsilon)$ or $L_{III} + M_{II,III}$	$K\alpha_1, K - L_{III} = 592.7$		68.4		661.1
300.8	586.3	11.1	$2M'''_{II,III} + \epsilon$ or $2M_{II,III}$	$K\beta_1, K - M_{II,III} = 655.9$	5.6			661.5
				Mean	5.5	68.4	80.7	661.3

The three values of K -level in Table 3 are fairly in agreement with one another and also with the value determined from the absorption edge, 661.59⁽³⁾.

From the above considerations, it may be said that plausible absolute values of X-ray levels can be determined by the method of magnetic spectroscopy. In the computations, however, some assumptions are necessary and the errors introduced by this cannot be warranted to be within the limit of the experimental error.

As such is the circumstance, a suitable method will be desirable, which can meet with the above requirement. The present investigation was carried out for this purpose, and a satisfactory result could be obtained at least in the case of copper.

II. Survey of the present author's previous investigations on copper

The four nondiagram lines, L_{β}' , β''' , α'' and α'''' of copper were selected from those measured by E. Gwinner⁽⁶⁾ and considered to be due to the transitions shown in Table 4. E_1 , E_2 , E_3 and E_4 are the energy levels associated with the valence electron; the suffixes refer to the order of the energy as illustrated in Fig. 1. These transitions were combined respectively with the transitions, $K\alpha_2, K - L_{II}$; λ 1541.232 X, ν/R 591.258 and $K\alpha_1, K - L_{III}$; λ 1537.397 X, ν/R 592.733, these numerical

(6) E. Gwinner, Zeits. Phys., 108 (1938), 523.

Table 4. X-ray L-line associated with valence electron.

X-ray line	(Å)	ν^*/R	Transition
$L\beta'$	12.989	70.157	$L_{II}-E_1$
$L\beta''''$	12.911	70.581	$L_{II}-E_3$
$L\alpha''$	13.233	68.863	$L_{III}-E_2$
$L\alpha''''$	13.176	69.161	$L_{III}-E_4$

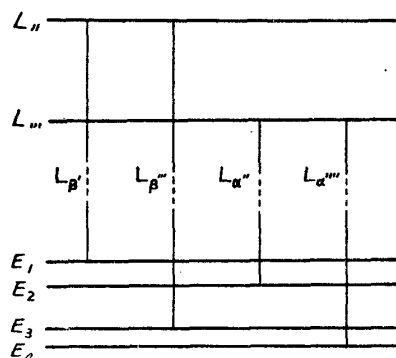


Fig. 1. Diagram, illustrating the X-ray levels associated with valence electrons in copper.

Table 5. Accurate wave-lengths of Cu, $K\alpha_1$ and $K\alpha_2$ (X-unit)

	$K\alpha_1(K-L_{III})$	$K\alpha_2(K-L_{II})$	Reference
λ (X.U)	1537.396	1541.243	(7)
	1537.395	1541.232	(8)
	1537.400	1541.220	(9)
	1537.397	1541.232	Mean
ν/R	592.733	591.258	

Table 8. Relations between the levels, $E_1 \sim E_4$ and photoelectric effect.

X-ray		Photoelectric effect		Reference
Level difference	λ (Å)	λ (Å)	Remark	
E_1-E_4	1900	1900	Selective max. in \perp comp.	(11)
E_1-E_3	2150	2000	Selective max. in \parallel comp.	
E_2-E_4	3036	3033	Threshold wave-length	(12)
E_2-E_3	3750	3750	Threshold wave-length for the surface(111), computed in reference to that for (100), the latter being assumed to be 3033 Å.	(13)

From the above it was inferred that the photo-electric phenomena were caused by

- (7) M. Siegbahn, Ark. Astr. O. Fys. (A) **21** Nr. 21 (1929).
 (8) I. Wennerlöf, Ark. Mat., Astr. O. Fys. (A) **22** Nr. 8 (1930).
 (9) J. A. Bearden and C. H. Shaw, Phys. Rev., **48** (1935), 18.
 (10) M. Satō, Sci. Rep. RITU, A2 (1950), 725.
 (11) F. Hlčuka, Zeits. Phys., **92** (1934), 359.
 (12) Lukirsky und Priležev, Zeits. Phys., **49** (1928), 236.
 (13) N. Underwood, Phys. Rev., **47** (1935), 502.

values being obtained in the way shown in Table 5. Thus, the energy distances from K to $E_1 \sim E_4$ were determined as shown in Table 6.

Further, from the above result, the relative positions of $E_1 \sim E_4$ were computed as shown in Table 7.

Table 6. Energy distances of $E_1 \sim E_4$ from K .

	$K-E_1$	$K-E_2$	$K-E_3$	$K-E_4$
ν/R	661.415	661.596	661.839	661.894

Table 7. Relative positions of $E_1 \sim E_4$.

	$\Delta\nu/R$	eV	(Å)
E_1-E_2	0.181	2.46	5030
E_1-E_3	0.424	5.76	2150
E_1-E_4	0.479	6.50	1900
E_2-E_3	0.243	3.00	3750
E_2-E_4	0.298	4.05	3036
E_3-E_4	0.055	0.75	16600

The above results were compared with the phenomena of the photoelectric effect⁽¹⁰⁾ and the results shown in Table 8 were obtained.

the electron transitions between the energy levels(E_1, E_2) and (E_3, E_4). Further, it was expected that the threshold λ 3033Å was really that for (100).

Next, the results in Table 7 were compared with the absorption spectrum of copper thin film ($d = 3.76 \text{ cm}^{-6}$) given by A. Smakula⁽¹⁴⁾ and illustrated in Fig. 2. The results of the comparison are shown in Table 9.

From the results shown in Tables 8 and 9, it was concluded that the energy levels E_1, E_2, E_3 and E_4 are of real existence. Hence, if the absolute value of any one of them can be found, that of K will be computed from Table 6 and, consequently, others will be obtained by suitable combinations.

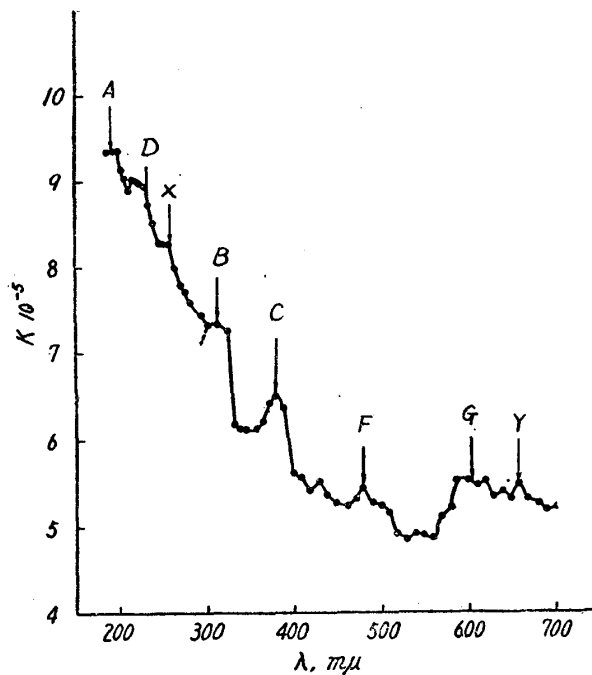


Fig. 2. Light absorption by copper thin film ; the value of K was given by A. Smakula.

Table 9. Relation between energy levels, $E_1 \sim E_4$ and light absorption in copper thin film.

X-ray		Light absorption		
Level difference	(Å)	Symbol	$\lambda(\text{m}\mu)$	Remark
$E_1 - E_4$	1900	A	193	Two maxima are not separated.
$E_1 - E_3$	2150			
$E_2 - E_4$	3036	B	312	Absorption maximum but not threshold.
$E_2 - E_3$	3750	C	380	

On the other hand, it has been reported⁽¹⁵⁾, that 4 anomalous temperatures exist in copper, having the regularities as shown in Table 10, in which the number in parenthesis is the interval ratio.

Table 10. Anomalous temperatures in copper.

Group	Anomalous temperatures in copper (°K)				
I	0	(3)	503	(2)	823
II	0	(2)	553	(3)	1356(mp)

From the regularities, it was inferred that the energy states of the valence electrons associated with the crystal structure of copper could be denoted by the spectroscopic notations as follows :

$$E_1 : (A_1, B_1)^3D_{1,2,3}, \quad \cdot \quad E_2 : (A_2, B_2)^3D_{3,2,1}.$$

(14) A. Smakula, Zeits. Phys., 86 (1933), 185.

(15) M. Satô, Sci. Rep. RITU, A3 (1951), 661.

Here, E_1 and E_2 (in Roman type) are electron shells, each consisting of two electrons supplied equally from the atoms A_1 and B_1 , and A_2 and B_2 , respectively; they correspond respectively to the energy levels, E_1 and E_2 (in Italic type). The suffixes denoting the values of J are written in the order of energy height, that is, the former is regular and the latter is inverted triplet. Thus, each pair of atoms were considered to be bound by the energy of LS -coupling of their valence electrons and the anomalous changes at the temperatures in Table 10 were explained to be attributed to the transitions between the adjacent components in the above triplet. The conduction states E_3 and E_4 (in Roman type) corresponding to the energy levels E_3 and E_4 (in Italic type) were supposed to exist. Further, it was assumed that the oscillations $E_1 \rightleftharpoons E_3$ and $E_2 \rightleftharpoons E_4$ took place, respectively, in the molecules (A_1, B_1) and (A_2, B_2) . To be kept from damping, these oscillations were, further, assumed to be in resonances with the neighbours in the group of the same kind of the molecule. Accordingly, the molecules in the group must be bound by the energy of the resonating oscillations, probably forming molecules chains.

The above assumptions were confirmed by the analysis of the critical potentials of the soft X-rays and of secondary electrons from copper⁽¹⁶⁾. The treatment was as follows: First, the critical potentials 5.3 and 9.2 V, which are those of the soft X-ray but not of the secondary electron emission, were assumed respectively to be the dissociation potentials of the molecules (A_2, B_2) and (A_1, B_1) . Then, the critical potentials higher than 5.3 V were explained as the dissociation potentials of the chained molecules into atoms with or without accompanying ionization. The assumptions concerning the two critical potentials were subsequently confirmed by the activation energies in self-diffusion and in sintering, and by the sublimation energy at 0°K⁽¹⁷⁾.

On the other hand, the critical potential 2 V, at which an inelastic collision of electron begins to occur but not secondary electron emission, was explained to be the dissociation potential of the molecule (A_1, B_1) in the state E_3 . The potential 2.6 V, which might be considered to be the critical potential of the soft X-ray and the secondary electron emission, was explained to be the dissociation potential of the molecule (A_1, B_1) in the state E_3 , accompanying ionization.

Next, we will repeat the explanation of the absorption spectrum in Fig. 2. From

Table 11. Explanation of the absorption spectrum.

Symbol	λ (m μ)	Explanation
A	193	$\begin{cases} E_1-E_4 \\ E_1-E_3 \end{cases}$
D	232	$E_2 : (A_2, B_2), Cu_2 \rightarrow 2Cu$
B	312	E_2-E_4
C	380	E_2-E_3
F	470	$E_3 : (A_1, B_1), Cu_2 \rightarrow Cu + Cu^+$
G	605	$E_3 : (A_1, B_1), Cu_2 \rightarrow 2Cu$

the above, the breaking point, D, λ 232 m μ or 5.34 eV, and the maxima, F, λ 470 m μ or 2.63 eV and G, λ , 605 m μ or 2.05 eV, were explained respectively to be the dissociation potential of the molecule (A_2, B_2) in the state E_2 , and the dissociation potentials of the molecule (A_1, B_1) in the state E_3 with and without accompanying ionization.

(16) M. Satô, Sci. Rep. RITU, A5 (1953), 533.

(17) M. Satô, Sci. Rep. RITU, A6 (1954), 458.

The explanation of the absorption spectrum given above are shown in Table 11, together with those in Table 9.

III. Determination of absolute values of X-ray level in copper

A sharp brecking X, λ 257 $m\mu$, ν/R 0.355 is found in the absorption spectrum in Fig. 2. From the fact that its energy is slightly higher than that of B, λ 312 $m\mu$, which corresponds to $E_2 - E_4$, it may be inferred that X corresponds to the ionization potential of E_2 -shell, that is, the absolute value of level E_2 . Further, a small maximum Y, though ambiguous, is found at the position λ 660 $m\mu$, ν/R 0.138. From the facts that its relative position referred to G, the dissociation potential of E_3 , (A_1, B_1), is similar to that of X referred to D, the dissociation potential of E_2 , (A_2, B_2), and that the difference

$$\begin{aligned}\Delta\nu/R &= 0.355 - 0.138 \\ &= 0.217\end{aligned}$$

is approximately equal to

$$\begin{aligned}\Delta\nu/R &= E_2 - E_3 \\ &= 0.243\end{aligned}$$

in Table 7, it may be inferred that Y corresponds to the absolute value of E_3 . Thus, our assertion that X corresponds to the absolute value of E_2 is supported more rigidly by the existence of Y. As a matter of fact, the absolute value of E_3 , 0.138, must be less accurate than that of E_2 , 0.355.

Then, from the above result,

$$E_2 = 0.355$$

and the difference

$$K - E_2 = 661.596$$

in Table 6, the absolute value of K can be obtained:

$$K = 661.951.$$

Accordingly, the absolute values of the other levels can be computed as follows:

1. L_{II} and L_{III}

From Table 5

$$K\alpha_2 : K - L_{II} = 591.258$$

$$K\alpha_1 : K - L_{III} = 592.733.$$

Hence, by inserting the above value of K into these equations,

$$L_{II} = 70.693 \quad \text{and} \quad L_{III} = 69.218.$$

2. M_{II} and M_{III}

For copper, the two lines, $K\beta_1 : K - M_{III}$ and $K\beta_3 : K - M_{II}$, were separated by Spencer⁽¹⁸⁾ (he used the notation $K\beta_2$ for $K\beta_3$), giving $\Delta\lambda = 0.38XU$. In the intensity curve recorded by him, $K\beta_3$ was very weak as compared with $K\beta_1$; hence, the wave lengths shown in Table 12 may be considered really to be those corresponding to the transition, $K - M_{III}$.

(18) Spencer, Phys. Rev., 38 (1931), 631.

Table 12. X-ray line $K\beta_1$ of copper.

$\lambda(\text{XU})$	1389.378 (2nd. order)	(19)
	1389.364 (1st. order)	(20)
	1389.349	(20)
	1389.364	Mean
ν/R	655.887	

Accordingly, by the equation,

$$\frac{\Delta\nu}{R} = -\frac{\Delta\lambda}{\lambda^2 R}$$

we obtain

$$M_{II} - M_{III} = 0.18$$

From Table 12

$$K - M_{III} = 655.887.$$

Hence, using the value

$$K = 661.951$$

it gives

$$M_{III} = 6.064 \quad \text{and} \quad M_{II} = 6.24.$$

3. L_I

The wave-lengths shown in Table 13 are generally assigned to $L\beta_{3,4}$. But, as $L\beta_4$

Table 13. X-ray line, $L\beta_{3,4}$.

$\lambda(\text{\AA})$	12.10	(21)(22)
	12.07	(23)
	12.09	Mean
ν/R	75.37	

is expected to be very weak as compared with $L\beta_3$, they may be considered really to be those for $L\beta_3$: $L_I - M_{III}$. Hence, using the relation

$$L_I - M_{III} = 75.37$$

and the value obtained in Article 2,

$$M_{III} = 6.064$$

it gives

$$L_I = 81.43.$$

4. M_I

Table 14. X-ray lines $L\eta$ and $L\iota$.

	$L\eta(L_{II}-M_I)$	$L\iota(L_{III}-M_I)$
$\lambda(\text{\AA})$	14.84(21)(22)	15.19(21)(22)
	14.87(23)	15.26(23)
	14.85	15.23
ν/R	61.36	59.83

(19) J.A. Bearden and C. H. Shaw, Phys. Rev., **48** (1936), 18.

(20) I. Wennerlof, Ark., Mat., Astr. O. Fysik (A) **22**, Nr. 8 (1930).

(21) M. Siegbahn and R. Thoraues, Ark. Mat., Astr. O. Fysik **18**, **24**, 6 (1924).

(22) R. Thoraues, Phil. Mag., (7) **2** (1926), 1007.

(23) A. Karlsson, Ark. Mat., Astr. O. Fysik (A), **22** (1930), Nr. 9.

From Table 14,

$$L_{II} - M_I = 61.36$$

$$L_{III} - M_I = 59.83$$

and

$$L_{II} = 70.693$$

$$L_{III} = 69.218$$

as obtained in Article 1. Hence

$$M_I = 70.69 - 61.36 = 9.33$$

or

$$M_I = 69.22 - 59.83 = 9.39$$

As the mean we have

$$M_I = 9.36.$$

5. M_{IV} and M_V

Table 15. Cu, X-rays associating with M_{IV} and M_V .

	$L\beta_1 (L_{II}-M_{IV})$	$L\alpha_{1,2} (L_{III}-M_{IV},\nu)$	$K\beta_5 (K-M_{IV},\nu)$
$\lambda(\text{\AA})$	13.029 ⁽²¹⁾ ⁽²²⁾ 13.027 ⁽²³⁾ 13.026 ⁽⁶⁾	13.306 ⁽²³⁾ 13.301 ⁽⁶⁾	1.378236 ⁽¹⁹⁾
	13.027	13.304	1.378236
ν/R	69.952	68.496	661.183

From Table 15,

$$L_{II} - M_{IV} = 69.952$$

and from Article 1,

$$L_{II} = 70.693.$$

Hence,

$$\begin{aligned} M_{IV} &= 70.693 - 69.952 \\ &= 0.741. \end{aligned}$$

From Table 15,

$$L_{III} - M_{IV,\nu} = 68.496$$

$$K - M_{IV,\nu} = 661.183$$

and as above stated

$$L_{III} = 69.218$$

$$K = 661.951.$$

Hence, we have

$$\begin{aligned} M_{IV,\nu} &= 69.218 - 68.496 \\ &= 0.722 \end{aligned}$$

$$\begin{aligned} \text{or } M_{IV,\nu} &= 661.951 - 661.183 \\ &= 0.768. \end{aligned}$$

These two values are not precisely in agreement with each other. This must have resulted from the fact that the precise measurement of $K\beta_5$ is difficult. As such

is the circumstance, the former value may be preferred to the latter. That is

$$M_{IV,V} = 0.722$$

Now, as the numbers of the electrons in the subshells M_{IV} and M_V are respectively 4 and 6, the above value of $M_{IV,V}$ may be considered to correspond to the energy position which divides the interval between M_V and M_{IV} in inversely proportional to 6:4.

From such a consideration, we have

$$M_V = 0.709.$$

6. E_1 , E_2 , E_3 and E_4

From $E_2 = 0.355$ and Table 7 or from $K = 661.951$ and Table 6, the absolute values of the levels E_1 , E_2 , and E_4 can be computed as follows:

$$E_1 = 0.536, \quad E_3 = 0.112, \quad \text{and} \quad E_4 = 0.057.$$

The results obtained in this section are summarized in Table 16.

Table 16. Absolute values of X-ray levels in copper.

Level	K	L_I	L_{II}	L_{III}	M_I	M_{II}	M_{III}	M_{IV}	M_V	E_1	E_2	E_3	E_4
Absolute value (ν/R)	661.951	81.43	70.693	69.218	9.36	6.24	6.064	0.741	0.709	0.536	0.355	0.112	0.057