The ΔE-Effect, Young's Modulus, and Magnetic Properties in Ferromagnetic Nickel-Copper Alloys

著者	YAMAMOTO Mikio
journal or publication title	Science reports of the Research Institutes, Tohoku University. Ser. A, Physics, chemistry and metallurgy
volume	6
page range	446-457
year	1954
URL	http://hdl.handle.net/10097/26657

The ΔE-Effect, Young's Modulus, and Magnetic Properties in Ferromagnetic Nickel-Copper Alloys*

Mikio YAMAMOTO

The Research Institute for Iron, Steel and Other Metals

(Received May 15, 1954)

Synopsis

The results of the dynamic measurements of Young's modulus and its change with magnetization (the ΔE-effect) as well as of the ballistic measurements of the ferromagnetic characteristics made at ordinary temperatures on annealed ferromagnetic nickel-copper alloys are described. The ΔE-effect in alloys containing 5 to 20 percent copper shows a negative small minimum at low fields or at low magnetization. It is shown that the observed values of the saturation ΔE-effect are in a quantitatively fairly good agreement with values computed from a formula:—

$$\left(\frac{\Delta E}{E_0}\right)_s = 0.7 \times \frac{\kappa_0 E_0 \lambda_s^2}{I_s^2} \Big/ \left(1 - 0.7 \times \frac{\kappa_0 E_0 \lambda_s^2}{I_s^2}\right),$$

where κ_0 is the initial susceptibility, I_s the saturation magnetization, and λ_s the saturation magnetostriction. Young's moduli at unmagnetized state as well as at magnetically saturated state reveal minima at 15 to 20 percent copper. Some new information is also given of the magnetic properties.

I. Introduction

Although a large number of researches, both experimental and theoretical, were already made on physical properties of nickel-copper alloys, not so much information is available concerning phenomena associated with the ferromagnetism of high-nickel alloys. In particular, the change of Young's modulus accompanied by magnetization, that is, the ΔE-effect had never been measured before we published in 1942 a report, written in Japanese, on the ΔE-effect measurements by the dynamic method of magnetostrictive vibration(1). Quite recently, Kouvelites and McKeehan(2) reported the data on the ΔE-effect of the alloys obtained by a similar method.

On the other hand, Young's moduli of ferromagnetic nickel-copper alloys have been the object of a large number of investigations(3,4,2), but the experimental data obtained are very diverse. Although for the composition dependency of Young's modulus most investigations gave monotonously descending or ascending

---

[*] The 768th report of the Research Institute for Iron, Steel and Other Metals.

(1) M. Yamamoto, Nippon Kinzoku Gakkai-shi (J. Japan Inst. Metals), **6** (1942), 249 (in Japanese).

(2) J. S. Kouvelites and L. W. McKeehan, Phys. Rev., **86** (1952), 808.

(3) N Kurnakow and J. Rapke, Z. anorg. Chem., **87** (1914), 269; Z. Nishiyama, Sci. Rep. Tôhoku Univ., **18** (1929), 359; K. Nakamura, ibid., **25** (1936), 415; S. Aoyama and T. Fukuroi, Bull. Inst. Phys. Chem. Res., **20** (1941), 809; H. Masumoto and H. Saito, Nippon Kinzoku Gakkai-shi, **8** (1944), 49; W. Köster and W. Rauscher, Z. Metallkde., **39** (1948), 111.

(4) T. Fukuroi and Y. Shibuya, Sci. Rep. RITU, **A2** (1950), 748; S. Umekawa, Nippon Kinzoku Gakkai-shi, **18** (1954), 387.

curves as well as a curve showing a maximum at intermediate composition (c.f. Fig. 10), we obtained for the first time a more complicated curve which has been confirmed again by recent investigations by Fukuroi and Shibuya(4a) and by Umekawa(4b).

In connection with the investigation of the ΔE-effect and Young's modulus, we have examined some magnetic properties of ferromagnetic nickel-copper alloys and found some new facts, of which accounts are also given in this report.

II. Specimens and Methods of Measurements

As specimens, two groups of nickel-copper alloys were used. Group A, consisting of five alloys, were selected out of the specimens used previously by the author for the density measurement(5). Chemically determined compositions of them are given in Table 1. Electrolytic nickel (impurities contained: Co 0.53%, Fe 0.05% and Al 0.02%) and electrolytic copper (impurities contained: Sb 0.015%, Fe 0.010% and S 0.007%) were melted together in a high-frequency induction furnace, casted, forged, and lathemachined into circular rods, dimensions of which are given in Table 1. These rods were annealed previously at 850°C in a vacuum for three hours and newly at 900°C for an hour in a vacuum.

Table 1. Composition, dimensions, and measured and calculated data of ferromagnetic nickel-copper alloys. ρ denotes the density, I_s the saturation magnetization, H_s the saturation field, K the ferromagnetic anisotropy constant, κ_0 the initial susceptibility, $(\Delta E/E_0)_-$ the nagative minimum value of the change of Young's modulus with magnetization relative to that at unmagnetized state, E_0, H_- the field at which $(\Delta E/E_0)_-$ is reached, H_m the field at which the differential susceptibility becomes maximum, $(\Delta E/E_0)_s$ the saturation ΔE-effect, $(I/I_s)_-$ the reduced magnetization at which $(\Delta E/E_0)_-$ is reached, and λ_s the saturation magnetostriction (measured by Masumoto, Shirakawa, and Kobayashi(26)).

Specimen No.	Composition	Dimension (cm)		ρ	I_s	H_s	$K\times10^{-4}$	κ_0	$(\Delta E/E_0)_-$	H_-	H_m	$(I/I_s)_-$	$(\Delta E/E_0)_s$	$E_0\times10^{-12}$	λ_s	$(\Delta E/E_0)_s$, calc.
	wt.-%Cu	Diameter	Length	g/cm³	C.G.S.	Oe	ergs/cm³		10^{-2}	Oe	Oe		10^{-2}	dynes/cm²	10^{-6}	10^{-2}
A0	0.6	0.5581	9.995	8.8755	502	340	−4.3	24	0.00	—	—	0.00	15.37	1.875	−36.9	16.6
A1	11.08	0.5498	10.015	8.8973	391	230	−2.2	13	−0.22	1.6	2.3	0.20	6.48	2.024	−22.8	5.6
A2	15.46	0.5516	10.015	8.8983	334	150	−1.2	13	−0.10	1.4	2.0	0.17	5.87	1.956	−17.4	4.3
A3	21.01	0.5510	10.020	8.9117	274	130	−0.9	11	−0.08	1.2	1.6	0.15	4.288	2.043	−11.2	2.2
A4	31.14	0.5506	10.010	8.9178	127	70	−0.2	10	0.00	—	—	0.00	1.313	2.018	− 2.4	1.3
B1	4.88	0.2515	9.950	8.870	453	275	−3.1	23	−0.78	—	—	—	11.59	1.934	−30.8	13.6
B2	9.77	0.2512	9.955	8.868	394	225	−2.2	16	−0.37	—	—	—	7.77	1.989	−24.4	8.4
B3	14.63	0.2534	9.955	8.896	330	175	−1.4	16	−0.26	—	—	—	6.71	1.939	−18.5	6.1
B4	34.42	0.2502	9.955	—	20	60	0.0	1	—	—	—	—	0.00	—	− 0.1	0.0

---

(5) M. Yamamoto, Nippon Kinzoku Gakkai-shi, 5 (1941), 375 (in Japanese).

Compositions of four B-group specimens as analysed chemically are also given in Table 1. The raw materials employed for the preparation of them are again electrolytic nickel (99.60 percent pure; impurities contained: C 0.02%, Fe 0.08%, Co 0.32%, Si 0.011%, P 0.001% and Cu none) and electrolytic copper (99.90 percent pure; impurities contained: Fe 0.008%, S 0.005%, Sb 0.005%, Si 0.009%, Ni none and Co none). We took the same procedure of preparation as in the case of group A and finally obtained round-bar specimens, dimensions of which are given in Table 1. These were annealed at 900°C for two hours in a vacuum.

Young's moduli and ΔE-effects were measured with the dynamic method of magnetostrictive vibration(6~8) and the magnetic properties were determined ballistically. The ΔE-effect as well as magnetic measurements were made in magnetic fields up to about 1000 oersteds produced by a water-jacketed solenoid, 40 cm long, of the coil constant of 63.1 Oe/A. The homogeneity of magnetic field produced was within one percent over the length of 18 cm in the central portion of the solenoid.

The experimental procedure adopted is as follows:— First of all, the proper fundamental frequency, f, of the longitudinal magnetostrictive vibration of a specimen rod is measured as a function of the externally applied magnetic field, H_{ex}, and the value for the unmagnetized state, f_0, is determined by extrapolating this relation in low fields to $H_{ex}=0$. Then, the change of Young's modulus relative to that for the unmagnetized state is computed by the formula $\Delta E/E_0=2\Delta f/f_0$. On the other hand, the effective magnetic field (which will be simply called magnetic field in the following), H, is determined by the formula $H=H_{ex}-NI$, where N and I are the demagnetizing factor and intensity of magnetization of the specimen rod, respectively, using the data for the magnetization curves as obtained ballistically. Finally, Young's modulus for the unmagnetized state is calculated by the formula $E_0=4l^2\rho f_0^2$, where l and ρ are the length and density of the specimen rod, respectively. The density measurements were made with the "weighing-in-water" method(5), the results of which are given in Table 1.

All the measurements were carried out at ordinary temperatures.

III. Results of Measurements

A. Magnetic properties

Magnetization curves are shown in Fig. 1. Values of the saturation magnetization, I_s, and of the saturation field, H_s, as determined directly from the magnetization curves, absolute values of the magneto-crystalline anisotropy constant, K, as computed by a following formula:

$$|K| = I_s H_s/4, \tag{1}$$

---

(6) M. Yamamoto, Sci. Rep. Tôhoku Univ., **27** (1938), 115; Nippon Kinzoku Gakkai-shi, **2** (1938), 495 (in Japanese).

(7) M. Yamamoto, Nippon Kinzoku Gakkai-shi, **5** (1941), 167 (in Japanese) Sci. Rep. Tôhoku Univ., **31** (1943), 101.

(8) M. Yamamoto, Nippon Kinzoku Gakkai-shi, **6** (1942), 331 (in Japanese); Sci. Rep. RITU, **A3** (1951), 308.

and values of the initial magnetic susceptibility, κ_0, as determined by seperate measurements at very low fields are given in Table 1 and plotted against the composition in Figs. 2~4. As the copper content increases, all of these four magnetic quantities decrease and eventually vanish at about 35 percent copper.

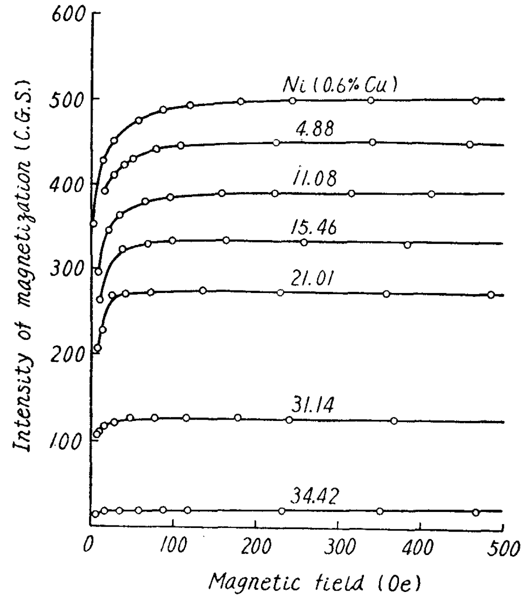

Fig. 1. Magnetization curves of nickel-copper alloys.

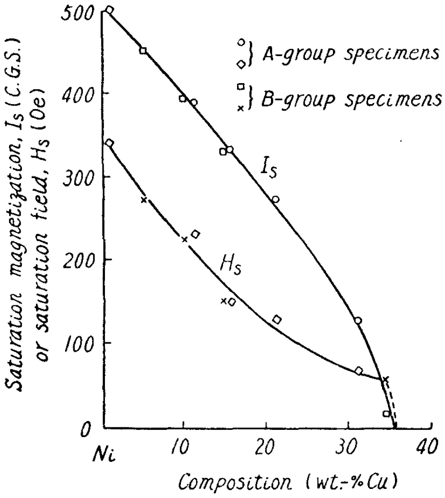

Fig. 2. Saturation magnetization and saturation field as functions of the composition in nickel-copper alloys.

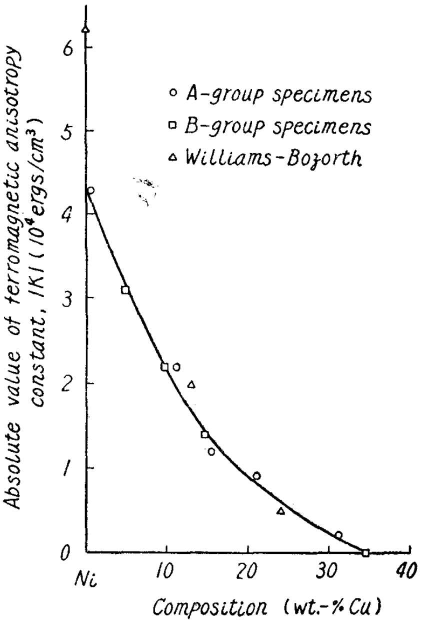

Fig. 3. Ferromagnetic anisotropy constant as dependnt upon the composition in nickel-copper alloys

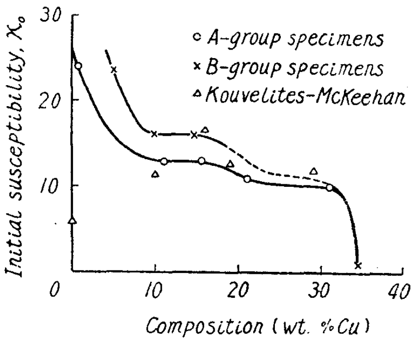

Fig. 4. The relation between the initial susceptibility and composition in nickel-copper alloys.

B. The ΔE effect as a function of the magnetic field

Figs. 5 and 6 show the change of Young's modulus relative to that for the unmagnetized state, $\Delta E/E_0$, as a function of the magnetic field, H. The general course of the change is roughly the same for all alloys ; Young's modulus increases quickly at low fields, but the rate of increase gradually diminishes as the field increases and finally the change saturates (Fig. 5). In alloys containing approximately 5 to 20 percent copper, however, Young's modulus once decreases slightly

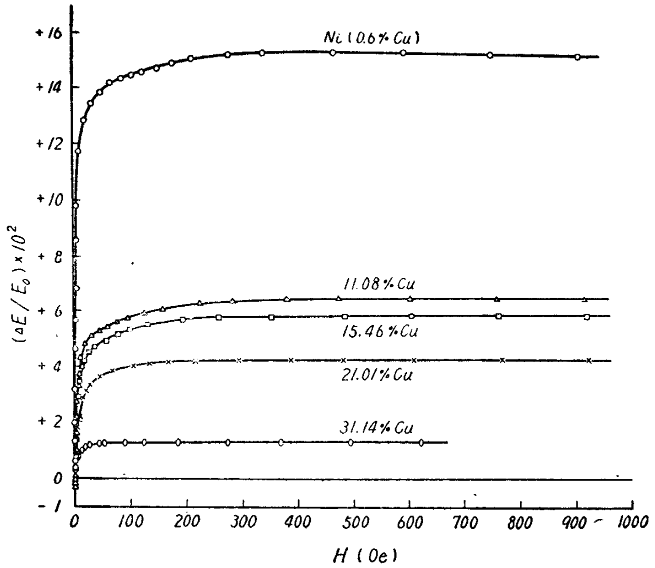

Fig. 5. Relative change of Young's modulus, $\Delta E/E_0$, as a function of the magnetic field, H, in nickel-copper alloys (A-group alloys).

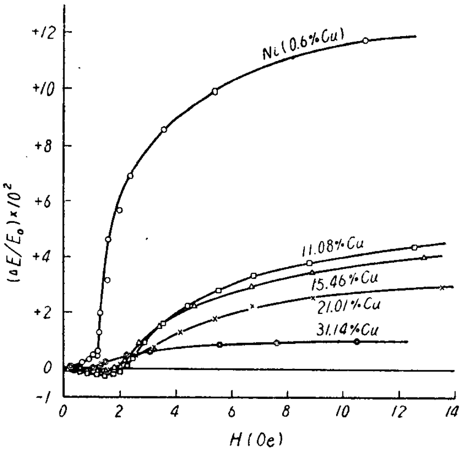

Fig. 6. ΔE-effect at low fields in nickel-copper alloys (A-groups alloys).

at low fields below 1~2 oersteds till it reaches a minimum, and then begins to increase (Fig. 6). It is to be noted that the field where this minimum locates, H_-, is approximately the same as that where the differential susceptibility is maximum, H_m(c.f. Table 1). Further, especially in nickel (containing 0.6 percent copper) and alloys containing less than about 15 percent copper, Young's modulus increases again more or less quickly just before the saturation (Fig. 5).

C. The ΔE-effect as a function of the intensity of magnetization

In Fig. 7 the relative change of Young's modulus is plotted as a function of the reduced magnetization, namely, the ratio of the intensity of magnetization, I, to the saturation magnetization, I_s (Table 1). General course of the $\Delta E/E_0-I/I_s$ curves is roughly the same irrespective of the composition of alloys; Young's modulus increases more and more rapidly as the magnetization proceeds, but just before the saturation the rate of increase once diminishes slightly and then increases very abruptly. This rapid rise of the $\Delta E/E_0-I/I_s$ curve corresponds to the above-cited secondary increase of the $\Delta E/E_0-H$ curve. Young's moduli of alloys of intermediate compositions, however, show a temporary slight decrease at low magnetizations (below 0.15~0.20 of I/I_s) corresponding to the above-mentioned temporary decrease of Young's modulus at low fields.

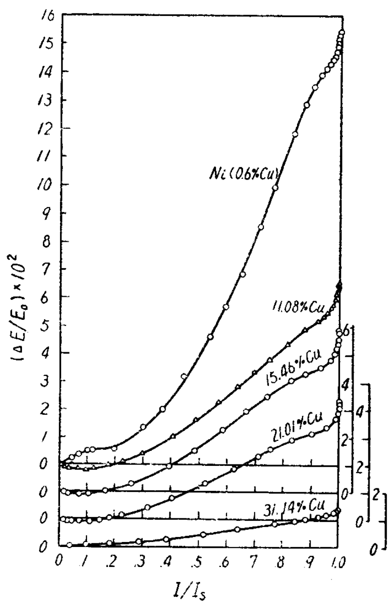

Fig. 7. Relative change of Young's modulus, $\Delta E/E_0$, as dependent on the reduced magnetization, I/I_s, in nickel-copper alloys (A-group alloys).

D. The Saturation value and the magnitude of the temporary decrease at low fields of the ΔE-effect as functions of the composition

The satulation value of the relative change of Young's modulus, $(\Delta E/E_0)_s$, is tabulated in Table 1 and plotted as a function of the composition in Fig. 8. The large ΔE-effect of pure nickel (containing 0.6% Cu) amounting to more than 15 percent decreases quickly by an addition of copper and falls below a half of the initial value at 10 percent copper. But, the rate of decrease diminishes once in the composition range from 10 to 15 percent copper, and then $(\Delta E/E_0)_s$ decreases quickly again (but more slowly than before). If the $(\Delta E/E_0)_s$ vs. composition curve is extrapolated to $(\Delta E/E)_s=0$, then it becomes that the ΔE-effect would vanish at about 34 percent copper. Indeed, the ΔE-effect could not be observed with a B-group alloy containing 34.42 percent copper. The magnitude of the negative ΔE-effect at low fields, $(\Delta E/E_0)_-$ as observed only for alloys of intermediate compositions, is given in Table 1, which indicates that $(\Delta E/E)_-$ decreases with an increase of copper content.

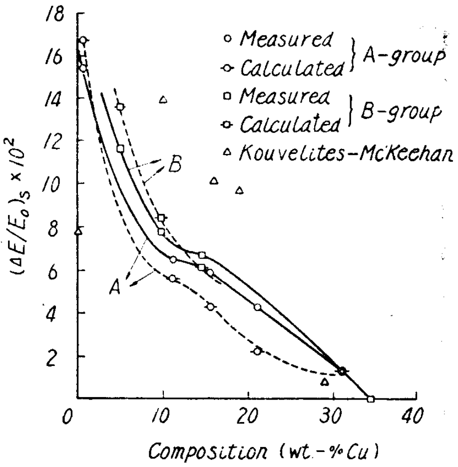

Fig. 8. Saturation value of the ΔE-effect, $(\Delta E/E_0)_s$, as a function of the composition in nickel-copper alloys.

E. Young's moduli at unmagnetised and at magnetically saturated states as functions of the composition

Young's moduli at unmagnetized state as well as at magnetically saturated state, E_0 and E_s, were determined by using values for the length of the specimens, values of density measured previously by the author⁽⁵⁾, and the data for $(\Delta E/E_0)_s$ as given in Table 1. They are plotted against the composition in Fig. 9.

Young's modulus of nickel at unmagnetized state increases, while that at magnetically saturated state decreases slightly, first, by the addition of copper. Both make minima near 15 percent copper, and then begins to decrease with the increase of copper content beyond about 20 percent.

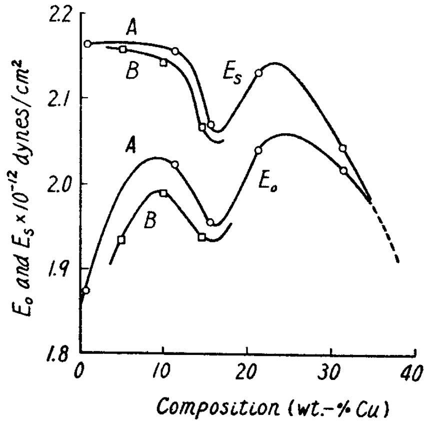

Fig. 9. Young's moduli at unmagnetized and at magnetically saturated states, E_0 and E_s, as functions of the composition in nickel-copper alloys.

IV. Discussion

A. Magnetic properties

The magnetization curves (Fig. 1) and the saturation magnetization, I_s, as a function of the composition (Fig. 2) in nickel-copper alloys are well known.

The saturation field, H_s, decreases with an increase of copper content, but it approaches to a finite value of about 60 oersteds near about 34 percent copper, and eventually would vanish abruptly (Fig. 2). This asymptotic value of H_s may be regarded as due to the strain energy of the internal stress.

The magnetocrystalline anisotropy constants, K, of nickel-copper alloys were determined previously by Williams and Bozorth(9) from the measurements on single crystal speciments. Their data for 20°C are also plotted for comparison in Fig. 3. As seen from Fig. 3, the data for K as computed by us from Eq. (1) on the basis of the measurements with polycrystalline specimens coincide very well with Williams and Bozorth's ones, except for nickel. For nickel, our value is equal to, but Williams and Bozorth's value is higher than (in the absolute magnitude), the accepted value. It is to be added that values for K computed from Eq. (1) have been found to be in good agreement with observed values for nickel-cobalt rlloys(10) and iron-aluminium alloys(11).

Further, the composition dependence of the initial susceptibility, κ_0, in nickel-copper alloys have been little known. Our data (Fig. 4) indicate that κ_0 decreases with increasing copper content in a not so simple way. This seems to have an intimate connection with the above-mentioned composition dependence of the ferromagnetic anisotropy constant.

B. The negative ΔE-effect at low magnetization

The decrease of Young's modulus or the negative ΔE-effect at low fields or at low magnetization as observed with nickel-copper alloys containing approximately 5 to 20 percent copper (Fig. 6) was a new phenomenon discovered by a dynamic method of measurement at that time, though the same and even more marked effects were observed later with iron(12), and iron-cobalt(12), nickel-iron(13, 14, 15), and nickel-cobalt alloys(16), and an iron-nickel-cobalt alloy(13). A similar phenomenon was early observed by Honda and Terada(17) in the measurement of the ΔE-effect of nickel with the static method of bending. This phenomenon will be considered in a separate paper.

It is to be noted that Kouvelite and McKeehan's(2) measurements by the method similar to ours of the ΔE-effect in annealed nickel-copper alloys do not show any negative ΔE-effect at low magnetization, which may be due to the roughness of their measured points.

---

(9) H. J. Williams and R. M. Bozorth, Phys. Rev., **55** (1939), 673; R. M. Bozorth, *Ferromagnetism*, D. van Nostrand, (1951), p. 573.

(10) M. Yamamoto, Nippon Kinzoku Gakkai-shi, **11** (1947), No. 11–12; **13** (1949), No. 6 (both in Japanese); Sci. Rep. RITU, **A4** (1952), 14.

(11) M. Yamamoto and S. Taniguchi, Nippon Kinzoku Gakkai-shi, **17** (1953), 532 (in Japanese).

(12) M. Yamamoto, Nippon Kinzoku Gakkai-shi, **6** (1942), 581 (in Japanese).

(13) H. J. Williams, R. M. Bozorth, and H. Christensen, Phys. Rev., **59** (1941), 1005.

(14) M. Yamamoto, Nippon kinzoku Gakkai-shi, **7** (1943), 467 (in Japanese).

(15) N. Kunitomi, J. Phys. Soc. Japan, **8** (1953), 76.

(16) M. Yamamoto, Nippon Kinzoku Gakkai-shi, **12** (1948), No. 2–3; M. Yamamoto and S. Taniguchi, ibid, **B15** (1951), 337 (both in Japanese).

(17) K. Honda and T. Terada, Phys. Z., **6** (1905), 622; Phil. Mag., **13** (1907), 36.

C. The secondary rise of the ΔE-effect just before the saturation

A more or less rapid rise of the $\Delta E/E_0-H$ curve just before the saturation observed with nickel and nickel-rich alloys containing less than 16 percent copper (Fig. 5) is the phenomenon first found by the author(7) in iron and nickel. As noted previously(7), this may be due to the fact that the marked and abrupt increase of the ΔE-effect just before the saturation for the principal crystal directions in single crystal are not completely smoothed out by being averaged over all crystal directions in polycrystal. Then, as the ferromagnetic anisotropy becomes smaller, this phenomenon should occur at more and more lower fields and over more and more narrower field range, and eventually it should mingle into the primary increase at low fields. This consideration is in accordance with our observations (Fig. 5). It is needless to say that the secondary rise of the $\Delta E/E_0-H$ curve corresponds to that of the $\Delta E/E_0-I/I_s$ curve (Fig. 8).

It is to be noted that the secondary rise of the ΔE-effect just before the saturation may be detected in Kouvelites and McKeehan's(2) data on the $\Delta E/E_0-I$ curves of nickel-copper alloys, although they did not note this effect but draw simply rising curves through their measured points.

D. The saturation ΔE-effect as a function of the composition

As to the saturation ΔE-effect in pseudo-isotropic polycrystalline specimens of cubic metals and solid-solution alloys, such as iron, nickel, and nickel-copper alloys, various theories(18) give formulae of similar forms for the case where the ferromagnetic anisotropy energy predominates over the magnetic strain energy, that is, the contribution from the non-180° wall displacements predominates over that from the continuous rotation of magnetization vectors. These formulae are written in the form:

$$(\Delta E/E_0)_s = C\kappa_0\lambda_e^2E_0/I_s^2/(1-C\kappa_0\lambda_e^2E_0/I_s^2)\,, \qquad (2)$$

where C is a factor associated with the elastic constants, λ_e the saturation magnetostriction for the direction of easy magnetization, and κ_0 is the (isotropic) initial susceptibility contributed only from the non-180° wall displacements. C is given by

$$C = C_1 \qquad (3a)$$

when the ΔE-effects are averaged over all crystal grains in a polycrystal under an assumption that the stress is uniform throughout the grains (the Reuss method of average), and by

$$C = C_1\{5c_{44}/(c_{11}+c_{12}+3c_{44})\}^2 \qquad (3b)$$

when, for face-centered cubic metals and alloys, of which the ferromagnetic anisotropy constant is negative as in nickel and nickel-copper alloys, the ΔE-effects are averaged under an assumption that the strain is uniform throughout all the grains (the Voigt method of average), where C_1 is 3/5 according to the statistical

---

(18) Cf. M. Yamamoto, Nippon Kinzoku Gakkai-shi, **5** (1941), 167 (in Japanese); Sci. Rep. Tôhoku Univ., [1] **31** (1943), 101.

domain theories by Akulov and Kondorsky(19), Brown(20), and Takagi(21), while it is 4/5 according to the orthodox domain theory by Döring(22), and c_{11}, c_{12} and c_{44} are the principal elastic parameters.

Now, with nickel-copper alloys excluding pure nickel the saturation magnetostriction for the direction of easy magnetization, namely for the [111] direction, λ_e, and the principal elastic parameters, c_{11}, c_{12}, and c_{44}, have never been measured. As for λ_e, since the magnetostriction in nickel is known to be comparatively isotropic, it may be highly probable that the magnetostriction is more isotropic than that of nickel, and hence λ_e may be replaced by the saturation magnetostriction of a polycrystalline specimens, λ_s. On the other hand, the elastic anisotropy is considerable for nickel(24) and it is rather more conspicuous for copper(25), but we may assume that the elastic parameters of nickel-copper alloys containing less than 35 percent copper are roughly the same as those of nickel. For pure nickel, $c_{11} = 2.44\times10^{12}$, $c_{12} = 1.58\times10^{12}$, and $c_{44} = 1.02\times10^{12}$ dynes/cm^2(24), so that $\{5c_{44}/(c_{11}+c_{12}+3c_{44})\}^2 = 0.256$. Further, C_1 is taken to be equal to 0.7, the average of the above-cited theoretical values. Then, from Eqs. (2) and (3a), we get, excluding $\{5c_{44}/(c_{11}+c_{12}+3c_{44})\}^2$,

$$\left(\frac{\Delta E}{E_0}\right)_s = 0.7\times\frac{\kappa_0\lambda_s^2E_0}{I_s^2}\Big/\left(1-0.7\times\frac{\kappa_0\lambda_s^2E_0}{I_s^2}\right). \tag{4}$$

$(\Delta E/E_0)_s$ for nickel-copper alloys calculated from this formula using our measured values for κ_0, E_0 and I_s and Masumoto, Shirakawa and Kobayashi's(26) ones for λ_s as given in Table 2 are shown in the same table and Fig. 8, which shows that the calculated and measured data are in a qualitatively good, and also in a quantitatively fairly good, agreement with each other. But it is to be noticed that this agreement may be rather fortuitous, since Eq. (4) represents only the contribution from the non-180° domain wall displacements and the measured data for κ_0 are the resultant of contributions from both the 180° and non-180° wall displacements. It is needless to say that Eq. (4) multiplied by $\{5c_{44}/(c_{11}+c_{12}+3c_{44})\}^2$ yields too low $(\Delta E/E_0)_s$ values.

The data for $(\Delta E/E_0)_s$ calculated from Eq. (4) using the data obtained recently by Kouvelites and McKeehan(2) are also shown in Fig. 8. Kouvelites-McKeehan's data agree with ours qualitatively, except for nickel. The large discrepancy for nickel may primarily be due to the large difference in the values for the initial susceptibility, as may be seen from the subsequent discussion.

---

(19) N. Akulov and E. Kondorsky, **78** (1932), 801; **85** (1933), 661.

(20) W. F. Brown, Phys. Rev., **52** (1937), 335.

(21) M. Takagi, Sci. Rep. Tôhoku Univ., **28** (1939), 85.

(22) W. Döring, Z. Phys., **114** (1939), 579.

(24) M. Yamamoto, Nippon Kinzoku Gakkai-shi, **6** (1942), 331 (in Japanese); Sci. Rep. RITU., **A3** (1951), 308.

(25) K. Kimura, Sci. Rep. Tôhoku Univ., [I] **22** (1933), 553.

(26) H. Masumoto, Y. Shirakawa and T. Kobayashi, Read at the autumn meeting of the Japan Institute of Metals in 1931. Masumoto-Shirakawa-Kobayashi's data are in better agreement with those of Went (Physica, **17** (1951), 98), and they are higher than those of Kouvelite and McKeehan(2).

As shown in Fig. 8, the $(\Delta E/E_0)_s$ vs. composition curve of the A-group alloys take a course similar to that of the B-group alloys, but the latter lies always above the former. This quantitative difference may naturally be caused by the difference in the origin and purity of specimens, as described in II. From the phenomenological point of view, however, it may be said that it is caused mainly by the difference in the initial susceptibility, κ_0, as shown in Fig. 4 and Table 1. Because, as discussed above, $(\Delta E/E_0)_s$ of nickel-copper alloys may be given approximately by Eq. (4), and for the same metal or alloy, only κ_0 may remarkably be influenced by impurities and mechanical and thermal treatments. In fact, the relations between $(\Delta E/E_0)_s$ and κ_0 for two sets of specimens which have similary compositions taken from the A-group and B-group alloys are as shown in Table 2, which indicates that $(\Delta E/E_0)_s$ is approximately proportional to κ_0 in each case.

Table 2. Parallel relation between the saturation value of the ΔE-effect, $(\Delta E/E_0)_s$, and initial susceptibility, κ_0.

Specimen No.	Composition wt.-%Cu	Ratio of $(\Delta E/E)_s$	Ratio of κ_0
A 1 A 2	11.08 9.77	1.20	1.2
A 2 B 3	15.46 14.63	1.14	1.2

E. Young's moduli at unmagnetised and at magnetically saturated states as functions of the composition

A large number of investigators[3,4,2] measured Young's moduli at unmagnetized state, E_0, of ferromagnetic nickel-copper alloys and obtained very diverse results, as shown in Fig. 10. Most investigations gave monotnously descending or ascending E_0 vs. composition curves as well as a curve showing a maximum at an inter-

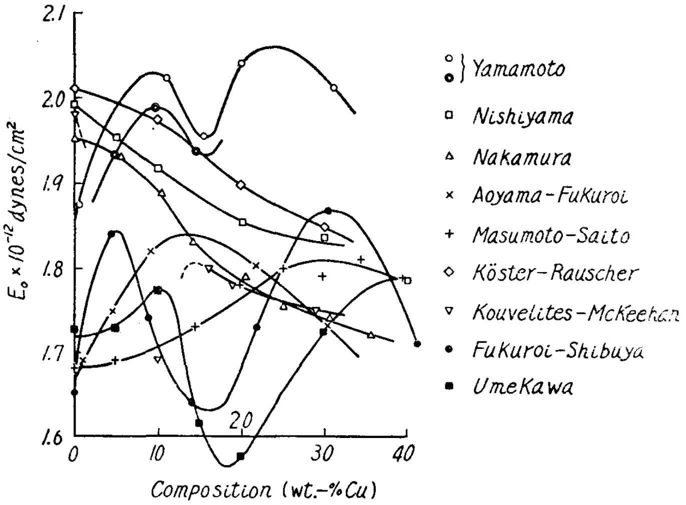

Fig. 10. Literature data on the relation between Young's modulus at unmagnetized state, E_0, and composition in nickel-copper alloys.

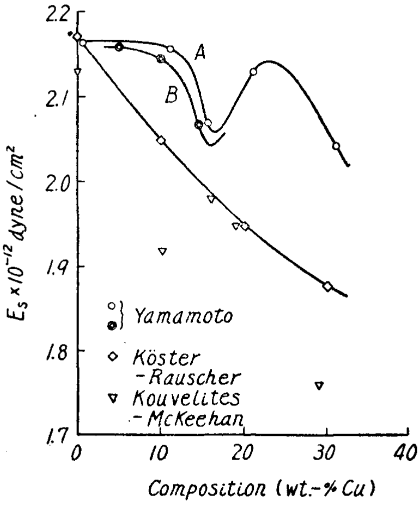

Fig. 11. Literature data on the relation between Young's modulus at magnetically saturated state, E_s, and composition in nickel-copper alloys.

mediate composition, but we obtained for the first time a more complicated curve, which has been confirmed, though qualitatively, recently by Fukuroi and Shibuya's$^{(4a)}$ static measurement and by Umekawa's$^{(4b)}$ dynamic measurement.

A minimum at 15~20 percent copper still remains in Young's modulus at magnetically saturated state, E_s, vs. composition curve (Fig. 11). This indicates that the cohesion of a ferromagnetic substance may contain an additive contribution from its ferromagnetism.

Summary

Young's modulus and its change with magnetization (the ΔE-effect) in annealed ferromagnetic nickel-copper alloys have been measured at ordinary temperatures by the method of magnetostrictive oscillation. The measured range of magnetic field for the ΔE-effect is up to about 1000 oersteds. The magnetic characteristics of the alloys have also been measured ballistically.

The general course of the change of Young's modulus accompanied by an increase of magnetic field or of magnetization, is similar for all alloys. The modulus increases quickly at weak fields, but the rate of increase diminishes gradually with growing field, finally the change of the modulus attaining a saturation. As for the relation between the change of the modulus and intensity of magnetization, the modulus increases more and more rapidly from the low to the high magnetization range, but once the rate of its increase diminishes slightly, and afterwards the modulus shows a rapid increase in the magnetization range adjacent to its saturation.

The detailed observation of the ΔE-effect reveals, however, the following two particulars. In the first place, Young's modulus in alloys of intermediate compositions (5~21 weight percent copper) decreases, that is, $\Delta E/E_0$ (the change of the modulus relative to that at unmagnetized state) becomes negative, though slightly, at first at very weak fields or in the low magnetization range, and makes a minimum before it increases rapidly with further increasing field or magnetization. Secondly, in nickel and alloys of high nickel contents (below 15.5 percent copper), the modulus increases somewhat rapidly again in the field range just preceding the saturation.

The saturation value of the relative change of Young's modulus, $(\Delta E/E_0)_s$, decreases with increasing copper content, but the course of decrease is by no means montonous, making an inflexion in the composition range of 10~15 percent copper. An extrapolation of the $(\Delta E/E_0)_s$ vs. composition curve shows that the ΔE-effect vanishes at the composition of about 34 percent copper, where the ferromagnetism disappears at ordinary temperatures. It has been found that the curve agreeing quantitatively fairly well with the measured one may be calculated from

$$\left(\frac{\Delta E}{E_0}\right)_s = 0.7\times\frac{\kappa_0\lambda_s^2E_0}{I_s^2}\bigg/\left(1-0.7\times\frac{\kappa_0\lambda_s^2E_0}{I_s^2}\right),$$

where κ_0 is the initial susceptibility, I_s the saturation magnetization, and λ_s the saturation magnetostriction. It is also shown that a quantitative difference recognized between $(\Delta E/E_0)_s$ vs. composition curves of two sets of alloys of different origins can be explained in terms of the difference in their initial susceptibilities.

The relation between Young's modulus at unmagnetized state and composition shows a fairly complicated aspect. The modulus increase at first with an addition of copper to nickel, makes a maximum at about 10, a minimum at about 15, and again a maximum at about 20 percent copper, and then it decreases with further increasing copper content. The existence of the minimum at about 15 percent copper remains unchanged by magnetization.

Values of the ferromagnetic anisotropy constant, K, computed from a formula $|K| = I_s H_s / 4$ (H_s = the saturation field) were found to be in good agreement with Williams and Bozorth's data obtained from the measurements on single crystal specimens. We have also found that the composition dependence of the initial susceptibility is not so simple.

In conclusion, the author expresses his thanks to Messrs. T. Washida, K. Endô, K. Sakai, and K. Hoshi, and to Miss H. Saitô for their assistance.