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# Measurement of Viscosities of Gases at High Pressure. I

Viscosity of air at 50°, 100° and 150°C

up to 200 atmospheres.

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## Synopsis

A modified oscillating disc viscometer was constructed to study the viscosities of gases at high pressures over a wide range of temperatures.

Detail accounts are given of the apparatus and of the method for calculating the viscosity from the experimental data.

Results are given for the viscosity of air at 50°, 100° and 150°C up to 200 atmospheres to an accuracy of about 1 percent.

## I. Introduction

Measurements have been made by several observers<sup>(1-9)</sup> on the viscosity of gases at high pressure from the kinetic theoretical point of view and also with the purpose to study the properties in the critical region. Recently, with the development of high pressure chemical process measurements<sup>(10-25)</sup> have been reported, especially on the low hydrocarbon gases.

These measurements are not only limited to several gases and restricted within the narrow experimental conditions, but also present results which differ by large percentage with observers; so these are still insufficient for theoretical and engineering purposes.

- (1) E. Warburg and L. von Babo, *Wied. Ann.*, **17** (1882), 390: CO<sub>2</sub>.
- (2) P. Phillips, *Proc. Roy. Soc.*, **A87** (1912), 48: CO<sub>2</sub>.
- (3) J. H. Boyd, *Phys. Rev.*, **35** (1930), 1284: N<sub>2</sub>, H<sub>2</sub>, N<sub>2</sub>-H<sub>2</sub>.
- (4) A. Michels and R. O. Gibson, *Proc. Roy. Soc.*, **A 134** (1931), 288: N<sub>2</sub>.
- (5) A. G. Nasini and G. Pastonesi, *Gazz. Chim. Ital.*, **63** (1933), 821; *Landolt, Erg. IIIa* 189: air.
- (6) S. G. Mason and O. Maass, *Can. J. Research*, **18 B** (1940), 128: C<sub>2</sub>H<sub>4</sub>.
- (7) S. N. Naldrett and O. Maass, *Can. J. Research*, **18 B** (1940), 322: CO<sub>2</sub>.
- (8) E. Schröer and G. Becker, *Z. Phys. Chem.*, **A 173** (1935), 178: CO<sub>2</sub>, (C<sub>2</sub>H<sub>5</sub>)<sub>2</sub>O.
- (9) Kh. Khalilov, *J. Expt. Theoret. Phys. (U. S. S. R.)*, **9** (1939), 335; *Chem. Abst.*, **33** (1939), 8069: C<sub>2</sub>H<sub>5</sub>OH.
- (10) H. Stakelbeck, *Z. ges. Kälte-Ind.*, **40** (1933), 33; *Chem. Abst.*, **27** (1933), 3016: CH<sub>3</sub>Cl, SO<sub>2</sub>, NH<sub>3</sub>, CO<sub>2</sub>.
- (11) B. H. Sage and W. N. Lacey, *Am. Inst. Min. Met. Eng. Tech. Pub.*, No. 845 (1937), 16: air, CH<sub>4</sub> and two natural gases.
- (12) I. F. Golubev, *J. Tech. Phys. U. S. S. R.*, **8** (1938), 1932; *Chem. Abst.*, **33** (1939), 6672: air.
- (13) B. H. Sage, W. D. Yale and W. N. Lacey, *Ind. Eng. Chem.*, **31** (1939), 223: Butane and iso-butane.

The present investigation was undertaken with the purpose of refining and extending the results to wide ranges of temperature and pressure. It was necessary to devise a new experimental technique of measuring the viscosities of high temperatures and pressures to a degree of 1 percent in precision. This has been done by means of modified oscillating disc viscometer, whereby the viscosity of air could be measured at 50°C, 100°C, and 150°C, and up to 200 atm to an accuracy of about 1 percent.

The measurements have not been carried to higher pressure owing to the lack of reliable compressibility data needed in the calculation of the results.

Investigations of the viscosity of air have been described by Nasini and Pastonesi<sup>(5)</sup> at 14°C up to 200 atm by means of the efflux method, Sage and Lacey<sup>(11)</sup> up to 160°F and 2,500 lb/□" using rolling ball method and by Golubev<sup>(12)</sup> up to 100°C and 300 atm with the capillar flow method. Wildhagen<sup>(25)</sup> and Moulton and Beuschlein<sup>(16)</sup> studied air at room temperatures up to 200 and 300 atm, respectively, using the capillary flow method, but owing to the limited accuracy of the several experimental techniques the results must be regarded as being of a preliminary nature.

Sage and Lacey found from their measurement at 73.9°, 100° and 160°F that all viscosities increase with pressure and the increase at lower temperatures is more rapid than at higher temperatures, causing the isotherms to intersect at pressures below 500 lb/□".

Since I could not obtain the full papers of Nasini and Pastonesi and Golubev the experimental procedures and the accuracy of their experiments are not clear to me.

## II. Method and Apparatus

On the viscosity of gases at ordinary pressure the measurements have been made by many observers by means of Maxwell's Oscillating Disc.<sup>(26)</sup> On the viscosity of compressed gases Maass *et al.* studied the viscosities of ethylene<sup>(6)</sup> and carbon dioxide<sup>(7)</sup> in the critical region using the same method.

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- (14) C. Codegone, *Energia Termica*, 7 (1939), 54; *Chem. Abst.*, 34 (1940), 4952: steam.  
 (15) G. A. Hawkins, H. L. Solberg and A. A. Potter, *Trans. Am. Soc. Mech. Engrs.*, 62 (1940), 677: steam.  
 (16) R. W. Moulton and W. L. Beuschlein, *Trans. Am. Inst. Chem. Engrs.*, 36 (1940), 113: air.  
 (17) E. W. Comings and R. S. Egly, *Ind. Eng. Chem.*, 33 (1941), 1224: C<sub>2</sub>H<sub>4</sub>, CO<sub>2</sub>.  
 (18) W. L. Sibbitt, G. A. Hawkins and H. L. Solberg, *Trans. Am. Soc. Mech. Engrs.*, 65 (1943), 461; *Chem. Abst.*, 37 (1943), 5632: N<sub>2</sub>.  
 (19) A. S. Smith and G. G. Brown, *Ind. Eng. Chem.*, 35 (1943), 705: C<sub>2</sub>H<sub>6</sub>, C<sub>3</sub>H<sub>8</sub>.  
 (20) L. B. Bicher and D. L. Katz, *Ind. Eng. Chem.*, 35 (1943), 754: CH<sub>4</sub>-C<sub>3</sub>H<sub>8</sub>.  
 (21) R. M. Hubbard and G. G. Brown, *Ind. Eng. Chem.*, 35 (1943), 1276: C<sub>5</sub>H<sub>12</sub>.  
 (22) W. A. Felsing and F. Blankenship, *Proc. Oklahoma Acad. Sci.*, 24 (1944), 90; *Chem. Abst.*, 39 (1945), 849: C<sub>2</sub>H<sub>4</sub>.  
 (23) E. W. Comings, B. J. Mayland and R. S. Egly, *Univ. Ill. Eng. Expt. Sta. Bull.*, 354 (1944), 7; *Chem. Abst.*, 39 (1945), 2439: CO<sub>2</sub>, CH<sub>4</sub>, C<sub>2</sub>H<sub>4</sub>, C<sub>2</sub>H<sub>6</sub>, C<sub>3</sub>H<sub>8</sub>.  
 (24) M. G. Gonikberg and L. F. Vereshchagin, *Compt. Rend. acad. Sci. U. S. S. R.*, 55 (1947), 801; *Chem. Abst.*, 41 (1947), 7183: C<sub>2</sub>H<sub>4</sub>.  
 (25) Max Wildhagen, *Z. Angew. Math. Mechanik*, 3 (1923), 181: air.  
 (26) C. Maxwell, *Phil. Trans. Roy. Soc. London*, 156 (1866), 249.

As the accuracy of the above method is affected by the small distance between two fixed plates which form the space for the suspended disc to oscillate, it is difficult to obtain the results in constant accuracy over wide experimental conditions, especially wide pressure range.

Felsing and Blankenship,<sup>(22)</sup> and Gonikberg and Vereshchagin<sup>(24)</sup> measured the viscosities of ethylene, the former at 30~50°C up to 115 atm and the latter at 24°C up to 1000 atm, with the oscillating disc method. These two measurements were, however, taken from Chemical Abstracts, thus the details are not clear to me.

Macwood<sup>(27)</sup> has developed fully the theory of oscillating disc not merely on the case of small separation but also of the large separation which case corresponds to the fixed plates at an infinite distance from the disc.

Keesom and Macwood<sup>(28)</sup> measured satisfactorily the viscosity of liquid helium and hydrogen using the method based on the latter case.

I concluded from the calculation using the viscosity data reported in literature that the viscosities of gases can be measured precisely over wide ranges of temperature and pressure by means of the Macwood's method of large separation by giving a suitable height to the space in the vessel in which the disc is left to suspend and made to swing, and decided to measure the viscosity of gases at high pressure by means of the method based on the Macwood's oscillating disc (large separation).

The theory, which is fully treated in his papers, leads to the following relations for the viscosity ( $\eta$ ) of fluids

$$\frac{\eta' - \eta}{\eta'} = \frac{2\sqrt{\frac{T\eta'}{\pi\rho} - 4d}}{2\sqrt{\frac{T\eta'}{\pi\rho} + \left(\frac{R}{2} - 2d\right)}}, \quad (1)$$

where

$$\sqrt{\eta'} = C(\tau^2 A - \tau A_0) \left(1 - \frac{A}{2}\right) \sqrt{\frac{2\gamma}{\rho}}. \quad (2)$$

The above relations are applicable for measuring viscosities in accuracy within 1 percent, if

$$D\sqrt{\frac{\gamma}{2\nu}} > 4, \quad (3)$$

where

$C$  ; apparatus constant

$\tau = T/T_0$  ;  $T$  : period

$A = \delta/2\pi$  ;  $\delta$  : logarithmic decrement of free oscillations

$A_0, T_0$  : values of  $A$  and  $T$  measured in vacuum, respectively\*

$\gamma = 2\pi/T$

$\nu$  : kinetic viscosity

(27) G. E. Macwood, *Physica*, 5 (1938), 374, 763.

(28) W. H. Keesom and G. E. Macwood, *Physica*, 5 (1938), 737, 749.

\* whereby, of course, the suspension wire should be as it was under the experimental conditions for measurement of the viscosity of gas.

$\rho$  : density of fluid

$D$  : the separation between the oscillating disc, and fixed plates

$2d, R$  : thickness and radius of oscillating disc, respectively.

When  $C, T_0$  and  $\Delta_0$  are known, the viscosity ( $\eta$ ) can be calculated from the measurements of logarithmic decrements and periods using Eq. (2) and (1).

In addition, if using the experimental results equation (3) is fulfilled satisfactorily in calculation, it is found that the measurements of viscosities were carried out with an accuracy within 1 percent.

The main parts of the viscometer are shown in Fig. 1.

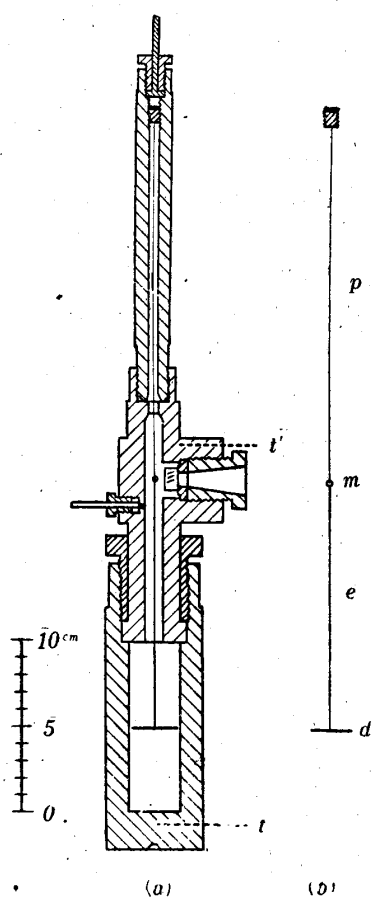


Fig. 1. Viscometer.

It consists of two parts, that is, the oscillating system and the pressure bomb. The former drawn to scale is shown in Fig. 1(b). The disc is 28.0 mm in diameter, 0.925 mm thick, and is made of V2A stainless steel. To the centre of this is fastened perpendicularly a steel rod, electrically plated with nickel, 154 mm long and 0.6 mm in diameter, fixed to a platinum suspension wire 0.05 mm in diameter and 182 mm in length. This is sealed in turn to a screw, which is used for supporting the suspension system and adjusting its position in the containing vessel described below. The oscillations are observed by means of a small V2A stainless steel mirror, 1.5 mm  $\times$  1 mm, fastened to the steel rod.

In Fig. 1 (a) the bomb and the assembly of the viscometer are shown. The main body was machined from the 18-8 stainless steel and equipped with glass or fused quartz window.

The window is based on the method of Toriumi and Hara<sup>(29)</sup>, which is the modification of that of Poulter<sup>(30)</sup>. This window mounting consists of a piece of glass (or fused quartz), 13 mm in diameter and 7 mm in thickness and having one of its flat surfaces placed and baked by the adhesive over a 8 mm hole in a polished surface of a stainless steel disc. At this time there is no need to make the surfaces of steel and glass optically flat. The piece of stainless steel is held in place by means of a large screw.

The viscometer assembly was set in the double chambered air thermostat and the connections made to the filling system are as shown in Fig. 2.

(29) T. Toriumi and R. Hara, J. Soc. Chem. Ind. Japan, 47 (1944), 502.

(30) Poulter, Phys. Rev., 40 (1932), 860.

The thermostat was constructed from the inner and outer chambers fitted with glass windows, heating coils, and temperature regulating systems, respectively. The inner chamber was also fitted with efficient mechanical stirrer arranged so that no vibration was transmitted to the suspension. The bomb was held perpendicularly on the small steel ball (*b*) through the short cylinder (*c*) clamped to the stand (*s*). The cylinder was locked

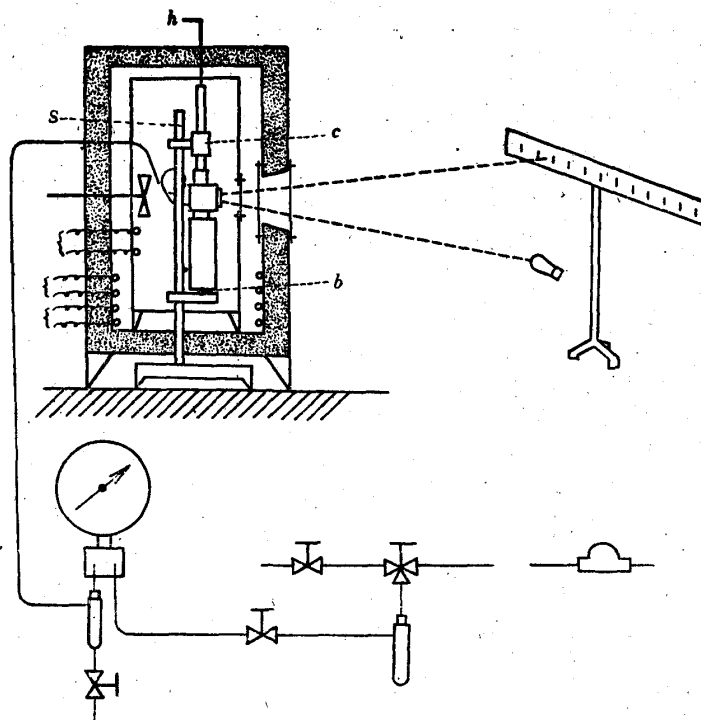


Fig. 2. The assembly.

smoothly with the bomb and used for the rotation of it as the guide. Since the suspension is delicate and susceptible to minute vibration, the air thermostat is placed on the stout table which was put on the large concrete blocks isolated from the floor of the laboratory.

The temperature of the viscometer was held by the automatic temperature controllers to  $\pm 0.2^\circ\text{C}$  and this was improved during the test period, so that the variation was seldom more than  $\pm 0.1^\circ\text{C}$ .

The temperature was measured by means of two copper constantan thermocouples calibrated with the standard Reichanstadt mercury thermometer, which were inserted in the viscometer at *t*, *t'* (Fig. 1a) and read with the suspension-type millivoltmeter, and controlled finely so that the temperature difference between the two parts was not found with the above meter.

The pressure in the viscometer was measured to 0.1 atm by 8" diameter Bourdon-type pressure gauge covered the range from 0 to 300 atm, which was calibrated with the standard pressure balance.<sup>(29)</sup>

By rotating the bomb slightly by the handle (*h*), the deflection of the disc could be induced smoothly. The disc oscillations were observed on a 50 cm ground glass scale placed apart 1.5 m from the mirror. The filament of a lamp was focused through a lens on the scale via the mirror, and its position was observed through two magnifying glasses sliding on the upper edge of the scale over its complete range.

### III. Experimental procedure

The test gas purified through the purifying system was forced in the viscometer

through the stainless bomb packed with silica gel cooled in solid carbon dioxide and alcohol from the gas compressor to a pressure of 25 atm and then purged out to 1 atm. This procedure was repeated three times to ensure that the other gas was thoroughly flushed out and the test gas was, then, forced in to a measuring pressure. The observations were made after about one hour.

An amplitude of about 50 cm was built up by means of rotating the viscometer to a small extent as described above. Oscillations were recorded successively from the amplitude of about 30 cm. Logarithmic decrement was calculated from twenty complete oscillations. At the same time the period  $T$  was determined by measuring the time of 21 oscillations with a stopwatch which could be read to 0.02 sec.

A final logarithmic decrement and period for a given experimental condition were obtained by averaging the results from 3 to 4 runs.

Nitrogen used for calibrating the viscometer was purified by passing 96.0 percent  $N_2$  from the cylinder through  $NH_3-NH_4Cl$ -solution packed with copper spirals, water, concentrated sulfuric acid and solid potassium hydroxide and was 99.2 percent in purity.

Air, on which a number of measurements were made, was purified by passing through 30 percent potassium hydroxide solution, concentrated sulfuric acid and solid potassium hydroxide.

#### IV. Calibrations

To calculate the viscosity from the experimental data by means of equations (1) and (2), it is necessary first to determine  $C$ ,  $\Delta_0$  and  $T_0$ .

Among these, the constant  $\Delta_0$  and  $T_0$  are values of  $\Delta$  and  $T$  in vacuum with the platinum wire possessing its nature as it was under the experimental condition for viscosity measurement and may, therefore, vary not only with temperature and by leaving the system in suspension,<sup>(31)</sup> but also with pressure. With  $\Delta_0$ , however, the effects of the latter two were neglected, since the damping factor  $\Delta_0$  was small compared with  $\Delta$ . Then, the variation of the damping factor  $\Delta_0$  with temperature only was measured. It was achieved under high vacuum of 1/100,000 mm Hg at several temperatures by means of the glass apparatus in place of the steel bomb and the damping system suspended in it under the same condition as the viscosity measurement.

With  $T_0$ , on the contrary, its deviation, even if it may be small, offers a large influence to the calculation and, therefore, the effects of the other two as well as temperature on  $T_0$  must be considered. Then,  $T_0$  was derived as shown in the following instead of measuring in vacuum.

Under this experimental conditions, the slip coefficient<sup>(27)</sup>  $\lambda$  is naturally zero, that is,

$$\lambda = -C' \cdot (1/\rho) \cdot [(1-2\Delta)\tau^2 - 1 + 2\tau\Delta_0] = 0,$$

where  $C'$  is constant.

(31) G. B. Benerjea and B. Plattanaik, *Z. Phys.*, 110 (1938), 676; K. Iokibe and S. Sakai, *Sci. Rep. Tohoku Imp. Univ. Japan*, 10 (1921), 1.

Using the above relation  $T_0$  at a certain circumstance was calculated from the data of  $A$ ,  $T$  and  $A_0$  measured under the same experimental condition.  $T_0$  derived as above was constant within the experimental accuracy in a series of experiments at a certain temperature as shown in the table described later (Table I~V). On the other hand,  $T_0$  for platinum wire (A) obtained from the vacuum experiment were 7.434 sec (June 3, 1949) and 7.474 sec (June 2, 1949) at 50°C and 100°C, respectively, and showed the difference of about 0.6~0.7 percent from the values derived as above (Table I, IV). Further, in a series of experiments at 50°C (Table I-III)  $T_0$  changed slightly its value with time as follows:  $T_0 = 7.489$  sec (Sept. 5(1949)),  $T_0 = 7.511 \sim 7.515$  sec (Oct. 7, 8 (1949)),  $T_0 = 7.516$  (Nov. 11 (1949)); and showed the tendency to approach a certain constant value. Thus, the above method for the calculation of  $T_0$  is considered to be rational.

The apparatus constant  $C$  was calibrated by means of nitrogen at 16.2, 21.4, 33.1 and 44.3 atm and 50.0°C using the values obtained from the interpolation of measurements which were given by Michels and Gibson<sup>(4)</sup> in their critical survey and estimated by them to have an accuracy not greater than 0.5 percent. The results of the calibration are shown in Table I.

The pressure ( $p$ ) given in the international atmospheres, the densities ( $\rho$ ) in grams per cubic centimeters, the times ( $T$  and  $T_0$ ) in seconds and the viscosities ( $\eta$ ) in c. g. s. units.

Table I.

$$A_0(A) = 0.000017$$

Date	Exp.No.	Press.	$\rho$	$A$	$T$	$T_0$	$T_{0\text{mean}}$	$\eta' \cdot 10^6$	$C$	$C_{\text{mean}}$
Sept. 5, 1949	N-1	16.2	0.0171 <sub>0</sub>	0.00286 <sub>6</sub>	7.510	7.488		234.1	0.540 <sub>6</sub>	
"	N-2	21.4	0.0226 <sub>1</sub>	0.00318 <sub>1</sub>	7.512	7.489	7.489	221.2	0.543 <sub>9</sub>	0.543 <sub>2</sub>
"	N-3	33.1	0.0349 <sub>6</sub>	0.00381 <sub>4</sub>	7.519	7.490		206.4	0.546 <sub>4</sub>	
"	N-4	44.3	0.0466 <sub>6</sub>	0.00434 <sub>7</sub>	7.522	7.489		199.7	0.542 <sub>0</sub>	

where  $\eta'$  were calculated by Eq. (I), using the values obtained from the interpolation of Michels and Gibson's data  $\eta_{16.18\text{atm}} = 191.5 \times 10^{-6}$ ,  $\eta_{21.40\text{atm}} = 192.0 \times 10^{-6}$ ,  $\eta_{33.13\text{atm}} = 193.6 \times 10^{-6}$  and  $\eta_{44.28\text{atm}} = 195.5 \times 10^{-6}$ .

The apparatus constant  $C$  obtained above is for 50°C. The values for any other temperatures are calculated from the knowledge of the expansion coefficient data of V2A stainless steel using the relation  $C \propto I/R^4$  where  $I$  is the moment of inertia of the oscillating system; that is,  $C$  for 100°C is 0.542<sub>2</sub> and for 150°C 0.541<sub>2</sub>.

After an elapse of about one month from the determination of apparatus constant the viscosities of nitrogen at 50°C were measured at 32.4, 42.5 and 57.8 atm, showing desirable agreement with Michels and Gibson's values as shown in Table II.

The compressibility isotherms used in the above calculations were given by Holborn and Otto.<sup>(32)</sup>

This fact shows that the above treatments for the calibrations were rationally

(32) L. Holborn and J. Otto, Z. Physik, 33 (1925), 1.



explained and the value of the apparatus constant was correct within the limits of the experimental accuracy.

Table II.

$$\Delta_0(A) = 0.00001_7$$

Date	Exp. No.	Press.	$\rho$	$\Delta$	$T$	$T_0$	$T_{0\text{mean}}$	$\eta \cdot 10^6$	$\eta^* \cdot 10^6$
Oct. 8, 1949	N-5	32.4	0.0341 <sub>9</sub>	0.00386 <sub>7</sub>	7.538	7.509		194. <sub>6</sub>	193. <sub>5</sub>
"	N-6	42.5	0.0448 <sub>0</sub>	0.00434 <sub>7</sub>	7.545	7.512	7.511	196. <sub>2</sub>	195. <sub>5</sub>
"	N-7	57.8	0.0607 <sub>6</sub>	0.00500 <sub>2</sub>	7.550	7.512		199. <sub>1</sub>	198. <sub>2</sub>

$\eta^*$  are the values of interpolation of Michels and Gibson's results.

### V. Results and Discussion

The results of the measurements made with air at 50°, 100° and 150°C up to 200 atm are given in Table III to V.

In the course of the experiment, the platinum wire was reassembled several times and  $\Delta_0(A)$ ,  $\Delta_0(B)$  and  $\Delta_0(C)$  are damping factors of platinum wires A, B and C, respectively.

Table III.

50°C

$$\Delta_0(A) = 0.00001_7$$

Date	Exp. No.	Press.	$\rho$	$\Delta$	$T$	$T_0$	$T_{0\text{mean}}$	$\eta \cdot 10^6$
Oct. 7, 1949	A- 1	22.7	0.0248 <sub>8</sub>	0.00345 <sub>2</sub>	7.538	7.515		199. <sub>6</sub>
"	A- 2	43.6	0.0477 <sub>0</sub>	0.00456 <sub>3</sub>	7.549	7.511		203. <sub>9</sub>
"	A- 3	72.2	0.0788 <sub>1</sub>	0.00574 <sub>1</sub>	7.562	7.517	7.515	208. <sub>0</sub>
"	A- 4	86.8	0.0944 <sub>4</sub>	0.00625 <sub>0</sub>	7.562	7.517		209. <sub>9</sub>
"	A- 5	98.1	0.1065	0.00665 <sub>1</sub>	7.571	7.518		213. <sub>5</sub>
Nov. 11, 1949	A- 6	109.3	0.1187	0.00698 <sub>0</sub>	7.572	7.528		218. <sub>8</sub>
"	A- 7	123.1	0.1332	0.00740 <sub>2</sub>	7.574	7.518		222. <sub>8</sub>
"	A- 8	163.9	0.1748	0.00851 <sub>4</sub>	7.575	7.510	7.516	230. <sub>3</sub>
"	A- 9	178.3	0.1889	0.00894 <sub>7</sub>	7.580	7.512		237. <sub>4</sub>
"	A-10	193.6	0.2037	0.00933 <sub>7</sub>	7.584	7.513		239. <sub>9</sub>

Table IV.

100°C

$$\Delta_0(A) = 0.00007_3$$

Date	Exp. No.	Press.	$\rho$	$\Delta$	$T$	$T_0$	$T_{0\text{mean}}$	$\eta \cdot 10^6$
Nov. 17, 1949	A-11	21.7	0.0204 <sub>4</sub>	0.00340 <sub>0</sub>	7.543	7.517		221. <sub>4</sub>
"	A-12	44.7	0.0419 <sub>1</sub>	0.00454 <sub>0</sub>	7.553	7.519		223. <sub>4</sub>
Nov. 18, 1949	A-13	62.3	0.0581 <sub>7</sub>	0.00520 <sub>5</sub>	7.559	7.520		223. <sub>4</sub>
"	A-14	86.0	0.0797 <sub>4</sub>	0.00603 <sub>7</sub>	7.566	7.521	7.519	228. <sub>9</sub>
"	A-15	108.9	0.1006	0.00674 <sub>4</sub>	7.574	7.523		233. <sub>1</sub>
Nov. 19, 1949	A-16	140.1	0.1279	0.00762 <sub>9</sub>	7.575	7.518		240. <sub>3</sub>
"	A-17	162.9	0.1473	0.00818 <sub>1</sub>	7.578	7.516		243. <sub>2</sub>
"	A-18	177.5	0.1594	0.00856 <sub>1</sub>	7.578	7.513		247. <sub>5</sub>
"	A-19	196.1	0.1746	0.00900 <sub>5</sub>	7.590	7.522		252. <sub>7</sub>

Table V.  
150°C

Date	Exp. No.	Press.	$\rho$	$\Delta$	$T$	$T_0$	$T_{0\text{mean}}$	$\eta \cdot 10^6$
						$\Delta_0(\text{B}) = 0.00004_4$		
March 17, 1950	A-20	21.7	0.0179 <sub>8</sub>	0.00326 <sub>8</sub>	7.254	7.230		240. <sub>6</sub>
" 16, 1950	A-21	44.8	0.0368 <sub>7</sub>	0.00438 <sub>1</sub>	7.262	7.230	7.229	240. <sub>5</sub>
"	A-22	62.7	0.0513 <sub>0</sub>	0.00504 <sub>9</sub>	7.265	7.228		242. <sub>7</sub>
						$\Delta_0(\text{C}) = 0.00025_7$		
Sept. 20, 1950	A-23	44.6	0.0367 <sub>1</sub>	0.00535 <sub>8</sub>	9.571	9.522		240. <sub>1</sub>
"	A-24	109.3	0.0881 <sub>0</sub>	0.00773 <sub>8</sub>	9.596	9.525		248. <sub>1</sub>
"	A-25	139.7	0.1112	0.00859 <sub>7</sub>	9.610	9.530	9.515	252. <sub>3</sub>
"	A-26	163.0	0.1284	0.00919 <sub>1</sub>	9.614	9.528		255. <sub>2</sub>
Sept. 27, 1950	A-27	87.2	0.0708 <sub>7</sub>	0.00701 <sub>5</sub>	9.554	9.489		243. <sub>6</sub>
"	A-28	186.8	0.1456	0.00985 <sub>8</sub>	9.588	9.495		261. <sub>9</sub>

In the calculation use was made of the compressibility isotherms given by Holborn and Otto<sup>(33)</sup> at the pressures below 100 atm. At the pressures above 100 atm those were calculated from the equation of state developed by Beattie and Bridgeman.<sup>(34)</sup>

In the experiments of the calibration of apparatus and determinations of the viscosity of air it was found that the maximum variation among the measurements of the logarithmic decrement from the 3~4 runs for the given set of oscillations was not greater than 0.5 percent.

In addition, it may be well to calculate<sup>(27)</sup> from the observations of the amplitude and period, and the viscosity data that the amplitude of oscillation was at all times so small that the motion of the fluid was laminar. It was quite certain on the basis of the fact that the logarithmic decrement appears to be independent to the amplitude. Furthermore, using the viscosity data obtained it was found that the relation (3) was satisfactorily fulfilled in all measurements. From these, it is rationally explained that the measurements were carried out under satisfactory conditions.

The results may be extrapolated to 1 atm: 50°C,  $197.3 \times 10^{-6}$ , 100°C,  $219.5 \times 10^{-6}$  and 150°C,  $239.0 \times 10^{-6}$ . The values compare favourably with that obtained from the interpolation of values measured by Titani<sup>(35)</sup>: 50°C,  $195.0 \times 10^{-6}$ , 100°C,  $216.9 \times 10^{-6}$  and 150°C,  $237.5 \times 10^{-6}$ . The values of the viscosity are plotted against the pressure in Fig. 3.

It will be seen that the viscosity has a positive temperature coefficient,  $(\partial\eta/\partial\theta)_p > 0$ , and  $(\partial\eta/\partial p)_\theta$  becomes progressively smaller as the temperature increases. There are, however, no intersections of isotherms over all ranges of this measurement, contrary to Sage and Lacey's report.<sup>(11)</sup>

Golubev<sup>(12)</sup> has reported the values at 0°, 16°, 50° and 100°C up to 300 kg/cm<sup>2</sup>. The values at 50° and 100°C, and 100 kg/cm<sup>2</sup> are shown in Fig. 3. The accuracy of

(33) L. Holborn and J. Otto, Z. Physik, 23 (1924), 77.

(34) J. A. Beattie and O. C. Bridgeman, J. Amer. Chem. Soc., 50, (1928), 3133.

(35) T. Titani, Bull. Chem. Soc. Japan, 4 (1929), 277; 8 (1933), 255.

these values is not appreciated owing to the lack of the knowledge of its details, but these values show the deviation of about 3~4% higher from my results and Nassini and Pastonesi's.<sup>(5)</sup>

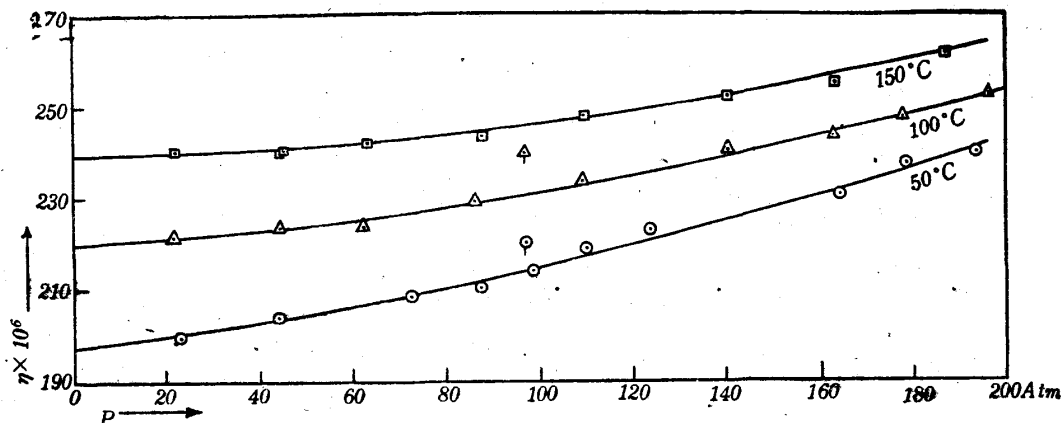


Fig. 3. Viscosity pressure curves.

⊙, △—Values at 50° and 100°C, respectively, by Golubev.

The viscosities at every decimal atmospheres and each temperature are obtained from the curves in Fig. 3 and given in Table VI.

Table VI.

atm	$\eta$	$\eta_{50^\circ\text{C}} \cdot 10^6$	$\eta_{100^\circ\text{C}} \cdot 10^6$	$\eta_{150^\circ\text{C}} \cdot 10^6$
1		(195.0)	(216.9)	(237.5)
1		197.3*	219.5*	239.0*
10		198.2*	220.3*	239.5*
20		199.6	221.0	240.0
30		201.0	221.9	240.1
40		202.6	222.7	240.5
50		204.3	223.6	241.2
60		206.0	224.7	242.0
70		207.7	226.0	242.8
80		209.6	227.7	243.6
90		211.9	229.2	244.9
100		214.2	230.9	245.6
110		216.9	232.8	247.6
120		219.4	234.8	249.0
130		222.0	236.7	250.6
140		224.9	238.9	252.0
150		227.6	241.0	253.8
160		230.5	243.2	255.8
170		233.4	245.8	257.6
180		236.7	248.0	259.8
190		240.0	250.7	261.9
200		243.7*	253.7*	264.0*

\* , values obtained from the extrapolation of curves.

( ) , values obtained by T. Titani.

Furthermore, the results for air obtained above were compared with Enskog's theory for the viscosity of compressed gases, showing good agreement over all experimental conditions. It will be fully described in other paper.

In conclusion, I wish to express my thanks to Prof. T. Toriumi for his kind guidance during the course of this investigation. I am also indebted to several assistants of the Chemical Research Institute of Non-Aqueous Solutions who have assisted me with the experimental work.

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