The Change of the Electric Resistance of Bismuth Crystals in Strong Magnetic Fields. Part IV : The Discussion and the Interpretation of the Experimental Results

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The Change of the Electric Resistance of Bismuth Crystals in Strong Magnetic Fields. Part IV
The Discussion and the Interpretation of the Experimental Results

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Synopsis

A discussion and the interpretaion of the experimental results in the change of resistance of bismuth single crystals and polycrystals in strong magnetic fields up to 200 kilo-oersted at room temperature were treated in the cases of the transverse phenomenon in Part II, the longitudinal phenomenon in Part III, and the phenomenon obtained by the case of the different directions between the current and the lines of force of the magnetic field relative to the orientation of the crystal in Part III.

The origin of the exceptionally large difference in the change of resistance of bismuth crystal in strong magnetic fields has been cleared by the special character of the conduction electrons in bismuth crystal based on Jones' model of the ènergy level in the bismuth crystal.

Tht value obtaiued in the experiment and the theory are practically the same, The complicated change in a strong magnetic field with respect to the relative directions among the magnetic field, the current and the orientation of the crystal, is not thoroughly explained quantitatively.

If these problems are explained by a futher study of the theory of solids, it will throw light on the peculiar character of the conduction phenomena in the bismuth crystal.

I. The general consideration of the experimental results

The change of electric resistance of bismuth crystals and polycrystals in strong magnetic fields up to 200 kilo-oersted have been studied at room temperature.

The results obtained were treated in Part II(1) and Part III(2). It is well known that in a magnetic field bismuth shows a greater change in resistance than any other substance.

The general view of the phenomenon in a bismuth polycrystal is that the increase of resistance is largest in the case of transverse phenomenon and smallest in the case of longitudinal phenomenon. In transverse phenomenon it has been shown that at the beginning up to about 30 kilo-oersted the resistance change is proportional to the square of the field but the latter is proportional to the field. In longitudinal phenomenon the resistance change is not only very small as compared with transverse phenomenon but shows a tendency to reach a limiting

(1) Y. Tanabe: Sci. Rep. RITU., A1 (1949), 275.
(2) Y. Tanabe: Sci. Rep, RITU., A2 (1950), 341.

value at stronger fields beyond 100 kilo-oersted.

In the case of a crystal the results from the point of view of magnetude and general character are exactly similar to those obtained in the case of a poly-crystal, but the phenomenon varies considerably with the orientation of the crystal.

We first consider the experimental results obtained in the transverse phenomenon. The maximum change in resistance of bismuth was observed in the case of crystal $P_{\perp}$ where the current flows along the trigonal axis i. e., perpendicular to the perfect cleavage plane and one of the binary axis on the perfect cleavage plane which corresponds to the intersection of one of the three imperfect cleavage planes with the perfect cleavage plane, is perpendicular to the lines of force of the magnetic field, and the other two lines make an angle of 30° with the magnetic field.

The small change in resistance is generally obtained in the case where the trigonal axis is parallel to the lines of force of the magnetic field.

It is also known that the impurities and imperfections in the crystal lattice greatly influence the change in resistance of bismuth.

The fact that under the same condition the extremum change in resistance can be observed in the case of the perfect crystal, has already been studied by Kapitza(3).

It is well known that the effect of the temperature on the resistance change is very great. The lower the temperature, the larger is the resistance change.

The complicated change of resistance at different field strengths has been studied at very low temperature by Schubnikow and de Haas(4).

Next we consider the experimental results obtained in the longitudinal phenomenon. The change in resistance of bismuth crystal in this case is smaller than in the case of a polycrystal and quite definitely reaches a limiting value at stronger fields beyond 100 kilo-oersted; in this case a smaller change is observed by the case of the perfect crystal under the same condition.

The change in resistance becomes larger as the temperature becomes lower but there appears no great change as in the case of the transverse effect.

Finally we consider the case where the relative direction between the lines of force of the magnetic field and the current is changed from the direction parallel to the direction perpendicular to each other for the different orientations of a crystal.

The current always flows along the axis of the crystal rod which is at first parallel to the axis of the magnetizing coil and then the direction of the rod axis is turned in a certain crystallographic plane to the direction perpendicular to the axis of the magnetitzing coil and the measurements were made at each 15°.

The general way in which the resistance changes, is as follows: in the cases of the range from 90° to 45°, for the angle between the directions of the lines of

(3) P. Kapitza: Proc. Roy. Soc. London., A 119 (1928), 401.
(4) L. Schubnikow and W. J. de Haas: Comm. Leiden., 207a, 207b, 210a, 210b (1930).

force of the magnetic field and the current, the resistance changes as the square of the field at weak fields and at stronger fields it is proportional to the field as in the case of the transverse phenomenon. In the neighbourhood of 45°, the resistance changes proportional to the field so that the curve of the change in resistance with the field is practically a straight line. In the case of an angle blow 30°, the resistance change is not only very small as compared with the greater angle but has a tendency to reach asymptotically a saturation value at fields above 100 kilo-oersted.

When the direction of the lines of force of the field is parallel or perpendicular to the special crystallographic line or direction, the anormalous change in resistance with respect to its magnitude is observed.

There appears a relative maximum or minimum value of the magnitude of the change at the same field strength when the direction of the lines of force of the field make a special angle to the direction of the current.

This anisotropy of the magnitude of the change is very marked at stronger fields.

The change in resistance of bismuth crystals in strong magnetic fields up to 320 kilo-oersted has previously been studied by Kapitza. He studied the transverse and longitudinal phenomenon with those crystals which have the same orientation as our crystals $P_{\perp}$ and $P_{\parallel}$. The measurements have been made at 290°, 193° and 90° absolute temperature. The present results obtained at room temperature coincide with those gained by Kapitza at the same temperature.

II. Discussion of the results for the experiments on the change of resistance when the current is perpendicular to the magnetic field

From the results obtained in Part II, it is seen that when the current is flowing perpendicular to the magnetic field there is an analogous change of resistance in the field for crystals differently orientated.

At the beginning the change of resistance $\Delta R/R$ is proportional to the square of the magnetic field H^2 but the latter is directly proportional to the field H.

The general character can be expressed by:

$$\frac{\Delta R}{R} = \alpha H_2 \qquad H_k \geqq H \geqq 0, \tag{1}$$

$$\frac{\Delta R}{R} = \beta(H-H_k)+\alpha H_k \qquad H \geqq H_k. \tag{2}$$

It is convenient to describe each curve by the constants α and β, and also by the strength of the magnetic field H_k at which the curve ceases to follow the square law (1) and changes to the linear low (2). As these formulae represent a continuous line, the straight line (2) will intersect the axis of H's at a field H_0 equal from (1) to (2) to

$$H_0 = \frac{H_k}{2}$$

and we get

$$\alpha = \frac{\beta}{4H_0}.$$

Inserting the values H_0 and α in (1) and (2) the following two expressions can be obtained:

$$\frac{\Delta R}{R} = \frac{\beta}{4H_0} H^2 \qquad H_k \geqq H \geqq 0, \tag{3}$$

$$\frac{\Delta R}{R} = \beta(H - H_0) \qquad H \geqq H_k. \tag{4}$$

In Tables 1 and 2 we give the values for α, β, H_0 and H_k for single crystals and polycrystals. In Fig. 1 aoa′ and ob represent the directions of the trigonal axis and the binary axis respectively, and oc represents the direction perpendicular to one of the binary axis in the perfect cleavage plane so that a plane boc corresponds to the perfect cleavage plane.

Table 1.

Specimens		Relative directions between H and I	H_0 (kilo-oersted)	H_k (kilo-oersted)	α	β
Single crystal	$P_{\perp}$	H ⊥ Binary axis ob* I ∥ Principal axis oa	17	34	0.235×10^{-8}	1.600×10^{-4}
	$P_{\perp}$	H ∥ Binary axis ob I ∥ Principal axis oa	17	34	0.132	0.900
	$P^{\perp}_{\parallel}$	H ∥ Binary axis ob I ⊥ Binary axis oc	25.5	51	0.089	0.885
	$P^{\perp}_{\parallel}$	H ∥ Principal axis oa I ⊥ Binary axis oc	28.5	57	0.070	0.781
	$P^{\parallel}_{\parallel}$	H ⊥ Binary axis oc I ∥ Binary axis ob	20	40	0.152	1.213
	$P^{\parallel}_{\parallel}$	H ∥ Principal axis oa I ∥ Binary axis ob	21	42	0 102	0.854
Polycrystal	No. 1	H ⊥ I	20	40	0.123	0.983
	No. 2	H ⊥ I	17	34	0.126	0.854

※ Fig. 1.

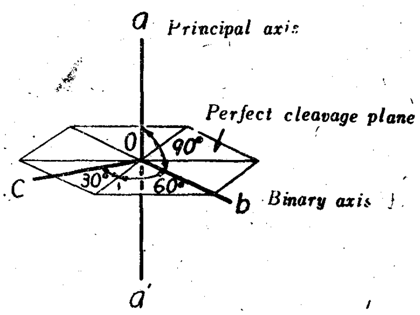

Fig. 1

From the Tables it is seen that the directions of the field and the current relative to the orientation of the crystal chiefly affect the constants α and β.

The values for α and β of the polycrystals have the intermidiate amounts of the extremum cases of single crystal in Table 1. The extremum values for α, β of single crystal are $\alpha = 0.235\times10^{-8}$, $\beta = 1.600\times10^{-4}$ in the maximum case, and $\alpha = 0.070\times10^{-8}$, $\beta = 0.781\times10^{-4}$ in the minimum case.

The influence of orientation of the crystal on these constants has been previously studied by Kapitza at three different temperatures. He assumed from the results

Table 2.

Speci-mens	Relative directions between H and I		φ_H (degree)	H_0 (kilo-oersted)	H_k (kilo-oersted)	α	β
$P_{\perp}$	Fig. 2	ob	0	17	34	0.132×10^{-8}	0.900×10^{-4}
			15	15	30	0.210	1.257
			30	17	34	0.235	1.600
$P^{\perp}_{\parallel}$	Fig. 3	ob	0	20	40	0.111	0.885
			15	14	28	0.161	0.900
			30	15	30	0.164	0.983
			45	24	48	0.141	1.351
			60	24	48	0.133	1.280
			75	12.5	25	0.266	1.130
		oa	90	26	52	0.075	0.781
$P^{\parallel}_{\parallel}$	Fig. 4	oa	0	21	42	0.102	0.854
			10	20	40	0.111	0.885
			20	21.5	43	0.110	0.949
			40	26.5	53	0.112	1.192
			65	22	44	0.151	1.327
		oc	90	20	40	0.152	1.213
			120	13	26	0.199	1.036
			150	14.5	29	0.124	0.727
			165	19	38	0.109	0.824
		oa′	180	21	42	0.102	0.854

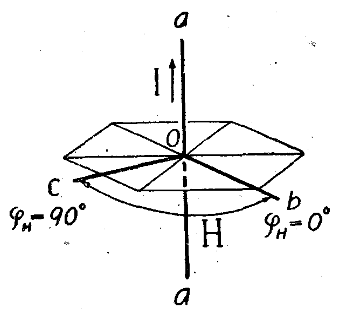

Fig. 2.

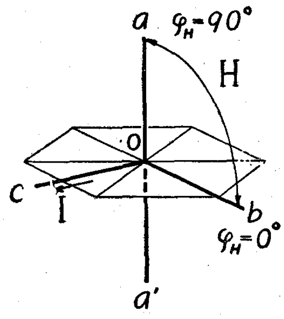

Fig. 3.

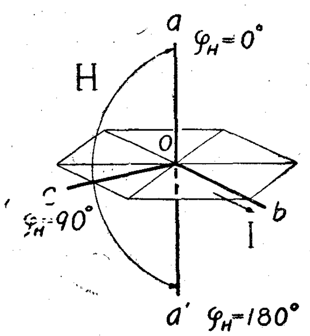

Fig. 4.

obtained at low temperature that for a definite temperature the orientation of the crystal axis relative to the lines of force of the magnetic field chiefly affected the constant α of the square law but the constant β of the linear law is the same for all orientations.

He reached the conclusion from the consideration above mentioned that the difference of magnitude in the change of resistance in a magnetic field with respect to the orientation of the crystal relative to the magnetic field is mainly affected in the range of the field strength where the square law is held and at fields above H_k there appears no such differece in magnitude of the change of resistance, corresponding to the same value of the β of the linear law for all orientations. But the values for α and β deduced from his experimental results at room temperature, have the same order difference in magnitude as the values listed in Table 1.

The theoretical calculation of the constant α of the square law has been made by Hirone and Matsuda[5] based on the crystal model used by Jones[6] in which the circular symmetry with respect to the trigonal axis of the bismnth crystal has been assumed to calculate the anisotropy of the change of resistance in a magnetic field. They obtained the following values for B in the equation

$$\frac{\Delta R}{R} = BH^2 \tag{5}$$

in three cases:

$H \parallel$ Trigonal axis, $I \perp$ Trigonal axis, $B = 0.124 \times 10^{-8}$
$H \perp$ Trigonal axis, $I \perp$ Trigonal axis, $B = 0.154 \times 10^{-8}$
$H \perp$ Trigonal axis, $I \parallel$ Trigonal axis, $B = 0.157 \times 10^{-8}$

It is readily seen that these theoretical values for B in each special case are practically the same as those for α obtained from the experimental results listed in Table 1.

It is also seen from these results that the anisotropy in the change of resistance is chiefly affected by the relative orientations in the current, the field and the trigonal axis or the perfect cleavage plane of the crystal.

The magnitude of the anisotropy in the change of resistance perviously obtained by many observers at weak fields are practically the same as those obtained herein.

From the consideration of the values for α and β in Tables 1 and 2, and the curves of crystals in Part II, it may be recognized that the change of resistance is markedly more affected by the orientation of the imperfect cleavage plane relative to the direction of the lines of force of the magnetic field at stronger than at weak fields.

The conduction electrons in a bismuth crystal which enforeed the perpendicular motion to the direction of the magnetic field constrained the orbital motion by the magnetic field. When the orbital plane of the motion of the conduction electrons is perpendicular to the perfect cleavage plane, the resistance change is large and small when it is parallel to the perfect cleavage plane.

But the results obtained in the case of the relative orientation of the orbital plane to the imperfect cleavage plane are completely opposed to those in the case of the perfect cleavage plane. That is, the resistace change is large when the imperfect cleavage plane is parallel to the field and small when it is perpendicular to the field.

The effect of the perfect cleavage plane to the orbital motion of the conduction electron by the action of the magnetic field is exactly opposite to the effect of the imperfect cleavage plane.

The effect of the perfect cleavage plane has been made clear by Jone's theory but the effect of the imperfect cleavage plane has not been cleared by the model of Jones' theory. It may be suggested that it is necessary to verify this

(5) T. Hirone and S. Matsuda : Rikagaku Kenkyûsho Ihô., 18 (1939), 711.
(6) H. Jones : Proc. Roy. Soc. London., A 155 (1936), 653.

fact from the analysis of the more complicated crystal moded in which the effect of the higher order plane other than the imperfect cleavage plane may of course be considered. But an adequate knowledge about the effect of the higher order planes can not be reduced from experiments only at room temperature as in the case of our study. At room temperature the crystal lattice is mainly affected by the atomic vibration owing to the disturbance of the thermal motion. Therefore the secondary disturbance from the impurity and imperfection of a crystal, and the effect of the higher order plane are generally thought to be of the same order of magnitude as the effect of the thermal agitation.

Next we consider the meaning of the critical field strength H_k. The change of resistance of bismuth in the transverse effect is conveniently described by the expressions (1) and (2). In these expressions the critical field strength H_k is assumed. Below the critical field the change of resistance follows the square of the field but after the critical field the change is proportional to the field.

This critical field is not defined as in most curves no definite bends exist. But the general character of the change of resistance of crystals is clearly alterad in the neighbourhood of the field strength between 30 and 50 kilo oersted.

Kapitza found from his observation at low temperature that the coefficient α much depends on the orientation of the crystal but the coefficient β is the same for all orientations of the crystal made of the same bismuth when observed at the same temperature.

From the above mentioned he suggested that in the region of the linear change of resistance after the critical field, the condition for the motion of the conduction electrons is similar in all crystals for all orientations and above the critical field the influence of the neighbouring atoms on the conduction electrons, which depends on the symmetry of the crystalline lattice, is lost.

This change or reconstruction as he called it, produced by the field, whatever it may be, probably takes place during the increase of the magnetic field from O to H_k during the square law part of the curve. The reconstruction takes place at relatively weaker fields in the case of bismuth crystal than in the case of polycrystal.

Impurities and imperfection in the crystal which disturb the lattice, makes the reconstruction easier and reduces the value of H_k. But from the present experiments the critical field strength is practically the same order of magnitude, whether the specimen is a single crystal or a polycrystal, in the range between 30 and 50 kilo-oersted.

Milner(7) studied the change of resistance of Cadmium at the temperature of liquid nitrogen and reduced the same value of H_k in the cases of a single crystal and a polycrystal.

From Table 1 and 2, it is seen that the coefficient β is not of the same value which is reduced from the experiments above H_k at room temperature, distributed

(7) C. J. Miner: Proc. Cam. Phil. Soc., 33 (1937), 145.

in the range between 0.727×10^{-4} and 1.600×10^{-4}.

From this order in magnitude of the distridution of the coefficient β, it is very doubtful to suppose that the reconstruction takes place during the increase of the magnetic field up to H_k.

III. Discussion of the results for the experiments on the change of resistance when the current is parallel to the magnetic field

The change of resistance in the longitudinal phenomenon makes a marked difference compared with the change of resistance in the transverse phenomenon in such a way as the change of resistance is not only very small but quite definitly reaches a limiting value. The resistance of a polycrystal change is larger than all cases of single crystals but it may be suggested from our experimental results that there is a tendency to reach a saturation value which is not very clearly claimed.

In the change of resistance of two polycrystals used in our experiment, there appears not only very noticable difference between them but also a large change of resistance when a specimen is stressed.

The magnitude of the increase of resistance with the field in cases of single crystals $P_{\perp}$, $P_{\parallel}^{\perp}$, P_{23}^{7} and $P_{\parallel}^{\parallel}$ is large in the range of the field strength up to about 100 kilo-oersted but it becomes very small at stronger fields.

The change of resistance above 120 kilo-oersted shows a tendency to reach a definite value in the experimental error. The saturation value of each crystals is practically in the same order of magnitude.

The effect of the change of resistance in the longitudinal phenomenon depends only on the perpendicular component of the current with respect to the magnetic field.

It is readily seen that in the case of the pure and perfect crystal, the component of the current meandering in the crystal grains. becomes small. According to this fact the specific resistance of a crystal may also become small.

Therefore the change in resistance of a crystal in a magnetic field is also reduced and the rapid change in the direction of the motion of conduction electrons to the direction of the main current which is set parallel to the magnetic field shows a saturation effect. From these considerations it would appear that the change of resistance in the case of a polycrystal has large value compared with the case of a single crystal.

Kapitza showed that the resistance in a bismuth crystal changes increase when it is stressed; this is also true in the case of a polycrystal as seen from present experiments. The effect of stress in which the resistance changes increased in the longitudinal phenomenon, is contrary to the transverse phenomenon in which the resistance change is reduced.

Theoretically no change in resistance will be found in the longitudinal phenomenon if studied with an ideal perfect crystal with no impurities and stress where the current flows exactly parallel to the lines of force of the magnetic field.

In practice a small change in resistance is probable and a saturation effect is shown in a certain field even if the ideal perfect crystal is used, because of the many factors which affect the change of resistance to increase in a magnetic field in experiments.

The most probable factors in the usual crystal are (1) the imperfection of a crystal, (2) the stress near the sectioned place, (3) the stress due to soldering the rod to lead wires, (4) the defective alignment of a rod axis and (5) the effect of impurity etc..

Strict comparison of the experimental results for each specimen is difficult owing to the effects of the above factors. In the transverse phenomenon, it was possible to explain the difference in the change of resistance for crystals $P_{\perp}$, $P_{\parallel}^{\perp}$ and $P_{\parallel}^{\parallel}$ by comparing the effect of the perfect cleavage plane and the imperfect cleavage plane. Similarly the experimental results for the case of longitudinal phenomenon may be explicable.

According to Fig. 1 in Part III, the magnitude in the least change of resistance below the field strength of about 50 kilo-oersted occured in the crystal $P_{\perp}$ and the largest in the crystal $P_{\parallel}^{\perp}$ and the median one in the crystals P_{23}^{7} and $P_{\parallel}^{\parallel}$.

In the case of the smallest change of resistance for a crystal $P_{\perp}$, the direction of the magnetic field and the current are at the same time perpendicular to the perfect cleavage plane which compels the orbital plane of the conduction electrons possessing the perpendcular components of the motion with respect to the direction of the magnetic field to set in the perfect cleavage plane. Therefore the increase of resistance may be expected to be small but the results obtained was contrary.

In the crystals $P_{\parallel}^{\perp}$ and $P_{\parallel}^{\parallel}$ the direction of the magnetic field and the current are at the same time parallel to the perfect cleavage plane, so that the orbital plane of the conduction electrons cross at right angle to the perfect cleavage plane.

Therefore the magnitude of the change of resistance compared with the case of a crystal $P_{\perp}$ may be increased.

Moreover in the case of a crystal $P_{\parallel}^{\perp}$ the orbital plane is perpendicular to the perfect cleavage plane and is inclined to one of the three imperfect cleavage planes at an angle of 18° 22′ which is most parallel to the imperfect cleavage plane but in the case of crystal $P_{\parallel}^{\parallel}$ the orbital plane is perpendicular to the perfect cleavage plane and imperfect cleavage plane. From these reasons the change of resistance of the crystal $P_{\parallel}^{\perp}$ may be the greatest and that in the crystal $P_{\parallel}^{\parallel}$ next. The magnitude in the change of resistance for the crystal P_{23}^{7} is slightly larger but the character of its change is the same as crystal $P_{\parallel}^{\parallel}$.

This reason is explained as follows: in the case of crystal P_{23}^{7} the algebraic sum of the decrease in the change of resistance owing to the effect of the perfect cleavage plane inclined at an angle of 23° to the axis of the rod and the increase in the change owing to the effect of the inclination of the orbital plane with respect to the imperfect cleavage plane as compared with the case of crystal $P_{\parallel}^{\parallel}$ affects the total increase in the change of resistance. It is not very clear to confirm the above results from only present experimental data, in strong magnetic field, although the same pheonmenon was already stated by Stierstadt(8).

The larger increase in the change of resistance in the crystal $P_{\perp}$ rather than in the crystal $P_{\parallel}^{\parallel}$ at fields above 50 kilo-oersted may be considered as the greater stress may be set in the crystal $P_{\perp}$ than in the crystal $P_{\parallel}^{\parallel}$.

The crystal $P_{\perp}$ can easily cleave the plane perpendicular to the axis of the rod axis as the perfect cleavage plane is perpendicular to the rod axis and easily receives a stress by the external force. From the fact that the curve of the change of resistance in the case of crystal $P_{\perp}$ considerably imcreases to reach a saturation value, this increase may be attributed to the effect of stress.

If the change of resistance in the crystal $P_{\perp}$ may be observed at the perfect and stress-free crystal, it is permissible to suppose that its change of resistance is smallest in comparison with other orientations of the crystals up to stronger fields.

IV. Disscussion of the results for the experiments on the change of resistance when the current is inclined to the magnetic field

From the results obtained in Part III, it is seen that except for some special cases, the largest change of resistance occured in the case of $\varphi_H=90°$, the smallest in the case of $\varphi_H=0°$ and the decrease of the magnitude in the change of resistance from $\varphi_H=90°$ to 0° follows with the decrease in the value of φ_H.

In the range of $\varphi_H=90°$ to 45° the change of resistance is in its character similar to those obtained in the transverse phenomenon. At weak fields the resistance changes as the square of the field and at fields above the critical field strength H_k it is proportional to the field.

In the neighbourhood of $\varphi_H=45°$ to 30° the change of resistance is proportional to the field up to 180 kilo-oersted which is the strongest field in these experiments, thus the curve for the change of resistance with the field is practically a straight line.

In the case of smaller φ_H below 30° the curve has a tendency similar to those obtained in the longitudinal phenomenon.

The general way in which the change of resistance alters with φ_H in the same strength of the field is not very clear at fields below 30 kilo-oersted but at stronger fields it become very marked.

Table 3 shows the values for α, β, H_0 and H_k for the case similar in character to the transverse phenomenon by the assumption that these curves can be ex-

(8) O. Stierstadt: Zeit. Physik., 85 (1933), 697.

pressed by the formulae (3) and (4). In this Table the case in which the critical field strength $H_k=0$ means that the curve of the change of resistance is practically a straight line, thus in this case $\alpha = 0$.

Table 3.

Speci-mens	Relative directions between H and I	φ_H (degree)	H_0 (kilo-oersted)	H_k (kilo-oersted)	α	β
$P_{\perp}$	Fig. 5 ob	45	0	0	0.000×10^{-8}	0.315×10^{-4}
		60	22.5	45	0.081	0.727
		75	20	40	0.099	0.795
		90	17	34	0.132	0.900
$P^{\perp}_{\parallel}$	Fig. 6 oa	30	0	0	0.000	0.364
		45	25	50	0.064	0.613
		60	15	30	0.164	0.983
		75	25	50	0.102	1.018
		90	26	52	0.075	0.781
	Fig. 7 ob	30	30	60	0.019	0.231
		60	22	44	0.066	0.577
		75	18	36	0,094	0.675
		90	20	40	0.111	0.885
$P^{\parallel}_{\parallel}$	Fig. 8 oa	45	15	30	0.073	0.435
		60	15	30	0.094	0.566
		75	17	34	0.113	0.767
		90	21	42	0.102	0.845
	Fig. 9 cc	60	0	0	0.000	0.456
		75	14	28	0.101	0.566
		90	20	40	0.152	1.213

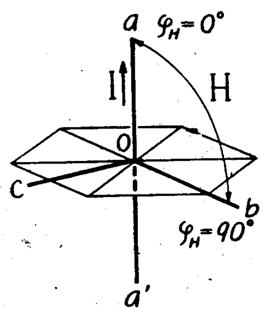

Fig. 5.

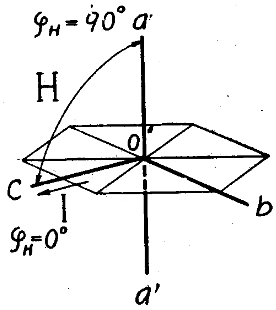

Fig. 6.

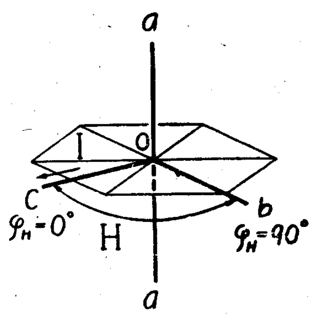

Fig. 7.

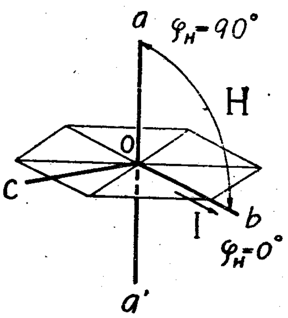

Fig. 8.

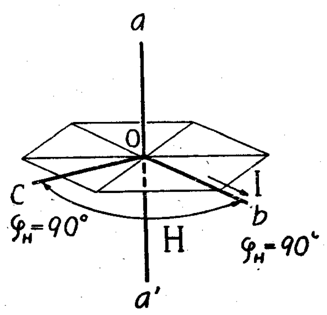

Fig. 9.

It is seen that the value of the critical field strength H_k which is taken from the observed curves, have similar character to the transverse phenomenon.

The order of magnitude is roughly settled in the range of the field strength from 30 to 50 kilo-oersted.

It is supposed therefore that the critical field strength is not affected by φ_H and it is decided by the impurity of the specimen.

V. The theory of the change of resistance of bismuth crystal with a magnetic field

To explain the exceptionally large change of resistance of bismuth in a magnetic field, Kapitza adopted the following assumptions: first, that the magnitude of the interchange of energy between a conduction electron and an atom may be much smaller than may be expected from the result of calculation on the basis of a collision theory. Secondly, the free electrons in the metal make the most efficient collisions with the atoms of the crystalline lattice when they have an unsymmetical structure. Finally, as it has been suggested by Ehrenfest from a consideration of the large diamagnetism of bismuth, the electrons move round several nucli forming large orbits, which will make the crystall lattice behave as though it were made up of molecules having a very unsymmetrical structure.

By means of these assumptions Kapitza has explained the high specific resistance of bismuth and the exceptionally large increase of resistance in a magnetic field as the magnetic field gives the disturbing effect on an atom in changing the velocity of the electrons and orienting the orbits, making it thereby less symmetrical, and thus increasing its efficiency of collision with electrons. Specially owing to the large size of the electronic orbits in bismuth the magnetic field will be much more efficient in spoiling the symmetry of the molecules and in this way very easily account for the large increase of resistance.

With respect to the observed influence of the orientation of the crystalline lattice relative to the magnetic field on the change of resistance in bismuth he stated that these large electronic orbits must have a definite orientation in the crystal. These orbits are therefore differentlly affected by the magnetic field when it is applied at a different angle and the efficiency of collision is changed leading to a different increase in the resistance.

Finally he explained the existance of a critical field H_k in the transverse phenemenon by the fact that above H_k the critical value for the orientation effect of the magnetic field on the orbits is stronger than that of the inter-molecular forces arising from the symmetrical distribution of the atoms in the lattice, and the orbits reach a steady orientation. In this state the orbits are probably less affected by the field and have a smaller change of resistance. So that the change of resistance after the critical field is reached in most cases does not depend on the initial orientation of the crystal, i. e., β=constant. This is strongly supported by his observations of the change of resistance at low temperature.

The temperature effect may be described as follows: the atoms in a metals suffer a thermal disturbance. This disturbance is evidently equivalent to making the atom less symmetrical and increases the efficiency of the collision with the free electrons. The disturbance evidently being larger at higher temperature leading to have the large specific resistance at room temperature. At room temperature the disturbances due to the temperature motion of the atoms in bismuth is more prevalent than that due to the magnetic field. The lower the temperature the more prominent are the disturbances produced by the field. So that the change of resistance of bismuth in a magnetic field at room temperature is smaller than that at lower temperature.

The impurity and the imperfections of the lattice not only affects the symmetry of the molecule, but may also break the large orbits into small ones leading to the smaller change of resistance in a mrgnetic field than that in the case of the perfect crystal.

According to the above mentioned qualitative explanation adopted by Kapitza the main parts of the present experimental results can be accounted.

In the case of the change of resistance of bismuth in the transverse phenomenon at room temperature the values for coefficient β in the linear law above the critical field H_k when the magnetic field and the current are applied to the different orientation of the crystal, are not practically of the same value as compared with those previously obtained by the several cases in Kapitza's experiments.

The diffeaence in the values of coefficient β may be explained by the following consideration: at room temperature the disturbing effect of the thermal motion of the atoms is more prevalent than the orientation effect of the electron orbits by the action of the magnetic field, and these additive disturbances may differ from each orientations of the crystal.

With respect to the quantitative explanation of the exceptionally large influence in the orientation of the crystalline lattice relative to the magnetic field on the change of resistance of bismuth in the transverse nhenomenon, Hirone and Matsuda have calculated the value of coefficient α in the region of the weak fields where the square law holds, according to the model of electronic levels of a bismuth crystal postulated by Jones in which the conduction electrons of bismuth occupy the electronic energy states in the crystalline lattice. The values calculated on the basis of their assumption are practically in accordance with those obtained by the experiments.

The origin of the exceptionally large differece of the change of resistance relative to the orientation of the crystal has been cleared by the fact that the conduction electrons of a bismuth crystal possess a very small effective mass relative to the special direction of the crystal.

According to the theory of Jones, 78 electrons of an atoms of bismuth which contained 83 electrons, constitute the closed shell from K to O energy levels and the upper levels in the first energy zone of a bismuth crystal are nearly fully occupied by the residual 5 electrons, i. e. $(6s)^2$ and $(6p)^3$. Owing to the fact that the upper value of the energy in the first energy zone is slightly greater than the lower value of it in the second energy zone, few of the conduction electrons is therefore situated in the second energy zone.

In the case of pure bismuth the current may be carried by the electrons in the second energy zone and by the same numbers of the positive holes in the first energy zone even if a part of the current carried by the positive holes is very small.

From the above mentioned consideration it is clear that the electrons concerning the change of resistance in a magnetic field are those in the neighbourhood of the upper levels in the first energy zone and those of the neighbourhood of the lowest levels in the second energy zone.

The equi-energy surface in the wave vector space corresponding to above mentioned electrons are the ellipsoid of rotation which have the exceptionally large eccentricity.

The equi-energy surface of the ellipsoid of rotation in the neighbourhood of the Fermi surface in the second energy zone is therefore the ellipsoid of rotation with the eccentricity of about 1:100 elongated along the direction of the trigonal axis of the bismuth crystal. Therefore the effective mass of the electron in the direction of the trigonal axis is in the same order of magnitude as the ordinry mass but that of the perfect cleavage plane, perpendicular to the trigonal axis, becomes to about 1/100 of the normal electron mass thus becoming very effective by the action of the external force in this direction.

On the contrary in the neighbourhood of the upper levels in the first energy zone the electrons are easily moved to the trigonal axis.

In this case when the magnetic field is applied to the driection of the trigonal axis, i. e. Z-axis and the electric field is at the same time applied to the perpendicular direction, i. e. X-axis, the change of resistance in the direction of X-axis $(\Delta R/R)_x$ is expressed by the following formula;

$$\left(\frac{\Delta R}{R}\right)_x = \frac{\dfrac{\sigma_y^{(-)}\sigma_y^{(+)}}{\sigma_x\sigma_y}\left(\dfrac{\sigma_x^{(+)}}{n^{(+)}}+\dfrac{\sigma_x^{(-)}}{n^{(-)}}\right)^2\left(\dfrac{cH}{e}\right)^2}{1+\dfrac{\sigma_x^{(-)}\sigma_y^{(-)}\sigma_x^{(+)}\sigma_y^{(+)}}{(n^{(+)}\,n^{(-)})^2\,\sigma_x\sigma_y}\left(n^{(-)}-n^{(+)}\right)^2\left(\dfrac{cH}{e}\right)^2} \qquad (6)$$

where σ_x, σ_y are the components of the conductivity along the direction of X and Y axis respectively, $\sigma_x^{(-)}$, $\sigma_y^{(-)}$ are the components of the conductivity carried by the electrons in the second energy zone along the direction of X and Y axis respectively, $\sigma_x^{(+)}$, $\sigma_y^{(+)}$ are the components of the conductivity carried by the positive holes in the first energy zone along the direction of X and Y axis respectively, and $n^{(-)}$, $n^{(+)}$ are the numbers of the electrons in unit volume in the second energy zone and that of the positive holes in unit volume in the first energy zone respectivity.

It is seen from the equation (6) that the change of resistance in a magnetic field does not neccessary obey to the square law. But in the case of the pure bismuth, $n^{(+)}=n^{(-)}=n_B$, so that the equation (6) becomes as follows,

$$\left(\frac{\Delta R}{R}\right)_x=\left(\frac{\sigma_y^{(-)}\sigma_y^{(+)}}{\sigma_x\ \sigma_y}\right)(\sigma_x^{(+)}+\sigma_x^{(-)})^2\left(\frac{cH}{en_B}\right)^2 \tag{7}$$

The equation (7) is exactly of the same expression as the square law. To calculate the anisotropy of the change of resistance of bismuth crystal with the equation (7), it is assumed that in place of the 3-fold symmetry of the trigonal axis, the perfect circular symmetry is used leading to $\sigma_x=\sigma_y$. If $\sigma_{\parallel}$ and $\sigma_{\perp}$ is the components of the conductivity parallel and perpendicular to the trigonal axis respectively, then

$$\sigma_{\parallel}=\sigma_{\parallel}^{(-)}+\sigma_{\parallel}^{(+)},$$
$$\sigma_{\perp}=\sigma_{\perp}^{(-)}+\sigma_{\perp}^{(+)}.$$

The coefficient B in the formula (5) may be expressed as previously given by Jones.

If H, I and A represent the directions of the magnetic field, the current and the trigonal axis,

$$\left.\begin{aligned}
&(1)\quad H \parallel A,\ I\perp A,\ H\perp I,\\
&\qquad B_1=\frac{\sigma_{\perp}^{-)}}{\sigma_{\perp}}\left(1-\frac{\sigma_{\perp}^{(-)}}{\sigma_{\perp}}\right)\left(\frac{\sigma_{\perp}c}{en_B}\right)^2,\\
&(2)\quad H\perp A,\ I\perp A,\ H\perp I,\\
&\qquad B_2=\sigma_{\parallel}^{(-)}\sigma_{\perp}\left(1-\frac{\sigma_{\parallel}^{(-)}}{\sigma_{\parallel}}\right)\left(\frac{c}{en_B}\right)^2,\\
&(3)\quad H\perp A,\ I \parallel A,\ H\perp I,\\
&\qquad B_3=\sigma_{\perp}^{(-)}\sigma_{\parallel}\left(1-\frac{\sigma_{\perp}^{(-)}}{\sigma_{\perp}}\right)\left(\frac{c}{en_B}\right)^2.
\end{aligned}\right\} \tag{8}$$

Hirone and Matsuda used the following numerical values for n_B, $\sigma_{\parallel}$ and $\sigma_{\perp}$ etc., to compare the experimental and the calculated value based on the expression (8),

$$n_B=0.88\times10^{20}\ \text{cm}^{-3},$$

and at room temperature,

$$\sigma_{\parallel}=\sigma_{\parallel}^{(-)}+\sigma_{\parallel}^{(+)}=1.27\times10^{-5}\ \text{cgs e. m. u.},$$
$$\frac{\sigma_{\parallel}^{(-)}}{\sigma_{\parallel}^{(+)}}=1.3,$$
$$\sigma_{\perp}=\sigma_{\perp}^{(-)}+\sigma_{\perp}^{(+)}=1.0\times10^{-5}\ \text{cgs e. m. u.},$$
$$\frac{\sigma_{\perp}^{(-)}}{\sigma_{\perp}^{(+)}}=1.2.$$

Finally they obtained the following calculated values of the coefficients B,

$$B_1=0.124\times10^{-8},$$
$$B_2=0.154\times10^{-8},$$
$$B_3=0.157\times10^{-8}.$$

It is seen from a comparision with the values of coefficients α in Table 1 that the calculated values of B are practically the same those of α obtained in the experiments.

All the conductivity phenomena of bismuth crystal are more sensitive to imperities and imperfections of the lattice than those of any other metal.

If there are added the atoms of 4 or 6 valency as the impurity, the change of resistance in this case may be expressed as[9]

$$\frac{\Delta R}{R}=\frac{BH^2}{1+Z^2CH^2}\,, \tag{9}$$

where Z is the ratio of the amount of impurities and the total number of atoms and C is the quantity depending upon the values $\sigma_{\parallel}^{(-)}/\sigma_{\parallel}$, $\sigma_{\perp}^{(-)}/\sigma_{\perp}$ and at the same time the temperature in such a way that C decreases rapidly with the rise of temperature. As the number of electrons and positive holes per atom of bismuth is about 10^{-4}, the amcunt of about 0.001 atomic percent of the impurity already lead to departure from the square law.

The sensitiveness of impurity in bismuth as the general character of the change of resistance is strongly supported by these factors.

If $Z^2CH^2 \ll 1$, in the expression (9), the curve of the change of resistance draws the parabolic curve with respect to H axis and if $Z^2CH^2 \gg 1$, the curve is the linear asymptote parallel to H axis. Therefore it may be suggested that the curves of experimental results in the transverse phenomenon in both cases of Kapitza and the present study, are to be considered the initial part of the total curve which tend to reach this asymptote.

Moreover it is seen from the equation (9) that the transverse phenomenon for the curves of the change of resistance from the square to the linear law may occur not only by the effect of the magnetic field but also by the amount of the term Z^2CH^2 in (9).

If Z is increased by adding the impurities or C which is independent from the magnetic fieid, is increased by lowering the temperature, the values of Z^2CH^2 is considered to satisfy the same condition of the increase of the magnetic field. Therefore it is possible to review the transition phenomena in the weak as in the strong fields. This is strongly supported by the study of Thompson[10] in such a way that Pb, Sn and Ge are used as the impurities of 4 valency atoms which act to decrease the number of the electrons and to increase the number of the positive holes, and on the contrary Te and Se are used as the impurities of 6 valency atoms which act contrary to those of 4 valency atoms. He examined the effect of the impurity in the transverse phenomenon in the magnetic field up to 20 kilo-oersted and found that at 16°K the characteristic curves of the transition phanomenon were observed when 0.0004% Pb or 0.0003% Se was added to the pure bismuth.

In our experiment the bismuth used contained about 0.006% S but it may be considered that S may perhaps be considerably lost during crystallization prossess.

(9) N. Thompson: Proc. Roy. Soc. London., A **155** (1936), 111.
(10) N. Thompson: Proc. Roy. Soc. London., A **164** (1938), 24.

Moreover as our experiment was done at room temperature $Z^2 C$ is not so great so that the value of the amount of $Z^2 C H^2$ may be generally determined as the strength of the magnetic field.

With respect to the model of Jones the change of resistance of the pure bismuth in a magnetic field in the longitudinal phenomenon is expected to be zero, but the resistance shows its change in the experimental result however. small it may be.

To explain the roigin of this fact Jones assumed that the equi-energy surface in the wave vector space is not the surface of a quadratic function of the energy but changes its value on the Fermi surface.

But a saturation effect in a strong magnetic field is not completely explained by Jones' hyposis.

As mentioned above the origin of the exceptionally large difference in the change of resistance in a magnetic field relative to the orientation of a crystal, and the effect of the direction of the trigonal axis and the perfect cleavage plane in the transverse phenomenon has clearly been explained.

But the effect of imperfect cleavage plane which is observed in the present experiment and the quantitative explanation of the transition phenomenon in the change of resistance in which below the critical field H_k the resistance changes as the square of the field and above H_k it is proportional to the field, are not sufficiently explained.

Moreover the remarkable effect of the impurity in the conduction phenomenon of bismuth is very clearly explained but a saturation effect in the change of resistance of bismuth in the longitudinal phenomenon is not explained quantitatively.

Futher theoretical studies is expected in the near future to explain these phenomena.

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Summary

A discussion has been presented on the experimental results obtained in the change of resistance of bismuth single crystals and polycrystals in a strong magnetic field up to 200 kilo-orested at room temperature in the cases of the transverse phenomenon (Part II), the longitudinal phenomenon (Part III), and the different directions between the current and the lines of force of the magnetic field relative to the orientations of a crystal (Part III).

The exceptionally large difference in the change of resistance of bismuth crystal in a magnetic field has been interpretatad.

The origin of the large anisotropy in bismuth crystal has been cleared by the special character of the conduction electrons in a bismuth crystal, theoretically.

The complicated change of resistance in a magnetic field with respect to the relative directions among the field, the current and the orientation of the crystal, and also the special phenomenon in the change of resistance in a strong field at the transverse and the longitudinal phenomena have not been thoroughly explained quantitatively.

These problems must be resumed for a study of the theory of solids.

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