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Internal Stress due to Quenching in Cylindrical Steel Ingots *†

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The internal stresses in cylindrical steel ingots induced by the thermal treatment differ according to the difference of the cooling conditions, and those stresses sometimes become an origin for the occurence of cracks in large steel ingots. If the relations between stresses and the conditions of the thermal treatment are made clear from the theoretical or experimental point of view, we shall be able to find a way for prevention of thermal cracks in the ingots by controlling the conditions of quenching. Now there are some essential points to prescribe the internal stress due to cooling. In a case of the quenching of large cylindrical steel ingots into the quenching medium of constant temperature after heating uniformly in a high temperature, the main factors which specify the stresses are (1) the size or diameter of the cylinder, (2) the plastic deformation during the heat treatment and (3) the temperature before quenching, excepting the characteristic material constants of the steel such as thermal conductivity, elastic constants and the martensite or pearlite transformation temperature. Herein we discuss the influence of the above mentioned three factors upon the distribution and amount of the residual stress due to quenching on the basis of our previous theoretical consideration of the said stress.

The method of calculation of the internal stress of steel ingots due to cooling is given in the previous paper[1]. The residual stress of cylindrical steel ingots are, according to our previous discussions, given by

$$\left.\begin{aligned}
\widehat{rr} &= \frac{E}{(1-\sigma)(5-4\sigma)}\Big[3(1-\sigma)\Big\{2\int_0^1 y\rho d\rho - \frac{2}{\rho^2}\int_0^\rho y\rho d\rho\Big\} - \frac{1-2\sigma}{2}\Big\{2\int_0^1 x\rho d\rho - \frac{2}{\rho^2}\int_0^\rho x\rho d\rho\Big\}\Big].\\
\widehat{\varphi\varphi} &= \frac{E}{(1-\sigma)(5-4\sigma)}\Big[3(1-\sigma)\Big\{2\int_1^0 y\rho d\rho + \frac{2}{\rho^2}\int_0^\rho y\rho \alpha\rho - 2y\Big\}\\
&\quad - \frac{1-2\sigma}{2}\Big\{2\int_0^1 x\rho d\rho + \frac{2}{\rho^2}\int_0^\rho x\sigma\alpha\rho - 2x\Big\}\Big].\\
\widehat{zz} &= \frac{E}{(1-\sigma)(5-4\sigma)}\Big[3(1-\sigma)\Big\{2\int_0^1 y\rho d\rho - y\Big\} + (2-\sigma)\Big\{2\int_0^1 x\rho d\rho - x\Big\}\Big].
\end{aligned}\right\} \quad (1)$$

In this expressions $\widehat{rr}$, $\widehat{zz}$, $\widehat{\varphi\varphi}$ are respectively the radial, axial and tangential components of the stress, $x(\rho)$ and $y(\rho)$ are the strain parameters, the definition of which are given in the previous paper and ρ is the ratio of the radius vector r and the external radius b of the cylindrical ingots, i.e. $\rho=r/b$. Now it must be taken into account that above a certain temperature steel makes a plastic flow while it is elastic at low temperature. It is assumed that such a critical temperature is exsisting and it is denoted by θ_m. Hereafter we may equate it to the temperature of the end of transformation, i.e. $\theta_m=600°C$. (Fig. 1: the thermal expansion curve). Then we consider an instant of the temperature in which the portion $r=a$ $(0<a<b)$ is θ_m. So the temperature of the point $0<r<a$ is higher than θ_m and is elastic. At the instant we put the mean thermal

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expansion of this inner portion to $\tilde{\omega}_\alpha$ and outer to $\bar{\omega}_\alpha$, here α is the normalized radius $\alpha=a/b$. The stress parameters $x(\rho)$ and $y(\rho)$ are then given by

$$\left.\begin{aligned} x(\rho)&=\tilde{\omega}_\rho-\omega_m-2\int_\rho^1\frac{\omega_m-\tilde{\omega}_\alpha}{1-\alpha^2}\alpha d\alpha \\ y(\rho)&=\tilde{\omega}_\rho-\omega_0+2\int_\rho^1\frac{\omega_m-\bar{\omega}_\alpha}{\alpha}d\alpha \end{aligned}\right\} \qquad (2)$$

where ω_0 is the thermal expansion for initial temperature θ_0 of the ingots, and ω_m that for θ_m. For the calculation of strain parameters, it becomes necessary to know the mean thermal expansions $\bar{\omega}_\alpha$ and $\tilde{\omega}_\alpha$ for each stage of cooling of the steel ingot. The temperature distribution at any instants of the infinite long circular cylinder with radius b after quenching into the medium of temperature 0°C, which has been heated uniformly to the temperature ϑ_0, is

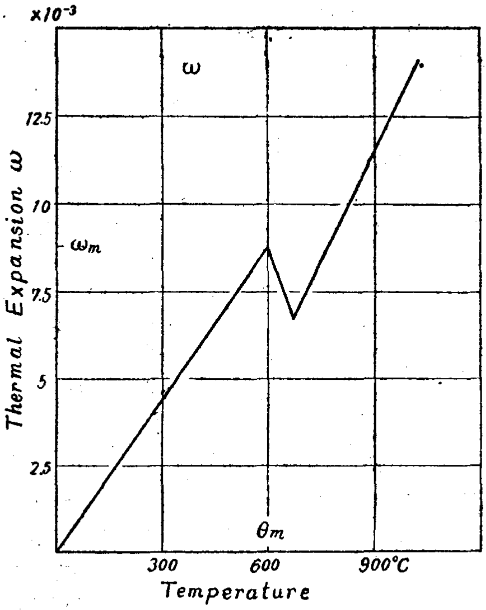

Fig. 1.

given by the theory of thermal conduction as follows: (Fig. 2)

$$\vartheta=\vartheta_0\sum_{n=1}^{\infty}2J_0(x_n\rho)exp(-\kappa x_n^2 t/b)/x_nJ_1(x_n),\quad(3)$$

where κ is the thermal diffusion coefficient $\kappa=K/c\rho'$, (K being the thermal conductivity, c specific heat and ρ' specific density). x_n is the n-th zero point of the Bessel function of the first kind of zero-th order $J_0(x)$. On the basis of the above cited thermal expansion curve (cf. Fig. 1), $\bar{\omega}_\alpha$ and $\tilde{\omega}_\alpha$ is calculated analytically and the results are shown in Figs. 3 and 4. The parameter m in the Figures is ϑ_m/ϑ_0. As those figures show, the mean expansion $\tilde{\omega}_\alpha$ for the ela-

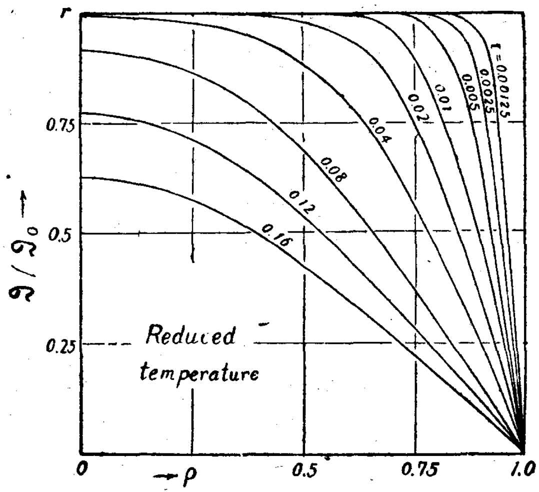

Fig. 2.

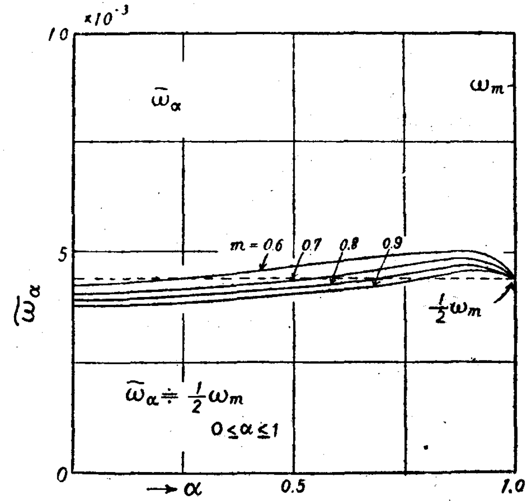

Fig. 3.

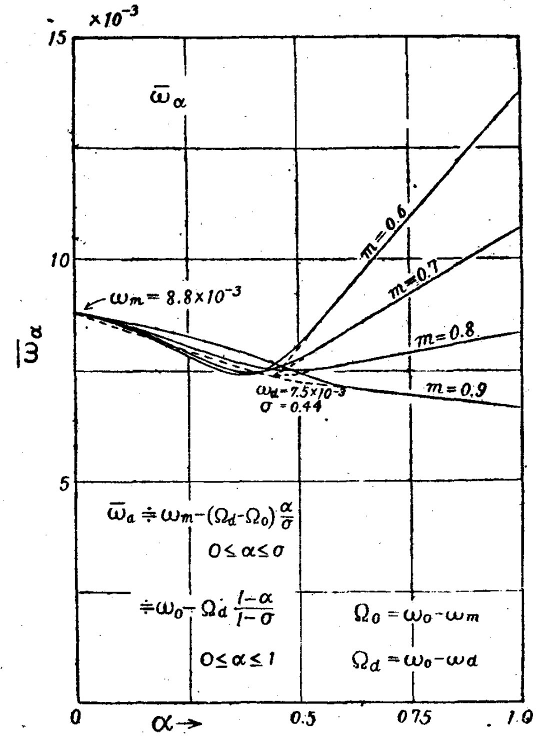

Fig. 4.

stic portion are nearly equal to the constant value $\omega_m/2$, and $\overline{\omega}_\alpha$ for the elastico-viscous portion take forms which are appropriately taken as a family of straight lines radiating from one point $\alpha=0.44\equiv\alpha_{\bar{a}}$, $\overline{\omega}_\alpha=7.5\times10^{-3}\equiv\overline{\omega}_{\bar{a}}$. So that for the purpose of obtaining the analytical form of x and y, those curves for $\overline{\omega}_\alpha$ are replaced by straight lines given by the broken lines in the figure. On the basis of these considerations we obtain the following forms of $x(\rho)$ and $y(\rho)$

$$
\left.
\begin{aligned}
x(\rho) &= -\Omega+\left\{1+\log(1-\rho^2)\right\} \quad 0\leq\rho<1 \\
y(\rho) &= \left.\begin{aligned} &-3\Omega_0+3(\Omega_0-\Omega_{\bar{a}})\frac{\rho}{\alpha_{\bar{a}}} \\ &+2\left(\Omega_0-\frac{\Omega_{\bar{a}}}{1-\alpha_{\bar{a}}}\right)\log\alpha_{\bar{a}} \end{aligned}\right\} \quad 0\leq\rho\leq\alpha_{\bar{a}} \\
&= \left.\begin{aligned} &-3\Omega_{\bar{a}}\frac{1-\rho}{1-\alpha_{\bar{a}}}+ \\ &2\left(\Omega_0-\frac{\Omega_{\bar{a}}}{1-\alpha_{\bar{a}}}\right)\log\rho \end{aligned}\right\} \quad \alpha_{\bar{a}}\leq\rho\leq1
\end{aligned}
\right\} \quad (4)
$$

where Ω_0 is $\omega_0-\omega_m$ and $\Omega_{\bar{a}}$ is $\omega_0-\omega_{\bar{a}}$. Here it is to be noted that these equations do not contain m explicitly, but actualy ω_0 contained in the equation is a function of m, because θ_m takes a constant value 600° C. The calculated residual stress distribution $\widehat{rr}$, $\widehat{\varphi\varphi}$ and $\widehat{zz}$, using the above formulaes with the constant $E=2.18\times10^4$ kg/mm² and $\sigma=0.25$ are shown in Figs. 5~8 and the exact treatment for $m=0.7$ is also shown in Fig. 6. As Fig. 6 shows, the approximate value of $\widehat{zz}(r=0)$ is slightly small compared with the exact one, on the other hand $\widehat{rr}(r=0)$ and $\widehat{\varphi\varphi}(r=0)$ is rather large. On the maximum values of $\widehat{rr}_{max}$, $\widehat{\varphi\varphi}_{max}$ and $\widehat{zz}_{max}$, however, it is clear that the above approximation is fairly good. In these figures, $m=0.6$, 0.7, 0.8 and 0.9 correspond to the case of quenching from the temperature θ_0 1000°C, 854°C, 750°C and 667°C respectively.

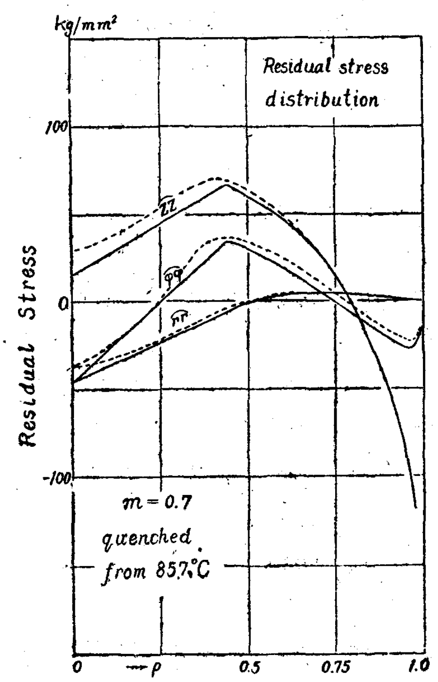

Fig. 6.

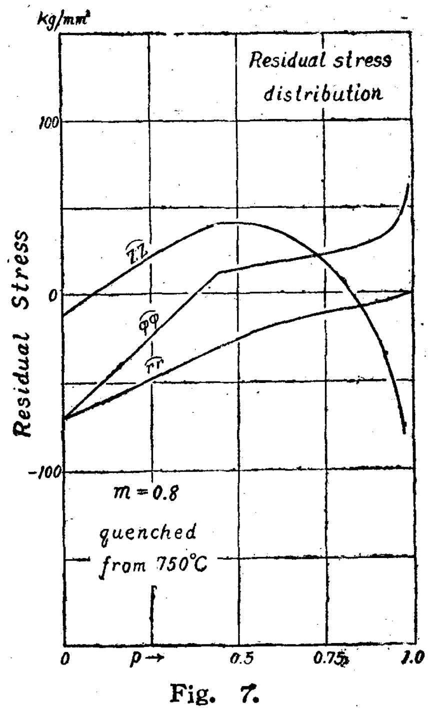

Fig. 7.

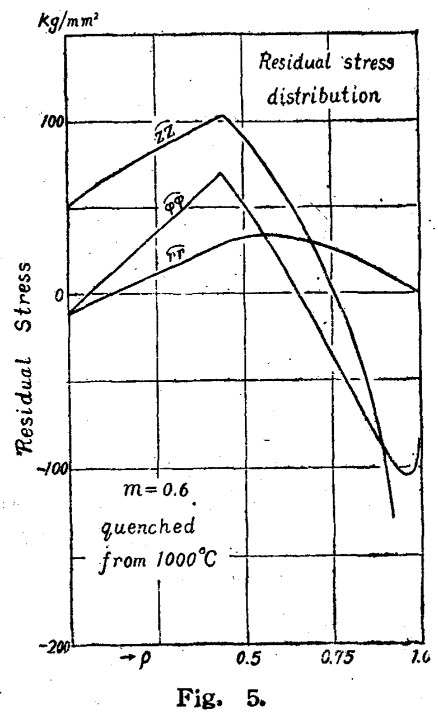

Fig. 5.

Summarizing the above mentioned affairs

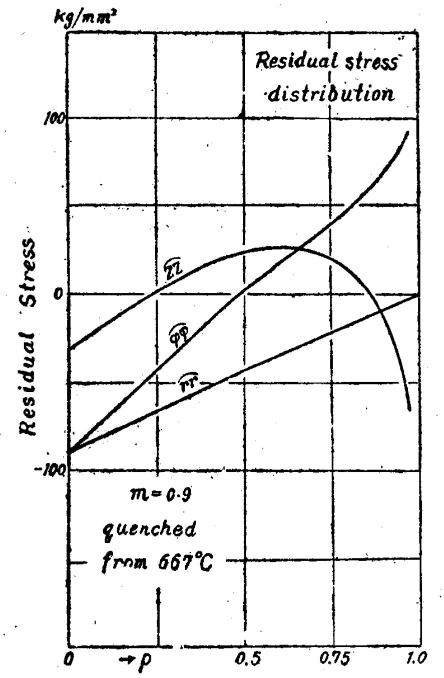

Fig. 8.

the stress comonents $\widehat{rr}$, $\widehat{\varphi\varphi}$ and $\widehat{zz}$ at $r=0.95$, and its maximum values are plotted against the quenching temperature in Fig. 9. As the figure shows, the thick lines, i.e. the maximum values of stress component $\widehat{rr}_{max}$, $\widehat{\varphi\varphi}_{max}$ and $\widehat{zz}_{max}$ decrease parallel to each other as the quenching temperature decreases, $\widehat{\varphi\varphi}_{max}$, however, in-

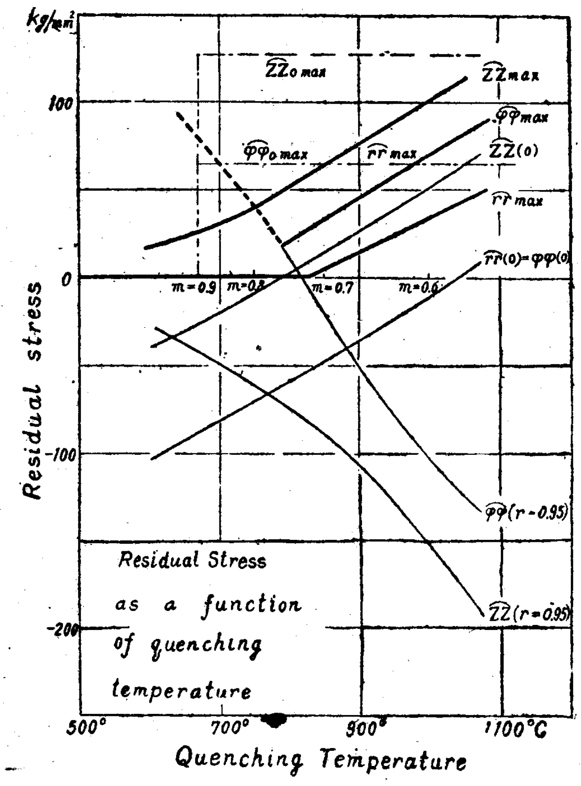

Fig. 9.

creases rapidly and $\widehat{rr}_{max}$ becomes zero when the said temperature passes through a certain temperature to a lower one. So that we are able to get the maximum values of stress component in the case of quenching from any temperature.

In addition, in Fig. 9, two parallel chain lines show the maximum stress of the ingot which has a little hole at the center. This affair provides us a conception about the the stress of the ingot in which cracks exist before the thermal treatment. This stress amount is nearly equal to the constant with respect to the quenching temperature and is considerably larger than that with no hole, so that may become an origin of growth of cracks. This circumstance is very different from the slow cooling, for example, in the case of cooling by radition, and can compare these two cases by referring to the previous paper and Fig. 9, of this report.

The logarithmic divergence of the stress component near $\rho=1$ in Fig. 5~8 is due to the non-convergency of the series (3) at $t=0$, $\rho=1$. $\vartheta(\rho=1, t=0)$ is ϑ_0 by the initial condition but if time t takes an infinitesimal value, $\vartheta(\rho=1)$ becomes discontinuously zero. Therefore $x(\rho)$ in (2) becomes meaningless at $\rho=1$, so that integration can not be carried over all interval, but $0<\rho<1-\Delta$, in which Δ is the infinitesimal value. The stress components, however, are not influenced by Δ because $x(\rho)$ is added by only a constant and this constant is subtracted through the calculation of stress components. In the actual cases, the temperature at $\rho=1$ becomes zoro continuously from ϑ_0 during a small time interval, so that this infinity disappears and in this case the stress components will become similar to the form given in Figs. 5~8. Morever, it is to be emphasized here that the case of quenching has the following property, i.e. the amount of stress and the distribution of it is independent of the radius b of the ingot. This is due to the fact that in equation (3) time t and radius b are contained in a form $\tau=k^2t/b$, therefore temperature ϑ becomes a function of τ as such as $\vartheta(\tau, \rho)$ and implicitly of b. Obviously the cooling velocity is affected by b, but stress components are the function of $\widetilde{\omega}_\alpha$ and $\overline{\omega}_\alpha$

and those are independent of cooling velocity (Fig. 10). However, since in the case

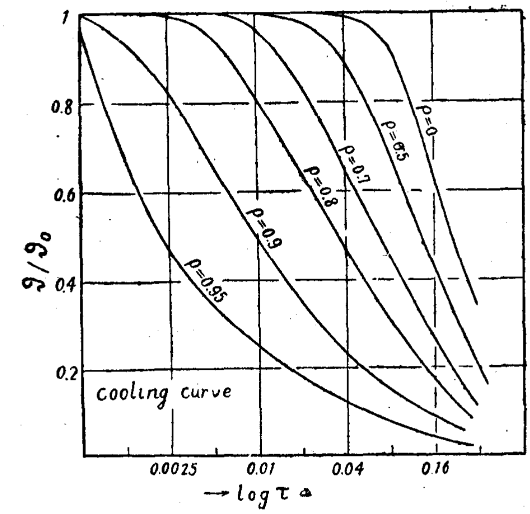

Fig. 10.

of too small ingot and too large cooling velocity the conception that steel becomes elastic abruptly from the elasticoviscos state, looses its meaning so that this calculation is not applicable for these small ingots.

Summary

The relations between residual internal stresses due to quenching and quenching temperature for large cylindrical steel ingots are obtained on the basis of the theory of elasticity, by taking into acount the thermal stresses and the plastic flows of steel during their heat treatment. The results thus obtained are summarized in Fig. 9.

References

(1) T. Hirone, Sci. Rep. Tohoku Univ., **26** (1937), 214. T. Hirone and N. Tsuya, Sci. Rep. RI TU, A **1** (1949), 111. For a complete list of literatures, readers should refer to the reports above cited.