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Anisotropy of flux-line lattice in cubic superconductors

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The magnetic field due to a single vortex in anisotropic type-II superconductors with cubic crystal symmetry is calculated at $T = 0\,°\mathrm{K}$ by means of the boson method. The results for the magnetic field are used in our analysis of stable orientation of the flux-line lattice with respect to the crystal lattice. The theoretical results agree well with the stable orientation of the flux-line lattice observed in weak-field region in Nb and Pb-Tl at low temperature. The boson characteristic function for cubic superconductors is also presented.

I. INTRODUCTION

Many type-II superconductors in the mixed state have anisotropic properties which are caused by the microscopic anisotropies of the superconducting electron system.[1] In this paper we present a theoretical analysis of low-κ type-II superconductors of cubic symmetry. It is well known that the flux lines form hexagonal two-dimensional lattices in the mixed state of isotropic type-II superconductors. However, the experiments with the decoration technique[2,3] and neutron diffraction[4–7] show that the flux-line lattice (FLL) correlates with the crystal lattice (CL) (see Fig. 1). Such correlations indicate that the interaction among vortices depends on their orientations. In the case of low-κ type-II superconductors, the interaction among vortices has an attractive part which causes the first-order transition at $H = H_{c1}$. The anisotropy of this attractive interaction gives rise to a variety of anisotropic forms of FLL structure in the weak-field region.

The vortex interaction energy, in the low-flux density limit, is given by

$$\epsilon_{\mathrm{int}} = \frac{\phi}{8\pi} \sum_{i \neq j} h(\vec{r}_{ij}) \quad .$$

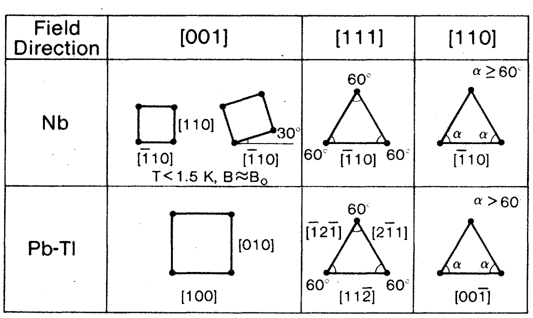

FIG. 1. Structures of the vortex lattices observed in Nb and Pb-Tl.

Here ϕ is the unit flux, and $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ where $\vec{r}_i$ means the position of the ith vortex. The function h is defined as follows: $h(\vec{r})$ means the magnetic field at $\vec{r}$ created by a single vortex at origin. Note that the vortex interaction energy is proportional to the magnetic field h and, therefore, that a field reversal creates an attractive interaction. Fisher and Teichler[8] estimated ϵ_{int} using the asymptotic form of $h(\vec{r})$ for $r \to \infty$. The FLL structure in the weak-field region is, however, affected mostly by the region where $h(\vec{r})$ takes its minimum value. Therefore, a precise knowledge of $h(\vec{r})$ in that region is needed when we want to understand the stable structure of the FLL in the low-κ anisotropic superconductors.

A detailed calculation of the magnetic field due to a single flux in isotropic superconductors was given in Ref. 9, in which use was made of the boson method. In this paper we calculate the magnetic field $h(\vec{r})$ for the cubic superconductors with an anisotropic Fermi surface, and discuss the preferential orientations of the FLL. Since various kinds of FLL structure appear only at low temperatures, we assume $T = 0\,°\mathrm{K}$ in the present paper. Our results show that the preferential orientations of the FLL agree with experiments for Nb and Pb-Tl.

In order to discuss the stability of the basic cell structure of the FLL, we need an accurate calculation of the free energy in the mixed state. Takanaka,[10] and Roger, Kahn, and Delrieu[11] calculated the mixed state energy in the high-field region ($H \sim H_{c2}$), extending the Ginzburg-Landau (GL) theory to the case of cubic superconductors with anisotropic Fermi surface. Their results, except the ones for $\vec{H} \parallel [001]$, agree with experiments for Nb. In a forthcoming paper we will present a calculation of the mixed state energy in the entire range of the magnetic field and temperature, using the boson method.

The boson theory of isotropic superconductivity is given in Refs. 12 and 13. The analysis of the mixed state of isotropic type-II superconductors by means of this theory was presented in Ref. 14. The first formulation of the boson theory for the anisotropic superconductors was presented in Refs. 15 and 16 in which we studied the mixed state of A-15 structure superconductors. There, we considered the case in which the superconducting electrons have anisotropic effective mass. The general form of the boson theory, for anisotropic superconductors was given in Ref. 17.

The macroscopic equations for the static case are

$$V_{ij}(-i\partial)\partial_i\partial_j f(\vec{x})=0 \ , \tag{1.1}$$

$$(\nabla^2\delta_{ij}-\partial_i\partial_j)A_j(\vec{x})=\frac{1}{\lambda_L^2}\int d^3y\, c(\vec{x}-\vec{y})V_{ij}(-i\partial)\times\left[A_j(\vec{y})-\frac{\hbar c}{e}\partial_j f(\vec{y})\right] , \tag{1.2}$$

$$J_i(\vec{x})=-\frac{1}{\lambda_L^2}\int d^3y c(\vec{x}-\vec{y})V_{ij}(-i\partial)\times\left[A_j(\vec{y})-\frac{\hbar c}{e}\partial_j f(\vec{y})\right] , \tag{1.3}$$

where λ_L is the London penetration depth, and $A_i(\vec{x})$ and $J_i(\vec{x})$ are the ith components of the vector potential and electric current, respectively. In Eqs. (1.1)–(1.3), use was made of the notation $a_ib_i \equiv \sum_{i=1}^{3} a_ib_i$. A superconducting state is characterized by the function $f(\vec{x})$ which satisfies Eq. (1.1). Note that $f(x)$ is equal to half the phase of the order parameter. In the above equations $c(\vec{x})$ is a function normalized by the condition

$$\int d^3x\, c(\vec{x})=1 \tag{1.4}$$

and has been called the boson characteristic function or c function. The symbol $V_{ij}(-i\partial)$ denotes a derivative tensor operator and is δ_{ij} for isotropic case. The c function for isotropic superconductor is given in Refs. 18–20. Note that $c(\vec{x})$ and $V_{ij}(-i\partial)$ become anisotropic when the superconducting electron system has an anisotropic property.

In Sec. II we solve the Eqs. (1.1) and (1.2) and calculate the magnetic field $h(\vec{r})$ due to a single vortex. Making use of this result for $h(\vec{r})$ we will discuss the anisotropic arrangements of FLL in the weak-field region. The calculation of $h(\vec{r})$ requires the knowledge of $c(\vec{x})$ and $V_{ij}(-i\vec{\nabla})$. In Sec. III, we calculate these quantities for cubic superconductors with anisotropic Fermi surface.

II. FLUX-LINE LATTICE STRUCTURE

In this section we calculate the magnetic field due to a single vortex and discuss the preferential orientations of the FLL. To calculate the magnetic field by means of the macroscopic equations (1.1) and (1.2), we need the knowledge of c function and tensor V_{ij}. But the evaluation of c function and V_{ij} for the anisotropic superconductors is a very tedious task. The derivation of the macroscopic equations and the calculations of the c function and V_{ij} for the cubic superconductors with a Fermi-surface anisotropy will be summarized in Sec. III. In this section we make use of this knowledge of the c function and V_{ij} and calculate the magnetic field.

Let us first solve Eq. (1.1) for the single vortex state. According to Eq. (1.2), we need only the knowledge of $\vec{\nabla}f(\vec{x})$, instead of $f(\vec{x})$ itself, in order to obtain the vector potential $\vec{A}$ or the magnetic field. In the following we shall consider the case in which the vortex lies on the $(1\bar{1}0)$ plane. Calculation of $\vec{\nabla}f(\vec{x})$ for this case is given in Appendix A, where we use the coordinate system in which the third axis is directed along the vortex line and the first and second axes are situated symmetrically with respect to the $(1\bar{1}0)$ plane (see Fig. 2). The results are

$$\partial_1 f(\vec{x})=\int\frac{d^2q}{(2\pi)^2}F_1(\vec{q})\exp(i\vec{q}\cdot\vec{x}) \ , \tag{2.1}$$

$$\partial_2 f(\vec{x})=\int\frac{d^2q}{(2\pi)^2}F_2(\vec{q})\exp(i\vec{q}\cdot\vec{x}) \ , \tag{2.2}$$

$$\partial_3 f(x)=0 \ , \tag{2.3}$$

where

$$F_1(\vec{q})=\pi[V_{21}(\vec{q},\theta)q_1+V_{22}(\vec{q},\theta)q_2]/q^2 \ , \tag{2.4}$$

$$F_2(\vec{q})=-\pi[V_{11}(\vec{q},\theta)q_1+V_{12}(\vec{q},\theta)q_2]/q^2 \ , \tag{2.5}$$

and

$$\vec{q}=(q_1,q_2) \ , \quad \vec{x}=(x_1,x_2) \ . \tag{2.6}$$

In Eqs. (2.4) and (2.5), the tensor $V_{ij}(\vec{q},\theta)$ is related

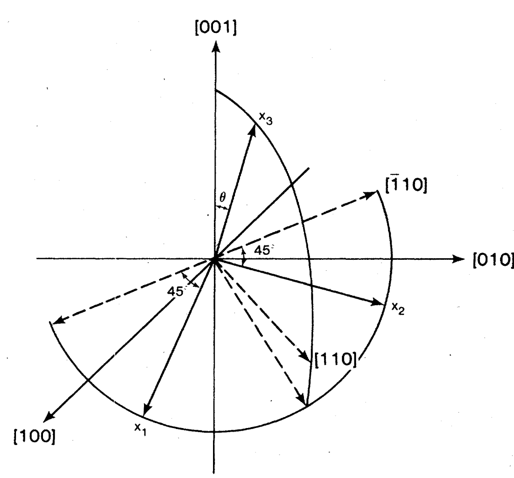

FIG. 2. Coordinate system used in calculation of the magnetic field. The vortex is directed along the x_3 axis.

to the one in the coordinate system given by the principal axes of the crystal as

$$V_{ij}(\vec{q},\theta)=u_{il}(\theta)\,V_{lm}\,u_{mj}^{-1}(\theta)\quad, \tag{2.7}$$

where

$$u_{ij}(\theta)=\begin{pmatrix} \frac{1}{2}(1+\cos\theta) & -\frac{1}{2}(1-\cos\theta) & -\frac{1}{2}(2)^{1/2}\sin\theta \\ -\frac{1}{2}(1-\cos\theta) & \frac{1}{2}(1+\cos\theta) & -\frac{1}{2}(2)^{1/2}\sin\theta \\ \frac{1}{2}(2)^{1/2}\sin\theta & \frac{1}{2}(2)^{1/2}\sin\theta & \cos\theta \end{pmatrix} . \tag{2.8}$$

Here θ is the angle between the vortex and [001] axis.

Let us now solve the macroscopic equation (1.2) for the vector potential. We see from Eqs. (2.1)–(2.3) that the solution of the macroscopic equation (1.2) should not depend on the variable x_3. Then by introducing the Fourier transforms of $\vec{A}(\vec{x})$ and $c(\vec{x})$ as

$$\vec{A}(\vec{x})=\int\frac{d^2k}{(2\pi)^2}\vec{A}(\vec{k})\exp(i\vec{k}\cdot\vec{x})\quad, \tag{2.9}$$

$$\bar{c}(\vec{x})=\int dx_3\,c(\vec{r})=\int\frac{d^2k}{(2\pi)^2}c(\vec{k})\exp(i\vec{k}\cdot\vec{x})\quad, \tag{2.10}$$

we can rewrite the macroscopic equation (1.2)

$$[-k^2A_i(\vec{k})+k_ik_jA_j(\vec{k})]=\frac{1}{\lambda_L^2}c(\vec{k})\,V_{ij}(\vec{k},\theta)\left(A_j(\vec{k})-\frac{\phi}{\pi}F_j(\vec{k})\right)\quad, \tag{2.11}$$

i.e.,

$$\begin{gathered}\left(k_2^2+\frac{c(\vec{k})V_{11}^{\theta}}{\lambda_L^2}\right)A_1(\vec{k})-\left(k_1k_2-\frac{c(\vec{k})V_{12}^{\theta}}{\lambda_L^2}\right)A_2(\vec{k})+\frac{c(\vec{k})V_{13}^{\theta}}{\lambda_L^2A_3(\vec{k})}=\frac{\phi c(\vec{k})(V_{11}^{\theta}F_1+V_{12}^{\theta}F_2)}{\lambda_L^2}\quad,\\ -\left(k_2k_1-\frac{c(\vec{k})V_{21}^{\theta}}{\lambda_L^2}\right)A_1(\vec{k})+\left(k_1^2+\frac{c(\vec{k})V_{22}^{\theta}}{\lambda_L^2}\right)A_2(\vec{k})+\frac{c(\vec{k})V_{23}^{\theta}}{\lambda_L^2A_3(\vec{k})}=\frac{\phi c(\vec{k})(V_{21}^{\theta}F_1+V_{22}^{\theta}F_2)}{\lambda_L^2}\quad,\\ \frac{c(\vec{k})V_{31}^{\theta}}{\lambda_L^2A_1(\vec{k})}+\frac{c(\vec{k})V_{32}^{\theta}}{\lambda_L^2A_2(\vec{k})}+\left(k^2+\frac{c(\vec{k})V_{33}^{\theta}}{\lambda_L^2}\right)A_3(\vec{k})=\frac{\phi c(\vec{k})(V_{31}^{\theta}F_1+V_{22}^{\theta}F_2)}{\lambda_L^2}\quad.\end{gathered} \tag{2.12}$$

Here V_{ij}^{θ} stands for $V_{ij}(\vec{k},\theta)$.

We now have

$$A_1(\vec{k})=\frac{\phi c(\vec{k})}{\lambda_L^2}\frac{\det\Gamma_1(\vec{k})}{\det\Gamma_0(\vec{k})}\;,\quad A_2(\vec{k})=\frac{\phi c(\vec{k})}{\lambda_L^2}\frac{\det\Gamma_2(\vec{k})}{\det\Gamma_0(\vec{k})}\;,\quad A_3(\vec{k})=\frac{\phi c(\vec{k})}{\lambda_L^2}\frac{\det\Gamma_3(\vec{k})}{\det\Gamma_0(\vec{k})}\quad, \tag{2.13}$$

where

$$\Gamma_0(\vec{k})=\begin{pmatrix} k_2^2+c(\vec{k})V_{11}^{\theta}/\lambda_L^2 & -k_1k_2+c(\vec{k})V_{12}^{\theta}/\lambda_L^2 & c(\vec{k})V_{13}^{\theta}/\lambda_L^2 \\ -k_2k_1+c(\vec{k})V_{21}^{\theta}/\lambda_L^2 & k_1^2+c(\vec{k})V_{22}^{\theta}/\lambda_L^2 & c(\vec{k})V_{23}^{\theta}/\lambda_L^2 \\ c(\vec{k})V_{31}^{\theta}/\lambda_L^2 & c(\vec{k})V_{32}^{\theta}/\lambda_L^2 & k^2+c(\vec{k})V_{33}^{\theta}/\lambda_L^2 \end{pmatrix}\quad, \tag{2.14}$$

$$\Gamma_1(\vec{k})=\begin{pmatrix} V_{11}^{\theta}F_1+V_{12}^{\theta}F_2 & -k_1k_2+c(\vec{k})V_{12}^{\theta}/\lambda_L^2 & c(\vec{k})V_{13}^{\theta}/\lambda_L^2 \\ V_{21}^{\theta}F_1+V_{22}^{\theta}F_2 & k_1^2+c(\vec{k})V_{22}^{\theta}/\lambda_L^2 & c(\vec{k})V_{23}^{\theta}/\lambda_L^2 \\ V_{31}^{\theta}F_1+V_{32}^{\theta}F_2 & c(\vec{k})V_{32}^{\theta}/\lambda_L^2 & k^2+c(\vec{k})V_{33}^{\theta}/\lambda_L^2 \end{pmatrix}\quad, \tag{2.15}$$

$$\Gamma_2(\vec{k})=\begin{pmatrix} k_2^2+c(\vec{k})V_{11}^{\theta}/\lambda_L^2 & V_{11}^{\theta}F_1+V_{12}^{\theta}F_2 & c(\vec{k})V_{13}^{\theta}/\lambda_L^2 \\ -k_2k_1+c(\vec{k})V_{21}^{\theta}/\lambda_L^2 & V_{21}^{\theta}F_1+V_{22}^{\theta}F_2 & c(\vec{k})V_{23}^{\theta}/\lambda_L^2 \\ c(\vec{k})V_{31}^{\theta}/\lambda_L^2 & V_{31}^{\theta}F_1+V_{32}^{\theta}F_2 & k^2c(\vec{k})V_{33}^{\theta}/\lambda_L^2 \end{pmatrix}\quad, \tag{2.16}$$

$$\Gamma_3(\vec{k})=\begin{pmatrix} k_2^2+c(\vec{k})V_{11}^{\theta}/\lambda_L^2 & -k_1k_2+c(\vec{k})V_{12}^{\theta}/\lambda_L^2 & V_{11}^{\theta}F_1+V_{12}^{\theta}F_2 \\ -k_2k_1+c(\vec{k})V_{21}^{\theta}/\lambda_L^2 & k_1^2+c(\vec{k})V_{22}^{\theta}/\lambda_L^2 & V_{21}^{\theta}F_1+V_{22}^{\theta}F_2 \\ c(\vec{k})V_{31}^{\theta}/\lambda_L^2 & c(\vec{k})V_{32}^{\theta}/\lambda_L^2 & V_{31}^{\theta}F_1+V_{32}^{\theta}F_2 \end{pmatrix}\quad. \tag{2.17}$$

The magnetic field $\vec{h}(\vec{x})$ due to a single vortex can be obtained by feeding the vector potential (2.13) into the relation $\vec{h}(\vec{x}) = \vec{\nabla} \times \vec{A}(\vec{x})$, i.e.,

$$h_1(\vec{x}) = i \int \frac{d^2k}{(2\pi)^2} k_2 A_3(\vec{k}) \exp(i\vec{k}\cdot\vec{x}) \ , \tag{2.18}$$

$$h_2(\vec{x}) = -i \int \frac{d^2k}{(2\pi)^2} k_1 A_3(\vec{k}) \exp(i\vec{k}\cdot\vec{x}) \ , \tag{2.19}$$

$$h_3(\vec{x}) = i \int \frac{d^2k}{(2\pi)^2} [k_1 A_2(\vec{k}) - k_2 A_1(\vec{k})] \exp(i\vec{k}\cdot\vec{x}) \ , \tag{2.20}$$

or

$$h_1(r,\chi) = i \left[\frac{\phi}{\lambda_L^2}\right] \int_0^\infty \frac{dk}{(2\pi)^2} k^2 \int_0^{2\pi} d\phi \sin\phi\, c(k,\phi) \frac{\det\Gamma_3(k,\phi)}{\det\Gamma_0(k,\phi)} \exp[ikr\cos(\chi-\phi)] \ , \tag{2.21}$$

$$h_2(r,\chi) = -i \left[\frac{\phi}{\lambda_L^2}\right] \int_0^\infty \frac{dk}{(2\pi)^2} k^2 \int_0^{2\pi} d\phi \cos\phi\, c(k,\phi) \frac{\det\Gamma_3(k,\phi)}{\det\Gamma_0(k,\phi)} \exp[ikr\cos(\chi-\phi)] \ , \tag{2.22}$$

$$h_3(r,\chi) = i \left[\frac{\phi}{\lambda_L^2}\right] \int_0^\infty \frac{d^2k}{(2\pi)^2} k^2 \int_0^{2\pi} d\phi \frac{c(k,\phi)}{\det\Gamma_0(k,\phi)} [\cos\phi \det\Gamma_2(k,\phi) - \sin\phi \det\Gamma_1(\chi,\phi)] \exp[ikr\cos(\phi-\chi)] \ , \tag{2.23}$$

where (r,χ) and (k,ϕ) are defined through the relations,

$$\begin{cases} x_1 = r\cos\chi \ , \\ x_2 = r\sin\chi \ , \end{cases} \quad \begin{cases} k_1 = k\cos\phi \ , \\ k_2 = k\sin\phi \ . \end{cases} \tag{2.24}$$

Recalling that the third axis in this coordinate system is directed along the vortex, we find that the nonvanishing components, h_1 and h_2, of the magnetic field $\vec{h}$ indicate that the microscopic magnetic field due to the vortex is not generally along the vortex direction.

In order to evaluate the integrals (2.21)–(2.23), we need c function and V_{ij} for the anisotropic superconductors. It will be shown in Sec. III that the Fourier amplitudes, $c(\vec{q})$ and $V_{ij}(\vec{q})$, for the cubic superconductors with Fermi-surface anisotropy and the isotropic BCS coupling depend, not only on $VN(0)$ and $q\xi_0$, but also on the parameters a_l $(l=4,6,8,\ldots)$ which measure the Fermi-surface anisotropy. Here V is the BCS coupling constant, $N(0)$ is the total density of states at the Fermi level, ξ_0 is the average coherence length at $T=0$ K defined as $\xi_0 = v_0/\pi\Delta$ where v_0 is the mean Fermi velocity. The parameter a_l is defined by

$$a_l = \int d^2k_F\, \rho(\vec{k}_F) H_l(\hat{v}_F) \ , \tag{2.25}$$

where $\rho(\vec{k}_F)$ is the direction-dependent density of states in direction of the Fermi momentum $\vec{k}_F$ and $H_l(\hat{v}_F)$ is the cubic harmonics of lth order. Here $\hat{v}_F$ denotes the direction cosines of the Fermi velocity $\vec{v}_F$. Taking into account only the two parameters a_4 and a_6 and assuming $VN(0)=0.32$, we obtain the following expressions for $c(\vec{q})$ and $V_{ij}(\vec{q})$ in the coordinate system given by the principal axes in the crystal:

(a) The Fourier amplitude of the c function

$$c(\vec{q}) = c_0(q) + c_4(q)H_4(\hat{q}) + c_6(q)H_6(\hat{q}) \ , \tag{2.26}$$

where

$$c_0(q) = \exp[-0.4228(q\xi_0)^{1.956}] - 0.3621 a_4^2 \{(q\xi_0)^{8.488}\exp[-0.9681(q\xi_0)^{2.312}] - 0.7458(q\xi_0)^{6.537}\exp[-0.4096(q\xi_0)^3]\} \ , \tag{2.27}$$

$$c_4(q) = 0.3535 a_4 (\exp\{-0.7631(q\xi_0)^{1.294}\tanh[0.4939(q\xi_0)^{0.6460}]\} - \exp\{-0.4228(q\xi_0)^{2.281}\tanh[0.3664(q\xi_0)^{1.243}]\}) \ , \tag{2.28}$$

$$c_6(q) = -0.5679 a_6 (\exp\{-0.4196(q\xi_0)^{1.720}\tanh[0.6569(q\xi_0)^{1.220}]\} - \exp\{-1.733(q\xi_0)^{1.012}\tanh[0.1168(q\xi_0)^{1.732}]\}) \ , \tag{2.29}$$

and $H_4(\hat{q})$ and $H_6(\hat{q})$ are defined as

$$H_4(\hat{q}) = (\tfrac{525}{16})^{1/2}(\hat{q}_1^2 + \hat{q}_2^2 + \hat{q}_3^2 - \tfrac{3}{5}) \ , \tag{2.30}$$

$$H_6(\hat{q}) = \tfrac{1}{8}(11 \times 21)(26)^{1/2}[\hat{q}_1^2\hat{q}_2^2\hat{q}_3^2 + \tfrac{1}{22}(\hat{q}_1^4 + \hat{q}_2^4 + \hat{q}_3^4 - \tfrac{3}{5}) - \tfrac{1}{105}] \ . \tag{2.31}$$

Here $\hat{q}$ denotes the direction cosines of $\vec{q}$.

(b) The function $V_{ij}(\vec{q})$

$$V_{11}(\vec{q}) = 1 + 5(21)^{1/2} a_4\gamma_4(q)\{\hat{q}_1^2 - \tfrac{3}{5} - [H_4(\hat{q})]\} + \tfrac{1}{4}[231(26)^{1/2}]a_6\gamma_y(q)\{\hat{q}_2^2\hat{q}_3^2 + \tfrac{1}{22}[H_4(\hat{q})] + \tfrac{1}{11}(\hat{q}_1^2 - \tfrac{3}{5}) - \tfrac{1}{35} - 3[H_6(\hat{q})]\} \ , \tag{2.32}$$

$$V_{22}(\vec{q}) = 1 + 5(21)^{1/2} a_4\gamma_4(q)\{\hat{q}_2^2 - \tfrac{3}{5} - [H_4(\hat{q})]\} + \tfrac{1}{4}[231(26)^{1/2}]a_6\gamma_6(q)\{\hat{q}_3^2\hat{q}_1^2 + \tfrac{1}{22}[H_4(\hat{q})] + \tfrac{1}{11}(\hat{q}_2^2 - \tfrac{3}{5}) - \tfrac{1}{35} - 3[H_6(\hat{q})]\} \ , \tag{2.33}$$

$$V_{33}(\vec{q}) = 1 + 5(21)^{1/2} a_4\gamma_4(q)\{\hat{q}_3^2 - \tfrac{3}{5} - [H_4(\hat{q})]\} + \tfrac{1}{4}[231(26)^{1/2}]a_6\gamma_6(q)\{\hat{q}_1^2\hat{q}_2^2 + \tfrac{1}{22}[H_4(\hat{q})] + \tfrac{1}{11}(\hat{q}_3^2 - \tfrac{3}{5}) - \tfrac{1}{35} - 3[H_6(\hat{q})]\} \ , \tag{2.34}$$

$$V_{ij}(\vec{q}) = 0 \ , \text{ for } i \neq j \ , \tag{2.35}$$

where

$$\gamma_4(q) = -\frac{1}{15}\frac{2\pi^2(q\xi_0)^2}{42 + \pi^2(q\xi_0)^2/VN(0)} \ , \tag{2.36}$$

$$\gamma_6(q) = \frac{1}{3465}\frac{\pi^2(q\xi_0)^4}{26 + \pi^2(q\xi_0)^2/66\,VN(0)} \ , \tag{2.37}$$

and

$$[H_4(\hat{q})] = \hat{q}_1^4 + \hat{q}_2^4 + \hat{q}_3^4 - \tfrac{3}{5} \ , \tag{2.38}$$

$$[H_6(\hat{q})] = \hat{q}_1^2\hat{q}_2^2\hat{q}_3^2 + \tfrac{1}{22}(\hat{q}_1^4 + \hat{q}_2^4 + \hat{q}_3^4 - \tfrac{3}{5}) - \tfrac{1}{105} \ . \tag{2.39}$$

We now summarize the results of numerical computations of the magnetic field in the cases in which the vortex is in [001], [111], and [110] directions, respectively. In these three cases, Eqs. (2.32)–(2.35) and (2.17) lead to

$$h_1(\vec{x}) = h_2(\vec{x}) = 0 \ . \tag{2.40}$$

In Figs. 3, 4, and 5 we present $h_3(\vec{x})$, the distribution of the magnetic field around the field reversal regions in the plane perpendicular to the vortex, for the different sets of the parameters (κ_0, a_4, a_6) where $\kappa_0 \equiv \lambda_L/\xi_0$. The angles given in these figures are defined in such a way that the $[1\bar{1}0]$ direction corresponds to 0°. We tried three choices of parameters (κ_0, a_4, a_6)—

choice (i):

$$\kappa_0 = 0.75, \quad a_4 = 0.2, \quad a_6 = -0.1 \ ,$$

choice (ii):

$$\kappa_0 = 0.8, \quad a_4 = 0.22, \quad a_6 = -0.08 \ ,$$

choice (iii):

$$\kappa_0 = 0.8, \quad a_4 = -0.2, \quad a_6 = 0.1 \ .$$

Note that choices (i) and (ii) are very similar to each other. The Figs. 3, 4, and 5 respectively correspond to the choices (i), (ii), and (iii). Comparing Fig. 3 with Fig. 4, we see that the magnetic field is quite sensitive to the choice of parameters.

Let us now consider the preferential orientations of the FLL in the weak-field region (the intermediate region), i.e., $B \lesssim B_0$. Here B is the magnetic induction and B_0 denotes the drop of the magnetization at $H = H_{c1}$. In the case of $\vec{h} \parallel [001]$, where the CL has fourfold symmetry, the FLL prefers a square lattice. According to Figs. 3(a) and 4(a), the magnetic field for the choices (i) and (ii) takes a minimum value when it is in the direction of $[1\bar{1}0]$ or [110]. Therefore, when we use the crudest approximation in which the lattice constant of the square lattice is equal to the distance where the magnetic field takes its minimum value, the nearest-neighbor direction in the cases of choices (i) and (ii) are parallel to [110] and $[1\bar{1}0]$. This is the situation which corresponds to the first figure for the (Nb, [001]) case in Fig. 1. A similar consideration shows that the choice (iii) leads to the square lattice with the nearest-neighbor direction parallel to [100] and [010]; this situation is similar to the (Pb-Tl, [001]) case in Fig. 1. However, these results can easily be modified when we take into account also the next-nearest-neighbor interactions. For example, Fig. 4(a) shows that, in the case of the choice (ii), the magnetic field at the next nearest neighbor is highest (i.e., least attractive) when the nearest neighbor is parallel to [110] and $[1\bar{1}0]$. We thus expect that the nearest-neighbor direction may somewhat deviate from [110] and $[1\bar{1}0]$. Situation is different in the case of the choice (i). In this case Fig. 3(a) shows that the magnetic field at the next nearest neighbor is lowest when the nearest-neighbor direction is parallel to [110] or $[1\bar{1}0]$. Moreover, an approximate estimation of the

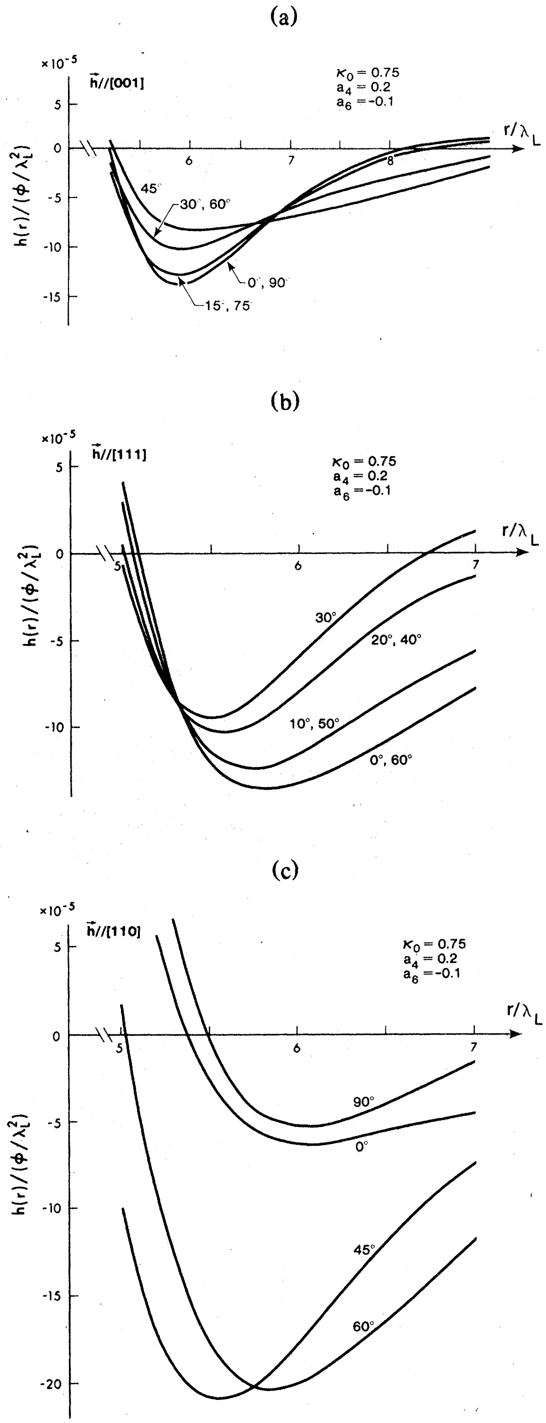

FIG. 3. Magnetic field distribution in the plane perpendicular to the single vortex line, (a) $\vec{h} \parallel [001]$, (b) $\vec{h} \parallel [111]$, and (c) $\vec{h} \parallel [110]$, for the parameters ($\kappa_0 = 0.75$, $a_4 = 0.2$, and $a_6 = -0.1$). The angles are measured from the $[1\bar{1}0]$ direction.

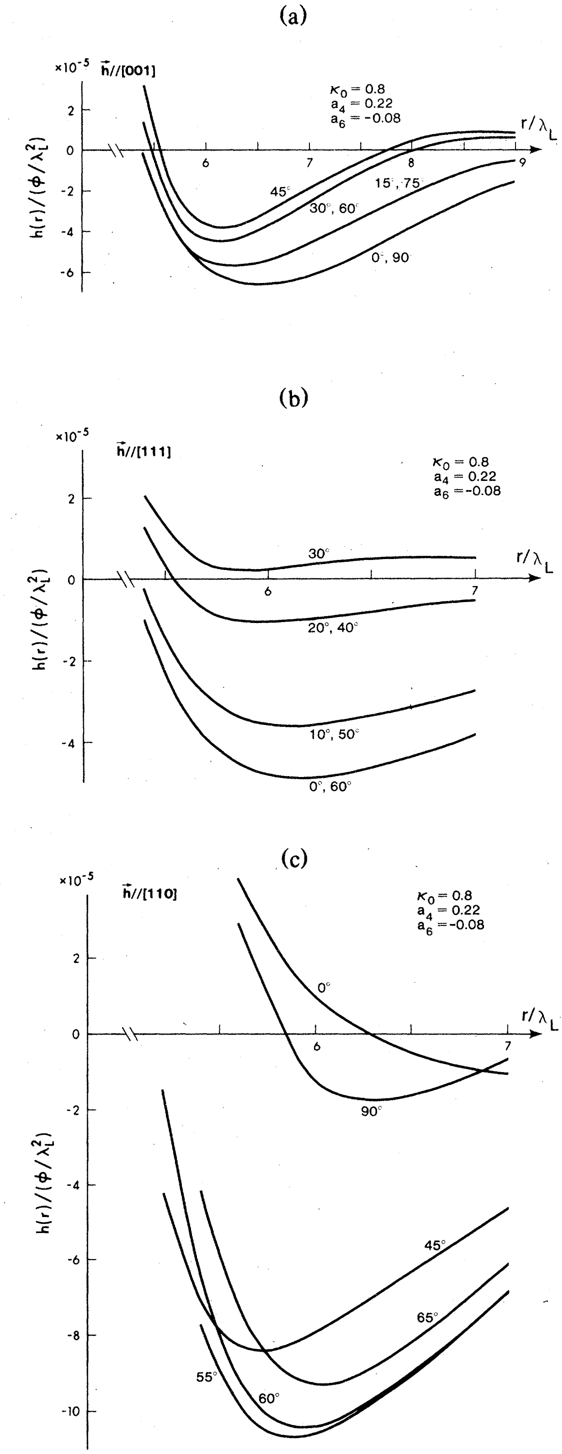

FIG. 4. Magnetic field distribution in the plane perpendicular to the single vortex line, (a) $\vec{h} \parallel [001]$, (b) $\vec{h} \parallel [111]$, and (c) $\vec{h} \parallel [110]$, for the parameters ($\kappa_0 = 0.8$, $a_4 = 0.2$, $a_6 = -0.1$). The angles are measured from the $[1\bar{1}0]$ direction.

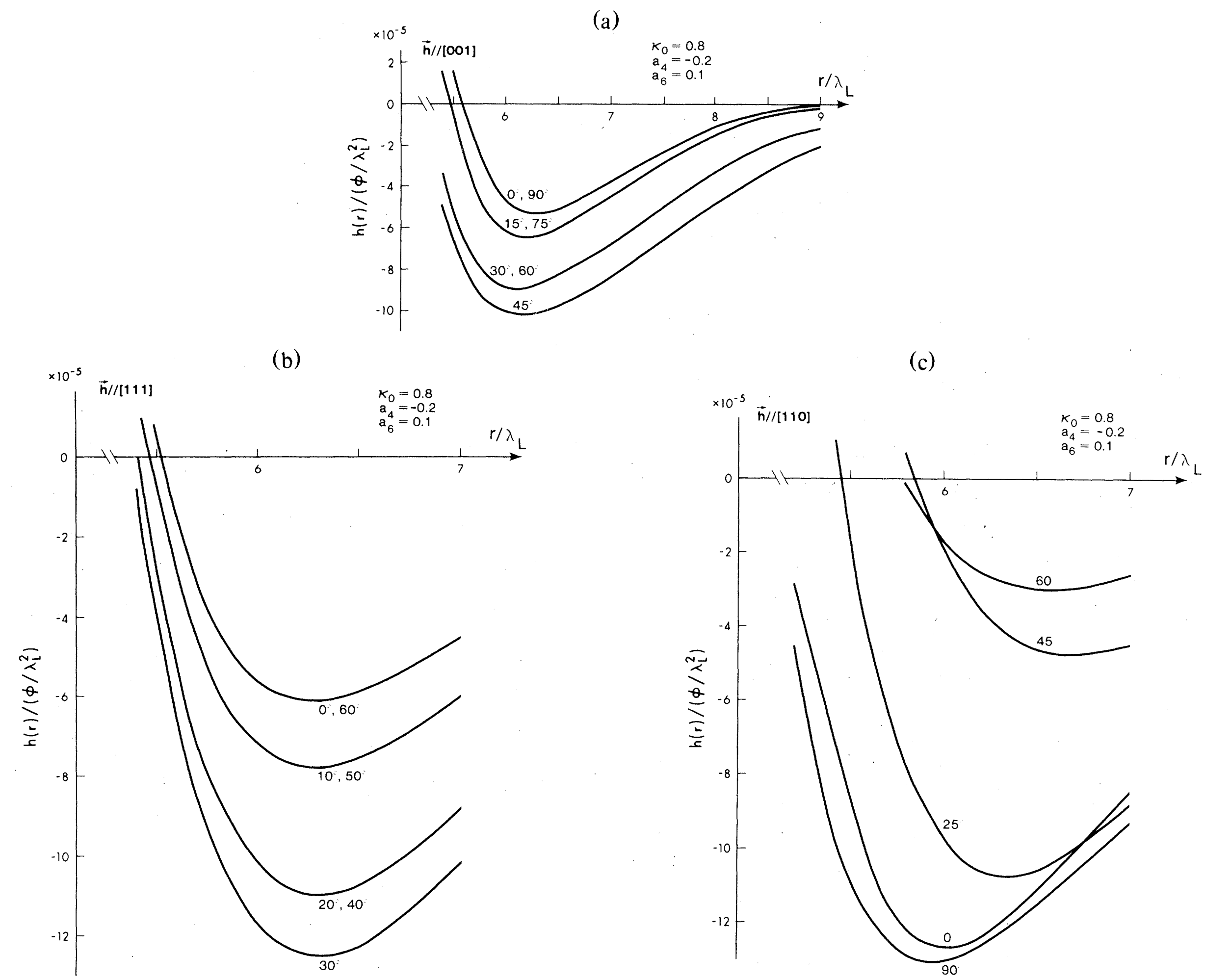

FIG. 5. Magnetic fields in the plane perpendicular to the single vortex line, (a) $\vec{h} \parallel [001]$, (b) $\vec{h} \parallel [111]$, and (c) $\vec{h} \parallel [110]$, for the parameters ($\kappa_0=0.8$, $a_4=-0.2$, $a_6=0.1$). The angles are measured from the $[1\bar{1}0]$ direction.

vortex-lattice interaction energy in these cases shows that the energy difference caused by different orientations of the square lattice is very small when the next-nearest-neighbor interaction is taken into account. We may thus expect the appearance of the domains with different orientations of the square lattice. This situation is similar to the one observed[4] experimentally in Nb, where the two FLL's are tilted with respect to one another by the angle of 30°.

Let us now turn our attention to the case $\vec{h} \parallel [111]$. When we use the choice (i) or (ii) for the parameters, Figs. 3(b) and 4(b) show that the magnetic field takes the minimum value in the direction $[1\bar{1}0]$ and also in the directions tilted by angles of 60° and 120° with respect to $[1\bar{1}0]$ direction. This agrees with the experiments of Nb. In the case of the choice (iii), the minimum of the magnetic field appears in the $[11\bar{2}]$ direction and those tilted by angles of 60° and 120° with respect to $[11\bar{2}]$ direction. This agrees with the experiment of Pb-Tl. The stable FLL is a hexagonal lattice in which the nearest neighbors are located at the minimum positions of the magnetic field. This is consistent with the fact that the crystal has the threefold symmetry.

In the case of $\vec{h} \parallel [110]$, the situation is quite complicated. According to the Figs. 3(c), 4(c), and 5(c), the directions, in which the magnetic field takes minimum value, in general, do not agree with any crystal symmetry direction. However when we consider the reflection symmetries with respect to the $(1\bar{1}0)$ and (110) planes, we find that the basic cell of the FLL forms an isosceles triangle, whose base is either in the $[\bar{1}10]$ direction or in the $[00\bar{1}]$ direction, depending on the choice of parameters (κ_0, a_4, a_6).

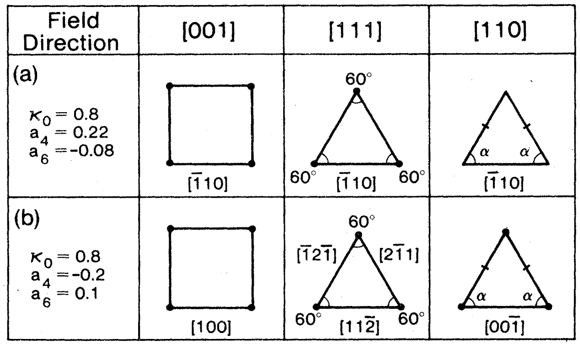

FIG. 6. Structures of the vortex lattices predicted by the calculated magnetic field for the two sets of parameters ($\kappa_0=0.8$, $a_4=0.22$, $a_6=-0.08$) and ($\kappa_0=0.8$, $a_4=-0.2$, $a_6=0.1$).

The base angle α also depends on these parameters. For example, when the choice (ii) for the parameters [see Fig. 4(c)] is used, the base of the triangle is in the direction [$\bar{1}$10], agreeing with the experimental observations in Nb (see Fig. 1). On the other hand, in the case of the choice (iii) [see Fig. 5(c)], the base of the triangle is in the direction of [00$\bar{1}$]. This is the situation which has been observed in the case of Pb-Tl (see Fig. 1). The observed base angle of the isosceles triangle in Nb is a little larger than 60°, while our estimation of the vortex lattice energy shows that the choice (ii) for the parameters leads to the base angle somewhat smaller than 60°. We feel that this minor discrepancy may diminish when we take into account the temperature effect.

Our theoretical results for the basic cell of FLL discussed in this section is summarized in Fig. 6. Comparing this with Fig. 1, we find a reasonable agreement between the theoretical and experimental results when we use the choice (ii) for Nb and choice (iii) for Pb-Tl.

Recently, we calculated H_{c1} and H_{c2} of Nb by means of the choice (ii) (i.e., $\kappa_0=0.8$, $a_4=0.22$, $a_6=-0.08$). The results well agree with experiments. A detailed account of the study of anisotropic behavior of H_{c1} and H_{c2} at various temperatures will be published in a forthcoming paper.

III. DERIVATION OF THE MACROSCOPIC EQUATIONS FOR ANISOTROPIC SUPERCONDUCTORS WITH CUBIC SYMMETRY

In this section we derive the macroscopic equations (1.1)–(1.3) for the cubic superconductors with anisotropic Fermi surface.

Let us consider the BCS Hamiltonian,

$$H_{\mathrm{BCS}} = \hbar \int d^3x\, [\psi_\uparrow^\dagger \epsilon(-i\partial)\psi_\uparrow + \psi_\downarrow^\dagger \epsilon(-i\partial)\psi_\downarrow - V\psi_\uparrow^\dagger\psi_\downarrow^\dagger\psi_\downarrow\psi_\uparrow - \mu(\psi_\uparrow^\dagger\psi_\uparrow + \psi_\downarrow^\dagger\psi_\downarrow)] \ , \tag{3.1}$$

where $\psi_{\uparrow,\downarrow}$ are the Heisenberg fields for the electrons, μ is the chemical potential, and V is the coupling constant ($V>0$) among electrons. It was assumed that the superconducting interaction is an isotropic contact interaction. In Eq. (3.1) $\epsilon(-i\partial)$ is the derivative operator defined by

$$\epsilon(-i\partial)\exp(i\vec{k}\cdot\vec{x}) = \epsilon(\vec{k})\exp(i\vec{k}\cdot\vec{x}) \ , \tag{3.2}$$

where $\epsilon(\vec{k})$ is the one electron energy measured from the Fermi level. It has a cubic symmetry.

In the boson formulation the superconducting state is described in terms of the free fields $\phi_{\uparrow,\downarrow}(x)$ and $B(x)$, where $\phi_{\uparrow,\downarrow}(x)$ are the quasielectron fields and $B(x)$ is the boson field which is the collective mode. These fields can be written in terms of creation and annihilation operators

$$\phi(x) = \int \frac{d^3k}{(2\pi)^{3/2}}[\alpha_{k\uparrow}u_k \exp(i\vec{k}\cdot\vec{x} - iE_k t) + \alpha_{k\downarrow}^\dagger v_k \exp(-i\vec{k}\cdot\vec{x} + iE_k t)] \ , \tag{3.3}$$

$$B(x) = \int \frac{d^3q}{(2\pi)^{3/2}}[B_q \exp(i\vec{q}\cdot\vec{x} - i\omega_q t) + B_q^\dagger \exp(-i\vec{q}\cdot\vec{x} + i\omega_q t)]/(2\omega_q)^{1/2} \ , \tag{3.4}$$

where $\phi(x)$ is the doublet field for the quasielectrons

$$\phi(x) = \begin{pmatrix} \phi_\uparrow(x) \\ \phi_\downarrow^\dagger(x) \end{pmatrix} \ . \tag{3.5}$$

The creation and annihilation operators $\alpha_{k\uparrow,\downarrow}$, $\alpha_{k\uparrow,\downarrow}^\dagger$ and B_q, $B_q^\dagger$ satisfy the fermionlike and bosonlike commutation relations, respectively. We use the Fock space of these quasiparticles. Then any physical operator is expressed in terms of a linear combination of normal products of these free fields.

The quasielectron field $\phi(x)$ satisfies the free-field equation,

$$\left[\epsilon(-i\partial)\tau_3 + \frac{1}{i}\frac{\partial}{\partial t} - \Delta\tau_1\right]\phi(x) = 0 \ , \tag{3.6}$$

where the τ matrices are the Pauli matrices. This equation can be obtained by imposing the condition $\Delta = -V\langle 0|\psi_\uparrow\psi_\downarrow|0\rangle \neq 0$ on the Heisenberg equation for the electron fields. Equation (3.6) determines u_k, v_k, and E_k as

$$u_k=\begin{pmatrix}\cos\theta_k\\ \sin\theta_k\end{pmatrix}, \quad v_k=\begin{pmatrix}-\sin\theta_k\\ \cos\theta_k\end{pmatrix}, \tag{3.7}$$

$$E_k=[\epsilon^2(\vec{k})+\Delta^2]^{1/2} . \tag{3.8}$$

In Eq. (3.7) the parameter θ_k is defined through the relations

$$\cos 2\theta_k=\epsilon(\vec{k})/E_k , \quad \sin 2\theta_k=-\Delta/E_k , \tag{3.9}$$

where the energy gap Δ is obtained from the gap equation

$$1=\frac{1}{2}V\int\frac{d^3k}{(2\pi)^3}\frac{1}{E_k} . \tag{3.10}$$

Next, we expand bilinear products of the Heisenberg fields ψ in terms of the free particles;

$$\begin{aligned}T[\psi(x)\psi^\dagger(y)]=\chi(x-y)&+\int d^3q\int d^3p\, F^{(1)}(p,q,x,y)\alpha_{q\uparrow}\alpha_{p\downarrow}+\int d^3q\int d^3p\, F^{(2)*}(p,q,x,y)\alpha_{q\downarrow}^\dagger\alpha_{p\uparrow}\\ &+\int d^3q\int d^3p\, F^{(3)}(p,q,x,y)\alpha_{q\uparrow}^\dagger\alpha_{p\uparrow}+\int d^3q\int d^3p\, F^{(4)}(p,q,x,y)\alpha_{q\downarrow}^\dagger\alpha_{p\downarrow}\\ &+\int d^3l\, G^{(1)}(l,x,y)B_l+\int d^3l\, G^{(2)*}(l,x,y)B_l^\dagger+\cdots .\end{aligned} \tag{3.11}$$

Here T is the chronological operator. In this expansion the conservation of spin of quasielectrons is assumed. The coefficients in the right-hand side of Eq. (3.11) are defined through the following relations:

$$\chi(x-y)\equiv\langle 0|T[\psi(x)\psi^\dagger(y)]|0\rangle , \tag{3.12}$$

$$F^{(1)}(p,q,x,y)\equiv\langle 0|T[\psi(x)\psi^\dagger(y)]|\alpha_{q\uparrow}\alpha_{p\downarrow}\rangle , \tag{3.13}$$

$$G(x,y;\vec{l})\equiv\langle 0|T[\psi(x)\psi^\dagger(y)]|B_l\rangle , \quad \text{etc.} \tag{3.14}$$

These quantities are called the Bethe-Salpeter (BS) amplitudes. The equations for these quantities can be obtained from the Heisenberg equations. When the pair approximation is used, the equation for the BS amplitude (3.14) is the following homogeneous integral equation[12]:

$$G(x,y;\vec{q})=i\int d^4z\, S(x-z)\tilde{G}(z)S(z-y) . \tag{3.15}$$

Here

$$S(x-y)\equiv\langle 0|T[\phi(x)\phi^\dagger(y)]|0\rangle , \tag{3.16}$$

which is the Green's function of Eq. (3.6), and

$$\tilde{G}(x)\equiv\begin{pmatrix}-\lambda G_{22}(x,x;\vec{q}) & \lambda G_{12}(x,x;\vec{q})\\ \lambda G_{21}(x,x;\vec{q}) & -\lambda G_{11}(x,x;\vec{q})\end{pmatrix} . \tag{3.17}$$

By using the Fourier representation,

$$G(x,x)=G(\vec{q})\exp(i\vec{q}\cdot\vec{x}-i\omega_q t) , \tag{3.18}$$

$$S(x)=i\int\frac{d^3k\,dE}{(2\pi)^4}S(\vec{k})\exp(i\vec{k}\cdot\vec{x}-ikt) , \tag{3.19}$$

we can rewrite Eq. (3.15) with $x=y$ as follows:

$$G(\vec{q})=i\int\frac{d^3k\,dE}{(2\pi)^4}S(\vec{k},E)\tilde{G}(\vec{q})S(\vec{k}-\vec{q},E-\omega_q) . \tag{3.20}$$

From Eq. (3.20) follow the relations between the BS amplitude and the frequency ω_q of the collective mode[18];

$$G^{(1)}(\vec{q})=G^{(2)}(\vec{q})=[\Delta\omega_q/(\omega_q^2-\bar{\omega}_q^2)]g(\vec{q}) , \tag{3.21}$$

$$\begin{aligned}\omega_q^2=\bar{\omega}_q^2&+Q^{12}(\vec{q},\omega_q)(\omega_q^2-\bar{\omega}_q^2)\\ &+2\Delta^2R(\vec{q},\omega_q)(\omega_q^2+\bar{\omega}_q^2) ,\end{aligned} \tag{3.22}$$

where

$$\begin{aligned}&G^{(1)}(\vec{q})\equiv G_{12}(\vec{q}) , \quad G^{(2)}(\vec{q})\equiv -G_{21}(\vec{q}) ,\\ &g(\vec{q})\equiv G_{22}(\vec{q})-G_{11}(\vec{q}) .\end{aligned} \tag{3.23}$$

The quantities $R(\vec{q},\omega_q)$, $\bar{\omega}_q^2$, and $Q^{12}(\vec{q},\omega_q)$ are defined by

$$R(\vec{q},E)=\frac{1}{4}\int\frac{d^3k}{(2\pi)^3}(E_++E_-)f(\vec{k},\vec{q},E) , \tag{3.24}$$

$$\begin{aligned}\bar{\omega}_q^2(\vec{q},E)=\frac{1}{4R(\vec{q},E)}&\int\frac{d^3k}{(2\pi)^3}(\epsilon_+-\epsilon_-)^2\\ &\times(E_++E_-)f(\vec{k},\vec{q},E) ,\end{aligned} \tag{3.25}$$

$$\begin{aligned}Q^{12}(\vec{q},E)=-\frac{1}{2}&\int\frac{d^3k}{(2\pi)^3}(E_+E_--\epsilon_+\epsilon_-)\\ &\times(E_++E_-)f(\vec{k},\vec{q},E) ,\end{aligned} \tag{3.26}$$

where

$$E_{\pm}=E_{\vec{k}\pm(1/2)\vec{q}}, \quad \epsilon_{\pm}=\epsilon(\vec{k}\pm\tfrac{1}{2}\vec{q}) \ , \tag{3.27}$$

$$f(\vec{k},\vec{q},E)=-V/E_{+}E_{-}[(E_{+}+E_{-})^2-E^2] \ . \tag{3.28}$$

Equation (3.22) is the relation which determines ω_q self-consistently. The BS amplitude $g(q)$ was obtained in Ref. 18 as

$$g^2(\vec{q})=\frac{\omega_q^2-\bar{\omega}_q^2}{V\omega_q[1-\Omega_1(\vec{q},\omega_q)]} \ , \tag{3.29}$$

where

$$\Omega_1(\vec{q},\omega_q)=\frac{\partial^2}{\partial E^2}\Omega(\vec{q},E^2)\Big|_{E^2=\omega_q^2} \ , \tag{3.30}$$

with

$$\Omega(\vec{q},E^2)=\bar{\omega}_q^2+Q^{12}(\vec{q},E)(E^2-\bar{\omega}_q^2)+2\Delta^2R(\vec{q},E)(E^2+\bar{\omega}_q^2) \ . \tag{3.31}$$

Let us now study the boson contribution to the electric current. The boson current can be written

$$\vec{j}^{B}(x)=\int\frac{d^3q}{(2\pi)^{3/2}}\langle 0|\vec{j}(x)|B_q\rangle B_q+\text{H.c.} \ . \tag{3.32}$$

Here H.c. means the Hermitian conjugate. The matrix element in Eq. (3.32) can be put in the form

$$\langle 0|\vec{j}|B_q\rangle=-\tfrac{1}{2}\{[\vec{v}(-i\partial_x)-\vec{v}(-i\partial_y)]\times[G_x(x,y;\vec{q})+G_{22}(x,y;\vec{q})]\}_{x=y} \ , \tag{3.33}$$

where the operator $\vec{v}(-i\partial)$ is defined by

$$\vec{v}(-i\partial)\exp(i\vec{q}\cdot\vec{x})=\vec{v}(\vec{q})\exp(i\vec{q}\cdot\vec{x}) \ , \tag{3.34}$$

with $\vec{v}$ being the group velocity of the electrons, i.e.,

$$\vec{v}(\vec{q})=\frac{\partial\epsilon(\vec{q})}{\partial\vec{q}} \ . \tag{3.35}$$

The expression (3.33) can be written

$$\langle 0|\vec{j}|B_q\rangle=\exp(i\vec{q}\cdot\vec{x}-i\omega_q t)\int\frac{d^3k}{(2\pi)^3}\vec{v}(\vec{k})\frac{1}{2E_{+}E_{-}}\frac{-V}{(E_{+}+E_{-})^2-\omega_q^2}\times\left[(\epsilon_{+}-\epsilon_{-})(E_{+}+E_{-})\frac{2\Delta^2}{\omega_q^2-\bar{\omega}_q^2}+(\epsilon_{-}E_{+}-\epsilon_{+}E_{-})\right]\omega_q g(\vec{q}) \ , \tag{3.36}$$

where uses were made of Eqs. (3.20) and (3.21). Calculation of this quantity in the case of isotropic superconductors is rather easy because this quantity should be in the direction of $\vec{q}$: The result is

$$\langle 0|\vec{j}|B_q\rangle=\exp(i\vec{q}\cdot\vec{x}-i\omega_q t)\vec{q}g(\vec{q})\omega_q/q^2 \tag{3.37}$$

for the isotropic case. However, in the case of anisotropic superconductors, the direction of $\langle 0|\vec{j}|B_q\rangle$ does not need to be in the $\vec{q}$ direction. Therefore, we introduce a tensor a_{ij} and write $\langle 0|\vec{j}|B_q\rangle$

$$\langle 0|j_i|B_q\rangle=\exp(i\vec{q}\cdot\vec{x}-i\omega_q t)g(\vec{q})\omega_q a_{ij}q_j \ , \tag{3.38}$$

where

$$a_{ij}q_j=\int\frac{d^3k}{(2\pi)^3}v_i(\vec{k})\frac{1}{2E_{+}E_{-}}\frac{-V}{(E_{+}+E_{-})^2-\omega_q^2}\left[(\epsilon_{+}-\epsilon_{-})\frac{2\Delta^2}{\omega_q^2-\bar{\omega}_q^2}+(\epsilon_{-}E_{+}-\epsilon_{+}E_{-})\right] \ . \tag{3.39}$$

The current conservation law, $\partial_\mu j_\mu=0$, together with Eq. (3.38) leads to

$$a_{ij}q_iq_j=1 \ . \tag{3.40}$$

Let us now introduce the tensor $V_{ij}(\vec{q})$ defined by

$$V_{ij}(\vec{q})=\vec{q}^{\,2}a_{ij}(\vec{q}) \ . \tag{3.41}$$

Then Eq. (3.40) gives

$$V_{ij}q_iq_j=\vec{q}^{\,2} \ . \tag{3.42}$$

Note that this does not necessarily lead to $V_{ij}=\delta_{ij}$. Use of Eqs. (3.38) and (3.41) leads to the following expression for the boson current in the anisotropic superconductors;

$$\begin{aligned} j_i^B(x)&=\int\frac{d^3q}{(2\pi)^{3/2}}\omega_q g(\vec{q})V_{ij}(q)q_jB_q/\vec{q}^{\,2}\exp(i\vec{q}\cdot\vec{x}-i\omega_q t)+\text{H.c.}\\ &=d(-i\nabla)V_{ij}(-i\partial)\partial_j\int\frac{d^2q}{(2\pi)^{3/2}}B_q/(2\omega_q)^{1/2}\exp(i\vec{q}\cdot\vec{x}-i\omega_q t)+\text{H.c.}\\ &=d(-i\partial)V_{ij}(-i\partial)\partial_jB(x) \ , \end{aligned} \tag{3.43}$$

where

$$d(\vec{q}) = -[2i/(2\omega_q)^{1/2}](\omega_q^2/\vec{q}^{\,2})g(\vec{q}) \ . \tag{3.44}$$

It was shown in Ref. 12 that when the electromagnetic field is introduced, the replacement

$$\partial_i B \to \partial_i B - \frac{e}{c}\eta(-i\partial)A_i \tag{3.45}$$

should be performed everywhere in the theory. Here

$$\eta(\vec{q}) \equiv [2i/(2\omega_q)^{1/2}]g(\vec{q}) \tag{3.46}$$

and $\vec{A}$ is the vector potential. Such replacement makes the theory invariant under the gauge transformation $[\vec{A} \to \vec{A} + \vec{\nabla}\chi,\ B \to B + (e/c)\eta(-i\partial)\chi]$. We thus have

$$\begin{aligned} j_i^B &= d(-i\vec{\nabla})V_{ij}(-i\vec{\nabla})\partial_j B(x) - \frac{e}{c}d(-i\nabla)\eta(-i\nabla)V_{ij}(-i\nabla)A_j(x) \\ &= d(-i\nabla)V_{ij}(-i\nabla)\partial_j B(x) - \frac{c\hbar}{4\pi\lambda_L^2 e}\int d^3y\, c(\vec{x}-\vec{y})V_{ij}(-i\nabla)A_j(\vec{y}) \end{aligned} \tag{3.47}$$

for the boson current. The following notations are used:

$$c(\vec{x}) = \int \frac{d^3k}{(2\pi)^3}c(\vec{k})\exp(i\vec{k}\cdot\vec{x}) \ , \tag{3.48}$$

$$c(\vec{k}) \equiv \frac{d(\vec{k})\eta(\vec{k})}{d(0)\eta(0)} \ , \tag{3.49}$$

$$\lambda_L^2 \equiv \frac{1}{d(0)\eta(0)} \ . \tag{3.50}$$

Using Eqs. (3.44), (3.46), and (3.29), we get

$$d(\vec{k})\eta(\vec{k}) = \frac{2}{q^2}\frac{\bar{\omega}_q^2 - \omega_q^2}{V[1-\Omega_1(\vec{q},\omega_q)]} \tag{3.51}$$

and

$$d(0)\eta(0) = \tfrac{2}{3}VN(0)v_0^2 \ , \tag{3.52}$$

where it was considered that, in the cubic superconductors, the anisotropy vanishes at $q=0$. In Eq. (3.52) $N(0)$ is the total density of states at the Fermi level and v_0^2 is the mean value of $v_F^2(\vec{k})$ over the Fermi surface [$v_F(\vec{k})$ means the Fermi velocity]. Now Eqs. (3.49), (3.51), and (3.52) lead to the c function

$$c(\vec{q}) = \frac{3(\bar{\omega}_q^2 - \omega_q^2)}{VN(0)v_0^2q^2[1-\Omega_1(\vec{q},\omega_q)]} \tag{3.53}$$

for the superconductors with anisotropic Fermi surface (FS). Although this expression is the same as the one for isotropic superconductors, $c(\vec{q})$ for anisotropic superconductors is not a function of $\vec{q}^{\,2}$, because the Fermi surface is anisotropic.

We computed the c function (3.53) numerically by evaluating the integrals (3.24)–(3.26). We disregarded the terms higher than the sixth order in the cubic harmonics expansion. The numerical result was presented in Sec. II. A detailed account of the calculation is given in Appendix B.

Let us now derive the explicit form of V_{ij}. Equations (3.39) and (3.41) lead to

$$\begin{aligned} V_{ij}(\vec{q})q_j = q^2\int \frac{d^3k}{(2\pi)^3}v_i(\vec{k})\frac{1}{2E_+E_-}\frac{-V}{(E_++E_-)^2-\omega_q^2} \times \Bigg[&(\epsilon_+-\epsilon_-)(E_++E_-)\frac{2\Delta^2}{\omega_q^2-\bar{\omega}_q^2} \\ &+(\epsilon_-E_+-\epsilon_+E_-)\Bigg] \ . \end{aligned} \tag{3.54}$$

The integration element can be written

$$\frac{d^2k}{(2\pi)^3} = N(0)\,d\epsilon\,d^2k_F\,\rho(\vec{k}_F) \ , \tag{3.55}$$

where

$$\rho(\vec{k}_F) = 1/(2\pi)^3N(0)v_F(\vec{k}_F) \ , \tag{3.56}$$

which is the direction-dependent density of states along the $\vec{k}_F$ direction and is normalized as

$$\int_{\mathrm{FS}} d^2k_F\,\rho(\vec{k}_F) = 1 \ . \tag{3.57}$$

Following the arguments in Ref. 18, we now introduce a new integration variable w;

$$\begin{aligned} w &= (E_++E_-)^2-(\epsilon_+-\epsilon_-)^2 \\ &\equiv (E_++E_-)^2-(\vec{q}\cdot\vec{v}_F)^2 \ . \end{aligned} \tag{3.58}$$

We then find

$$d\epsilon\,d^2k_F\,\rho(\vec{k}_F)\frac{E_++E_-}{E_+E_-} = dw\,d^2k_F\,\rho(\vec{k}_F) \times \frac{1}{w^{1/2}}\frac{1}{(w-4\Delta^2)^{1/2}} \ . \tag{3.59}$$

The integration domain for w runs from $4\Delta^2$ to w_m defined by

$$w_m = 4(\omega_D^2 + \Delta^2) \ , \tag{3.60}$$

with the Debye frequency ω_D. Substituting Eq. (3.59) into Eq. (3.54), we have

$$V_{ij}(\vec{q})q_j = -2q^2\Delta^2 VN(0)\int_{4\Delta^2}^{w_m} dw \frac{1}{w^{1/2}(w-4\Delta^2)^{1/2}}\left[\frac{1}{\omega_q^2-\bar{\omega}_q^2}-\frac{1}{w}\right]\int_{\rm FS} d^2k_F\,\rho(\vec{k}_F)\frac{v_i(\vec{q}\cdot\vec{v}_F)}{w-\omega_q^2+(\vec{q}\cdot\vec{v}_F)^2} \ . \tag{3.61}$$

Let us now study the integral over the Fermi surface in Eq. (3.61)

$$\vec{\sigma}(z,\vec{q}) = \int_{\rm FS} d^2k_F\,\rho(\vec{k}_F)\frac{\vec{v}_F(\vec{q}\cdot\vec{v}_F)}{z+(\vec{q}\cdot\vec{v}_F)^2} = \frac{1}{2}\frac{\partial}{\partial\vec{q}}\int_{\rm FS} d^2k_F\,\rho(\vec{k}_F)\ln[z+(\vec{q}\cdot\vec{v}_F)^2] \ . \tag{3.62}$$

We can evaluate this integral by using the cubic harmonics expansion (see Appendix B). The result is

$$\vec{\sigma}(z,\vec{q}) = \frac{1}{2}\sum_l a_l \frac{\partial}{\partial\vec{q}}[\gamma_l(z,qv_0)H_l(\hat{q})] \ , \tag{3.63}$$

where

$$a_l = \langle H_l(\vec{v}_F)\rangle \equiv \int_{\rm FS} d^2k_F\,\rho(\vec{k}_F)H_l(\hat{v}_F) \ , \tag{3.64}$$

$$\gamma_l(z,qv_0) = \int_0^1 dx\,P_l(x)\ln(z+q^2v_0^2x^2) \ . \tag{3.65}$$

Here $P_l(x)$ is the Legendre polynomial of the lth order. Equation (3.63) can be divided into two parts;

$$\vec{\sigma}(z,\vec{q}) = \vec{\sigma}^{(1)}(z,\vec{q}) + \vec{\sigma}^{(2)}(z,\vec{q}) \ , \tag{3.66}$$

where $\vec{\sigma}^{(1)}$ and $\vec{\sigma}^{(2)}$ are the components parallel and perpendicular to $\vec{q}$, respectively, that is,

$$\vec{\sigma}^{(1)}(z,\vec{q}) = \sum_l a_l\int_0^1 dx\,P_l(x)\frac{q^2v_0^2x^2}{z+q^2v_0^2x^2}H_l(\vec{q})\frac{\vec{q}}{q} \ , \tag{3.67}$$

$$\vec{\sigma}^{(2)}(z,\vec{q}) = \frac{1}{2}\sum_l a_l\int_0^1 dx\,P_l(x)\ln(z+q^2v_0^2x^2)\frac{\partial}{\partial\vec{q}}H_l(\vec{q}) \ . \tag{3.68}$$

Substituting Eq. (3.66) into Eq. (3.61), we obtain

$$V_{ij}(\vec{q})q_j = V_{ij}^{(1)}q_j + V_{ij}^{(2)}q_j \ , \tag{3.69}$$

where

$$V_{ij}^{(1)}q_j = -2q^2\Delta^2 VN(0)\int_{4\Delta^2}^{w_m} dw\frac{1}{w^{1/2}(w-4\Delta^2)^{1/2}}\left[\frac{1}{\omega_q^2-\bar{\omega}_q^2}-\frac{1}{w}\right]\sigma_i^{(1)}(w-\omega_q^2,\vec{q}) \ , \tag{3.70}$$

$$V_{ij}^{(2)}q_j = -2q^2\Delta^2 VN(0)\int_{4\Delta^2}^{w_m} dw\frac{1}{w^{1/2}(w-4\Delta^2)^{1/2}}\left[\frac{1}{\omega_q^2-\bar{\omega}_q^2}-\frac{1}{w}\right]\sigma_i^{(2)}(w-\omega_q^2,\vec{q}) \ . \tag{3.71}$$

Since $q_i\sigma_i^{(2)}=0$, Eq. (3.42) leads to

$$V_{ij}^{(1)}q_j = q_i \ , \tag{3.72}$$

which gives

$$V_{ij}^{(1)} = \delta_{ij} \ . \tag{3.73}$$

When we use the coordinate system given by the principal axes of the cubic crystal, we find

$$\begin{aligned}
\frac{\partial H_4(\hat{q})}{\partial q_1} &= 5(21)^{1/2}\frac{q_1}{q^2}\{\hat{q}_1^2-\tfrac{3}{5}-[H_4(\hat{q})]\} \ ,\\
\frac{\partial H_4(\hat{q})}{\partial q_2} &= 5(21)^{1/2}\frac{q_2}{q^2}\{\hat{q}_2^2-\tfrac{3}{5}-[H_4(\hat{q})]\} \ ,\\
\frac{\partial H_4(\hat{q})}{\partial q_3} &= 5(21)^{1/2}\frac{q_3}{q^2}\{\hat{q}_2^2-\tfrac{3}{5}-[H_4(\hat{q})]\} \ ,
\end{aligned} \tag{3.74}$$

and

$$\frac{\partial H_6(\hat{q})}{\partial q_1}=\tfrac{1}{4}[231(26)^{1/2}]\frac{q_1}{q^2}\{\hat{q}_2^2\hat{q}_3^2+\tfrac{1}{22}[H_4(\hat{q})]+\tfrac{1}{11}(\hat{q}_1^2-\tfrac{3}{5})-\tfrac{1}{35}-3[H_6(\hat{q})]\}\ ,$$
$$\frac{\partial H_6(\hat{q})}{\partial q_2}=\tfrac{1}{4}[231(26)^{1/2}]\frac{q_2}{q^2}\{\hat{q}_3^2\hat{q}_1^2+\tfrac{1}{22}[H_4(\hat{q})]+\tfrac{1}{11}(\hat{q}_2^2-\tfrac{3}{5})-\tfrac{1}{35}-3[H_6(\hat{q})]\}\ , \tag{3.75}$$
$$\frac{\partial H_6(\hat{q})}{\partial q_3}=\tfrac{1}{4}[231(26)^{1/2}]\frac{q_3}{q^2}\{\hat{q}_1^2\hat{q}_2^2+\tfrac{1}{22}[H_4(\hat{q})]+\tfrac{1}{11}(\hat{q}_3^2-\tfrac{3}{5})-\tfrac{1}{35}-3[H_6(\hat{q})]\}\ .$$

Here $\hat{q}$ means the direction cosine of $\vec{q}$; and $[H_4]$ and $[H_6]$ are defined by

$$[H_4(\hat{q})]=\frac{4}{5(21)^{1/2}}H_4(\hat{q})\ ,\quad [H_6(\hat{q})]=\frac{8}{231(26)^{1/2}}H_6(\hat{q})\ . \tag{3.76}$$

Then Eqs. (3.69)–(3.76) lead to the following expressions for $V_{ij}(q)$ in the coordinate fixed to the principal axes:

$$\begin{aligned}
V_{11}(\vec{q})&=1+5(21)^{1/2}a_4\gamma_4(q)\{\hat{q}_1^2-\tfrac{3}{5}-[H_4(\hat{q})]\}\\
&\quad+\tfrac{1}{4}[231(26)^{1/2}]a_6\gamma_6(q)\{\hat{q}_2^2\hat{q}_3^2+\tfrac{1}{22}[H_4(\hat{q})]+\tfrac{1}{11}(\hat{q}_1^2-\tfrac{3}{5})-\tfrac{1}{35}-3[H_6(\hat{q})]\}+\cdots\ ,\\
V_{22}(\vec{q})&=1+5(21)^{1/2}a_4\gamma_4(q)\{\hat{q}_2^2-\tfrac{3}{5}-[H_4(\hat{q})]\}\\
&\quad+\tfrac{1}{4}[231(26)^{1/2}]a_6\gamma_6(q)\{\hat{q}_3^2\hat{q}_1^2+\tfrac{1}{22}[H_4(\hat{q})]+\tfrac{1}{11}(\hat{q}_2^2-\tfrac{3}{5})-\tfrac{1}{35}-3[H_6(\hat{q})]\}+\cdots\ ,\\
V_{33}(\vec{q})&=1+5(21)^{1/2}a_4\gamma_4(q)\{\hat{q}_3^2-\tfrac{3}{5}-[H_4(\hat{q})]\}\\
&\quad+\tfrac{1}{4}[231(26)^{1/2}]a_6\gamma_6(q)\{\hat{q}_1^2\hat{q}_2^2+\tfrac{1}{22}[H_4(\hat{q})]+\tfrac{1}{11}(\hat{q}_3^2-\tfrac{3}{5})-\tfrac{1}{35}-3[H_6(\hat{q})]\}+\cdots\ ,\\
V_{ij}(\vec{q})&=0\quad\text{for } i\neq j\ .
\end{aligned} \tag{3.77}$$

Here

$$\gamma_l(q)=-q^2\Delta^2VN(0)\int_{4\Delta^2}^{w_m}dw\,\frac{1}{w^{1/2}(w-4\Delta^2)^{1/2}}\left[\frac{1}{\omega_q^2-\bar{\omega}_q^2}-\frac{1}{w}\right]\int_0^1 dx\,P_l(x)\ln(w-\omega_q^2+q^2v_0^2x^2)\ . \tag{3.78}$$

The above integral can be calculated analytically in the two limiting cases, i.e., we have

$$\gamma_4(\vec{q})\sim\frac{\pi^2}{315}(q\xi_0)^2\ ,$$
$$\gamma_6(\vec{q})\sim\frac{2\pi^2}{45045}(q\xi_0)^4\ , \tag{3.79}$$

for $q\to 0$, and

$$\gamma_4(\vec{q})\sim\tfrac{2}{15}VN(0)\ ,\quad \gamma_6(\vec{q})\sim\tfrac{1}{105}VN(0)\ , \tag{3.80}$$

for $q\to\infty$.

The results of numerical computation of $\gamma_l(q)$ can be approximately expressed by the following relations:

$$\gamma_4(\vec{q})=-\frac{1}{15}\frac{2\pi^2(q\xi_0)^2}{42+[\pi^2/VN(0)](q\xi_0)^2}\ , \tag{3.81}$$

$$\gamma_6(\vec{q})=\frac{1}{3465}\frac{\pi^2(q\xi_0)^4}{26+[\pi^2/66VN(0)](q\xi_0)^4}\ . \tag{3.82}$$

These expressions were used in our analysis in Sec. II.

We are now ready to derive the macroscopic equations (1.1)–(1.3) for the cubic superconductors. The persistent current $\vec{J}$ is the vacuum expectation value of the boson current $\vec{j}_B$. In the inhomogeneous ground state $\vec{j}_B$ is modified by the boson transformation

$$B(x)\to B(x)+\eta(-i\partial)f(x)\ , \tag{3.83}$$

which regulates the boson condensation. Here $f(x)$ is a c-number function which satisfies the free-field equation for B. The boson transformation induces the persistent current

$$\begin{aligned}
J_i(x)&=\frac{e}{c}\langle 0|j_i^B|0\rangle\\
&\equiv-\frac{1}{4\pi\lambda_L^2}\int d^3y\,c(\vec{x}-\vec{y})V_{ij}(-i\nabla)\\
&\quad\times\left[A_j(\vec{y})-\frac{\hbar c}{e}\partial_j f(\vec{y})\right]\ .
\end{aligned} \tag{3.84}$$

The Maxwell equation then gives

$$(\nabla^2\delta_{ij}-\partial_i\partial_j)A_j(\vec{x})=\frac{1}{\lambda_L^2}\int d^3y\, c(\vec{x}-\vec{y})\,V_{ij}(-i\partial)\times\left[A_j(\vec{y})-\frac{\hbar c}{e}\partial_j f(\vec{y})\right] . \tag{3.85}$$

We thus obtained Eqs. (1.2) and (1.3). It should be noted that, according to the general theory of anisotropic superconductors in Ref. 17, the quantity $V_{ij}q_iq_j$ is proportional to ω_q^2 where ω_q is the boson frequency. Since $f(x)$ satisfies the free-field equation for B, we find

$$V_{ij}(-i\partial)\partial_i\partial_j f(\vec{x})=0 \tag{3.86}$$

in static cases. This is Eq. (1.1). When we consider the relation (3.42), Eq. (3.86) is reduced to the Laplace equation. Summarizing we obtained the following macroscopic equations for the cubic superconductors:

$$\nabla^2 f(\vec{x})=0 \ , \tag{3.87}$$

$$(\nabla^2\delta_{ij}-\partial_i\partial_j)A_j(\vec{x})=\frac{1}{\lambda_L^2}\int d^3y\, c(\vec{x}-\vec{y})\,V_{ij}(-i\partial)\times\left[A_j(\vec{y})-\frac{\hbar c}{e}\partial_j f(\vec{y})\right] . \tag{3.88}$$

These equations hold in any coordinate system, although the expression of V_{ij} in Eq. (3.77) is true only when the coordinate system is given by the principal axes of the crystal. We can find V_{ij} in any other coordinate system by means of suitable transformations [see Eq. (2.7)].

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APPENDIX A: CALCULATION OF $\partial_i f(\vec{x})$ FOR SINGLE VORTEX

In the following we consider the single vortex state in which the vortex line lies in the $(1\bar{1}0)$ plane, and calculate $\partial_i f(\vec{x})$ by means of the method presented in Ref. 13. We use the coordinate system, where the third axis is in the direction of the vortex line and the first and second axes are situated symmetrically with respect to the $(1\bar{1}0)$ plane (see Fig. 2). To emphasize the fact that the derivative operator $V_{ij}(-i\partial)$ depends on θ which is the angle between the vortex line and the [001] direction, we denote it by $V_{ij}(-i\partial,\theta)$. Then, according to Eq. (1.1), the equation for $f(x)$ is given by

$$V_{ij}(-i\partial,\theta)\partial_i\partial_j f(\vec{x})=0 \ . \tag{A1}$$

We have

$$V_{ij}(-i\partial,\theta)=u_{il}(\theta)\,V_{lm}u_{mj}^{-1}(\theta) \ , \tag{A2}$$

where V_{lm} is the derivative operator in the coordinate system given by the principal axis of the crystal, and therefore, is given by Eq. (3.77). The transformation matrix u_{il} was defined by Eq. (2.8).

The solution $f(x)$ for the vortex under consideration possesses the topological singularity expressed by

$$\vec{\nabla}\times\vec{\nabla}f(\vec{x})=\pi\vec{e}_3\delta(x_1)\delta(x_2) \ , \tag{A3}$$

where $\vec{e}_3$ is the unit vector along the third axis. Introducing $g_{ij}(x)$ by

$$g_{ij}(\vec{x})=[\partial_i,\partial_j]f(\vec{x}) \ , \tag{A4}$$

we see from Eq. (A3) that

$$g_{12}(\vec{x})=-g_{21}(\vec{x})=\pi\delta(x_1)\delta(x_2) \tag{A5}$$

and other components of g_{ij} vanish. Since $\partial f(\vec{x})$ appears in the expression of the current, it should be single valued

$$[\partial_i,\partial_j]\partial_k f(\vec{x})=0 \ . \tag{A6}$$

Let us now define

$$D_i(\partial,\theta)\equiv V_{ij}(-i\partial,\theta)\partial_j \tag{A7}$$

and

$$D(\partial,\theta)=\partial_i D_i(\partial,\theta) \ . \tag{A8}$$

Then Eq. (A1) reads

$$D(\partial,\theta)f(\vec{x})=0 \ . \tag{A9}$$

Now, Eqs. (A4), (A6), and (A9) give

$$D_i(\partial,\theta)g_{ij}(\vec{x})=D(\partial,\theta)\partial_j f(\vec{x}) \ , \tag{A10}$$

which leads to

$$\partial_i f(\vec{x})=\int d^3y\, G(\vec{x}-\vec{y})D_j(\partial,\theta)g_{ji}(\vec{y}) \ , \tag{A11}$$

where $G(\vec{x}-\vec{y})$ is the Green's function defined by

$$D(\partial,\theta)G(\vec{x})=\delta^{(3)}(\vec{x}) \ .$$

Considering Eq. (A5) we find that

$$\partial_1 f(\vec{x})=\pi\int\frac{d^2k}{(2\pi)^2}[V_{21}(\vec{k},\theta)k_1+V_{22}(\vec{k},\theta)k_2]/k^2\exp(i\vec{k}\cdot\vec{x}) \ ,$$

$$\partial_2 f(\vec{x})=-\pi\int\frac{d^3k}{(2\pi)^2}[V_{11}(\vec{k},\theta)k_1+V_{12}(\vec{k},\theta)k_2]/k^2\exp(i\vec{k}\cdot\vec{x}) \ ,$$

$$\partial_3 f(\vec{x})=0 \ , \tag{A12}$$

where the relation $V_{ij}k_ik_j=k^2$ was used.

APPENDIX B: CALCULATION OF THE c FUNCTION AND $\vec{\sigma}(z,\vec{q})$

A. Calculations of $R(\vec{q},E)$, $\bar{\omega}_q^2(\vec{q},E)$, $Q^{12}(\vec{q},E)$, and c function

Let us first calculate the integrals R, $\bar{\omega}_q^2$, and Q^{12},

$$R(\vec{q},E)=\int\frac{d^3k}{(2\pi)^3}(E_++E_-)f(\vec{k},\vec{q},E) \ , \tag{B1}$$

$$\bar{\omega}_q^2(\vec{q},E)=\frac{1}{4R(\vec{q},E)}\int\frac{d^3k}{(2\pi)^3}(\epsilon_+-\epsilon_-)^2(E_++E_-)f(\vec{k},\vec{q},E) \ , \tag{B2}$$

$$Q^{12}(\vec{q},E)=-\tfrac{1}{2}\int\frac{d^3k}{(2\pi)^3}(E_+E_--\epsilon_+\epsilon_-)(E_++E_-)f(\vec{k},\vec{q},E) \ . \tag{B3}$$

Here

$$f(\vec{k},\vec{q},E)=-V/E_+E_-[(E_++E_-)^2-E^2] \ . \tag{B4}$$

By using Eqs. (3.55)–(3.60), we get the following expressions for Eqs. (B1)–(B3):

$$R(\vec{q},E)=-\tfrac{1}{2}\Lambda\int_{4\Delta^2}^{w_m}\frac{1}{w^{1/2}(w-4\Delta^2)^{1/2}}\int_{\rm FS}d^2k_F\,\rho(\vec{k}_F)\frac{1}{w-E^2+(\vec{q}\cdot\vec{v}_F)^2} \ , \tag{B5}$$

$$\begin{aligned}\bar{\omega}_q^2(\vec{q},E)=&-\frac{1}{2}\frac{\Lambda}{R(\vec{q},E)}\int_{4\Delta^2}^{w_m}\frac{dw}{w^{1/2}(w-4^2)^{1/2}}\\&+\frac{\Lambda}{2R(\vec{q},E)}\int_{4\Delta^2}^{w_m}dw\,\frac{w-E^2}{w^{1/2}(w-4\Delta^2)^{1/2}}\int_{\rm FS}d^2k_F\,\rho(\vec{k}_F)\frac{1}{w-E^2+(\vec{q}\cdot\vec{v}_F)^2} \ ,\end{aligned} \tag{B6}$$

$$\begin{aligned}Q^{12}(\vec{q},E)=&-2\Delta^2R(\vec{q},E)+2\Delta^2\Lambda\int_{4\Delta^2}^{w_m}dw\,\frac{1}{w^{3/2}(w-4\Delta^2)^{1/2}}\\&-2\Delta^2\Lambda\int_{4\Delta^2}^{w_m}dw\,\frac{w-E^2}{w^{3/2}(w-4\Delta^2)^{1/2}}\int_{\rm FS}d^2k_F\,\rho(\vec{k}_F)\frac{1}{w-E^2+(\vec{q}\cdot\vec{v}_F)^2} \ ,\end{aligned} \tag{B7}$$

where $\Lambda=VN(0)$. To calculate Eqs. (B5)–(B7), we must treat the following integral over the Fermi surface:

$$I(z,\vec{q})=\int_{\rm FS}d^2k_F\,\rho(\vec{k}_F)\frac{1}{z+(\vec{q}\cdot\vec{v}_F)^2} \ . \tag{B8}$$

To evaluate this we use the Teichler's method.[21] Let us expand Eq. (B8) in terms of the spherical harmonics Y_{lm}. We have the expansion

$$I(z,\vec{q})=4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\int_{\rm FS}d^2k_F\,\rho(\vec{k}_F)\,Y_{lm}^*(\hat{v}_F)\tfrac{1}{2}\int_{-1}^{1}dx\,P_l(x)\frac{1}{z+q^2v_F^2x^2}Y_{lm}(\hat{q}) \ , \tag{B9}$$

where $P_l(x)$ is the Legendre polynomial of l th order, $\hat{v}_F$ and $\hat{q}$ are the angular parts of $\vec{v}_F$ and $\vec{q}$, respectively. The expansion can be easily proved when we use the formulas

$$4\pi \sum_{m=-l}^{l} Y_{lm}^*(\hat{v}_F)\, Y_{lm}(\hat{q}) = (2l+1) P_l(\cos\chi) \quad , \tag{B10}$$

$$\frac{1}{2} \sum_{l=0}^{\infty} (2l+1) P_l(x) P_l(x') = \delta(x - x') \quad , \tag{B11}$$

where χ is the angle between $\vec{q}$ and $\vec{v}_F$, i.e.,

$$\vec{q} \cdot \vec{v}_F = q v_F \cos\chi \quad . \tag{B12}$$

We can rewrite the expansion (B9) in a compact form when we use the cubic harmonics $H_l(\hat{q})$. A symmetry consideration applied to the integral over the Fermi surface in Eq. (B9) leads to

$$I(z,\vec{q}) = \sum_l \left\{ \int_{FS} d^2k_F\, \rho(\vec{k}_F) H_l(\vec{v}_F) \times \int_0^1 dx\, P_l(x) \frac{1}{z + q^2 v_F^2 x^2} \right\} H_l(\hat{q}) \quad . \tag{B13}$$

Assuming that the anisotropy of Fermi surface is small (i.e., $\delta v_F^2 / v_0^2 << 1$ where $\delta v_F^2 = v_F^2 - v_0^2$), we replace v_F^2 with the average value v_0^2 over the Fermi surface in the above integral, neglecting the direction dependence of v_F^2. The average value v_0^2 is defined by

$$v_0^2 = \int_{FS} d^2k_F\, \rho(\vec{k}_F) v_F^2 \quad . \tag{B14}$$

Then we get

$$I(z,\vec{q}) = \sum_l a_l \Gamma_l(z, q v_0) H_l(\hat{q}) \quad , \tag{B15}$$

where

$$a_l = \langle H_l(\hat{v}_F) \rangle \equiv \int_{FS} d^2k_F\, \rho(\vec{k}_F) H_l(\hat{v}_F) \quad , \tag{B16}$$

$$\Gamma_l(z,\alpha) = \int_0^1 dx\, P_l(x) \frac{1}{z + \alpha^2 x^2} \quad . \tag{B17}$$

Equation (B17) leads to

$$\Gamma_0(z,\alpha) = \frac{1}{\alpha z} \tan^{-1}\frac{\alpha}{z} \quad , \tag{B18}$$

$$\Gamma_4(z,\alpha) = -\frac{55}{24}\frac{1}{\alpha^2} - \frac{35}{8}\frac{z^2}{\alpha^4} + \left[\frac{35 z^3}{\alpha^5} + \frac{30 z}{\alpha^3} + \frac{3}{\alpha z} \right] \tan^{-1}\frac{\alpha}{z} \quad , \tag{B19}$$

$$\Gamma_6(z,\alpha) = \frac{1}{16} \left[\frac{231}{5}\frac{1}{\alpha^2} + \frac{238 z^2}{\alpha^4} + \frac{231 z^4}{\alpha^6} - \left[\frac{231 z^5}{\alpha^7} + \frac{315 z^3}{\alpha^5} + \frac{105 z}{\alpha^3} + \frac{5}{\alpha z} \right] \tan^{-1}\frac{\alpha}{z} \right] \quad . \tag{B20}$$

Applying the formula (B15) to Eqs. (B5)–(B7), we obtain the cubic harmonics expansion of $R(\vec{q},E)$, $\bar{\omega}_q^2(\vec{q},E)$, and $Q^{12}(\vec{q},E)$,

$$R(\vec{q},E) = -\frac{1}{2}\Lambda \sum_l a_l \int_{4\Delta^2}^{w_m} dw \frac{\Gamma_l(q v_0, w - E^2)}{w^{1/2}(w - 4\Delta^2)^{1/2}} H_l(\vec{q}) \quad , \tag{B21}$$

$$\bar{\omega}_q^2(\vec{q},E) = -\frac{\Lambda}{2R(\vec{q},E)} \int_{4\Delta^2}^{w_m} dw \frac{1}{w^{1/2}(w-4\Delta^2)^{1/2}} + \frac{\Lambda}{2R(\vec{q},E)} \sum_l a_l \int_{4\Delta^2}^{w_m} dw \frac{(w - E^2)\Gamma_l(q v_0, w - E^2)}{w^{1/2}(w - 4\Delta^2)^{1/2}} H_l(\hat{q}) \quad , \tag{B22}$$

$$Q^{12}(\vec{q},E) = -2\Delta^2 R(\vec{q},E) + 2\Delta^2\Lambda \int_{4\Delta^2}^{w_m} dw \frac{1}{w^{3/2}(w-4\Delta^2)^{1/2}} - 2\Delta^2\Lambda \sum_l a_l \int_{4\Delta^2}^{w_m} dw \frac{(w-E^2)\Gamma_l(q v_0, w - E^2)}{w^{3/2}(w-4\Delta^2)^{1/2}} H_l(\hat{q}) \quad . \tag{B23}$$

The expansion parameters a_l defined by Eq. (B16) are the phenomenological parameters which measure the Fermi-surface anisotropy.

The frequency ω_q of the collective modes are determined by solving the self-consistent relation [Eq. (3.22)],

$$\omega_q^2 = \bar{\omega}_q^2 + Q^{12}(\vec{q},\omega_q)(\omega_q^2 - \bar{\omega}_q^2) + 2\Delta^2 R(\vec{q},\omega_q)(\omega_q^2 + \bar{\omega}_q^2) \quad .$$

From the knowledge of ω_q and Eqs. (B21)–(B22) we can compute the c function through the relation [Eq. (3.53)],

$$c(\vec{q}) = \frac{3(\bar{\omega}_q^2 - \omega_q^2)}{\Lambda q^2 v_0^2 [1 - \Omega_1(\vec{q},\omega_q)]} \quad .$$

The numerical results of the c function can be well approximated by

$$c(\vec{q}) = c_0(q,a_4,a_6) + c_4(q,a_4,a_6) H_4(\hat{q}) + c_6(q,a_4,a_6) H_l(\hat{q}) \quad , \tag{B24}$$

when we ignore a_l with $l \geqslant 8$. The coefficients c_0, c_4, and c_6 have the following form:

$$c_0(q,a_4,a_6)=c_0(q,0,0)+d_1^{(0)}(\tau)a_4^2+d_2^{(0)}(\tau)a_4^3+d_3^{(0)}(\tau)a_6^2+d_4^{(0)}(\tau)a_4a_6\ , \tag{B25}$$

$$c_4(q,a_4,a_6)=d_1^{(4)}(\tau)a_4+d_2^{(4)}(\tau)a_4^2+d_3^{(4)}(\tau)a_6^2+d_4^{(4)}(\tau)a_4a_6\ , \tag{B26}$$

$$c_6(q,a_4,a_6)=d_1^{(6)}(\tau)a_6+d_2^{(6)}(\tau)a_6^2+d_3^{(6)}(\tau)a_4^2+d_4^{(6)}(\tau)a_4a_6\ , \tag{B27}$$

where $\tau=q\xi_0$, $\xi_0=v_0/\pi\Delta$, and $d_i^{(l)}(\tau)$'s are functions depending only on τ. We retain only the leading terms in the right-hand sides of Eqs. (B25)–(B27):

$$c_0(q,a_4,a_6)\simeq c_0(1,0,0)+d_1^{(0)}(\tau)a_4^2\ , \tag{B28}$$

$$c_4(q,a_4,a_6)\simeq d_1^{(4)}(\tau)a_4\ , \tag{B29}$$

$$c_6(q,a_4,a_6)\simeq d_1^{(6)}(\tau)a_6\ . \tag{B30}$$

We have numerically computed c_0, c_4, and c_6 for $|a_4|,|a_6|\leqslant 0.3$ and $VN(0)=0.32$. The results are well approximated by the expressions in Eqs. (2.27)–(2.29).

B. Calculation of $\vec{\sigma}(z,\vec{q})$

Let us now calculate

$$\vec{\sigma}(z,\vec{q})\equiv\int_{\mathrm{FS}}d^2k_F\,\rho(\vec{k}_F)\frac{\vec{v}_F(\vec{q}\cdot\vec{v}_F)}{z+(\vec{q}\cdot\vec{v}_F)^2}=\frac{1}{2}\frac{\partial}{\partial\vec{q}}\int_{\mathrm{FS}}d^2k_F\,\rho(\vec{k}_F)\ln[z+(\vec{q}\cdot\vec{v}_F)^2] \tag{B31}$$

which was introduced in Eq. (3.62). To expand (B31) in terms of the cubic harmonics, we note that

$$\int_{\mathrm{FS}}d^2k_F\,\rho(\vec{k}_F)\ln[z+(\vec{q}\cdot\vec{v}_F)^2]=4\pi\sum_{l=0}^{\infty}\sum_{m=-l}^{l}\int_{\mathrm{FS}}d^2k_F\,\rho(\vec{k}_F)\,Y_{lm}^*(\hat{v}_F)\,\tfrac{1}{2}\int_{-1}^{1}dx\,P_l(x)\ln(z+q^2v_F^2x^2)\,Y_{lm}(\hat{q})$$
$$=\sum_l a_l\gamma_l(z,qv_0)H_l(\hat{q})\ , \tag{B32}$$

where

$$\gamma_l(z,qv_0)=\int_0^1 dx\,P_l(x)\ln(z+q^2v_0^2x^2) \tag{B33}$$

for small anisotropy case. Deriving Eq. (B32) we used the formulas (B10) and (B11). Equation (B32) leads to

$$\vec{\sigma}(z,\vec{q})=\tfrac{1}{2}\sum_l a_l\frac{\partial}{\partial\vec{q}}[\gamma_l(z,qv_0)H_l(\vec{q})]\ . \tag{B34}$$

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