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Quasi-particle Spectrum of Nano-scale Superconductors under External Magnetic Field

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Abstract. We have solved the Bogoliubov-de Gennes equation of nano scale superconducting square plate under an external magnetic field. The result shows that multiple-vortex states become stable at low temperature. In these states, vortex bound states of these vortices interfere with each other and the vortices form a kind of vortex molecule, and this is reflected in the local density of states.

Keywords: Nano-scale superconductors, Bogoliubov-de Gennes equation, Finite element method
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INTRODUCTION

Recently, mesoscopic or nanoscopic superconductors have been studied, especially vortices under the magnetic field. Though a single quantized vortex appears in a bulk superconductor, there are a double quantized vortex or giant vortices in a nanoscopic superconductor [1] [2]. Previous studies based on the phenomenological Ginzburg-Landau (GL) equations. In order to consider quasi-particle spectrum, which is accessible by Scanning Tunneling Microscopy or Scanning Tunneling Spectroscopy, we must solve a microscopic equation of superconductivity, Bogoliubov-de Gennes equation, for a superconductor under the perpendicular magnetic field.

BOGOLIUBOV-DE GENNES EQUATION

For a superconductor that has the s-wave symmetry of the order parameter $\Delta(\mathbf{r})$, the Bogoliubov-de Gennes equation is given as,

$$\left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla+\frac{e}{c}\mathbf{A}\right)^2-\mu\right]u_n(\mathbf{r})+\Delta(\mathbf{r})v_n(\mathbf{r})=E_n u_n(\mathbf{r}), \tag{1}$$

$$-\left[\frac{1}{2m}\left(\frac{\hbar}{i}\nabla-\frac{e}{c}\mathbf{A}\right)^2-\mu\right]v_n(\mathbf{r})+\Delta^*(\mathbf{r})u_n(\mathbf{r})=E_n v_n(\mathbf{r}), \tag{2}$$

where $u_n(\mathbf{r})$ and $v_n(\mathbf{r})$ are wave functions of nth quasi-particle state, and E_n is the eigenvalue of them and μ is the chemical potential. We assume that $u=v=0$ at the boundary. The order parameter is given as

$$\Delta(\mathbf{r})=g\sum_{n}^{|E_n|\le E_c}u_n(\mathbf{r})v_n^*(\mathbf{r})(1-2f(E_n)), \tag{3}$$

where g is the interaction constant, E_c is the cut-off energy of the attravtive ineration and $f(E)$ is the Fermi-Dirac distribution function. The vector potential is calculated by the Maxwell equation

$$\nabla\times(\nabla\times\mathbf{A}-\mathbf{H}_0)=\frac{4\pi}{c}\mathbf{j}, \tag{4}$$

where $\mathbf{H}_0$ is external magnetic field and $\mathbf{j}$ is the current in the superconductor, which is defined as

$$\mathbf{j}=\frac{\hbar e}{2mc}\sum_n\left[f(E_n)u_n^*\nabla u_n+(1-f(E_n))v_n\nabla v_n^*-h.c.\right] +\frac{e}{mc}\sum_n\left[f(E_n)|u_n|^2-(1-f(E_n))|v_n|^2\right]\mathbf{A}. \tag{5}$$

We impose particle number conservation,

$$N_e=\int\sum_n\left[f(E_n)|u_n(\mathbf{r})|^2+(1-2f(E_n))|v_n(\mathbf{r})|^2\right]d\mathbf{r}, \tag{6}$$

where N_e is the total number of normal electrons. From Eq. (6), the chemical potential μ is determined.

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For solving above equations, we use the finite element method [3].

We consider a square superconductor, of which length is L. We choose the GL parameter is $\kappa = 3.0$, the order parameter at T=0 is $\Delta/E_c = 0.2$, the Fermi wave number is $k_F\xi = 3.0$, the coherence length is $\xi/L = 0.2$ and the temperature is $T = 0.1T_c$. For Nb, $\xi \approx 200\,\mathrm{nm}$.

RESULT AND DISCUSSION

In the following, we show numerical results for multiple-vortex states. In Fig. 1, we show amplitude of the order parameter for four vortices at $H/L^2 = 13\Phi_0$ and five vortices at $H/L^2 = 14\Phi_0$. Increasing magnetic field, the four vortices configuration appears at $H/L^2 = 10\Phi_0$. Calculating the free energy, it is found that four vortices configuration is a stable state between $H/L^2 = 12\Phi_0$ and $H/L^2 = 15\Phi_0$ and that the five vortices configuration is a metastable state.

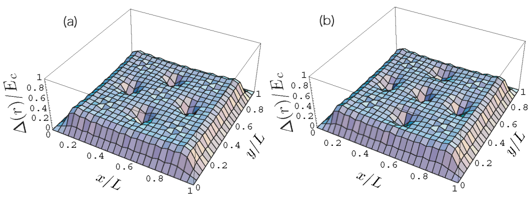

FIGURE 1. Spatial dependence of the order parameter for (a) four vortices and (b) five vortices states.

In Fig, 2, the local density of states (LDOS) of four vortices state is shown. At $E = 0.14E_c$, there are separate

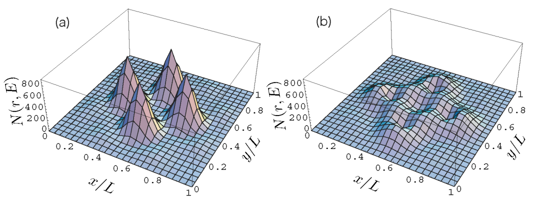

FIGURE 2. Local density of sates of four vortices at (a) $E = 0.14E_c$ and (b) $E = -0.14E_c$.

peaks from bound states of four vortices with angular momentum $m = 0$. At $E = -0.14E_c$, bound states of four vortices have angular momentum $m = 1$, are not separated and vortex bound states interfere mutually.

In Fig. 3, the LDOS of the five vortices state is shown. The peak at the center vortex is different from others. To understand the difference, we investigated five eigenfunctions $u_n(\mathbf{r})$ and $v_n(\mathbf{r})$ made from five vortex bound states. Four of five eigenstates show the interference of bound states, but one eigenstate is same as the bound state of a single vortex at center. At $H/L^2 = 14\Phi_0$, the energy eigenvalue of two kinds of eigenstates are $E = 0.13487E_c$, $E = 0.15677E_c$, $E = 0.15840E_c$ and

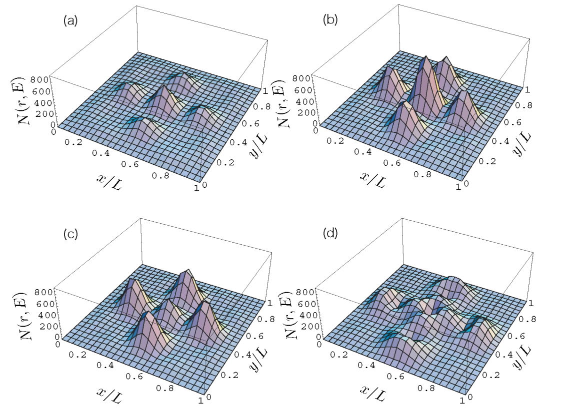

FIGURE 3. Local density of sates of five vortices at (a) $E = 0.20E_c$, (b) $E = 0.17E_c$, (c) $E = 0.13E_c$ and (d) $E = -0.17E_c$.

$E = 0.18368E_c$ for the former states and $E = 0.17601E_c$ for the latter state (and $E = 0.17814E_c$ for the vortex bound state of the single vortex). Therefore the height of peaks of the LDOS at $E \approx 0.17E_c$ becomes different between the center vortex and four vortices around it.

These results show the quasi-particle bound states form a molucular orbital-like states between different vortices. Therefore, we may consider that vortices form a kind of vortex-molecule.

CONCLUSION

We have solved Bogoliubov-de Gennes equation for a nano-scaled superconducting square plate under an external field. We have obtained the order parameter and the local density of states of four vortices and five vortices configurations. They form vortex-molecule-like state.

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