# One-Loop Correction to the Mean Field Theory for the Neutral BCS Superconductors



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# One-Loop Correction to the Mean Field Theory for the Neutral BCS Superconductors

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**Abstract.** To investigate the strong coupling effect in neutral BCS superfluids we construct a theory beyond the Hartree –Fock-Bogoliubov approximation, in which the self-energy correction is incorporated into the single-particle fermionic excitations. We consider the self-energy correction arising from the interaction between the Goldstone boson and the fermions. The Ward-Takahashi relations resulting from the U(1) symmetry are systematically utilized to ensure the consistency between the single- and two-particle channels.

**Keywords:** degenerate Fermi gas**,** neutral BCS super-fluid, self-energy correction.
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## INTRODUCTION

Recent observation of superfluidity in trapped fermion atomic gasses has explored a new challenging field of physics[1-3]. In these systems two neutral fermions of different hyperfine states form a BCS-like Cooper-pair or a molecule which can be condensed into the BEC state at low temperatures. The interaction between fermions in this system is tunable and is strong near the Feshbach resonance. Then, it has a meaning to construct a theory beyond the Hartree-Fock-Bogoliubov (HFB) approximation in the BCS superfluids. In this paper we investigate the self-energy correction to the HFB approximation.

## FORMULATION

In this paper we restrict ourselves to the uniform single-component superfluid system at $T = 0$ described by the conventional BCS Hamiltonian. In the following we use the Nambu's notation for the field operators. The single-particle Green function, which is a 2 by 2 matrix, is defined as

$$S(x-y) = -i < T\Psi(x)\Psi^{+}(y) > , \qquad (1)$$

where $\Psi(x)$ is the Nambu doublet field. The Fourier transform of the Green function is generally expressed in the superfluid state as

$$S^{-1}(p_0, \vec{p}) = p_0 - \varepsilon(\vec{p})\tau_3 - UM\tau_1 - \Sigma(p), \quad (2)$$

where $\tau_1$ and $\tau_3$ are the Pauri matrices, $\varepsilon(\vec{p})$ is the free fermion energy measured from the chemical potential, $U$ is the bare coupling constant, $M$ is the superconducting 'polarization' defined as

$$M = < \psi_{\uparrow}(x)\psi_{\downarrow}(x) > , \qquad (3)$$

and $\Sigma(p)$ is the self-energy function. In eq.(2) the order parameter is assumed real. In neutral superfluid fermionic systems the Goldstone mode appears in the region below twice the energy gap and brings about a pole singularity in the two-particle Green function,

$$D(x-y) = i < T\tau_2(x)\tau_2(y) > , \qquad (4)$$

where

$$\tau_2(x) = \Psi^{+}(x)\tau_2\Psi(x) . \qquad (5)$$

At low temperatures the self-energy correction in the low-energy region is expected to arise from the interaction between the Goldstone mode and the fermions, since the energy gap in the fermionic excitations is large and the Goldstone mode is well defined as the low energy excitation. From the detailed study of the Dyson equation we can show that the self-energy function is expressed in the one-loop order as

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$$\Sigma(p) = -\frac{ig_r^{\,2}}{2}\int \frac{d^4q}{(2\pi)^4}\tau_2 S(p+q)\tau_2 D(q), \quad (6)$$

where we introduce the renormalized coupling constant $g_r$, which is defined later. To evaluate eq.(6) we need the two-particle Green function. To obtain $D(q)$ one has to introduce the vertex function. We use the 3-point vertex function $\Gamma$ defined graphically in Fig.1. From the study of the Ward-Takahashi relation resulting from the U(1) symmetry we can prove the following rigorous relation for the vertex function,

$$S^{-1}(p)\tau_3 - \tau_3 S^{-1}(p) = 2iM\Gamma(p;p;0). \quad (7)$$

This relation can be utilized to ensure the gapless nature of the Goldstone mode in the calculation of $D(q)$, that is, we construct the approximate equation for $D(q)$ that satisfies eq.(7). The details of the calculation for $D(q)$ will be published elsewhere. In solving the above equations it is convenient to introduce the renormalized quantities. We choose a point on the Fermi surface of non-interacting Fermi gas as the renormalization point. i.e., $p_F = (0, \vec{p}_F)$. We define the renormalized coupling constant as

$$\Gamma(p_F; p_F; 0) = g_r\tau_2, \quad (8)$$

and the renormalized gap function and the renormalized chemical potential are defined as

$$\Delta_r = UM + \frac{1}{2}tr[\tau_1\Sigma(p_F)], \quad (9)$$

$$\mu_r = \mu - \frac{1}{2}tr[\tau_3\Sigma(p_F)], \quad (10)$$

In this formulation we can also prove that eq.(9) is rewritten as

$$\Delta_r = g_r M. \quad (11)$$

Let us present the numerical results briefly. In this paper we compare the results with those in the HFB approximation. The numerical procedure is as follows. For given values of $\mu_r$, $g_r$ and $\Delta_r$ we calculate the bare coupling constant $U$. Then, using the obtained bare coupling constant, we calculate the gap in the MFB approximation, $\Delta_{MF}$. In Fig.2 we present two cases as a function of the renormalized coupling constant.

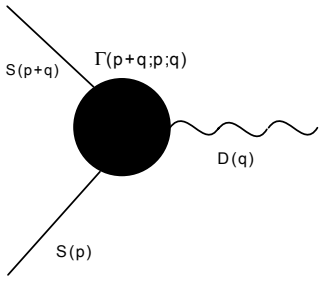


**FIGURE 1.** Definition of the 3-point vertex function. This vertex function describes the correction due to the interaction between the Goldstone mode and the fermions.

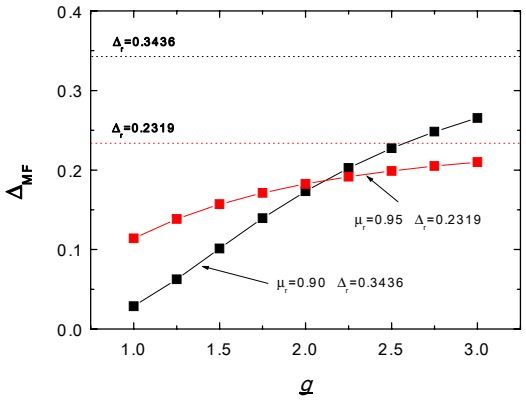


**FIGURE 2.** Comparison with the mean field results. The chemical potential and the gap are normalized by the chemical potential of the non-interacting system.

As seen in this figure, one understands that the self-energy correction due to the interaction between the Goldstone boson and the fermions enhances the gap function. However, as the renormalized coupling constant increases, the mean-field gap function approaches the renormalized one. This result indicates that in the strong coupling limit the mean-field approximation is justified. The details of the numerical results and discussions will be given in a separate paper.

## REFERENCES


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