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Sub- and overcritical stable states of composite high-T_c superconductors with different $E(J)$ dependences and their unavoidable overheating

V. R. Romanovskii[a)]
High Field Laboratory for Superconducting Materials, Institute for Materials Research, Tohoku University, Sendai 980-8577, Japan and Russian Research Center, Kurchatov Institute, Moscow 123182, Russia

S. Awaji, G. Nishijima, and K. Watanabe
High Field Laboratory for Superconducting Materials, Institute for Materials Research, Tohoku University, Sendai 980-8577, Japan

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To understand the underlying physical trends of the current instability in a composite high-T_c superconductor, the limiting margin of its current-carrying capacity is derived in dc magnetic fields in the framework of the macroscopic continuum approximation. A static zero-dimensional model was used to formulate the peculiarities of the nonisothermal electric field distribution in a composite in the fully penetrated current states. The power and exponential equations describing the $E(J)$ dependences of a superconductor are used. The boundary of the allowable stable values of the electric field, current, and temperature are investigated using qualitative and quantitative models. Permissible stable values of the electric field and current, which might be lower (subcritical states) or higher (overcritical states) than those determined by the critical voltage criterion, are discussed. It is stated that the subcritical quenching electric states are more probable in the operating regimes, which are observed in the high magnetic field. The overcritical stable quantities of the electric field exist, for example, if the superconducting composite has a relatively small volume fraction of the superconductor in a composite. In the meantime, the stable current modes may be both subcritical and overcritical when the permissible value of the electric field is overcritical. As a consequence of these features, an unavoidable increase in temperature of the composite superconductor occurs before its transition to the normal state. The latter depends on a broad shape of the $E(J)$ dependence of high-T_c superconductor and the current sharing between the superconducting core and the matrix. In the limiting case, a stable value of the composite temperature may equal the critical temperature of the superconductor. For such operating states, the criterion of the complete thermal stability condition is written taking into consideration the nonlinear character of the $E(J)$ dependence. Simultaneously, an allowable change in temperature of the superconducting composite leads to the thermal degradation of its current-carrying capacity. It depends on the critical current density of the superconductor at bath temperature, amount of a superconductor, and cross section of a composite under fixed cooling conditions. In particular, it is shown that the currents corresponding to the instability onset do not increase proportionally with relevant increase of the superconductor's amount. The estimates presented have general character and may be used to verify the operating states of low-T_c superconducting composite.
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I. INTRODUCTION

The studies of the vortex state stability conditions of type-II superconductors are of renewed interest after the discovery of high-T_c superconductors. These investigations are important for both the characterization of the performances of high-T_c superconducting materials and the understanding of the mechanisms limiting their current-carrying capacity, which has specific conditions of instability onset due to the huge flux-creep regimes.

The instabilities in superconductors and composite conductors based on them (multifilamentary superconductors sheathed by a normal metal) are caused by perturbations of different natures.[1] In particular, the magnetic and current instabilities limit their application. The magnetic instability phenomena in the high-T_c superconductors were intensively studied.[2–11] At the same time, the basic peculiarities of the current instability mechanisms, which limit the current-carrying capacity of high-T_c superconductors, have not been fully investigated.

To estimate the limiting currents that can stably flow in the superconductor without its transition into the normal state, the voltage-current characteristics are widely used. Consequently, one measures the voltage-current characteristic and then defines the critical current density J_c at a given operating temperature and applied magnetic field. The determination of J_c value may be based on various criteria. As a rule, this quantity is defined by a fixed electric field criterion. Usually, it is equal to $E_c=1\ \mu\text{V/cm}$. This technique is based

[a)]Electronic mail: vromanovskii@netscape.net

on Bean's model, which omits the nonlinear part of the voltage-current characteristic. It is a good approximation for low-T_c superconductors with sufficiently steep voltage-current characteristics. However, the voltage-current characteristics of high-T_c superconductors have a broad shape, which does not permit using the critical current criterion to define their true current-carrying capacity.

The nonlinear dependence of the electric field E on the current density J induced in high-T_c superconductors by any external disturbance is due to many reasons: pinning heterogeneity, vortex structure defects, thermal activation of flux, etc. Therefore, there exist various theoretical models explaining the observed $E(J)$ relations of high-T_c superconductors, taking into consideration the microscopic quantities of the superconductor. However, a uniform macroscopic theory describing the dependence of E on J is lacking. Therefore, the phenomenological equations, in particular, the power and exponential relations, are extensively used to describe the macroscopic electromagnetic properties of high-T_c superconductors. Numerous studies (see, for example, Refs. 12–20 and references cited therein) show that the following equations:

$$E = E_c(J/J_c)^n$$

and

$$E = E_c \exp[(J - J_c)/J_\delta]$$

can be used for the $E(J)$ description of both low- and high-temperature superconductors. Here, J_c is the current density at $E=E_c$, n is the creep exponent of $E(J)$ dependence, and J_δ is the creep current density. As known, the power equation corresponds to logarithmic current dependence of the potential barrier when the flux creep is determined by numerous spatial defects of the superconductor. The thermally activated model with a linear current dependence of the potential barrier lies at the basis of the $E(J)$ exponential relation. This model describes the flux-creep state of the superconductor with point defects of its structure. There are also some macroscopic reasons leading to an exponential form of the $E(J)$ relation. In particular, it may result from the bulk heterogeneity of superconducting properties inside the sample. Besides the bulk heterogeneity of critical parameters, the superconductor may have the longitudinal heterogeneity. However, the $E(J)$ relations of these superconductors are also approximated satisfactorily by the power equation.

Using these $E(J)$ approximations, it was shown that the macroscopic electrodynamics behavior of high-T_c superconductors has some particular peculiarities that cannot be explained in the framework of Bean's model (see, e.g., Refs. 20–27 and references cited therein). Moreover, the Bean model does not allow, in principle, explaining the physical reasons underlying the current instability phenomenon even in low-T_c composite superconductors.[17] Since the current instability investigations considering essentially the nonlinear shape of $E(J)$ dependence of high-T_c composite superconductor are not numerous,[28–31] in this paper, we investigate the key static features, which are initially characteristic of the current instability problem, and formulate the general criteria indicating the influence of the parameters of the superconductor and matrix on the stable thermal and current states.

Various types of high-T_c superconductors have been developed. At present, the most promising high-T_c superconductors are the Bi-based cuprates. In particular, $Bi_2Sr_2CaCu_2O_8$ is already superior to low-T_c superconductor at low cooling bath temperature, which allows one to create the high-field magnets, in particular, conduction-cooled ones.[32] Therefore, the peculiarities of the stable static current states of Ag-sheathed $Bi_2Sr_2CaCu_2O_8$ conductor are discussed below, considering the nonlinear $E(J)$ dependence that is described by the power and exponential equations mentioned above.

II. BASIC EQUATIONS

Let us consider an infinitely long composite superconductor. Assume that the applied magnetic induction B is constant and the twist pitch of a superconductor is not very small. In general, the evolution of the temperature and electric field inside the composite superconductor obeys the multidimensional Fourier and Maxwell equations. But this description is mathematically complicated. To understand the basic physical peculiarities of the current instability onset and to evaluate the stability conditions avoiding a large volume of computations, let us investigate the limiting case where the current is charged at an infinitely low rate $(dI/dt \to 0)$ in the fully penetrated regime. To simplify the analysis, let us also assume that (i) the superconductor is evenly distributed over a cross section of a composite with the volume fraction η $(0<\eta<1)$ and the macroscopic continuum approximation can be applied; (ii) the size of the superconducting filament is relatively small and magnetic instability is absent; (iii) the longitudinal magnetic field variation is negligible; (iv) the conduction heat exchange between the composite and the refrigerator occurs on the surface; (v) the transverse conduction heat flux essentially exceeds the heat flux to the coolant; and (vi) the n value of the creep exponent is only a function of the external magnetic field $[n=n(B)]$, as is usually applied in the thermal stability theory (see, for example, Refs. 28–31). The small influence of the temperature dependence of n value on the current stability conditions at low operating temperatures is proved in the Appendix. (vii) The $E(J)$ dependences are described by the relationships mentioned above considering that the current modes under investigation will not essentially exceed the corresponding value of the critical current [namely, the quantity J is not greater than $(1+1/n)J_c$].

Under these assumptions, the static electric field, current, and temperature distributions in the cross section of a composite are approximately uniform. Therefore, in terms of this zero-dimensional model,[1] the temperature of the composite superconductor can be found from the following equation:

$$EJ = \frac{hp}{S}(T - T_0). \tag{1}$$

Here, h is the heat transfer coefficient, p is the cooling perimeter, S is the cross section of a composite, T_0 is the operating temperature, and J is the total transport current density, which is equal to the sum of currents in the superconducting core J_s and matrix J_m, and is defined as follows:

$$J = \eta J_s + (1-\eta)J_m. \tag{2}$$

The steady electric field is generated by the parallel circuit on the superconducting core and matrix according to the relations

$$E = E_c\left[\frac{J_s}{J_c(T,B)}\right]^n = J_m\rho_m(T,B) \tag{3}$$

for the superconductor with the power $E(J)$ dependence and

$$E = E_c \exp\left[\frac{J_s - J_c(T,B)}{J_\delta}\right] = J_m\rho_m(T,B) \tag{4}$$

for the superconductor with the exponential $E(J)$ dependence. Here, ρ_m is the matrix resistivity.

The critical current density J_c depends on the temperature and magnetic field. This dependence can be described by the following relations.

First, to simplify the analysis, the well-known linear temperature-dependent model

$$J_c(T,B) = J_{c0}(B)\frac{T_{cB}(B) - T}{T_{cB}(B) - T_0} \tag{5}$$

will be used. Here, the current density J_{c0} and temperature T_{cB} are the constants at a given value of the applied magnetic induction.

Second, let us calculate the critical current density J_c of Bi-based superconductors in terms of the model reported in Ref. 33 as follows:

$$J_c(T,B) = J_0\left(1 - \frac{T}{T_c}\right)^\gamma\left\{(1-\chi)\frac{B_0}{B_0 + B} + \chi\exp\left[-\frac{\beta B}{B_{c0}\exp(-\alpha T/T_c)}\right]\right\}, \tag{6}$$

summarizing the results which were presented in Refs. 34 and 35. This formula considers the huge flux creep of Bi-based superconductors, which leads to strong temperature degradation of the critical current in the high magnetic fields. The constants T_c=87.1 K, $J_0 = 1.1\times10^6$ A/cm^2, B_{c0}=466 T, B_0=0.0075 T, α=10.3, β=5, γ=1.73, and χ=0.2 were used for Ag/$Bi_2Sr_2CaCu_2O_8$ tape. They were adapted from the data presented in Ref. 33. Formula (6) was used also to calculate the effective values of J_{c0} and T_{cB} by linear fitting to the corresponding nonlinear curves $J_c(T,B)$. For example, it was found that T_{cB}=26.2 K and $J_{c0} = 1.78\times10^5$ A/cm^2 at T_0=4.2 K and B=10 T. Figures 1 and 2(a) show the comparison between the experimental data and calculations, which were made using formulas (5) and (6).

The resistivity of silver as a function of the temperature and the magnetic field (Kohler's rule) was approached using the dependences presented in Refs. 36 and 37. In the simulation the characteristic values of the residual resistivity ratio RRR=ρ_m(273 K)/ρ_m(4.2 K) were used at ρ_m(273 K)=1.48 $\times10^{-6}$ Ω cm according to Ref. 36.

III. QUALITATIVE STATIC STABILITY ANALYSIS OF FULLY PENETRATED STATES

Equations (2)–(4) may be rewritten as

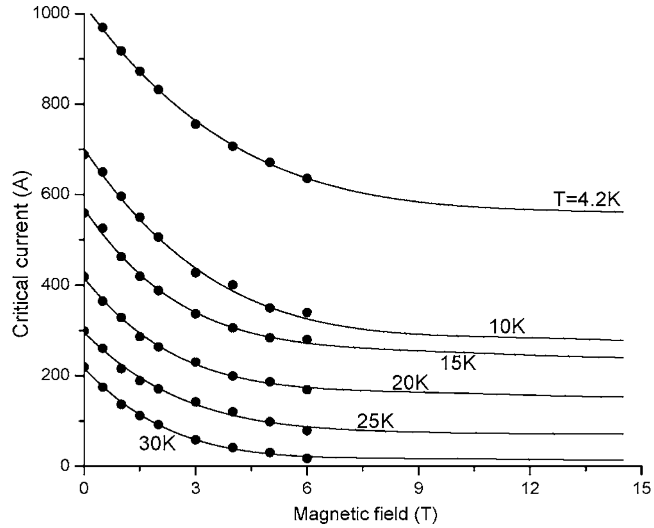

FIG. 1. Critical currents of Ag-sheathed $Bi_2Sr_2CaCu_2O_8$ superconductor vs applied magnetic field after the experiment (●) and fit calculations (—).

$$E = E_c\left\{\frac{J - E[(1-\eta)/\rho_m(T,B)]}{\eta J_c(T,B)}\right\}^n \tag{7}$$

for the superconductor with the power $E(J)$ dependence and

$$E = E_c \exp\left\{\frac{J - E[(1-\eta)/\rho_m(T,B)] - \eta J_c(T,B)}{\eta J_\delta}\right\} \tag{8}$$

for the superconductor with the exponential $E(J)$ dependence. These equations are transformed to the simple formulas when Eq. (5) is used and assuming that $\rho_m(T,B) \approx \rho_m(T_0,B)$=const. This approximation is reasonable in the temperature range up to 20 K (Fig. 3). Eliminating the temperature from Eqs. (7) and (8), the relevant dependence of the current flowing in a composite on the electric field is expressed by the following analytical formulas:

$$J = \frac{\eta J_{c0}(E/E_c)^{1/n} + [(1-\eta)/\rho_m]E}{1 + [\eta J_{c0}SE/hp(T_{cB}-T_0)](E/E_c)^{1/n}} = \eta J_{c0}\frac{(E/E_c)^{1/n} + (E/E_1)}{1 + (E/E_2)(E/E_c)^{1/n}} \tag{9}$$

for the superconductor with the power $E(J)$ dependence and

$$J = \frac{\eta J_{c0} + \eta J_\delta \ln(E/E_c) + [(1-\eta)/\rho_m]E}{1 + [\eta J_{c0}SE/hp(T_{cB}-T_0)]} = \eta J_{c0}\frac{1 + (J_\delta/J_{c0})\ln(E/E_c) + (E/E_1)}{1 + (E/E_2)} \tag{10}$$

for the superconductor with the exponential $E(J)$ dependence. Here,

$$E_1 = \frac{\eta J_{c0}\rho_m}{1-\eta}, \quad E_2 = \frac{hp(T_{cB}-T_0)}{\eta J_{c0}S}.$$

These formulas demonstrate the existence of two characteristic values of the electric field, which are rooted in the formation of the voltage-current characteristic of a composite superconductor. First, the main part of the applied current stably flows in the superconducting core $[\eta J_s \gg (1-\eta)J_m]$ under the condition of

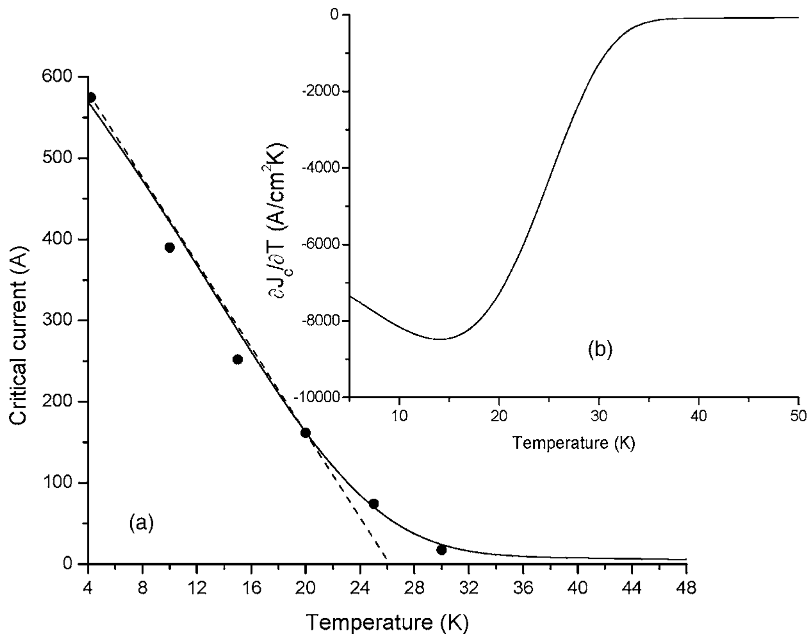

FIG. 2. (a) Critical current and (b) its temperature derivative of a Ag-sheathed $Bi_2Sr_2CaCu_2O_8$ superconductor vs temperature after the experiment (●), fit calculation (—), and linear approximation (----) at $B=10$ T.

$$E \ll E_c\left(\frac{E_1}{E_c}\right)^{n/(n-1)} \quad (11)$$

for the superconductor with the power $E(J)$ dependence and

$$\frac{E}{1+(J_\delta/J_{c0})\ln(E/E_c)} \ll E_1 \quad (12)$$

for the superconductor with the exponential $E(J)$ dependence. They are easily obtained by comparing formula (2) with (9) and (10). Second, the temperature behavior of the voltage-current characteristic of a composite superconductor depends on the second term in the denominator of formulas (9) and (10). Therefore, the nearly isothermal voltage-current characteristic ($T \sim T_0$) is defined by the condition

$$E \ll E_c\left(\frac{E_2}{E_c}\right)^{n/(n+1)} \quad (13)$$

for the superconductor with the power $E(J)$ dependence and

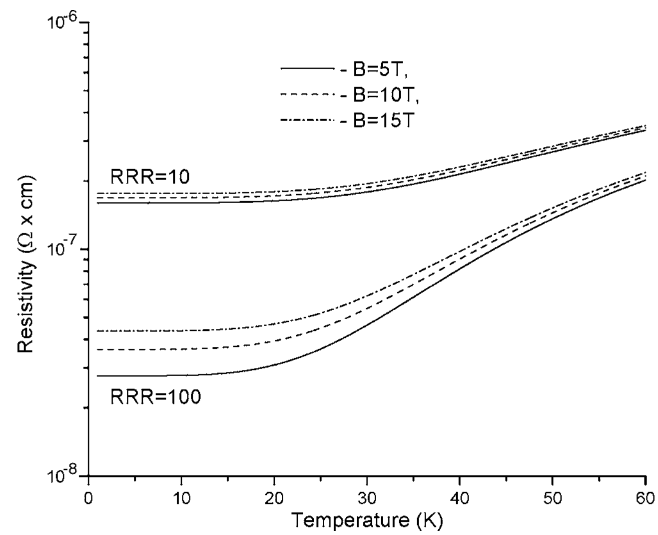

FIG. 3. Resistivity of silver as a function of temperature.

$$E \ll E_2 \quad (14)$$

for the superconductor with the exponential $E(J)$ dependence.

The quantities E_1 and E_2 depend on the composite properties and the cooling power. So, they may satisfy both $E_1 \gg E_2$ and $E_1 \ll E_2$ conditions. Let us estimate the possible values of E_1 and E_2 using the following estimates: $\eta \sim 0.5$, $\rho_m \sim 10^{-7}\ \Omega$ cm, $J_{c0} \sim 10^5$ A/cm^2, $T_{cB}-T_0 \sim 20$ K, $p \sim 0.1$ cm, and $S \sim 10^{-2}$ cm^2. Then $E_1 \sim 10^{-2}$ V/cm and $E_2 \sim 4\times 10^{-6}$ V/cm at the conduction-cooling condition [$h \sim 10^{-3}$ W/(cm^2 K)] Under this condition, the current sharing will have the nonisothermal nature.

If conditions (13) and (14) do not carry out, then the thermal dissipation in a superconducting composite occurs. Consequently, the corresponding temperature variation as a function of electric field is given by

$$T = T_0 + (T_{cB}-T_0)\frac{(E/E_c)^{1/n}+(E/E_1)}{(E/E_c)^{1/n}+(E_2/E)} \quad (15)$$

for the superconductor with the power $E(J)$ dependence and

$$T = T_0 + (T_{cB}-T_0)\frac{1+(J_\delta/J_{c0})\ln(E/E_c)+(E/E_1)}{1+(E_2/E)} \quad (16)$$

for the superconductor with the exponential $E(J)$ dependence. These formulas indicate that the thermal states of the superconductor with the power and exponential $E(J)$ dependences will differ in the high electric field region and at relatively small quantities n.

The written expressions are convenient for the analysis of the validity of different electric field criteria used for describing the critical current measurements, which must be defined at isothermal operating states. Besides, formulas (11) and (12) allow to estimate the effect of the matrix on the voltage-current characteristic of a composite defining the currents, which flow in the superconducting core and the matrix for the given electric field criterion. It is seen that the lower the volume fraction of a superconductor or matrix resistivity, the lower the electric field range at which the current-sharing mechanism will not be observed. This range also decreases with decreasing critical current density of a superconductor. At the same time, according to (13) and (14), the increase in the quantities J_{c0} and η decreases the possible values of the electric field when the stable operating states of the composite are practically isothermal. The existence of the isothermal mode of the composite superconductor depends also on its size: the langer the cross section of the composite, the higher the temperature effect, as shown by formulas (9) and (10).

Using (15) or (16), one can estimate the temperature of the sample during experiments. In particular, these formulas allow one to formulate the condition of the complete thermal stability in the framework of the linear approximation (5). In this case, the possible stable temperature of the composite is equal to the critical temperature of the superconductor T_{cB} and the applied current flows stably only in the matrix. As a result, such states satisfy the following inequalities:

$$\left(\frac{\eta J_{C0}}{E_C^{1/n}}\right)^2\left[\frac{S}{hp(T_{cB}-T_0)}\right]^{1-(1/n)}\left(\frac{\rho_m}{1-\eta}\right)^{1+(1/n)}<1$$

and

$$1-\frac{J_\delta}{J_{c0}}<\frac{E_f}{E_1}$$

for the superconductor with the power and exponential $E(J)$ dependences, respectively. Here, E_f is the solution of the following equation:

$$\frac{E_2}{E_f}-\frac{E_f}{E_1}=\frac{J_\delta}{J_{c0}}\ln\frac{E_f}{E_c}.$$

They have the limiting transition to the inequality

$$\alpha=\frac{\eta^2J_{c0}^2\rho_mS}{hp(1-\eta)(T_{cB}-T_0)}<1$$

at $n\to\infty$ and $J_\delta\to 0$, which is well known as the Stekly stability criterion.[1]

Thus, formulas (11)–(16) give the exact estimates of the current sharing and temperature effects when the applied current stably increases. However, they do not describe the boundary of the stable states. To define the stability parameters, let us define the differential resistivity of the composite, which determines the slope of its voltage-current characteristic in the uniform electric field distribution. According to (9) and (10), it is equal to

$$\frac{\partial E}{\partial J}=\frac{\rho_m}{1-\eta}\frac{[1+(E/E_2)(E/E_c)^{1/n}]^2}{1+(E_1/nE)(E/E_c)^{1/n}-(E_1/E_2)(E/E_c)^{2/n}-(E/nE_2)(E/E_c)^{1/n}} \tag{17}$$

for the superconductor with the power $E(J)$ dependence and

$$\frac{\partial E}{\partial J}=\frac{\rho_m}{1-\eta}\frac{[1+(E/E_2)]^2}{1+(J_\delta/J_{c0})(E_1/E)-(E_1/E_2)[1-(J_\delta/J_{c0})+(J_\delta/J_{c0})\ln(E/E_c)]} \tag{18}$$

for the superconductor with the exponential $E(J)$ dependence.

These formulas indicate that the differential resistivity of the composite may have both positive and negative values. The voltage-current characteristic of the composite superconductor with a positive derivative $\partial E/\partial J$ corresponds to the stable static states and the negative one determines the unstable states during uniform distributions of temperature and electric field. Therefore, it is easy to find the stable and unstable operating regimes by analyzing $\partial E/\partial J$. In principle, formulas (17) and (18) indicate that the current instability in superconductors occurs due to the unavoidable increase in their temperature. Namely, when the quantity E_2 is very large, i.e., when the temperature of the composite is practically constant during current charging and is equal to the cooling bath temperature, then the differential resistivity of the composite is positive in the wide electric field range. As a result, the charged currents will be stable in this electric field range. According to relationship (14), the influence of temperature on the current instability conditions will be redoubled with the increase in the quantities η, J_{c0}, and S under fixed cooling conditions. In other words, there exists a thermal degradation mechanism, which will decrease the current-carrying capacity of superconductors. The effect of η and J_{c0} on the thermal degradation of the current-carrying capacity is discussed below.

Let us discuss the basic static peculiarities that underlie the current instability onset in a composite superconductor with arbitrary temperature dependences of quantities J_c and ρ_m. Assume that the dependence of the density of the current flowing in a superconducting core on the electric field and the temperature may be described by the relation

$$J_s=J_c(T)V(E,T).$$

Here, $V(E,T)$ is the arbitrary function of the electric field and temperature depending on the microscopic mechanisms of vortex states. Let us substitute this relation into Eq. (2) and differentiate it with respect to E. It leads to the following expression:

$$\frac{\partial J}{\partial E} = \eta V \frac{\partial J_c}{\partial E} + \eta J_c \left(\frac{\partial V}{\partial E} + \frac{\partial V}{\partial T}\frac{\partial T}{\partial E} \right) + \frac{1-\eta}{\rho_m} - \frac{1-\eta}{\rho_m^2}\frac{\partial \rho_m}{\partial T}\frac{\partial T}{\partial E}E.$$

Using the additional relations

$$\frac{\partial J_c}{\partial E} = \frac{\partial J_c}{\partial T}\frac{\partial T}{\partial E}, \quad \frac{\partial T}{\partial E} = \frac{JS}{hp} + \frac{ES}{hp}\frac{\partial J}{\partial E},$$

the differential resistivity of a composite superconductor is written as follows:

$$\frac{\partial E}{\partial J} = \frac{1 - \{\eta V(\partial J_c/\partial T) + \eta J_c(\partial V/\partial T) - [(1-\eta)/\rho_m^2](\partial \rho_m/\partial T)E\}(ES/hp)}{[(1-\eta)/\rho_m] + \eta J_c(\partial V/\partial E) + \{\eta V(\partial J_c/\partial T) + \eta J_c(\partial V/\partial T) - [(1-\eta)/\rho_m^2](\partial \rho_m/\partial T)E\}\{\eta J_c V + [(1-\eta)/\rho_m]E\}(S/hp)}.$$

This formula demonstrates that the temperature variation in the $\partial J_c/\partial T$ term affects specifically the onset of the current instability. As known, the $\partial J_c/\partial T$ term is negative in many cases of practical interest and, therefore, the instability happens. However, its absolute value in the case of high-T_c superconductors becomes very small in the intermediate temperature region that is not close to the critical temperature. Figure 2(b) shows the corresponding curve describing the temperature derivative of the critical current density as a function of temperature in the high magnetic fields. As a consequence, the current charging may be stable and the composite temperature may stably increase noticeably in the intermediate temperature range because only one stable branch of the $E(J)$ dependence may exist due to the small value of $|\partial J_c/\partial T|$. This peculiarity depicts the intrinsic advantage of high-T_c superconductors over low-T_c superconductors, which leads to their high current stability. The given conclusions should be emphasized because this feature of high-T_c superconductor's stability is revealed by the static analysis and is not based on the known dynamics stability mechanism of the heat capacity of the superconductor and the matrix. Indeed, the heat capacity usually is used to explain the high stability properties of high-T_c superconductors. At the same time, the advanced stability conditions of high-T_c superconductors in high magnetic fields are also due to their nonlinear temperature dependence of $\partial J_c/\partial T$ term.

Thus, there exist a specific limiting current that is not equal to the critical current of a superconductor, and after which the irreversible transition of a superconductor into the normal state will occur. It is defined by the condition[17]

$$\partial E/\partial J \to \infty. \tag{19}$$

Under this condition, the instability boundary is described by so-called quenching values of the electric field E_q, current density J_q, and temperature T_q. In dimensionless variables, $\varepsilon_q = E_q/E_c$, $i_q = J_q/\eta J_{c0}$, $\theta_q = (T_q - T_0)/(T_{cB} - T_0)$, and within the framework of the linear $J_c(T)$ model defined by Eq. (5), these quantities satisfy the relations

$$\frac{1}{n\varepsilon_2}\varepsilon_q^{1+(1/n)} + \frac{\varepsilon_1}{\varepsilon_2}\varepsilon_q^{2/n} - \frac{\varepsilon_1}{n}\varepsilon_q^{(1/n)-1} = 1,$$

$$i_q = \frac{\varepsilon_q^{1/n} + \varepsilon_q/\varepsilon_1}{1 + \varepsilon_q^{1+(1/n)}/\varepsilon_2},$$

$$\theta_q = \frac{\varepsilon_q^{1+(1/n)} + \varepsilon_q^2/\varepsilon_1}{\varepsilon_q^{1+(1/n)} + \varepsilon_2} \tag{20}$$

for the superconductor with the power $E(J)$ dependence and

$$\frac{\varepsilon_q}{\varepsilon_1} + \delta = \frac{\varepsilon_q}{\varepsilon_2}(1 - \delta + \delta \ln \varepsilon_q),$$

$$i_q = \delta\frac{\varepsilon_2}{\varepsilon_q} + \frac{\varepsilon_2}{\varepsilon_1},$$

$$\theta_q = \delta + \frac{\varepsilon_q}{\varepsilon_1} \tag{21}$$

for the superconductor with the exponential $E(J)$ dependence. Here,

$$\delta = \frac{J_\delta}{J_{c0}}, \quad \varepsilon_1 = \frac{E_1}{E_c}, \quad \varepsilon_2 = \frac{E_2}{E_c}.$$

Formulas (20) and (21) show that the allowable increase in the composite temperature before the onset of instability, which is absent in the Bean model, is always finite because there exist two mechanisms leading to its unavoidable overheating. First, it is a broad shape of the $E(J)$ dependence of the high-T_c superconductors: the lower n or higher δ, the higher the increase in the temperature of the composite. Second, the thermal state of the composite depends on the current sharing, which is a function of the superconductor and matrix properties. The latter is described by the dimensionless parameter ε_1: the higher this value, the lower the current flowing in a matrix and the lower the stable temperature of the superconductor. Therefore, the formation of stable states of the composite superconductor is a result of relevant collective thermal and electrodynamics behavior of the superconductor and the matrix: the higher the charging current, the higher the induced electric field and temperature of the composite before the instability onset.

The connection between values ε_1 and ε_2 should be also noted. One can see that the above-mentioned thermal stability parameter α equals

$$\alpha = \frac{E_1}{E_2}.$$

As known, the physical meaning of α denotes the ratio of the characteristic Joule heat generation in the matrix to the heat flux transferring to the refrigerator.[1] It defines the thermal stability conditions of the composite superconductor with respect to the external temperature disturbances.[38] The introduced quantities E_1 and E_2 show that α is also equal to the ratio of the characteristic electric field identifying the existence of a current-sharing mechanism to the characteristic electric field determining the boundary of the isothermal states of the composite. Usually, the condition $\alpha \gg 1$ takes place in many experiments. Therefore, it corresponds to such thermal states of composite superconductor at which its quenching overheating may occur due to the increase in temperature of a superconductor without current sharing.

Using the above-written formulas (20) and (21), it is easy to find the conditions describing the boundary between stable values of the electric field and current, which might be lower (subcritical regimes) or higher (overcritical regimes) than those determined by the critical voltage criterion. Let us put $\varepsilon_q = 1$. Then the boundary between the subcritical and overcritical values of the quenching electric field is defined by the equation

$$\varepsilon_2 = \begin{cases} \dfrac{1 + n\varepsilon_1}{\varepsilon_1 + n} \\ \dfrac{\varepsilon_1(1-\delta)}{1+\delta\varepsilon_1} \end{cases}$$

for the superconductor with the power and exponential $E(J)$ dependences, respectively. With this value of the parameter ε_2, the relevant quenching currents are equal to

$$i_q = \begin{cases} \dfrac{n}{n+1} + \dfrac{1}{(n+1)\varepsilon_1} \\ 1-\delta, \end{cases}$$

i.e., they are less than the critical current of a superconductor. According to these formulas, the allowable values of the electric field and the current before the instability are subcritical ($E_c > E_q$, $I_c > I_q$) if the operating parameters satisfy the inequality

$$\frac{\eta J_{c0} E_c S}{hp(T_{cB} - T_0)} > \frac{\eta \rho_m J_{c0} + n(1-\eta)E_c}{n\,\eta\rho_m J_{c0} + (1-\eta)E_c}$$

for the superconductor with the power $E(J)$ dependence and

$$\frac{\eta^2 J_{c0}^2 E_c S \rho_m}{hp(T_{cB} - T_0)} > \frac{\eta\rho_m J_\delta + (1-\eta)E_c}{1 - J_\delta/J_{c0}}$$

for the superconductor with the exponential $E(J)$ dependence. If these conditions are broken, the quenching electric field exceeds the critical voltage criterion E_c. However, in these cases, the quenching current may be both subcritical and overcritical. The overcritical current mode ($I_c < I_q$) exists if the condition $E_c < E_{oc} < E_q$ takes place. Here, the value of E_{oc} satisfies the equation

$$1 + \frac{E_c}{E_2}\left(\frac{E_{oc}}{E_c}\right)^{1+(1/n)} = \left(\frac{E_{oc}}{E_c}\right)^{1/n} + \frac{E_{oc}}{E_1}$$

for the superconductor with the power $E(J)$ dependence and equals

$$E_{oc} = \frac{E_1 \delta}{E_1/E_2 - 1}$$

for the superconductor with the exponential $E(J)$ dependence. Consequently, the possible stable operating modes of the high-T_c composite superconductors will have the overcritical values of the electric field ($E_c < E_q$) and the subcritical values of the current ($I_c > I_q$) when $E_c < E_q < E_{oc}$ and

$$\frac{\eta J_{c0} E_c S}{hp(T_{cB} - T_0)} < \frac{\eta \rho_m J_{c0} + n(1-\eta)E_c}{n\,\eta\rho_m J_{c0} + (1-\eta)E_c}$$

for the superconductor with the power $E(J)$ dependence and

$$\frac{\eta^2 J_{c0}^2 E_c S \rho_m}{hp(T_{cB} - T_0)} < \frac{\eta\rho_m J_\delta + (1-\eta)E_c}{1 - J_\delta/J_{c0}}$$

for the superconductor with the exponential $E(J)$ dependence.

The results of the generalized analysis of the current instability onset based on the written formulas are presented in Figs. 4 and 5. They were obtained at $\varepsilon_1 = 10^4$, which is typical for the high-T_c superconducting composite. The values of n and δ were set so that the equality $n = 1/\delta$ was fulfilled.[9] Under this condition, the power and exponential $E(J)$ dependences touch each other at the prescribed point $\{E_c, J_{c0}\}$ and the calculated values of the electric field do not considerably differ. It allows to compare the effect of $E(J)$ dependences used on the simulation results.

Figure 4 shows the influence of the smoothness parameters of $E(J)$ dependences on the subcritical boundary of the admissible quantities of the electric field, current, and temperature at various values of ε_2. (Note that in the framework of the dimensionless analysis performed, the variation of ε_2 is caused by the change of the hp/S term, e.g., due to the possible modification of the heat transfer condition and/or the transverse size of the composite.) In this case, according to relations (20) and (21), the relevant subcritical quenching electric field and current may be estimated as follows:

$$\varepsilon_q \sim \left(\frac{\varepsilon_2}{n}\right)^{n/(n+1)}, \quad i_q \sim \frac{n}{n+1}\left(\frac{\varepsilon_2}{n}\right)^{1/(n+1)} \qquad (22)$$

for the superconductor with the power $E(J)$ dependence and

$$\varepsilon_q \sim \frac{\delta\varepsilon_2}{1-\delta}, \quad i_q \sim 1 - \delta + \delta \ln \frac{\delta\varepsilon_2}{1-\delta} \qquad (23)$$

for the superconductor with the exponential $E(J)$ dependence under the condition $\varepsilon_1 \gg 1$. These subcritical states depending on quantity ε_2 are characterized by the corresponding decrease in the quenching currents and increase in the quenching temperature. Therefore, the electric states of the composite superconductor before the instability onset tend to develop into the overcritical ones when the quality of the superconductor is degraded (with decreasing n or increasing δ) due to the finite increase in a composite temperature be-

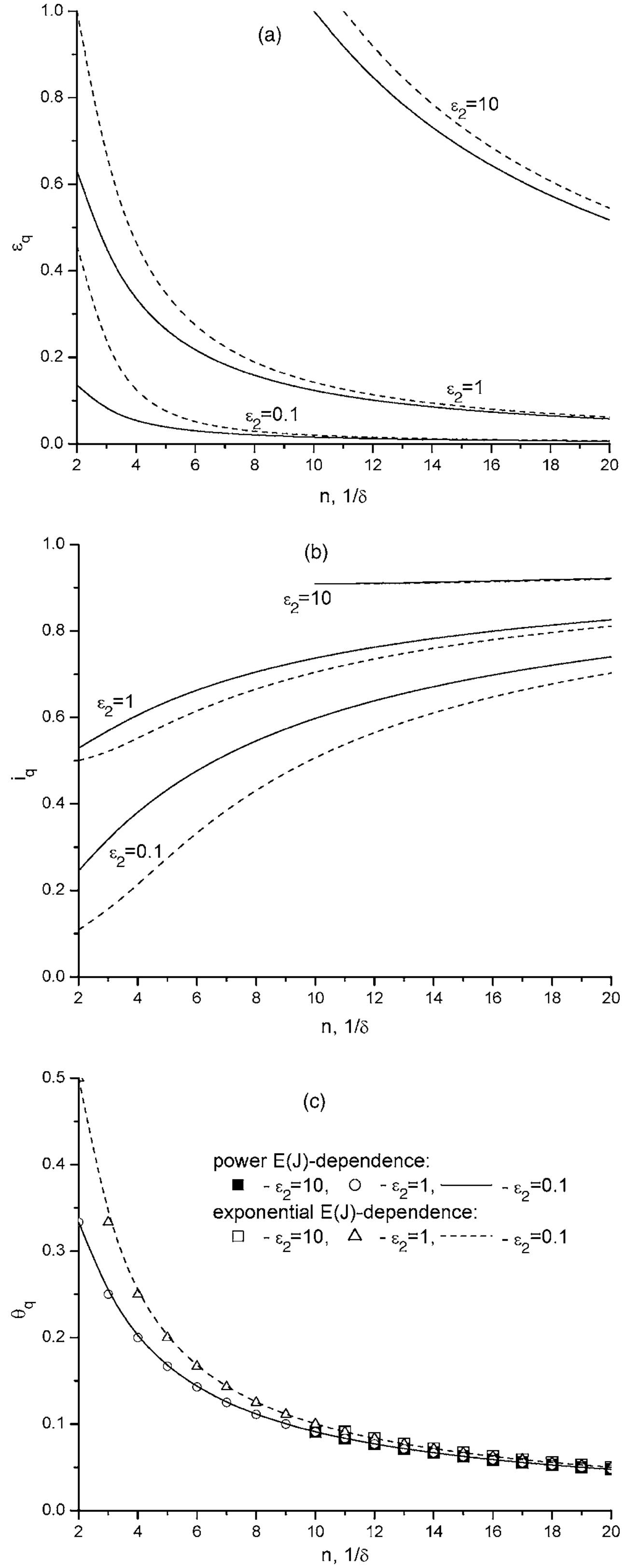

FIG. 4. (a) Stable subcritical electric field, (b) current, and (c) temperature vs smoothness parameters: (—) power $E(J)$ dependence and (----) exponential $E(J)$ dependence.

fore the instability onset. This subcritical overheating of a superconductor is unavoidable owing to a broad shape of $E(J)$ dependences and becomes noticeable when $E(J)$ dependences correspond to strong flux-creep states ($n<10$, $\delta>0.1$). This result shows that the sample may have finite overheating and the current modes may be unstable below the critical point $\{E_c, J_{c0}\}$ in the measurements of the critical currents. The importance of this conclusion should be emphasized because it is usually believed that the temperature of a sample equals the cooling bath temperature and the operating regime of the composite is stable as the charging current and induced voltage do not exceed the boundary defined by the parameters $\{E_c, J_{c0}\}$ used. At the same time, the proper current-carrying capacity of a composite may not satisfy such assumption.

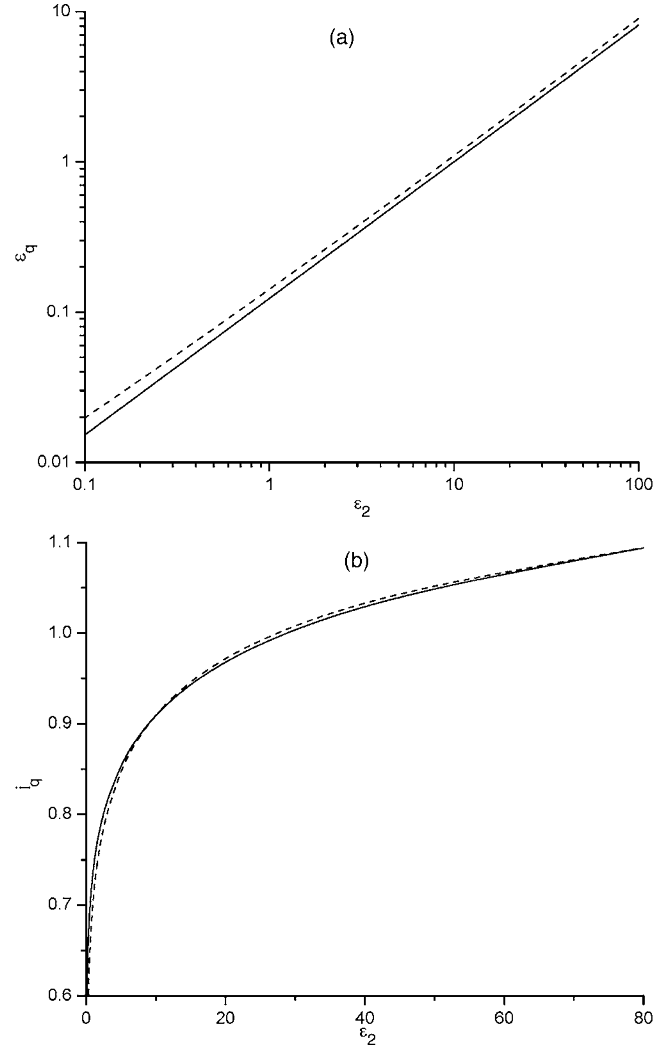

FIG. 5. (a) Stable electric field and (b) current as a function of dimensionless electric field ε_2 at $n=1/\delta=10$: (—) power $E(J)$ dependence and (----) exponential $E(J)$ dependence.

As a whole, the written estimates and Fig. 4 show that the composite superconductors with the power and exponential $E(J)$ dependences will have practically the same current stability conditions in the weak creep range ($n>10$, $\delta<0.1$). In the meantime, the difference increases with decreasing ε_2 due to the relevant temperature effect on the current modes of the composite, as discussed above and following Eqs. (15)–(18). As a result, the superconducting composite with power $E(J)$ dependence is more stable than that with exponential $E(J)$ dependence in the subcritical regimes. However, this tendency depends on the quantity ε_2. The effect of the parameter ε_2 on the operating regimes is depicted in Fig. 5. It indicates the existence of two characteristic regions of the quenching currents. First, the $i_q(\varepsilon_2)$ dependence has an area where the subcritical quenching currents depend essentially on increasing ε_2. These subcritical states are observed at small values of ε_2. In other words, this peculiarity will be detected at nonintensive cooling conditions or when the cross section of the composite superconductor is relatively large. As calculations show, this i_q value range also depends on the matrix resistivity. It is larger when the matrix has lower resistivity. Second, the quenching current has an area where their values do not change sharply with increasing ε_2. Under these conditions, the operating states become overcritical. As a result, in the intensive cooling conditions or in the case of a composite with a small cross section, the stable regimes may have only overcritical values of the electric field or overcritical quantities of the electric field and current.

IV. QUANTITATIVE CURRENT STABILITY ANALYSIS AT LOW OPERATING TEMPERATURE (T_0=4.2 K)

Let us discuss the possible change in the stability boundary of the Ag-sheathed $Bi_2Sr_2CaCu_2O_8$ superconductor. The corresponding quench parameters are depicted in Figs. 6 and 7 as a function of the volume fraction of the superconductor in a composite. (Note that the proposed dimensionless analysis does not allow one to understand directly the influence of η on the stability conditions.) The simulation was made for different resistivities of the matrix at the heat transfer coefficient $h=10^{-3}$ W/(cm^2 K) that is close to the conduction-cooling condition. The parameters of the superconducting composite were set as $n=1/\delta=11$, $S=0.0123$ cm^2, and $p=0.47$ cm. Linear and nonlinear temperature models of the $J_c(T)$ dependence were used. As a result, the curves presented in Fig. 6 were defined according to the written analytical formulas as $\rho_m(T,B)=\rho_m(T_0,B)$. The results plotted in Fig. 7 correspond to the numerical computations based on Eqs. (1) and (6)–(8) and condition (19) at which the resistivity of silver has taken into account its dependence on the temperature and magnetic field.

Figure 6 depicts the influence of $E(J)$ type and quantities J_{c0} and RRR on the allowable values of the electric field, temperature, and current before the instability onset that leads to the overcritical regimes for practically all operating modes considered. The calculations were made under the assumption that $B=10$ T, at which $\rho_m(T_0,B)=1.685\times10^{-7}$ Ω cm.

As follows from Figs. 6(a) and 6(b), the quenching values of the electric field and temperature increase with decreasing η. Therefore, the subcritical states are possible at high values of the volume fraction coefficient. Besides, Fig. 6(b) also indicates the effect of the broad shape of $E(J)$ dependences and current-sharing mechanism on thermal modes of the composite, which will be observed at the variable quantity η. It is seen that there are two characteristic regions of $T_q(\eta)$ dependence. Firstly, it has an area where the quench temperature depends weakly on η, particularly, when J_{c0} is high. The overheating of the composite in these states is mainly the result of the smooth nature of the $E(J)$ dependence. The second characteristic area existing due to the current sharing is the region where the change in η influences essentially the allowable increase in temperature. The

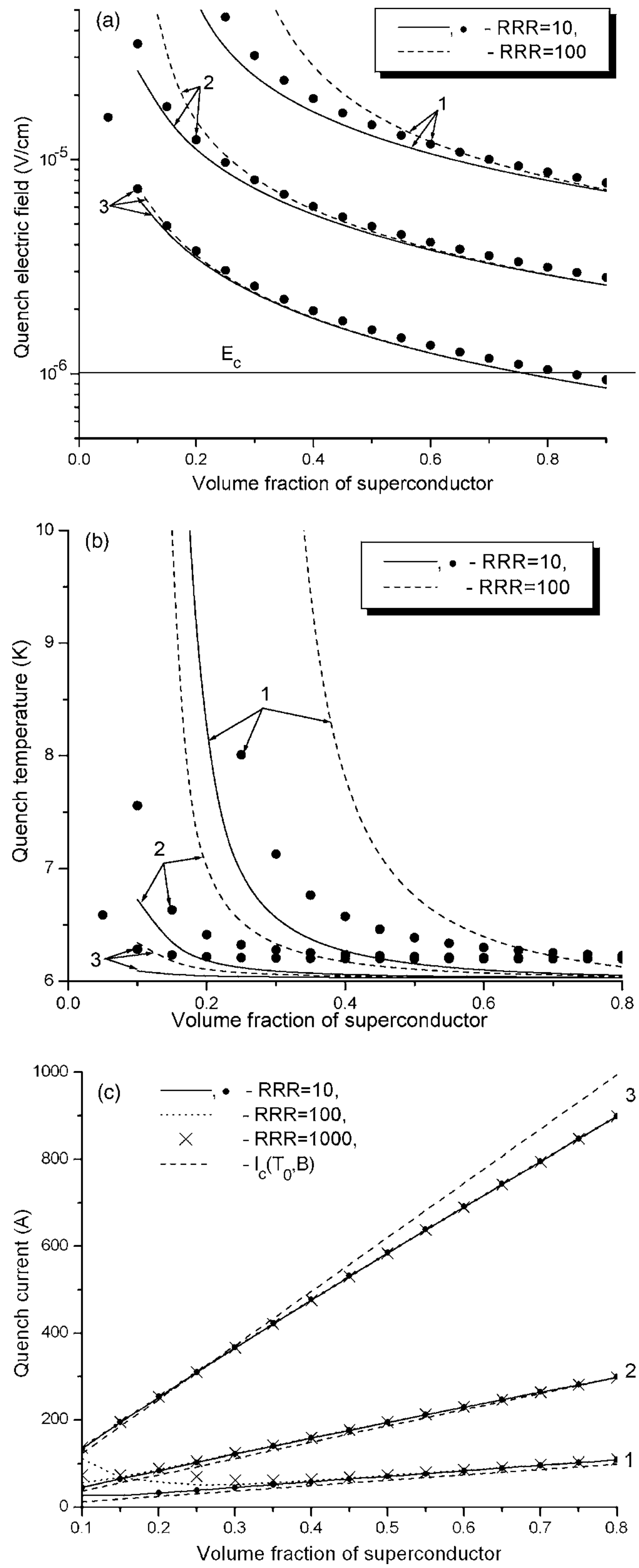

FIG. 6. (a) Stable electric field, (b) temperature, and (c) current vs volume fraction of superconductor with various $E(J)$ dependences [—, ○, ×: power $E(J)$ dependence; ●: exponential $E(J)$ dependence] under linear approximation (5): 1–$J_{c0}=10^4$ A/cm^2, 2–$J_{c0}=3\times10^4$ A/cm^2, and 3–$J_{c0}=10^5$ A/cm^2.

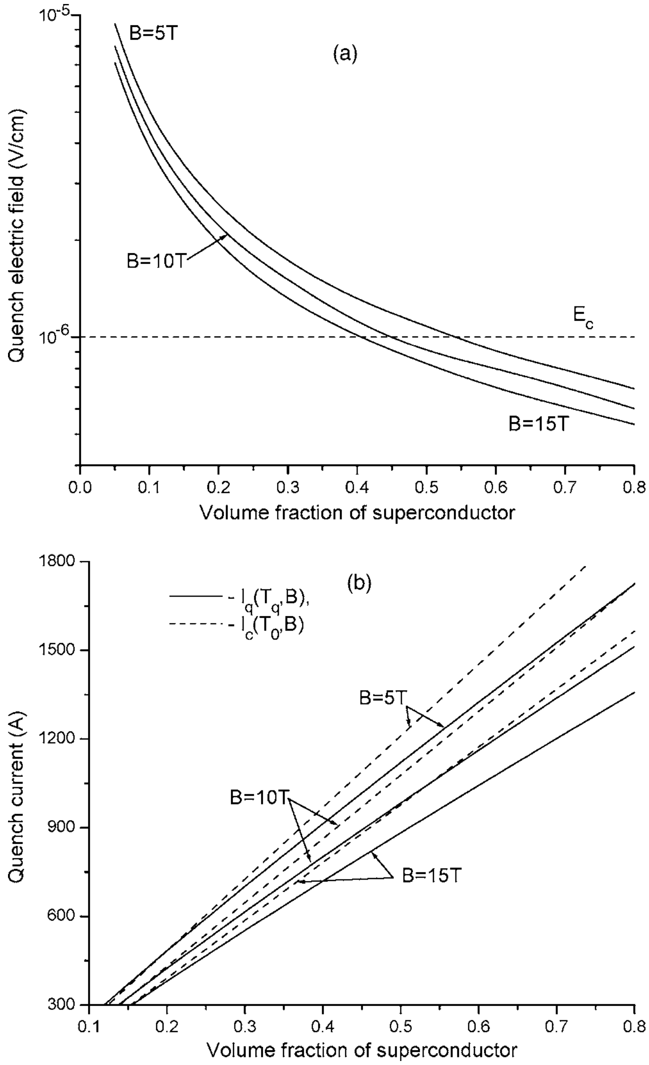

FIG. 7. (a) Stable electric field and (b) current as a function of volume fraction of a superconductor at various values of the applied magnetic field defined in the framework of the nonlinear temperature dependences J_c and ρ_m.

amount of the superconductor where these areas exist depends on the matrix resistivity, critical current density, and, as it is easy to understand, the cooling conditions. As a whole, the dominant role is played by the second area at a relatively low value of J_{c0}. In these cases, the allowable electric field and temperature before the instability onset may be high, namely, $E_q > 10^{-5}$ V/cm, and the quench temperature may exceed 10 K under the conduction-cooling conditions. For such superconducting composites, the allowable rise of the electric field and overheating becomes not only higher but shows a more drastic increase, as has been discussed above. Simulations demonstrate that these peculiarities are valid when the nonlinear model $J_c(T)$ is considered. These overcritical operating states having high values of the allowable electric field and overheating were experimentally observed in Refs. 29 and 39–41.

The η dependence of the current stability boundary is plotted in Fig. 6(c). It is seen that $I_q(\eta)$ dependences may have a minimum. The existence of the volume fraction $\eta_{\min}$, at which the minimum I_q exists, depends on the matrix resistivity and the value of J_{c0}. At the parameters considered, the value of $\eta_{\min}$ displaces to the area of lower values. The simulated analysis reveals that the existence of the minimum value of I_q is a result of the current-sharing mechanism. That is why, for these operating modes, the allowable increase in the temperature and electric field before the instability may be high. As a whole, Fig. 6(c) shows that the matrix resistivity does not practically change the quenching currents of a composite superconductor with the high value of the critical current under the conduction-cooling conditions. However, it should be noted that the value of the critical current density at the cooling bath temperature affects the region where sub- and overcritical currents exist. As a result, the difference between the critical and quench currents increases with increasing J_{c0} according to estimates (13) and (14).

Figure 6 also affirms the above-discussed tendency, which underlies the possible difference between the stability conditions of superconductors with the power and exponential $E(J)$ dependences. It is seen that the difference between the allowable increase in temperatures of such composite superconductors becomes more visible in the range of small values of η. This regularity follows from relationships (15) and (16), which indicate that the difference in the thermal states of superconducting composites with the power and exponential $E(J)$ dependences will be observed in the high electric field. The given conclusion should be taken into account when the equation of the $E(J)$ dependences is recovered from the experiments.

Figure 7 demonstrates the magnetic field effect on the subcritical and overcritical regimes of the composite superconductor with the power $E(J)$ dependence and high critical current. The latter also is depicted in Fig. 7(b). It is seen that the higher the magnetic field, the higher the range of η where the subcritical electric fields exist. In this case, the possible quenching currents tend to be lower than the corresponding value of the critical currents in a wide range of the η value. As a whole, the change in the depicted $I_q(\eta)$ curves are due to the following reasons. First, the high critical current density of the composite superconductor considered leads to current states at which the current sharing occurs only at small values of the volume fraction coefficient. Owing to this peculiarity, the quenching currents may exceed the critical ones, as discussed above, when the amount of superconductor is small. Second, the overheating of the composite superconductor is not zero due to the broad shape of the $E(J)$ dependence. This overheating leads to the relevant thermal degradation of the composite current-carrying capacity caused by the variation of the volume fraction coefficient. Accordingly, the following regularity is observed: the higher the volume fraction coefficient, the higher the difference between the critical and quenching currents. Therefore, the quenching currents do not increase proportionally to the proportional increase in the volume fraction coefficient. In particular, $I_q' = 424.6$ A and $I_q'' = 1512.7$ A at $\eta' = 0.2$ and $\eta'' = 0.8$, respectively, at $B = 10$ T, i.e., $\eta''/\eta' = 4$ but $I_q''/I_q' = 3.56$. According to formulas (9) and (10), this η degradation of the current-carrying capacity of the composite takes place due to the finite value of the characteristic quantity E_2 that decreases with increasing η. As a result, the influence of unavoidable overheating of the composite on the stable values of charging current increases despite the small overheating of the composite at high values of η.

V. CONCLUSIONS

The current instability problem has been investigated in the macroscopic static approximation for the composite superconductors having the power and exponential $E(J)$ dependences. The results of this study show that the possible stable increase in temperature of high-T_c composite superconductors should be taken into account for correct investigation of their critical currents and current stability conditions. As a result, the following peculiarities take place in the operating state formation of high-T_c superconducting composites:

(1) The electrodynamics states of high-T_c composite superconductors having power and exponential $E(J)$ dependences may not be equivalent. The noticeable difference will be seen in strong creep states ($n<10$, $J_\delta/J_{c0}>0.1$) due to the corresponding difference in the stable increase in temperature of a superconductor.

(2) The formulated conditions describing the boundary of the stable subcritical and overcritical states of superconducting composite with the nonlinear $E(J)$ dependences show that the subcritical electric fields (stable electric fields that cannot exceed the fixed critical quantity) are more probable in the high magnetic fields or when a composite with the high-resistivity matrix has a relatively high value of the volume fraction of the superconductor. In this case, the stable value of the current flowing in a composite may be lower or higher than those defined by the critical voltage criterion.

(3) The written analytical expression defining the differential resistivity of a composite with arbitrary temperature dependences of the critical current density proves that not only the negative value of dJ_c/dT but also its temperature variation in the temperature range, which is not close to the critical temperature of a superconductor, particularly affect the current instability onset. By that, the current instability in Bi-based composite superconductors may be absent at intermediate operating temperatures.

(4) The critical current density of a superconductor at coolant temperature, the volume fraction coefficient, and composite cross section or smoothness character of $E(J)$ dependences affect the range of the allowable rise in the electric field at which the operating state of a composite is practically isothermal.

(5) The unavoidable overheats of the superconductor before its transition to the normal state may be noticeable, in particular, when the composite has a low-resistivity matrix or at a relatively small value of the volume fraction coefficient. In these cases, the current sharing underlies the current-carrying capacity of a composite that may lead to minimum quenching currents. The overheating peculiarity takes place as well at subcritical regimes owing to a broad shape of the $E(J)$ dependence. This overheating becomes essential when the superconductor has strong flux-creep states. Therefore, the temperature of the composite is not equal to the cooling bath temperature before the instability onset at both subcritical and overcritical modes.

(6) The unavoidable overheats of the superconducting composite lead to the existence of the thermal degradation mechanism, which results to the relevant decrease in the composite's current-carrying capacity. As a result, the quenching currents do not increase proportionally to the proportional increase in the critical current of the superconductor.

The features discussed must be considered to make the correct investigation of the current instability problem of Bi-based composite superconductors.

ACKNOWLEDGMENT

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APPENDIX

Let us estimate the possible effect of the temperature dependence of n index on the conditions of the current instability onset, investigating the composite differential resistivity and assuming that $\rho_m(T,B)\approx\rho_m(T_0,B)=\text{const}$. The set of Eqs. (2) and (3) may be rewritten as follows:

$$J=\eta J_c(T)\left(\frac{E}{E_c}\right)^{1/n(T)}+\frac{1-\eta}{\rho_m}E.$$

Differentiating this equation with respect to E, one can get the following expression:

$$\frac{\partial J}{\partial E}=\eta\frac{\partial J_c}{\partial E}\left(\frac{E}{E_c}\right)^{1/n(T)}+\eta J_c\frac{\partial}{\partial E}\left(\frac{E}{E_c}\right)^{1/n(T)}+\frac{1-\eta}{\rho_m}.$$

Using the additional relations

$$\frac{\partial J_c}{\partial E}=\frac{\partial J_c}{\partial T}\frac{\partial T}{\partial E},\quad \frac{\partial T}{\partial E}=\frac{JS}{hp}+\frac{ES}{hp}\frac{\partial J}{\partial E},$$

$$\frac{\partial}{\partial E}\left(\frac{E}{E_c}\right)^{1/n(T)}=\left(\frac{E}{E_c}\right)^{1/n(T)}\left\{\frac{1}{nE}+\frac{d}{dT}\left[\frac{1}{n(T)}\right]\frac{\partial T}{\partial E}\ln\frac{E}{E_c}\right\},$$

the composite differential resistivity is defined by

$$\frac{\partial E}{\partial J}=\frac{1-\eta(ES/hp)(E/E_c)^{1/n(T)}(dJ_c/dT)\{1+[J_c/(dJ_c/dT)](d/dT)[1/n(T)]\ln(E/E_c)\}}{[(1-\eta)/\rho_m]+(\eta/n)(J_c/E)(E/E_c)^{1/n}+\eta(JS/hp)(E/E_c)^{1/[n(T)]}(dJ_c/dT)\{1+[J_c/(dJ_c/dT)](d/dT)[1/n(T)]\ln(E/E_c)\}}.$$

Thus, the effect of the temperature-dependent n index depends on the term

$$\Psi(T)=\frac{J_c}{(dJ_c/dT)}\frac{d}{dT}\left[\frac{1}{n(T)}\right]\ln\frac{E}{E_c}.$$

To estimate it, let us use the known relationship

$$n(T)=\frac{U_0(T,B)}{k_BT}.$$

Here, k_B is the Boltzmann constant and U_0 is the pinning potential, which in general form may be written as

$$U_0(T,B)=U_{00}(B)\left[1-\left(\frac{T}{T_{cB}}\right)^m\right]^p,$$

where m and p are the constants. Then it is easy to obtain that

$$\Psi(T)\sim\frac{k_BT_{cB}}{U_{00}(B)}.$$

Consequently, the temperature dependence of n index will have a small influence on the current stability conditions when the temperature of a composite is not too close to the critical temperature of a superconductor because the quantity U_{00} is very large.

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