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著者	渡辺　和雄
journal or publication title	IEEE Transactions on Applied Superconductivity
volume	17
number	2
page range	3133-3136
year	2007
URL	http://hdl.handle.net/10097/47178

doi: 10.1109/TASC.2007.898320

Steady and Unsteady Current Modes and Thermal Runaway Conditions of High-T_C Composite Superconductors

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***Abstract*—Transport current properties of the steady and unsteady operating modes and thermal runaway conditions of the Ag-sheathed $Bi_2Sr_2CaCu_2O_8$ composite superconductors are theoretically studied in DC magnetic fields. It is shown that the temperature of a composite is not equal to the coolant temperature before thermal runaway. As a result, the shape of the voltage-current characteristics of high-T_c superconducting composites has only a positive slope during continuous current charging both before and after thermal runaway. That is why the voltage-current characteristics of high-T_c superconductors do not define the boundary of the thermal runaway. This peculiarity has to be considered during experiments at which the critical or quench currents are defined.**

***Index Terms*—Applied current, composite superconductor, high temperature superconductors, overheating, thermal runaway.**

I. INTRODUCTION

As known, one of the main parameters required to design a superconducting magnet is the current-carrying capacity of superconductors. It is an operating current which can stably flow in a superconductor without its transition into the normal state. To estimate this value, the voltage-current characteristics (VCC) of a superconducting composite are usually used. In this case, the critical current I_c corresponding to the fixed electric field criterion, for example $E_c = 1\ \mu\mathrm{V/cm}$, is determined at a given operating temperature and external magnetic field. It is a good approximation for low-T_c superconductors with sufficiently steep VCC which are practically isothermal. However, the VCC of the high-T_c superconductors have a broad shape. As a result, the stable regimes of high-T_c superconductors may be non-isothermal, as it was proved experimentally and theoretically in [1]–[8]. However, characteristic thermal features of the VCC-formation have not been discussed in detail yet. Therefore, this study focuses on the investigation of the temperature effect on the formation of the steady and unsteady current modes of a Ag-sheathed $Bi_2Sr_2CaCu_2O_8$ composite superconductor.

Manuscript received August 28, 2006. This work was supported by the Grant-in-Aid from the Ministry of Education, Science and Technology, Japan.

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Digital Object Identifier 10.1109/TASC.2007.898320

II. MODEL

To discuss the basic properties of the thermal runaway phenomenon, let us consider a superconducting composite with a slab geometry ($-a < x < a$, $-\infty < y < \infty$, $-b < z < b$, $b \gg a$) under the continuum model. Let the slab be placed in an external magnetic field parallel to its surface in the z-direction, which has penetrated over its cross section ($S = 4ab$). Suppose that the applied current is charged in the y-direction increasing linearly from zero with the constant sweeping rate dI/dt, and its self field is negligibly less than the external magnetic field. Let us describe the VCC of a superconductor by a power law. Assume also that the magnetic field variation along the composite is negligible; the superconducting filaments having the transverse sizes, which do not lead to the magnetic instability, is evenly distributed over a cross section of a composite with the volume fraction coefficient η. According to these assumptions, the transient equations, which is independent of z and y coordinates and have the mirror symmetry, is defined as follows

$$C(T)\frac{\partial T}{\partial t} = \frac{\partial}{\partial x}\left(\lambda(T)\frac{\partial T}{\partial x}\right) + EJ, \quad \mu_0\frac{\partial J}{\partial t} = \frac{\partial^2 E}{\partial x^2} \quad (1)$$

where the total current density $J(x,t)$ depends on the currents flowing in the superconducting core J_s and the matrix J_m and is equal to

$$J = \eta J_s + (1-\eta)J_m \quad (2)$$

J_s and J_m satisfy the following relationship

$$E = E_c\left(\frac{J_s}{J_c(T,B)}\right)^n = J_m\rho_m(T,B) \quad (3)$$

The initial and boundary conditions may be given by

$$\begin{aligned} &T(x,0) = T_0, \quad E(x,0) = 0,\\ &\frac{\partial T}{\partial x}(0,t) = 0, \quad \lambda\frac{\partial T}{\partial x}(a,t) + h\left[T(a,t) - T_0\right] = 0\\ &\frac{\partial E}{\partial x}(0,t) = 0, \quad \frac{\partial E}{\partial x}(a,t) = \frac{\mu_0}{4b}\frac{dI}{dt} \end{aligned} \quad (4)$$

Here, C and λ are the specific heat capacity and thermal conductivity of a composite, respectively; h is the heat transfer coefficient; T_0 the cooling bath temperature, ρ_m the matrix resistivity; n the power law exponent of the $E - J$ curve; E_c the voltage criterion defining the critical current density of a superconductor. The dependence of the critical current on the temperature and magnetic field may be approximated in compliance with the linear relationship

$$J_c(T,B) = J_{c0}(B)\frac{T_{cB}(B)-T}{T_{cB}(B)-T_0} \tag{5}$$

where J_{c0} and T_{cB} are constant at the given external magnetic field B.

The finite difference method was used to solve the subjective above-mentioned. The simulation was made for a silver-sheathed $Bi_2Sr_2CaCu_2O_8$ superconductor at $T_0 = 4.2$ K assuming that the background magnetic field is equal to $B = 10$ T and the heat removal conditions on the surface of the composite are close to the conduction-cooling power applying $h = 10\ \mathrm{W/(m^2 \cdot K)}$. The following constants $n = 10$, $\eta = 0.2$, $\mathrm{RRR} = \rho_m(273\ \mathrm{K})/\rho_m(4.2\ \mathrm{K}) = 10$, $\rho_m(273\ \mathrm{K}) = 1.48 \times 10^{-8}\ \Omega \cdot \mathrm{m}$, $E_c = 10^{-4}$ V/m, $T_{cB} = 26.1$ K, $J_{c0} = 1.52 \times 10^8\ \mathrm{A/m^2}$ were set. The specific heat capacity of a composite was defined by the relation

$$C(T) = \eta C_s(T) + (1-\eta)C_m(T) \tag{6}$$

considering the relevant heat capacities of the superconductor C_s and the matrix C_m based on the additive law. Here, the temperature-dependent specific heat capacity of $Bi_2Sr_2CaCu_2O_8$ was given as [7]

$$C_s[\mathrm{J/m^3K}] = \begin{cases} 58.5T + 22T^3, & T \le 10\ \mathrm{K} \\ -10.54 \times 10^4 + 1.28 \times 10^4 T, & T > 10\ \mathrm{K} \end{cases} \tag{7}$$

The specific heat capacity of silver C_m was calculated in accordance with [9]. Its resistivity was approached by the relations proposed in [9], [10]. The matrix resistivity was also used to calculate the thermal conductivity of the composite in its transverse direction. It is written in the form of

$$\lambda(T) = 2.45 \times 10^{-8} T(1-\eta)/(1+\eta)/\rho_m(T,B) \tag{8}$$

according to the Wiedemann-Franz and additive laws.

III. Results

The variation of the electric field and temperature inside a superconducting slab having various thicknesses is shown in Fig. 1. The simulation was made for the partially and fully penetrated regimes at the fixed width ($b = 2.45 \times 10^{-3}$ m) and the current charging rate $dI/dt = 10$ A/s under the condition $\eta = \mathrm{const}$. Solid curves were obtained using (1)–(8). Dashed curves correspond to the static uniform state in the slab by the infinitely slow current charging ($dI/dt \to 0$). In this case, the set of (1) has the limiting transition to the zero-dimensional model [6]–[8] described by the equation

$$EJ = \frac{h}{a}(T - T_0) \tag{9}$$

together with the relationships (2) and (3).

The presented curves denote the following peculiarities of the electric and thermal states taking place during the current charging.

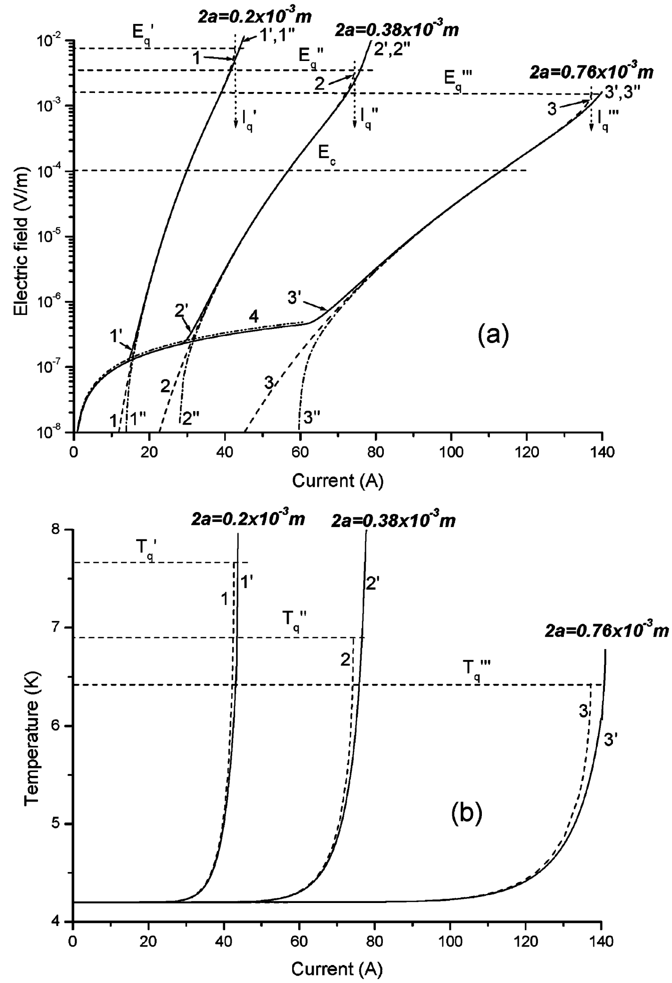

Fig. 1. Increase in the electric field (a) and temperature (b) during the current charging: 1, 2, 3—steady zero-dimensional state, 1′, 2′, 3′—unsteady one-dimensional states at $x = a$ (*surface*), 1″, 2″, 3″—unsteady one-dimensional state at $x = 0$ (*center*), 4—scaling approximation defined by eq. (10).

For understandable reasons, the current mode during incomplete unsteady states has the non-uniform distribution of the electric field. Therefore, the results obtained with unsteady one-dimensional and steady zero-dimensional models are different at the non-uniform stage. The calculations show that the temperature of the composite during this mode is close to the cooling bath temperature and the main part of the induced current flows in a superconductor. Thereby, the development of the unsteady incomplete penetration modes may be estimated using the scaling model proposed in [11]. Consequently, the electric field distribution inside the composite superconducting slab and the moving current penetration boundary x_p may be described by the approximate formulae

$$E(x,t) = \begin{cases} 0, & 0 \le x \le x_p \\ \frac{\mu_0}{4b}\frac{dI}{dt}(x - x_p), & x_p \le x \le a \end{cases}$$
$$x_p(t) = a - \left(\frac{n+1}{n}t\right)^{\frac{n}{n+1}} \left(\frac{\mu_0}{4b}\frac{dI}{dt}\right)^{\frac{n-1}{n+1}} \times \left[\frac{E_c}{\mu_0^n \eta^n J_{c0}^n}\right]^{\frac{1}{n+1}} \tag{10}$$

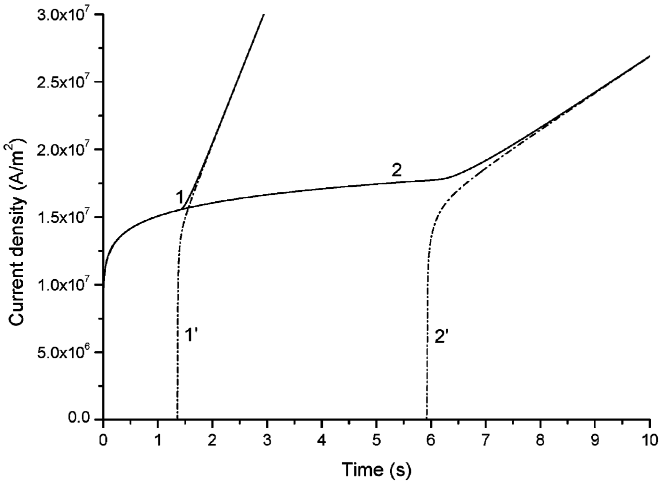

Fig. 2. Current density versus time on the surface of the slab (curves 1, 2) and in its centre (curves 1′, 2′) with different values of the thickness: 1, 1′—a $= 10^{-4}$ m; 2, 2′—a $= 0.19 \times 10^{-4}$ m.

in the weak flux-creep regimes ($n \geq 10$). Then the corresponding value of the induced electric field on the surface of the slab at the end of the incomplete penetration mode may be estimated as

$$E_f = \frac{\mu_0 a}{4b}\frac{dI}{dt} \tag{11}$$

This formula allows one to estimate the current evolution in the superconducting slab just after the incomplete penetration mode. Indeed, the current redistribution in the cross-section of the slab depends on the term $\partial^2 E/\partial x^2 \sim E_f/a^2$. So, the duration of the non-uniform character of the current distribution in a slab, which exists in the initial stage of the complete penetration mode, becomes more continued in the slab having the higher values of the thickness (Fig. 2).

These unsteady features of VCC-formation during the initial stage show that the low electric field (below about 10^{-5} V/m) should be carefully used to choose the voltage criterion allowing one to find the critical current of a superconductor during continuous current charging. To estimate the minimum voltage drop V_f below which the voltage criterion is not acceptable, one can use the scaling approximation. Then it is given by

$$V_f = \frac{\mu_0 a^2}{2b}\frac{dI}{dt}$$

After the transient incomplete and complete penetration modes, the evolution of the thermoelectric states has the following stages.

First, there is the current range at which the stable electric field distribution is close to the steady zero-dimensional one. Clearly, it occurs, if the condition $ha/\lambda \ll 1$ takes place and the current charging rate is relatively small. This stable mode precedes the onset of the current instability and is characterized by the steady growth of the electric field and, therefore, the temperature [Fig. 1(b)]. As a result, the high voltage range exceeding about 10^{-4} V/m should be carefully used to define the critical current of a superconductor due to its unavoidable stable increase in the temperature of a composite, which may lead to the thermal runaway. As it was shown in [6], the quenching values of the electric field, current and temperature follow from the conditions $\partial E/\partial J \to \infty$ or $\partial T/\partial J \to \infty$. When the quenching temperature is relatively small under which $\rho_m \sim const$, the relevant thermal runaway current I_q is described by the formula [6]

$$I_q = \frac{\frac{\eta J_{c0}}{E_c^{1/n}}E_q^{1/n} + \frac{1-\eta}{\rho_m}E_q}{1 + \frac{J_{c0}}{E_c^{1/n}}\frac{\eta S}{hp(T_{cB}-T_0)}E_q^{\frac{n+1}{n}}}S$$

where the quenching electric field E_q is the solution of the equation

$$\frac{\eta J_{c0}}{nE_c^{1/n}}E_q^{\frac{1-n}{n}} - \left(\frac{\eta J_{c0}}{E_c^{1/n}}\right)^2 \frac{S}{hp(T_{cB}-T_0)}E_q^{\frac{2}{n}} = \frac{1-\eta}{\rho_m}\left[\frac{\eta J_{c0}}{nE_c^{1/n}}\frac{S}{hp(T_{cB}-T_0)}E_q^{\frac{1+n}{n}} - 1\right]$$

The corresponding quenching values are indicated in Fig. 1 by the dotted curves.

Second, the fully penetrated parts of the curves depicted in Fig. 1, which were calculated in the framework of the one-dimensional model, are not equivalent to the static ones near the stability boundary defined by the zero-dimensional model. The main reason of the difference between steady and unsteady parts of the VCC has been discussed in [8]. It has been proved that the temperature-dependent heat capacity of a high-T_c superconducting composite affects its high voltage parts of the VCC in the continuous current charging. Therefore, the slope of the $E(I, dI/dt)$ and $T(I, dI/dt)$ curves become smaller when the heat capacity of a composite is higher at any finite value of the current charging rate. This unsteady feature occurs because of the finite overheating of a composite high-T_c superconductor [Fig. 1(b)]. As a result, the transient values of the differential resistivity of a superconducting composite not only have positive values but decrease with increasing the current.

It is clear that the temperature rise of a composite depends also on the current charging rate. In this case, influence of dI/dt on the non-isothermal part of the complete penetration state will be more noticeable when this quantity increases. In the meantime, the transient period of the initial complete penetration mode will decrease with increasing dI/dt, as follows from the relationship (11). The corresponding VCC are presented in Fig. 3. It is seen that the non-isothermal fully penetrated part of $E(I)$-curves are shifted toward the higher currents with increasing the current charging rate. As a result, the high voltage range exceeding about 10^{-3} V/m should be carefully used to define the critical current of a superconductor.

As a whole, Figs. 1 and 3 demonstrate that the unsteady high-voltage part of the VCC of high-T_c composite superconductors does not allow to find the boundary of the stable modes. Accordingly, the currents exceeding in Figs. 1 and 3 the corresponding static quenching values I_q are unstable. In experiments, to find the thermal runaway current, the pulsed current method is performed (see, for example, [4], [5]). They are owing to the existence of the steady operating states. Under the considered parameters, they are described by the simple zero-dimensional model. Generally, the static description must be based on the multi-dimensional stationary equations.

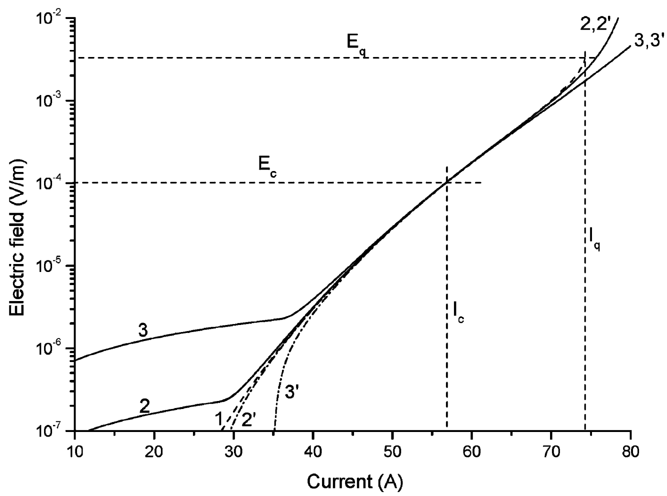

Fig. 3. Effect of the current charging rate on the electric field evolution on the composite surface (curves 2, 3) and in its centre (curves $2'$, $3'$): 1—$dI/dt \to 0$; 2, $2'$—$dI/dt = 10$ A/s; 3, $3'$—$dI/dt = 100$ A/s.

It should be noted that the pulsed current method has been formulated for the first time in [12] where the thermal runaway current was determined for low-T_c composite superconductors. It has been proved that the current charging in low-T_c composite superconductors causes its self-heating before instability due to the imbalance with cooling. Therefore, the thermal runaway phenomenon has a similar energy nature for both low- and high-T_c superconductors. This concept allows to solve the thermal runaway problem for all superconducting materials from the common point of view based on the steady or unsteady analysis of the energy balance conditions considering influence of the allowable increase in the temperature of a composite before thermal runaway on the stability conditions. In particular, above-presented results prove the existence of the size effect changing the conditions of the thermal runaway. Indeed, the curves pictured in Fig. 1(a) lead to the following relationships: $I_q''/I_q' = 1.74$ at $a''/a' = 1.9$ and $I_q'''/I_q' = 3.21$ at $a'''/a' = 3.8$ under the condition $\eta = \text{const}$. This degradation of the current-carrying capacity of a superconductor is due to its unavoidable overheating when the temperature of a superconducting composite before current instability is not equal to the coolant temperature [Fig. 1(b)]. According to this non-isothermal behavior of the VCC, the quenching currents will not increase proportionally to the corresponding increase of the thickness of a current-carrying element.

IV. Conclusion

The analysis performed indicates the existence of the steady and unsteady peculiarities of the voltage-current characteristics of high-T_c composite superconductors. It is shown that the low and upper voltage boundaries, inside which the critical current may be defined, depend on the thickness of a superconducting composite and the current charging rate. The allowable low-voltage boundary shifts to higher values, if the thicknesses of a tape or the current charging rate increase. The upper-voltage boundary depends on the permissible stable temperature variation of a tape that is also a function of its thickness and current charging rate. As a result, the allowable rise of the temperature before the thermal runaway intensifies the evolution of the electric states of composites with increasing the thickness or the current charging rate. Besides, the heterogeneity of the fully penetrated electric field distribution in its initial stage depends on the thickness of a tape or the current charging rate. Under these peculiarities, the possible value of the electric field, which may be used to define the critical current of high-T_c superconductors, should be close to the quantity 10^{-4} V/m. In this case, the operating modes of a composite are characterized by practically uniform distribution of the induced electric field and the existing stable overheating of a composite is small. At the same time, the dependence of the operating electric modes on the unavoidable temperature rise caused by the variation of the thickness of a superconducting tape is accompanied by the relevant thermal degradation of its current-carrying capacity: the higher the thickness of a tape, the lower the quenching current.

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