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Current-Spin Coupling for Ferromagnetic Domain Walls in Fine Wires

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The coupling between a current and a domain wall is examined. In the presence of a finite current and in the absence of a potential which breaks the translational symmetry, there is a perfect transfer of angular momentum from the conduction electrons to the wall. As a result, the ground state is in uniform motion and this remains the case even when relaxation is included. This is described by, appropriately modified, Landau-Lifshitz-Gilbert equations. The results for a simple pinning model are compared with experiment.

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Spintronic devices have great technological promise but represent a challenging problem at both an applied and a fundamental level. It has been shown theoretically [1,2] that the direction of a magnetic domain might be switched using currents alone. Devices designed to use this principle often consist of multilayers of magnetic and nonmagnetic conductors. The advantages of similar devices based upon the current induced displacement of a *domain wall* are simplicity and the fact that the switching current is much smaller [3–6]. Experimentally the current induced displacement of a domain wall has been clearly demonstrated and in recent experiments [5,6] the velocity of the wall was measured.

The current induced motion of a magnetic domain involves the transfer of angular momentum from the conduction electrons. The early theory [1,2] and most of the subsequent work [7] are based upon some type of assumption about this torque transfer process and there is no real consensus on how this should appear in the (Landau-Lifshitz-Gilbert) equations of motion [7]. The purpose of this Letter is to develop a *complete* theory of this process for a domain wall, based upon a specific model Hamiltonian and physically justified approximations.

The current carrying ferromagnetic wire lies along an easy z axis, and although similar conclusions are valid for the Stoner (and related) models, here attention will be focused upon the $s-d$-exchange model. The direction of the local moments, $\vec{S}_i$, which make up the domain wall, is specified by the Euler angles θ_i and ϕ_i. To make diagonal the interaction $\mathcal{H}_{tJ} = -J\vec{S}_i \cdot \vec{s}_i$, at site i, the axis of quantization of the conduction electron moments $\vec{s}_i$ is rotated along this same direction. Here, J is the conduction electron to local moment exchange constant. If $\psi(\vec{r}_i, t)$ is the conduction electron spinor field at the position $\vec{r}_i$, then this amounts to making a SU(2) gauge transformation, $\psi(\vec{r}_i, t) \rightarrow r(\theta_i, \phi_i)\psi(\vec{r}_i, t)$, where $r(\theta_i, \phi_i) \equiv e^{i\phi_i s_z/\hbar} \times e^{i\theta_i s_y/\hbar} e^{-i\phi_i s_z/\hbar}$ and where $\vec{s} \equiv \hbar \frac{\vec{\sigma}}{2}$ and $\vec{\sigma}$ are the Pauli matrices. This transformation introduces no less than *three* gauge fields. The *longitudinal* such field has been exploited in the development of theories of the Hall effect [8]. Bazaliy *et al.* [9] describe angular momentum transfer in terms of this same field. That a *transverse* field appears in their Landau-Litshiftz equations reflects a finite $\partial \mathcal{V}\{\theta_i, \phi_i\}/\partial \phi_i$, where $\mathcal{V}\{\theta_i, \phi_i\}$ is the energy as a function of the angles $\{\theta_i, \phi_i\}$. This current induced transverse field leads to a solution in which the wall moves with a finite velocity, but which cannot be the ground state since it is equally the case that the equilibrium conditions require both $\partial \mathcal{V}/\partial \phi_i$ and $\partial \mathcal{V}/\partial \theta_i$ to be zero. This state must relax into one which is stationary and for which $\partial \mathcal{V}/\partial \phi_i = 0$. If true, this demonstrates the existence of *intrinsic* pinning, as pointed out by Tatara and Kohno [10]. Here it will be shown that such pinning does *not* exist; i.e., the ground state has a finite velocity in the absence of *extrinsic* pinning.

In order to compare with experiment, damping due to extrinsic defect pinning is introduced on a phenomenological basis. The resulting predictions for the velocity of the wall are found to be consistent with experiment.

Given that the domain wall lies in z-x plane, i.e., that the $\phi_i = 0$ (see below), angular momentum transfer effects can be accounted for in a simpler U(1) theory for rotations about the perpendicular axis. The rotations $r(\theta_i) \equiv e^{i\theta_i s_y/\hbar} = [\cos(\theta_i/2) + i\sin(\theta_i/2)\sigma_y]$ are all that is needed to diagonalize $-J\vec{S} \cdot \vec{s}$. This simpler approach can *only* generate a single *transverse* gauge field, precisely that ignored in the earlier work [9], and this *alone* is found to be the origin of the transfer process.

The Hamiltonian is $\mathcal{H} = \mathcal{H}_e + \mathcal{H}_s + \mathcal{H}_{tJ}$, where

$$\mathcal{H}_e = -\sum_{\langle ij\rangle\sigma\sigma'} (c^\dagger_{i\sigma} t_{ij\sigma\sigma'} c_{j\sigma'} + \text{H.c.}) - \mu\hat{N} \quad (1)$$

is the electronic part, while the spin Hamiltonian

$$\mathcal{H}_s = -\sum_i (A^0 S^2_{iz} - K^0_\perp S^2_{iy}) - J^0_s \sum_{\langle ij\rangle} \vec{S}_i \cdot \vec{S}_j. \quad (2)$$

The $c^\dagger_{i\sigma}$ are the conduction electron creation operators for spin σ and site i. The uniform hopping integral is $t_{ij\sigma\sigma'} = t\delta_{\sigma\sigma'}$, and $\hat{N}$ is the number operator. The constant J^0_s reflects the direct exchange between local moments. The anisotropy constants A^0 and $K^0_\perp$ are positive, making the z

direction easy with the y axis hard. These reflect both intrinsic anisotropy *and* the demagnetizing field.

The Holstein-Primakoff transformation [11] $S_{iz} = \hbar(S - b_i^\dagger b_i)$, $S_i^+ = \hbar(2S - b_i^\dagger b_i)^{1/2} b_i \approx \hbar(2S)^{1/2} b_i$, using the direction defined by θ_i and ϕ_i as the axis of quantization for the local spins, leads to $\mathcal{H}_s = \mathcal{V}\{\theta_i, \phi_i\} + \mathcal{H}_b$. To within a constant, the usual energy functional $\mathcal{V}\{\theta_i, \phi_i\} = \mathcal{V}_\ell\{\theta_i, \phi_i\} + \mathcal{V}_J\{\theta_i, \phi_i\}$ comprises a single ion part,

$$\mathcal{V}_\ell\{\theta_i, \phi_i\} = -\hbar^2 A S^2 \cos^2\theta_i + \hbar^2 K_\perp S^2 \sin^2\theta_i \sin^2\phi_i, \quad (3)$$

where $A = [(2S-1)/2S]A^0$, $K_\perp = [(2S-1)/2S]K_\perp^0$. The interactions lead to

$$\mathcal{V}_J\{\theta_i, \phi_i\} = -\sum_{\langle ij\rangle} \hbar^2 J_s S^2 \cos\theta_{ij}. \quad (4)$$

In the continuum limit, dropping a constant, $\mathcal{V}_J \approx \hbar^2 J_s S^2 \sum_{\langle ij\rangle} (\vec{\nabla}\theta \cdot \vec{r}_{ij})^2$, where $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$. The remaining part $\mathcal{H}_b$, which describes magnons, is not relevant for the ground state properties. Minimizing $\mathcal{V}\{\theta_i, \phi_i\}$ gives $\theta(z_i) = 2\cot^{-1} e^{-(z_i/w)}$ with $\phi = 0$, where z_i is the z coordinate of site i. The wall width $w = a(J_s/2A)^{1/2}$, where a is the lattice spacing. This corresponds to the static solution without currents.

Including the diagonal part of $\mathcal{H}_{tJ}$, after performing the U(1) rotations $r(\theta_i) \equiv r(\theta(\vec{r}_i))$, the electronic part,

$$\mathcal{H}_{eJ} = -\sum_{\langle ij\rangle\sigma\sigma'} (c_{i\sigma}^\dagger t_{ij\sigma\sigma'} c_{j\sigma'} + \text{H.c.}) - \hbar J S \sum_i s_{iz} - \mu \hat{N}, \quad (5)$$

where $t_{ij} = t r^{-1}(\theta(\vec{r}_i)) r(\theta(\vec{r}_j))$ reduces to

$$t_{ij} = t[\cos(\vec{\nabla}\theta \cdot \vec{r}_{ij}) + i(\vec{\nabla}\theta \cdot \vec{r}_{ij})\sigma_y]. \quad (6)$$

Correct to second order in the gradient,

$$t_{ij} = e^{i\int_{\vec{r}_i}^{\vec{r}_j} \vec{A}\cdot d\vec{r}}\, t \cos(\vec{\nabla}\theta \cdot \vec{r}_{ij}); \qquad \vec{A} = \vec{\nabla}\theta s_y. \quad (7)$$

This is the key result of the formulation. The effect of the wall is to reduce the hopping matrix element by a factor of $\cos(\vec{\nabla}\theta \cdot \vec{r}_{ij})$, but most importantly it introduces a transverse, i.e., spin off diagonal, effective vector potential $\vec{A}$. As does any vector potential, this couples to the current (see below).

Dropping the off-diagonal parts of $\mathcal{H}_{tJ}$ implies that, as the electrons pass through the wall, they adiabatically follow the local spin magnetization. It is precisely the term of interest $\Delta t_{ij}^\perp = it(\vec{\nabla}\theta \cdot \vec{r}_{ij})\sigma_y$ which leads to deviations from the adiabatic limit and which also causes the transfer of angular momentum to the wall. Without $\Delta t_{ij}^\perp$, the ground state of $\mathcal{H}_{ej}$ indeed has $\langle \vec{S}_i \rangle$ and $\langle \vec{s}_i \rangle$ parallel. The adiabatic approximation is manifestly valid in the half metal limit when $J \gg t$. The ground state is then a mixture of states in which all sites are either singly occupied by an electron or unoccupied. The singly occupied sites with the maximum angular momentum $S + (1/2)$ have the lowest energy while other states, and those with two electrons per site, have an energy which is higher by $\sim\hbar JS$ and hence have negligible weight in the ground state. The Wigner-Eckart theorem then dictates that all the matrix elements of $\vec{s}_i$ are equal to those of $(\vec{S}_i/2S)$. However, a much weaker inequality suffices. The adiabatic theorem demands that the transverse field $\Delta t_{ij}^\perp$ be small compared to the longitudinal field $\hbar JS$. The wall rotates by π over a distance w so $\vec{\nabla}\theta_i \sim \pi/w$ and $\Delta t_{ij}^\perp \sim i\pi t(a/w)$, and required is

$$\pi t(a/w) \ll \hbar JS, \quad (8)$$

which since, e.g., for Permalloy $w/a \sim 10^3$, is typically well satisfied. The conduction electron magnetization comprises two components with, by definition, the (minority) majority conduction electrons (anti-)parallel to the axis of quantization, i.e., the direction of the local spin. In the *local frame* the majority (minority) electrons have $\sigma_z = +1$ ($\sigma_z = -1$), so that it follows that when the adiabatic theorem is satisfied,

$$\vec{s}_i = \sigma_z(\vec{S}_i/2S), \quad (9)$$

independent of the details of the electronic structure, etc.

When the transverse parts are ignored, $t_{ij} = t\cos(\vec{\nabla}\theta \cdot \vec{r}_{ij})$. Using $(\vec{\nabla}\theta \cdot \vec{r}_{ij}) \sim \pi a/w$, the correction is $\sim t\pi^2(a/w)^2$, which with $(a/w) \sim 10^{-3}$ might be safely ignored; i.e., there is a negligible *pressure* on the wall. The correction in the spin sector *is* important, since evaluating the coefficient of $(\vec{\nabla}\theta \cdot \vec{r}_{ij})^2$ leads to a renormalization $J_s = J_s^0 + (x't/2S^2)$ of the exchange coupling. The effective concentration $x' = \langle c_i^\dagger c_j \rangle$ [12].

Even when $\Delta t_{ij}^\perp$ is included, $\mathcal{H}_{eJ}$ is of single particle nature. Consider first $\mathcal{H}_{eJ}$ for majority spin electrons. In order to account for $\Delta t_{ij}^\perp$, use is made of Eq. (9). To this end, it is noted, for majority electrons, $\sum_{\sigma\sigma'} c_{i\sigma}^\dagger \times \sigma_{y\sigma\sigma'} c_{j\sigma'} = i(s_i^- c_{i\Uparrow}^\dagger c_{j\Uparrow} - c_{i\Uparrow}^\dagger c_{j\Uparrow} s_j^+)/\hbar$. Then by virtue of Eq. (9), e.g., $s_i^+ \approx \hbar(2S)^{-1/2} b_i$ and $\sum_{\sigma\sigma'} c_{i\sigma}^\dagger \sigma_{y\sigma\sigma'} c_{j\sigma'} = i(2S)^{-1/2} c_{i\Uparrow}^\dagger c_{j\Uparrow}(b_i^\dagger - b_j)$. Combining this with the similar result for the minority electrons, that part of $\mathcal{H}_{eJ}$ proportional to $(\vec{\nabla}\theta \cdot \vec{r}_{ij})$ defines

$$\mathcal{H}_\tau = -\frac{t}{2(2S)^{1/2}} \sum_{ij\sigma} \sigma(\vec{\nabla}\theta \cdot \vec{r}_{ij}) c_{i\sigma}^\dagger c_{j\sigma}(b_i - b_j^\dagger) + \text{H.c.}, \quad (10)$$

which couples the spin current to spin deviations and reflects the *entire* angular momentum transfer process. This sole remaining interaction between the spin and charge sectors is a perturbation. The effective interaction is obtained by taking the expectation value with respect to the conduction electrons. The final result is then

$$\mathcal{H}_\tau = -i\frac{\hbar j_s a^3}{2eS} \sum_i \frac{\partial\theta}{\partial z}[(2S)^{1/2}(b_i - b_i^\dagger)], \quad (11)$$

where j_s is the *spin* current. Here $b_{i+1}^\dagger$ is replaced by $b_i^\dagger$, an approximation valid in the continuum limit $w \gg a$. The quantity $\hbar(2S)^{1/2}(b_i - b_i^\dagger)/2i \approx S_{iy}$ and $\mathcal{H}_\tau$ corresponds to an effective field which is *strictly* transverse in spin space. The appearance of such a term linear in $(b_i - b_i^\dagger)$ signals that there is no time independent solution.

That the wall is in motion implies that the rotations $r(\theta(\vec{r}_i, t))$ are also time dependent. A time dependent rotating frame is generated by $R = e^{-i\theta(\vec{r}_i,t)M_{iy}}$, where $\hbar\vec{M}_i = \vec{S}_i + \vec{s}_i$ involves the *total* spin angular momentum, whence the equations of motion become $i\hbar(\partial/\partial t)R^{-1}\vec{M}R = [\vec{M}, \mathcal{H}]$, where $\vec{M}$ is now defined in the rotating frame. The effect is $\mathcal{H} \rightarrow \mathcal{H} - \hbar(\partial\theta/\partial t)M_{iy}$. This generates a second purely transverse field term in the effective Hamiltonian:

$$-\hbar\sum_i \frac{\partial\theta}{\partial t}M_{iy} = i\hbar\frac{M}{S}\sum_i \frac{\partial\theta}{\partial t}(2S)^{1/2}(b_i - b_i^\dagger),$$

using the fact that $(\vec{M}_i/M) = (\vec{S}_i/\hbar S)$, i.e., that all magnetizations are parallel. Thus when

$$\frac{\partial\theta}{\partial t} + v_0\frac{\partial\theta}{\partial z} = 0, \qquad v_0 = \frac{j_s a^3}{2eM}, \tag{12}$$

the transverse effective fields generated by the spatial and temporal derivatives cancel each other. Given $\theta(z_i) \equiv 2\cot^{-1}e^{-(z_i/w)}$ for $j_s = 0$, the new *ground state* has $\theta(z_i) \rightarrow \theta(z_i - v_0 t)$; i.e., the wall moves without distortion and without tilting or twisting. It is easy to show that the result $v_0 = (j_s a^3/2eM)$ reflects the conservation of the z component of the total angular momentum, i.e., that the net spin current, $2j_s$, carried towards the wall by the electrons equals the change in the angular momentum, $J_S = Mv_0/a^3$, of the wall due to its motion. Since the conduction electrons are polarized, $j_s = pj$ is related to the charge current j by some material determined parameter p.

For the slice of the wall between z and $z + dz$, the Landau-Lifshitz (LL) equations, with a divergence $\vec{\mathcal{D}}$ and a relaxation $\vec{\mathcal{R}}$ term, are

$$\frac{\partial\vec{M}}{\partial t} + \vec{\mathcal{D}} = -\frac{g\mu_B}{\hbar}\vec{M}\times\vec{B} - \vec{\mathcal{R}}, \qquad \vec{\mathcal{D}} = \vec{\nabla}\cdot\vec{\vec{j}}_s, \tag{13}$$

where $\vec{B} \equiv -(\partial\mathcal{V}/\partial\vec{M})$ and $\vec{\vec{j}}_s$ is the spin current tensor. As Gilbert recently emphasizes in connection with a single domain [13], for $\vec{\mathcal{D}} = 0$, his $\vec{\mathcal{R}}_g = (\alpha/M)\vec{M}\times(\partial\vec{M}/\partial t)$ and the original LL $\vec{\mathcal{R}}_\ell = (\lambda/M^2)\vec{M}\times(\vec{M}\times\vec{B})$ are, to within a renormalization of parameters, mathematically equivalent. Since $\phi = 0$, $M_x = M\sin\theta$ and $M_z = M\cos\theta$, whence Eq. (12) implies $(\partial\vec{M}/\partial t) + v_0(\partial\vec{M}/\partial z) = 0$. This permits the identification $\vec{\mathcal{D}} = v_0(\partial\vec{M}/\partial z)$. It is a *sufficient* condition for the *second law* of thermodynamics to be satisfied that $\vec{\mathcal{R}} = 0$ when $\mathcal{V}(\theta, \phi)$ corresponds to an energy minimum. For nonequibrium situations, e.g., when $\vec{\mathcal{D}} \neq 0$, this *is* manifestly the case for the LL, $\vec{\mathcal{R}}_\ell$ but *not* the Gilbert $\vec{\mathcal{R}}_g$. This follows since, with $\vec{B} \equiv -(\partial\mathcal{V}/\partial\vec{M})$, $\vec{M}\times\vec{B} = 0$ implies such a minimum *and* $\vec{\mathcal{R}}_\ell = 0$. The Landau-Lifshitz-Gilbert (LLG) equations with relaxation mathematically equivalent to that of LL are

$$\frac{D\vec{M}}{Dt} \equiv \frac{\partial\vec{M}}{\partial t} + v_0\frac{\partial\vec{M}}{\partial z} = -\frac{g\mu_B}{\hbar}\vec{M}\times\vec{B} - \frac{\alpha}{M}\vec{M}\times\frac{D\vec{M}}{Dt}. \tag{14}$$

The "particle derivative" $D\vec{M}/Dt$ (more generally $\equiv \frac{\partial\vec{M}}{\partial t} + \vec{v}_0\cdot\vec{\nabla}\vec{M}$) occurs in fluid flow and here is the time derivative taken at a fixed position in the *moving* wall. We contend that Eqs. (14) embody correctly the relaxation dynamics of a domain wall driven by currents and external fields when $\vec{v}_0(t)$ is suitably generalized. In general, this velocity field *does not* coincide with the actual local velocity. The usual LLG equations are recovered instantaneously during times when the wall is not *externally* driven, i.e., when this generalized $v_0(t) = 0$.

In the Landau-Lifshitz equations obtained by Bazaliy *et al.* [9], the term proportional to j_s (or v_0) arises from a finite $\partial\mathcal{V}/\partial\phi$, via $\vec{B} \equiv -\frac{\partial\mathcal{V}}{\partial\vec{M}}$, which implies that $\vec{\mathcal{D}} = 0$ and that $\frac{D\vec{M}}{Dt} = \frac{\partial\vec{M}}{\partial t}$. Relaxed states would have $|\frac{\partial\vec{M}}{\partial t}| = 0$, and intrinsic pinning would result. The relaxed solution of Eq. (14) has $\frac{D\vec{M}}{Dt} = 0$, which is the equivalent of Eq. (12); i.e., relaxation is absent for the uniform motion induced by a current and there is no intrinsic pinning [9,10].

That, in fact, $\partial\mathcal{V}/\partial\phi = 0$ has been verified by performing the full SU(2) transformations. After some algebra, the result is Eq. (7) with $\vec{A} = (1/2)(\vec{\nabla}\theta)\sigma'_y + (1/2)\times\sin\theta(\vec{\nabla}\phi)\sigma'_x + (1-\cos\theta)(\vec{\nabla}\phi)\sigma'_z$, where the $\vec{\sigma}'$ are defined in the local frame of reference. The last term generates $\mathcal{V}_s = v_0(1-\cos\theta)(\vec{\nabla}\phi)$ in the effective spin Lagrangian density. With a kinetic energy density $T = S\hbar(\cos\theta - 1)(d\phi/dt)$, this $\mathcal{V}_s$ reproduces Eq. (12) [14]. Substituting $\theta(x') = \theta(x - v_0 t)$ and $\phi(x') = \phi(x - v_0 t)$ then leads to a Lagangian density for $\theta(x')$ and $\phi(x')$, which is identical to that for $j_s = 0$, thereby establishing that the sliding state implied by Eq. (12) is indeed *the* ground state for $j_s \neq 0$.

The effect of *extrinsic* pinning depends very much on the details of the pinning potential $\mathcal{V}_p(z)$ and the value of $K_\perp$. It will be assumed that $K_\perp$ is large enough that the deviations in ϕ due to the pressure $P_z = -(\partial\mathcal{V}_p/\partial z)/\mathcal{A}$ are small. Such a pressure is equivalent to an applied magnetic field in the z direction and results in

$$\frac{\partial\phi}{\partial t} = \frac{a^3}{2M\hbar}P_z, \tag{15}$$

which illustrates that the wall momentum is $p = \frac{2\mathcal{A}M\hbar}{a^3}\phi$. The coordinate of the wall is $z = \frac{a^3}{2\mathcal{A}M\hbar}M_z$, and from Eq. (14) it follows that

$$\frac{\partial z}{\partial t} - v_0 = \frac{1}{m_D}p - \alpha'\frac{a^3}{2\hbar}P_z, \tag{16}$$

where $m_D \propto 1/K_\perp$ is the Doring mass and where this and $\alpha' \propto \alpha$ depend on the detailed wall structure. These are just the traditional equation which describe domain wall

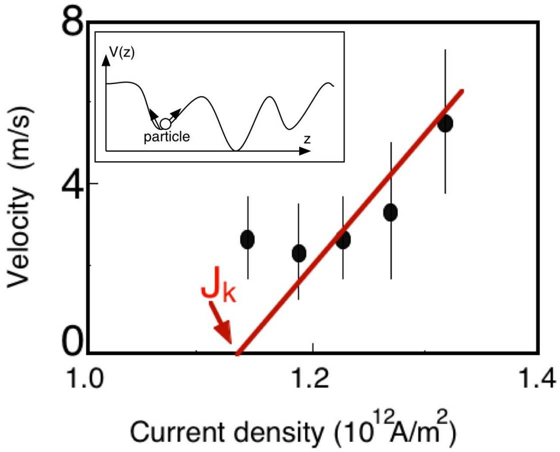

FIG. 1 (color online). The experimental points are taken from [5]. The solid line corresponds to the value of C from Eq. (18) with $p \approx 0.7$. The inset shows the equivalent particle problem.

motion with the relaxation modified as outlined above and with the angular momentum transfer term added [see, e.g., Eqs. (10.11) and (11.2) of Ref. [15]]. Without the current v_0 term, these are the homogeneous differential equations of motion for a "particle" moving in the potential $\mathcal{V}(z)$ as in the inset of Fig. 1. Assuming that v_0 and the relaxation are small corrections, P_z might be replaced by its average $\langle P_z \rangle$ over the motion. With this,

$$\frac{\partial z}{\partial t} - (v_0 - v_r) = \frac{1}{m_D} p, \tag{17}$$

where $v_r = \alpha' \frac{a^3}{2\hbar} \langle P_z \rangle$. The particular integral of this, i.e., the steady state solution, is obtained by eliminating the quantity $(v_0 - v_r)$ by $z \rightarrow z - (v_0 - v_r)t$. This causes the potential to become time dependent, i.e., the motion is that of a free particle driven by a time dependent force with zero average and no relaxation. In the original frame the resulting oscillations must be added to the constant velocity $(v_0 - v_r)$. The nonconservative driving term has placed the particle at an energy above the top of the maxima in the pinning potential. The average $\langle P_z \rangle$ is nonzero since the particle spends more time in the regions where the retarding effects of this same term are the greatest. This average depends strongly only on the velocity near the maxima in P_z, and these are far from the top of the well. The relevant velocities and therefore $\langle P_z \rangle$ are insensitive to small changes in the particle energy for the energies of interest. This justifies assuming that v_r is a constant near to the critical current. The kinetic critical current j_k is evidently given by $v_0 = v_r$. Near to this threshold current, the average velocity,

$$v = pC(j - j_k); \qquad C \equiv \frac{a^3}{2eM}. \tag{18}$$

Important is that the velocity near threshold is greatly reduced but that C is independent of j_k; i.e., above the critical current, the angular momentum not destroyed by the pinning is 100% converted into motion of the wall.

In Fig. 1 this prediction is compared with the experiments of Yamaguchi *et al.* [5]. Using the lattice constant for Permalloy, with $M = 1$, $C \approx 4.5 \times 10^{-11}\ \mathrm{m^3/C}$, and using $p \approx 0.7$ suggested in Ref. [5], $pC \approx 3.15 \times 10^{-11}\ \mathrm{m^3/C}$. This corresponds approximately to the gradient of the line shown in Fig. 1. Clearly, within a factor of 2 in either sense, this is consistent with the trend of the data points.

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