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Adaptive IIR Band-Pass/Band-Stop Filtering Using High-Order Transfer Function and Frequency Transformation

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This paper proposes a new unified framework for the adaptive IIR band-pass/band-stop filtering for detection and enhancement/suppression of an unknown narrowband signal immersed in a broadband signal. In most of the conventional methods, which are well-known as the adaptive notch filtering, the adaptive band-pass/band-stop filter is restricted to a low-order transfer function. On the other hand, our proposed method can be applied to arbitrary high-order band-pass/band-stop transfer functions in a simple manner. We derive this simple adaptive mechanism with the help of the frequency transformation and its block diagram representation. In addition, we prove that this result includes the conventional all-pass-based adaptive notch filters as special cases. Moreover, we demonstrate a significant property that the use of high-order adaptive band-pass/band-stop filters yields much better signal-to-noise ratio (SNR) improvement than the conventional low-order filters.

KEYWORDS: signal processing, adaptive digital filtering, narrowband signal, high-order transfer function, frequency transformation

1. Introduction

Adaptive band-pass filtering and adaptive band-stop filtering are important fundamental techniques for detection and enhancement/suppression of unknown narrowband signals immersed in a broadband (white) signal. These techniques play central roles in many practical applications of digital signal processing. Such practical applications include the adaptive line enhancer, the howling suppression in audio/speech precessing, the narrowband interference suppression in communication systems, and the detection of harmonic signals in power-electronic devices.

Over the years, a number of methods have been proposed for designing and realizing adaptive band-pass/band-stop filters. Among such methods, adaptive IIR notch filtering [1–11] has particularly attracted many researchers because it is very simple and thus easy to implement. This simplicity comes from the use of simple transfer functions. That is, the adaptive filter is given by the IIR band-pass/band-stop transfer functions of the *lowest order*. To be more precise, in order to detect a single narrowband signal, second-order functions are used for real coefficient filters and first-order functions are used for complex coefficient filters.

Although such adaptive band-pass/band-stop filtering based on low-order transfer functions is very tractable, it has a serious drawback from the viewpoint of the signal-to-noise ratio (SNR) at the filter output. Figure 1(a) illustrates this problem in the case of adaptive band-pass filtering for enhancement of an unknown narrowband signal. As is well-known, low-order band-pass filters cannot provide so sharp cutoff characteristic in the magnitude response. Due to this poor cutoff characteristic, low-order band-pass filters have poor ability to capture the entire narrowband signal and, in addition, they pass undesirable white noise out of the frequency region of the narrowband signal. Hence, the use of low-order transfer functions results in low SNR at the filter output. On the other hand, if we can use a higher-order filter as the adaptive filter, the output SNR is expected to be much more improved than the low-order case because of the sharper cutoff characteristic of the higher-order filter, as shown in Fig. 1(b).

Motivated by this observation, in this paper we focus on the approach of the high-order adaptive IIR band-pass/band-stop filtering. This approach has not been well studied so far: To the best of our knowledge, only Kumar and Pal [12–15] attempted this approach, and they proposed an adaptive filtering algorithm based on the fourth-order Butterworth band-pass/band-stop filter. Although they claimed that their method can be applied to any other filter, they also pointed out that mathematical description of higher-order filters is very complicated and, therefore, they stated that efficient description of high-order adaptive band-pass/band-stop filtering may not be possible. Due to this fact, efficient adaptive control of band-pass/band-stop filters of arbitrary order has still been an open problem, and thus no detailed discussion has been given regarding how the choice of the filter order is related to the SNR improvement.

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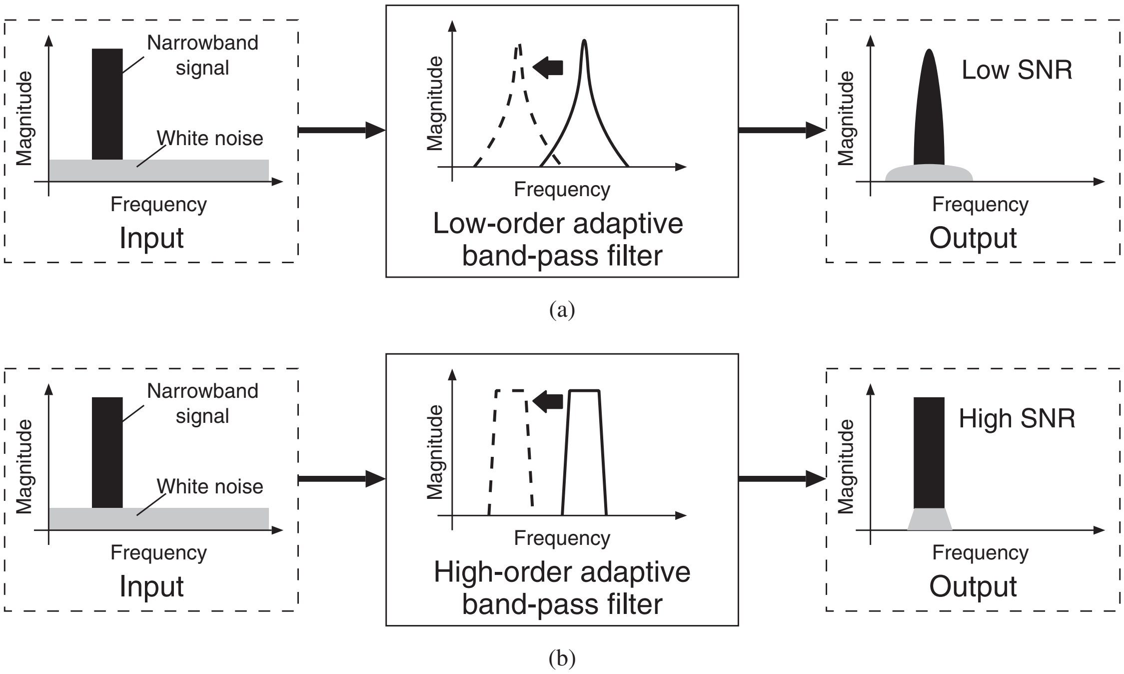

Fig. 1. Adaptive band-pass filtering: (a) based on low-order transfer function, and (b) based on high-order transfer function.

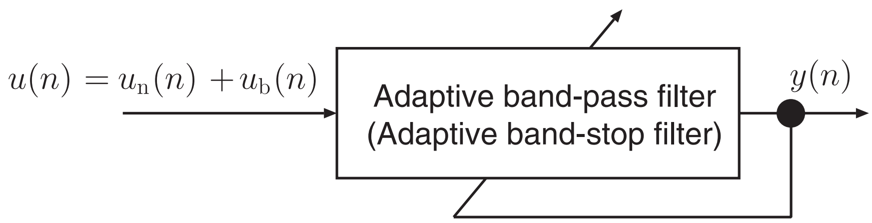

Fig. 2. Block diagram of adaptive band-pass/band-stop filtering.

In this paper, we achieve the above-mentioned challenging tasks. That is, the main contributions of this paper are the following.

(1) We propose a unified mathematical framework for adaptive filtering based on arbitrary band-pass/band-stop transfer functions. To this end, we first derive a simple time-domain filtering algorithm for arbitrary band-pass/band-stop filters with variable (tunable) center frequency. This simple algorithm is obtained with the help of the frequency transformation [16] and its block-diagram representation. Next we extend this result and obtain a gradient-based adaptive algorithm that can be applied to arbitrary band-pass/band-stop transfer functions. We also prove that our proposed method includes the conventional all-pass-based adaptive notch filters as special cases. These results are presented in §3.

(2) In §4, we give a simulation example and demonstrate that our proposed high-order adaptive band-pass/band-stop filtering attains higher output SNR at steady-state than the conventional low-order methods including the adaptive notch filtering.

2. Adaptive Band-Pass/Band-Stop Filtering

This section reviews the basic concepts of the adaptive band-pass/band-stop filtering.

The adaptive filter configuration to be considered is shown in Fig. 2, which is the same as in [12–15]. This configuration can be also seen in most of the literature on the adaptive notch filtering. The input signal $u(n)$ is given by

$$u(n) = u_\mathrm{n}(n) + u_\mathrm{b}(n) \tag{2.1}$$

where $u_\mathrm{n}(n)$ is a narrowband signal of which frequency component is unknown, and $u_\mathrm{b}(n)$ is a broadband signal that is white and uncorrelated with $u_\mathrm{n}(n)$. The output signal is denoted by $y(n)$, and its mean-square value $E[y^2(n)]$ is used as the cost function for adaptation.

As in [12–15], we assume that the bandwidth of the narrowband signal $u_\mathrm{n}(n)$ is known a priori, and that the adaptive band-pass/band-stop filter has the fixed pass-bandwidth/stop-bandwidth. The latter assumption means that only the center frequency of the filter pass-band/stop-band is variable and controlled by an adaptation mechanism.

3. Proposed Method

The strategy of our proposed adaptive IIR band-pass/band-stop filtering is outlined as follows.

Step 1. Design an IIR band-pass/band-stop filter with variable center frequency according to a prescribed filter specification.

Step 2. Derive a simple time-domain filtering algorithm for the transfer function of the designed variable band-pass/band-stop filter.

Step 3. Using this filtering algorithm, derive a gradient-based adaptive algorithm that controls the center frequency of the filter.

In the sequel, we present the details of this strategy for both the band-pass case and the band-stop case. Also, we prove that this method includes the conventional all-pass-based adaptive notch filters as special cases.

3.1 Proposed method: Band-pass case

In Step 1, we make use of the frequency transformation [16] to design an arbitrary IIR band-pass/band-stop filter with variable center frequency. This is a well-known technique for design of digital filters from a given prototype low-pass filter by means of a simple transformation. Hence we first need to give the transfer function $H_p(z)$ of a stable prototype low-pass filter as follows:

$$H_p(z) = \frac{\sum_{j=0}^{N} b_j z^{-j}}{1 + \sum_{i=1}^{N} a_i z^{-i}} \tag{3.1}$$

where N is the order of $H_p(z)$ and a_i's and b_j's are the denominator coefficients and the numerator coefficients, respectively. These coefficients are determined in such a manner that the pass-bandwidth of this low-pass filter is equal to that of the desired variable band-pass filter. Needless to say, this is easily achieved by using the classical theory of digital filter design. Then, by means of the frequency transformation, the transfer function of the desired band-pass filter is obtained as follows:

$$H_{BP}(z, \xi) = H_p(z)|_{z^{-1} \leftarrow T_{BP}(z,\xi)} \tag{3.2}$$

where $T_{BP}(z, \xi)$ is the second-order all-pass function of the form

$$T_{BP}(z, \xi) = -z^{-1} \frac{z^{-1} - \xi}{1 - \xi z^{-1}}, \quad \xi = \cos \omega_0 \tag{3.3}$$

and ω_0 is the center frequency of the variable band-pass filter. This type of frequency transformation is a specific low-pass-to-band-pass (LP-BP) transformation. Note that the order of $H_{BP}(z, \xi)$ is $2N$ because $T_{BP}(z, \xi)$ is the second-order function. Also, note that the stability of $H_{BP}(z, \xi)$ is ensured for any ω_0 satisfying $0 < \omega_0 < \pi$ (i.e., for any ξ satisfying $-1 < \xi < 1$) if the prototype filter $H_p(z)$ is stable. The center frequency ω_0 is related to the cutoff frequencies of $H_{BP}(z, \xi)$ as

$$\cos \omega_0 = \cos\left(\frac{\omega_2 + \omega_1}{2}\right) \Big/ \cos\left(\frac{\omega_2 - \omega_1}{2}\right) \tag{3.4}$$

where ω_1 and ω_2 denote the lower and upper cutoff frequencies of $H_{BP}(z, \xi)$, respectively. Note that the LP-BP transformation based on (3.3) forces the pass-bandwidth of $H_{BP}(z, \xi)$ to be equal to the pass-bandwidth of $H_p(z)$, and thus $H_{BP}(z, \xi)$ has the fixed bandwidth. In view of (3.2) and (3.3), it readily follows that the center frequency ω_0 is controlled by the parameter ξ. An example of $H_{BP}(z, \xi)$ is shown in Fig. 3.

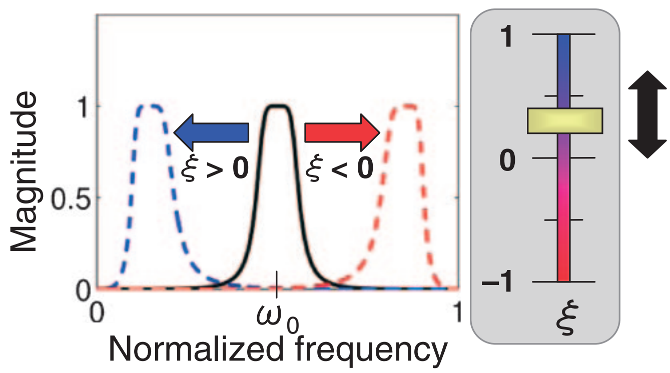

Fig. 3. Variable band-pass filter $H_{BP}(z, \xi)$ with fixed bandwidth and variable center frequency.

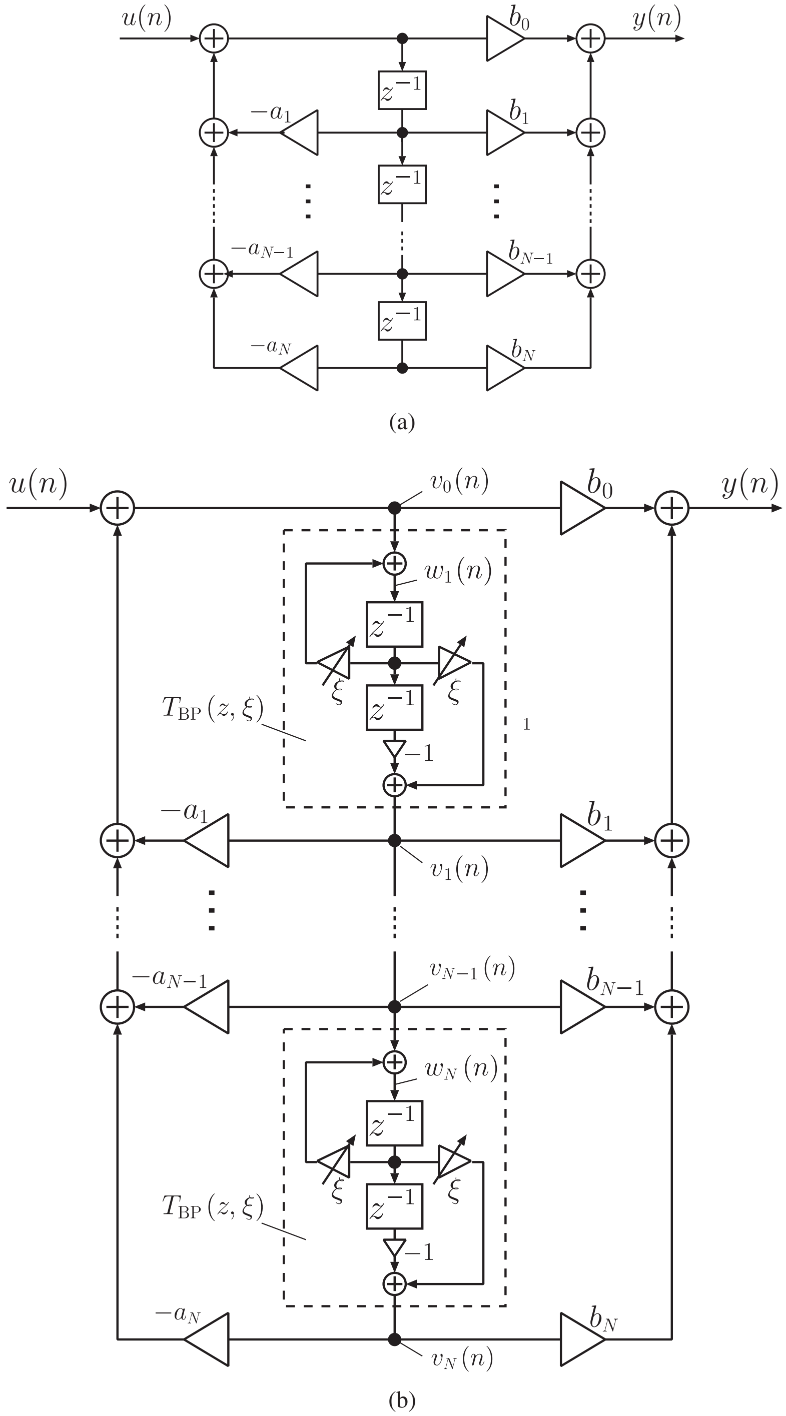

Fig. 4. LP-BP transformation: (a) prototype filter $H_{\mathrm{p}}(z)$, and (b) desired band-pass filter $H_{\mathrm{BP}}(z, \xi)$.

Next we present the details of Step 2. The key point here is that we will derive a time-domain filtering algorithm for $H_{\mathrm{BP}}(z, \xi)$ by using the components of $H_{\mathrm{p}}(z)$ and $T_{\mathrm{BP}}(z, \xi)$, rather than directly using the coefficients of the final composite function $H_{\mathrm{BP}}(z, \xi)$. To this end, we obtain the filter structure of $H_{\mathrm{BP}}(z, \xi)$ from the block-diagram representation of (3.2). Figure 4 shows this strategy based on the direct form II structure. As is well-known, the LP-BP transformation of (3.2)–(3.3) is realized by replacing each delay element of the prototype filter shown in Fig. 4(a) with the second-order all-pass filter $T_{\mathrm{BP}}(z, \xi)$, which yields the desired band-pass filter as in Fig. 4(b). Here, note that this replacement does not yield delay-free loops in the filter structure because the all-pass function given by (3.3) does not have a direct feedthrough term. Now, on the basis of Fig. 4 we derive the simple time-domain filtering algorithm for $H_{\mathrm{BP}}(z, \xi)$. First, we introduce internal variables $v_0(n)$, $v_i(n)$ and $w_i(n)$ for $1 \leq i \leq N$ into Fig. 4(b). In this figure, the variables $v_0(n)$ and $v_i(n)$ respectively correspond to the input to the all-pass filter at the first stage and the output of the i-th all-pass filter. The variable $w_i(n)$ is the input to the first delay element of the i-th all-pass filter. From these relationships we can easily obtain the following time-domain filtering algorithm:

$$\begin{aligned}
v_i(n) &= \xi w_i(n-1) - w_i(n-2) \quad \text{for } 1 \le i \le N; \\
v_0(n) &= -\sum_{j=1}^{N} a_j v_j(n) + u(n) \\
w_i(n) &= \xi w_i(n-1) + v_{i-1}(n) \qquad \text{for } 1 \le i \le N; \\
y(n) &= \sum_{i=0}^{N} b_i v_i(n).
\end{aligned} \tag{3.5}$$

Obviously, this filtering algorithm allows us to easily calculate the output $y(n)$ of the band-pass filter for arbitrary parameter ξ.

We next present the details of Step 3 for derivation of an adaptive algorithm. As was explained in §2, the mean-square value of the filter output, denoted by $E[y^2(n)]$, is considered here. In the band-pass case, the adaptation aims to find the optimal value of the tuning parameter ξ that *maximizes* $E[y^2(n)]$: This is the same approach as in [12–15]. Now, based on the concept of the LMS algorithm, the update equation for the adaptive band-pass filtering can be given by

$$\xi(n+1) = \xi(n) + \mu \frac{\partial y(n)}{\partial \xi(n)} y(n) \tag{3.6}$$

where μ is the adaptation step size and $\xi(n)$ is the center-frequency-tuning parameter at time n. From (3.5), the partial derivative $\partial y(n)/\partial \xi(n)$ is described as

$$\frac{\partial y(n)}{\partial \xi(n)} = \sum_{i=0}^{N} b_i \psi_{v_i}(n) \tag{3.7}$$

where

$$\begin{aligned}
\psi_{v_i}(n) &\equiv \frac{\partial v_i(n)}{\partial \xi(n)} \\
&= \frac{\partial}{\partial \xi(n)} (\xi(n) w_i(n-1) - w_i(n-2)) \\
&= w_i(n-1) + \xi(n) \frac{\partial w_i(n-1)}{\partial \xi(n)} - \frac{\partial w_i(n-2)}{\partial \xi(n)} \\
&\qquad \text{for } 1 \le i \le N;
\end{aligned} \tag{3.8}$$

$$\begin{aligned}
\psi_{v_0}(n) &\equiv \frac{\partial v_0(n)}{\partial \xi(n)} \\
&= -\sum_{j=1}^{N} a_j \psi_{v_j}(n).
\end{aligned} \tag{3.9}$$

Here, in a similar manner to the derivation of the LMS algorithm for IIR filters [17], we define

$$\begin{aligned}
\psi_{w_i}(n) &\equiv \frac{\partial w_i(n)}{\xi(n)} \\
&= \frac{\partial}{\partial \xi(n)} (\xi(n) w_i(n-1) + v_{i-1}(n)) \\
&= w_i(n-1) + \xi(n) \frac{\partial w_i(n-1)}{\partial \xi(n)} + \psi_{v_{i-1}}(n) \\
&\qquad \text{for } 1 \le i \le N;
\end{aligned} \tag{3.10}$$

and make the following approximation

$$\frac{\partial w_i(n-k)}{\partial \xi(k)} \simeq \frac{\partial w_i(n-k)}{\partial \xi(n-k)} = \psi_{w_i}(n-k) \quad \text{for } k = 1, 2; \tag{3.11}$$

under the assumption of a sufficiently small step size. Then we can rewrite (3.10) as the following recursive difference equation

$$\begin{aligned}
\psi_{w_i}(n) &\simeq w_i(n-1) + \xi(n) \psi_{w_i}(n-1) + \psi_{v_{i-1}}(n) \\
&\qquad \text{for } 1 \le i \le N;
\end{aligned} \tag{3.12}$$

from which we can also rewrite (3.8) as

$$\psi_{v_i}(n) \simeq w_i(n-1) + \xi(n)\psi_{w_i}(n-1) - \psi_{w_i}(n-2) \quad \text{for } 1 \le i \le N. \tag{3.13}$$

From (3.6)–(3.13), we now present the update equation for computing the center-frequency-tuning parameter as follows:

$$\begin{aligned}
\xi(n+1) &= \xi(n) + \mu\psi_y(n)y(n) \\
\psi_y(n) &= \sum_{i=0}^{N} b_i\psi_{v_i}(n) \\
\psi_{v_i}(n) &= w_i(n-1) + \xi(n)\psi_{w_i}(n-1) - \psi_{w_i}(n-2) \\
&\qquad \text{for } 1 \le i \le N; \\
\psi_{v_0}(n) &= -\sum_{j=1}^{N} a_j\psi_{v_j}(n) \\
\psi_{w_i}(n) &= w_i(n-1) + \xi(n)\psi_{w_i}(n-1) + \psi_{v_{i-1}}(n) \\
&\qquad \text{for } 1 \le i \le N.
\end{aligned} \tag{3.14}$$

Note that this adaptive algorithm can easily tackle the stability problem for the variable band-pass filter: As stated earlier, the frequency transformation ensures the stability of the variable band-pass filter for any $\xi(n)$ satisfying $-1 < \xi(n) < 1$, and thus it suffices to carry out the adaptation under the constraint of $-1 < \xi(n) < 1$. This constraint does not limit the tuning range of the center frequency because of the relationship of $\xi(n) = \cos\omega_0(n)$.

Before concluding this subsection, we discuss the advantages of our proposed algorithm in comparison with the conventional high-order-based methods [12–15]. We point out the advantages from the following two aspects.

(1) Our filtering algorithm (3.5) and our adaptive algorithm (3.14) can be easily applied to realization and adaptation of any kind of high-order variable band-pass filter. On the other hand, in the conventional methods [12–15] derivation of a different set of filtering algorithms is required for every other filter order and other filter type.

(2) Our filtering algorithm (3.5) can tune the center-frequency and the bandwidth of the band-pass filter independently. For example, if there is a need to change the filter bandwidth, only the coefficients a_i's and b_j's of the prototype low-pass filter $H_p(z)$ are recalculated* and thus we do not have to rewrite the expression of (3.5). Similarly, if the center-frequency is changed, only the parameter ξ is changed and the coefficients a_i's and b_j's remain fixed. On the other hand, in the conventional methods it was claimed in [15] that efficient description of high-order transfer functions with a single center-frequency dependent parameter may not be possible.

3.2 Proposed method: Band-stop case

The adaptive mechanism based on a variable band-stop filter can be constructed in an analogous fashion to the band-pass case. In Step 1, we prepare the transfer function $H_p(z)$ of an N-th order prototype low-pass filter as in (3.1), in such a manner that the stop-bandwidth of this filter is equal to the desired variable band-stop filter. Then we perform the following low-pass-to-band-stop (LP–BS) transformation

$$H_{BS}(z,\xi) = H_p(z)|_{z^{-1}\leftarrow T_{BS}(z,\xi)} \tag{3.15}$$

$$T_{BS}(z,\xi) = z^{-1}\frac{z^{-1}-\xi}{1-\xi z^{-1}}, \quad \xi = \cos\omega_0 \tag{3.16}$$

where $H_{BS}(z,\xi)$ is the transfer function of the desired variable band-stop filter of order $2N$, and ξ is the parameter that tunes the center frequency ω_0 of the stopband. As in the band-pass case, the center frequency ω_0 satisfies the relationship of (3.4), and the stop-bandwidth is fixed. In addition, if $H_p(z)$ is stable, $H_{BS}(z,\xi)$ is also stable for any ξ satisfying $-1 < \xi < 1$.

In Step 2, the time-domain filtering algorithm for $H_{BS}(z,\xi)$ can be easily obtained by simply replacing the block diagram of $T_{BP}(z,\xi)$ in Fig. 4(b) with that of $T_{BS}(z,\xi)$. Therefore, the resultant filtering algorithm is given as follows:

$$\begin{aligned}
v_i(n) &= -\xi w_i(n-1) + w_i(n-2) \quad \text{for } 1 \le i \le N; \\
v_0(n) &= -\sum_{j=1}^{N} a_j v_j(n) + u(n) \\
w_i(n) &= \xi w_i(n-1) + v_{i-1}(n) \qquad \text{for } 1 \le i \le N; \\
y(n) &= \sum_{i=0}^{N} b_i v_i(n).
\end{aligned} \tag{3.17}$$

*An easy way to achieve this is to apply the LP–LP transformation to $H_p(z)$. Since the choice of the filter bandwidth is beyond the scope of this paper, details on this topic are omitted here.

In Step 3, we consider the mean-square output $E[y^2(n)]$ as the cost function, which is the same as in the band-pass case. However, the adaptive algorithm to be derived here attempts to *minimize* this cost function, and thus we consider the following update equation

$$\xi(n+1) = \xi(n) - \mu \frac{\partial y(n)}{\partial \xi(n)} y(n). \tag{3.18}$$

Applying the relationship of (3.17) to (3.18) with the help of the derivation procedure in the band-pass case, we finally obtain the adaptive aglrotihm for computation of $\xi(n+1)$ as

$$\begin{aligned}
\xi(n+1) &= \xi(n) - \mu\psi_y(n)y(n) \\
\psi_y(n) &= \sum_{i=0}^{N} b_i \psi_{v_i}(n) \\
\psi_{v_i}(n) &= -w_i(n-1) - \xi(n)\psi_{w_i}(n-1) + \psi_{w_i}(n-2) \\
&\qquad\qquad\qquad\qquad\qquad \text{for } 1 \le i \le N; \\
\psi_{v_0}(n) &= -\sum_{j=1}^{N} a_j \psi_{v_j}(n) \\
\psi_{w_i}(n) &= w_i(n-1) + \xi(n)\psi_{w_i}(n-1) + \psi_{v_{i-1}}(n) \\
&\qquad\qquad\qquad\qquad \text{for } 1 \le i \le N.
\end{aligned} \tag{3.19}$$

It readily follows that the filtering algorithm (3.17) and the adaptive algorithm (3.19) have the same advantages as discussed in the band-pass case.

3.3 Relation to adaptive notch filtering

In this subsection we point out that our proposed method includes some class of the conventional adaptive notch filters as special cases. The adaptive notch filters to be discussed here are designed by second-order all-pass filters [1, 4, 7]. Such notch filters are divided into two types and we address their relationship to our method in the following two remarks.

Remark 3.1. A class of the all-pass-based adaptive notch filters makes use of the following transfer function [1, 4]

$$H_{\text{notch}}(z) = \frac{1+\eta}{2} \frac{1 - 2\xi z^{-1} + z^{-2}}{1 - (1+\eta)\xi z^{-1} + \eta z^{-2}} \tag{3.20}$$

where the parameters η and ξ determine the notch-bandwidth and the notch-frequency, respectively. This transfer function has been widely used in many methods on the adaptive notch filtering. This transfer function can be easily derived from the LP-BS transformation as follows. First, we choose the prototype low-pass filter as the following first-order transfer function

$$H_{\text{p}}(z) = \frac{1+\eta}{2} \frac{1 + z^{-1}}{1 + \eta z^{-1}}. \tag{3.21}$$

It is interesting to note that this transfer function coincides with the first-order Butterworth low-pass filter of which cutoff frequency (i.e., 3-dB bandwidth) is determined by the parameter η. Applying the LP-BS transformation (3.15)–(3.16) to (3.21), we obtain the following second-order Butterworth band-stop filter

$$H_{\text{BS}}(z, \xi) = \frac{1+\eta}{2} \frac{1 - 2\xi z^{-1} + z^{-2}}{1 - (1+\eta)\xi z^{-1} + \eta z^{-2}}. \tag{3.22}$$

Needless to say, this transfer function is equal to (3.20). Hence it follows that this type of adaptive notch filtering is a special case of our proposed method. Note that this type of all-pass-based adaptive notch filtering can construct the second-order band-pass filter as well as the aforementioned notch filter. This band-pass filter can be easily derived by applying the LP-BP transformation (3.2)–(3.3) to (3.21). Hence the conventional adaptive filtering based on the all-pass-based second-order band-pass filter is also included in our proposed method as a special case.

Remark 3.2. In [7], another type of all-pass-based adaptive notch filtering is proposed. This method uses the following transfer function

$$H_{\text{notch}}(z) = \frac{1 - 2\xi z^{-1} + z^{-2}}{1 - (1+\eta)\xi z^{-1} + \eta z^{-2}} \tag{3.23}$$

which is slightly different from (3.20). As is explained in [7], this notch filter is derived by applying the LP-BS transformation (3.15)–(3.16) to the following prototype filter

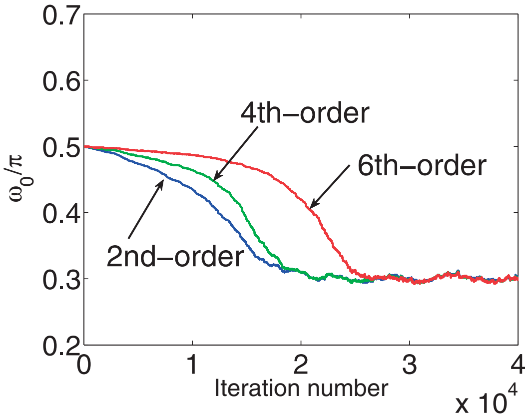

Fig. 5. Frequency estimates for 2nd-order, 4th-order, and 6th-order adaptive band-pass filters.

Table 1. Output SNR for 2nd-order, 4th-order, and 6th-order adaptive band-pass filters.

	2nd-order	4th-order	6th-order
Output SNR [dB]	6.5388	7.7153	8.0605

$$H_{\mathrm{p}}(z) = \frac{1 + z^{-1}}{1 + \eta z^{-1}}. \tag{3.24}$$

Therefore, this type of adaptive notch filtering is also included in our proposed method as a special case.

4. Simulation and Discussion

In this section, we first give a simulation example to demonstrate the utility of our proposed method in terms of the SNR. Next we give another comparison of our proposed method with conventional approaches in terms of the number of multiplications required to the adaptive filtering. We also discuss the trade off between the filter order and the complexity in our proposed method.

4.1 Simulation example

The simulation example to be given here is the adaptive band-pass filtering for detection and enhancement of a narrowband signal. Thus we consider the system of Fig. 2 based on an adaptive band-pass filter. The input signal $u(n)$ is given by (2.1), where the broadband signal $u_{\mathrm{b}}(n)$ is a zero-mean white Gaussian noise and the narrowband signal $u_{\mathrm{n}}(n)$ has the center frequency of 0.3π rad and the bandwidth of 0.15π rad. We generate this narrowband signal by passing a zero-mean white Gaussian sequence that is uncorrelated with $u_{\mathrm{b}}(n)$ through the FIR Hamming weighted band-pass filter of length 64, where its center frequency and bandwidth are set to be the same as those of the narrowband signal. The variances of these two white Gaussian sequences are determined in such a manner that the input SNR becomes 0 dB. For the transfer function of $H_{\mathrm{BP}}(z, \xi)$, we use the 2nd-order, 4th-order, and 6th-order Butterworth band-pass filters, and the bandwidth of these three filters are set to be 0.15π rad. This means that the corresponding prototype low-pass filters are respectively the 1st-order, 2nd-order, and 3rd-order Butterworth low-pass filters of which cutoff frequencies are 0.15π rad. The initial values of the tuning parameter of these filters are specified as $\xi(0) = 0$. Hence the corresponding initial center frequency ω_0 is 0.5π rad. The step size parameter is set to be $\mu = 0.0005$ for all of these filters.

Figure 5 shows the center-frequency estimates, i.e., the trajectories of ω_0 for the three adaptive band-pass filters. Each result is obtained from a single run of the adaptation algorithm. From this figure we see that all of the three adaptive band-pass filters successfully find the narrowband signal.

Table 1 shows the output SNR for the three adaptive band-pass filters at the steady state. These values are calculated from 10000 samples after convergence of the adaptive algorithm. As was pointed out in §1, this result is due to the fact that high-order filters yield sharper cutoff characteristics and thus they can process a narrowband signal more accurately than low-order ones. Therefore, from the viewpoint of the SNR improvement, it is quite advantageous to use high-order adaptive band-pass/band-stop filters.

However, we also see from Fig. 5 that the convergence speed becomes slower as the filter order increases. This is a drawback of our proposed method, and the reason for this is apparent: Higher filter order (i.e., sharper cutoff characteristic) yields almost a flat magnitude characteristic at the stopband, which unfortunately causes the magnitude of the gradient to be very small if ω_0 is far from the location of the narrowband signal. Therefore, improvement of the convergence speed is one of our future tasks.

4.2 Performance comparison on computational complexity

Here we first compare our proposed method with conventional low-order-based approaches from the viewpoint of computational complexity. To be specific, we analyze the number of multiplications that are required to the adaptive filtering in our proposed method. As is well-known in the field of digital filters, the number of multiplications highly dominates the computational time and the hardware complexity of a filter. Hence in this paper we focus on the comparison in terms of the number of multiplications per sampling interval.

In the band-pass case, the number of multiplications per sampling interval in our proposed adaptive filtering is determined from (3.5) and (3.14). It is not difficult to show that (3.5) requires $3N + 1$ multiplications and (3.14) requires $3N + 3$ multiplications, where N denotes the order of the prototype low-pass filter (i.e., the order of the adaptive band-pass filter is $2N$). Therefore, our proposed adaptive band-pass filtering requires $6N + 4$ multiplications. Here, it should be noted that the total number of multiplications in conventional approaches becomes 10 because the conventional approaches make use of a first-order prototype filter (i.e., a second-order adaptive band-pass filter). Although N becomes larger in our proposed method than conventional approaches, this is not a serious problem: The required number of multiplications is just $O(N)$, and the value of N does not become too large in practical situations because IIR filters are used.

We finally address the trade off between the filter order and the complexity in our proposed method. From the aforementioned analysis it is clear that the number of multiplications increases as $O(N)$. This fact shows that, as the filter order N increases, the computational time and the hardware complexity in our proposed method also increase as $O(N)$. However this result does not become a serious problem because of the reason mentioned above.

In the band-stop case, the same conclusions as the band-pass case can be easily derived.

5. Conclusion

This paper has presented a new approach to the adaptive IIR band-pass/band-stop filtering for detection and enhancement/suppression of an unknown narrowband signal. Most of the conventional methods had to rely on low-order transfer functions because mathematical description for adaptation of high-order band-pass/band-stop filters had been a very difficult task, as was earlier pointed out in [12–15]. On the other hand, our proposed method can be easily applied to arbitrary high-order band-pass/band-stop transfer functions, with the help of the frequency transformation and its block diagram representation. In addition, we have proved that our proposed method includes the conventional all-pass-based adaptive notch filters as special cases. Furthermore, in a simulation example we have demonstrated that the use of high-order adaptive filters yields better SNR improvement than low-order ones.

These results also show the utility of our proposed method from the viewpoint of practical applications such as audio/speech processing, communication systems, and power-electronic devices. These applications require enhancement or suppression of a narrowband signal in a broadband signal, and the conventional approaches based on low-order filters lead to low signal quality because of the low SNR. On the other hand, our proposed method gives higher performance in terms of the SNR than the conventional methods. Therefore, our proposed method can be considered as a promising technique for improvement of signal quality in these practical applications.

There are some open problems in our proposed method. As stated in the previous section, one of them is improvement of the convergence speed for high-order filters. Another problem that should be investigated is theoretical analysis of the output SNR with respect to the filter order. In addition, consideration of the quantization effects is also very important because our proposed method is currently applicable to only the direct-form structure, which is very sensitive to the quantization effects. We are now investigating all of these problems, and the results will appear in the future.

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