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# Empirical Study on Fair Allocation of Joint Project by Cooperative Game Theory —A Case of Local Bus Transportation Service in Japan—

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This study focused on real world conflicts over the allocation of costs in cases where several municipalities jointly maintain local bus transportation services. Traditionally, agreement over allocation of costs is reached through discussions between municipalities, which can be divided into two stages without any assistance from knowledge of game theory. Two points were analyzed during the study: the structure of the game and the concept of fairness behind each case. We described real situations using cooperative game theory and interpreted the games as consisting of two stages including the proposition of cost allocation methods followed by the seeking of acceptable allocations. The fair allocation concepts behind each case were identified and disruption nucleolus was shown to be among the most plausible concepts. These results were used to illustrate the procedure to allocate the costs, which was concluded as being useful to resolve conflicts over the allocation of costs of local bus transportation services jointly maintained by several municipalities.

KEYWORDS: cost allocation, cooperative game, fair allocation concept, bus transportation service, empirical study

## 1. Introduction

In the past several decades, fair cost allocation concepts have been studied extensively in cooperative game theory. Some well-known concepts include core (Gillies, 1959) and Shapley value (Shapley, 1953). Since their seminal studies, several variants and extensions have been developed (see the reviews by Krus and Bronisz, 2000; Monderer and Samet, 2002) and applied to the allocation of costs in joint projects (Young, 1985, Young, 1994). The relationships between these concepts and practical cost allocation methods such as the Separable Cost Remaining Benefit method (SCRB) have also been investigated (Nakayama, 1976; Legros, 1986; Driessen and Funaki, 1991). The results of these studies have helped to clarify the theoretical concept of fairness as held by practical allocation methods.

It remains a challenge to decision makers to select allocation concepts or methods for application in real joint projects in which the total costs must be allocated among the participants in the project. This is because the participants do not know which fair allocation concepts will be accepted by all participants, *a priori*. Alternatively, if we know the fair allocation concepts will be accepted by all participants before the allocation is discussed, it becomes useful to resolve the conflict by suggesting fair allocation to the participants. Each project has its own context and therefore acceptable fair allocation concept may differ from project to project. However, it is meaningful to identify the acceptable concepts in each specific project because it is possible to suggest the most plausible solution to be accepted if some concept could be identified among several cases.

This study focused on joint projects in which the shared costs are allocated among municipalities sharing local bus transportation services in Japan. A brief background of local bus transportation services in Japan is provided in Section 2. In order to utilize the service, multiple municipalities must allocate the costs of the subsidy to the local bus transportation company. How these costs are allocated among the municipalities usually results in conflicts. Persons responsible for determining the cost allocation were interviewed and the results were summarized. We then describe how the conflict can be divided into two stages. In Section 3, the situation of the cost allocation for local bus transportation services is described using cooperative game theory. The results of Section 2 indicate that the game consists of two stages, including the game preliminaries and then the cost allocation game. We then prove the theoretical relationship between the games in each stage in Section 3. In Section 4, a fair allocation concept is identified using the data from the interviews. We illustrate that a class of common concepts is identified over all cases. We illustrate the procedure used to allocate the costs, which may give the allocation with fairness, as identified in this study. In Section 5, we conclude the study.

## 2. Background of Local Bus Transportation Service —A Japanese Case Study—

In Japan, many people own their automobile and are therefore able to access anywhere, whenever they choose. Commonly, a household owns several cars, especially in rural areas. The ridership of the public transportation service in many areas has decreased as a result. Those who rely upon buses in rural areas now are the people who do not have or cannot drive a car such as elderly persons and students. Local bus transportation service is considered a necessary service for them because they would lose their mobility if the bus transportation service were to cease operating. Thus it is considered an important policy for the municipalities to maintain effective local bus transportation services.

It is often ineffective for municipalities to maintain their local bus transportation services independently because (1) residents often travel beyond the boundaries of individual municipalities and (2) redundancy can be eliminated by integrating the bus route between municipalities. In addition, because the financial situations of many municipalities have become less favorable recently, they are motivated to reduce expenditures. In many rural areas, no alternative public transportation such as railway is provided. Therefore, if not working together, the options the municipalities have is either to choose ineffective service by working alone or cease the service which incurs the extraordinary inconvenience to those who rely exclusively on bus transportation service. Thus the municipalities often agree to work together. This situation can be described well by cooperative game theory. With this in mind, there are several cases where neighboring municipalities jointly plan bus networks and subsidize the bus operation costs together. This study focused on these cases, where municipalities work together to create an effective local bus transportation service and allocate the costs between them.

We interviewed an individual involved in discussions regarding the planning of local bus transportation networks and the allocation of costs among municipalities in Japan. The person indicated that one of the more challenging considerations of planning local bus transportation services together were conflicts over the allocation of costs. In some cases, too much time was spent negotiating the allocation of costs among municipalities. Although the specific processes of conflict resolution were slightly different in each case, the overall processes were similar. The municipalities often propose practical cost allocation methods such as proportional allocation based on either the mileage or the ridership of the bus transportation. In effect, they try to select an allocation method or seek an agreeable allocation with the proposed allocation methods used as a reference. This implies that the process can be divided into two stages including the proposition of the methods and then deciding the allocation amounts. Finally, they reach an agreement. Note that, in any case, the process was not supported by tools such as game theory.

Using these traditional methods, agreement has generally been achieved because all participants assumed that the allocation was, at the least, fair to the participants. However, it is not fruitful to ask them which kind of fairness they had in mind during discussions because "fairness" can be ambiguous and/or considered implicit in the process. This emphasizes the usefulness of identifying "fairness" explicitly as a form of fair allocation concept in cooperative game theory. Once the concept of fairness is identified, it may be effective to apply the concept to other cases of planning local bus transportation services in order to resolve conflicts over cost allocation among municipalities.

## 3. Cooperative Game Model

### 3.1 Fair allocation concepts

Maschler (1992) stated that real-life data conflicts are complicated situations and encumbered with a lot of "noise", which is true in our case studies. In some cases, the municipalities sought to accept the allocation without resistance because it was expected that amalgamation of the municipalities in question would take place in the near future. In order to keep the option of amalgamation open between municipalities, keeping positive relations between them was thought to be a wise strategy. Thus, municipalities might play the game not only with the cost allocation of local bus transportation services in mind, but also the potential of future amalgamation. Therefore the perceived conflict is not confined to the cost allocation of local bus transportation services, making it difficult to formulate the conflict situation theoretically. It is no doubt that there are many extraneous factors that may affect the conflict. However, this does not mean that the fairness of the cost allocation is negligible during discussions, therefore fairness remains a critical issue. The fairness of the cost allocation is not, by itself, sufficient for the agreement, but represents a necessary condition for the agreement to occur.

There are many fair allocation concepts in cooperative game theory. Among them, we focus on the unique allocation concepts based on the core such as nucleolus (Schmeidler, 1969). It is known that the allocations by nucleolus and practical allocation method such as SCRB and Egalitarian Non-Separable Cost method (ENSC) may coincide (Nakayama, 1976; Legros, 1986; Driessen and Funaki, 1991). Therefore, it is likely that the concepts of nucleolus and its variants are acceptable in real joint projects including the allocating of costs for local bus transportation services by several municipalities. In this study, we utilized nucleolus and its variants as fair allocation concepts.

As noted previously, discussions regarding the allocation of costs can be divided into two stages. In the following sections, we describe the allocation of costs of local bus transportation services by a two-stage game.

### 3.2 The first stage: Proposing the cost allocation methods

Let us denote the set of the municipalities by $N = \{1, 2, \ldots, n\}$. Note that each municipality is a player in the game. An arbitrary player is represented by $i$ $(\in N)$ and the subgroup of the players, called a coalition, is denoted by $S$ $(\subset N)$.

Let $C_1(S)$ denote the cost of coalition $S$ if the coalition maintains local bus transportation services by himself/herself. Let $H$ represent the set of practical cost allocation methods recognized by the players. An arbitrary cost allocation method in $H$ is denoted by $h$ $(\in H)$. The allocated cost to player $i$ by allocation method $h$ is represented by $y_i^h$. If the allocated cost to some coalition $S$ is more than $C_1(S)$, it will be rejected by $S$ because he/she bears more cost than acting alone. Thus it is natural to assume that the proposed allocation methods which are not rejected by any of the players give the allocation in the core, defined with the cost function $C_1(S)$, which is given by:

$$\sum_{i \in S} y_i^h - C_1(S) \leq 0 \quad (\forall S \subset N) \tag{1}$$

$$\sum_{i \in N} y_i^h = C_1(N) \tag{2}$$

Let us call the core in the first stage as Core1. No allocation in Core1 discourages any coalition from working together because it is still advantageous to work together than to break away and act alone.

It is not necessary for the players to know the exact value of $C_1(S)$ when rejecting the methods. This seems unnatural because the allocation methods cannot be rejected without first knowing their value. However, in real-life situations, the methods are often intuitively rejected without first calculating the cost function. According to the interviews in this study, no player calculated the cost function, however they rejected the allocation methods. More critically, it is almost impossible to estimate the cost function tangibly due to the multitude of factors affecting the costs in the conflict. Thus, the following conjecture is persuasive: the reason that the players would choose to play the game with two-stages rather than one stage is that they would approach the exact value of the cost function by proposing (and rejecting) an allocation method which gives the cost allocation in Core1. In this sense, the first stage is a preliminary game to approach the cost function and the second stage is strictly a cost allocation game. If the players are rational, they will propose the allocation methods so that the second stage of the game is more advantageous for them. However, this discussion is not necessary in this study because it is sufficient for the modeling to define the set of allocation methods which satisfy Core1 in this stage.

### 3.3 The second stage: Seeking for the acceptable allocation

In this stage, the players decide now the costs will be allocated. We represent the set of the cost allocation methods which satisfy Core1 by $M$ $(\subset H)$. An arbitrary allocation method in $M$ is denoted by $m$ $(\in M)$. The allocated cost to player $i$ by allocation method $m$ is represented by $x_i^m$. Let us assume $|M| \geq 1$ hereafter where $|M|$ denotes the number of the elements of $M$.

The game in this stage is also described by cooperative game theory. The cost function in the second game is represented by $C_2(S)$, which is given by:

$$C_2(S) = \max_m \sum_{i \in S} x_i^m \tag{3}$$

The derivation of the definition in equation (3) is as follows. Assuming the non-cooperative game where the players are $S$ and $N \setminus S$. In this game, the players determine an allocation method to be applied, and the strategy of all players is to announce $m$. Therefore the strategy space is $M$. If both players announce $m$ unanimously, they work together with cost allocation method $m$. Otherwise player $S$ maintains the service separately and spends the cost $C_1(S)$. Assuming that for the simultaneous move, there are $|M|$ Nash equilibria. They are the outcomes of the game in which the arbitrary cost allocation method $m$ $(\in M)$ is announced unanimously. This result can be easily derived by recalling that we have $\sum_{i \in S} x_i^m \leq C_1(S)$ $(\forall m \in M)$. Note that we exclude the trivial case where $\sum_{i \in S} x_i^m = C_1(S)$ $(\forall m \in M)$. By giving the maximum cost for $S$ in possible Nash equilibria to the cost function of $S$, we derive the cost function in equation (3). Because the maximum cost is the highest cost that the coalition will bear, this method is one of the traditional definitions of the cost function (for example, Myerson, 1997). In other words, equation (3) shows that $C_2(S)$ is the cost under the most pessimistic result for each coalition in terms of the amount of the cost. The core in this stage, we call as Core2, is given by:

$$\sum_{i \in S} x_i \leq C_2(S) \quad (\forall S \subset N) \tag{4}$$

$$\sum_{i \in N} x_i = C_2(N) \tag{5}$$

**Proposition 1.** Core2 is included in Core1 and it is always non-empty. Note that Core2 is included in Core1 means any allocation in Core2 belongs in Core1.

*Proof.* At first, we prove that Core2 is included in Core1. For this proof, it is enough to show that we always have $C_2(S) \leq C_1(S)$. From the assumption that $|M| \geq 1$, there exists at least one allocation method $m$ ($\subset M$). Let us denote the allocation vector by allocation method $m$ ($\subset M$) by $X^m = (x_1^m, x_2^m, \ldots, x_n^m)$. Because $X^m$ is in Core1, we derive the following equation from equations (1) and (3).

$$C_2(S) = \max_m \sum_{i \in S} x_i^m \leq C_1(S) \quad (\forall S \subset N) \tag{6}$$

Therefore Core2 is included in Core1. Next, we will prove that Core2 is always non-empty. Let us consider an arbitrary cost allocation method $m$. Because the total cost allocated to the players is the same for the first and second stages of the game, we have the following equation:

$$\sum_{i \in N} x_i^m = C_2(N) = C_1(N) \quad (\forall m \in M) \tag{7}$$

From the definition of $C_2(S)$, we have:

$$\sum_{i \in S} x_i^m \leq \max_k \sum_{i \in S} x_i^k = C_2(S) \quad (\forall S \subset N) \tag{8}$$

Equations (7) and (8) show that $X^m$ is in Core2. Because there exists at least one cost allocation ($X^m$) in Core2, Core2 is always non-empty. □

Core2 approaches Core1 when $m \to \infty$. In this sense, Core2 is the core if the players are bounded in their ability to propose possible cost allocation methods. Thus Core2 is the approximated Core1 in terms of the bounded abilities. Proposition 1 therefore shows that the allocation in the approximated core does not fail to be in Core1. As long as the players decide the cost allocation in Core2, no allocation they will agree upon discourages them from at least working together.

### 3.4 Formulation of fair cost allocation concepts

**(1) Nucleolus**

Nucleolus is formulated by:

$$\min \varepsilon$$

$$\sum_{i \in S} x_i - C_2(S) \leq \varepsilon \quad (\forall S \subset N) \tag{9}$$

$$\sum_{i \in N} x_i = C_2(N) \tag{10}$$

The left side of equation (9) is called the dissatisfaction function because coalition $S$ makes do without spending it if acting separately. Nucleolus is unique allocation of total cost that lexicographically minimizes the dissatisfaction function among all coalitions.

**(2) Variants of nucleolus**

The variants of the nucleolus are defined by modifying the dissatisfaction function, which are given by:
Weak nucleolus or Average nucleolus (Shapley and Schubik, 1966)

$$\frac{\sum_{i \in S} x_i - C_2(S)}{|S|} \tag{11}$$

Proportional nucleolus (Young *et al.*, 1982)

$$\frac{\sum_{i \in S} x_i - C_2(S)}{C_2(S)} \tag{12}$$

Disruption nucleolus (Littlechild and Vaidya, 1976)[1]

$$\frac{\sum_{i \in N \setminus S} x_i - C_2(N \setminus S)}{\sum_{i \in S} x_i - C_2(S)} \tag{13}$$

[1] Originally, Littlechild and Vaidya named this nucleolus by propensity to disrupt. We follow the name disruption nucleolus by Krus and Bronisz (2000), in which equation (13) is rewritten in a different form.

Average disruption nucleolus (Charnes *et al.*, 1978)[2]

$$\frac{\sum_{i\in N\setminus S} x_i - C_2(N \setminus S)}{|N \setminus S|} - \frac{\sum_{i\in S} x_i - C_2(S)}{|S|} \tag{14}$$

Let $|S|$ represents the cardinality of coalition $S$. These concepts can be classified into two categories. Disruption nucleolus and average disruption nucleolus are in the same class because they consider the dissatisfaction not only of the coalition but also of his/her opponent. In this sense, these concepts are based on the disparity or "*relative dissatisfaction*." The others are in the same class.

**Proposition 2.** The nucleolus and its variants in the second stage game give a unique allocation in Core1.

*Proof.* In general, the nucleolus and its variants always give the allocation in the core if the core is non-empty. Thus they always give the allocation in Core2 because Core2 is non-empty, as shown in Proposition 1. From Proposition 1 again, Core2 is included in Core1. Thus the allocation by the nucleolus and its variants give unique allocation in Core1. □

According to Proposition 2, all coalitions are always motivated to work together if the nucleolus or its variant is applied.

## 4. Identifying Fair Allocation Concept

We described four cases to those who were interviewed during the cost allocation. Through the interviews, we gathered the information about proposed cost allocation methods and the data of the costs. A brief explanation of the four cases is provided below.

Case 1: Twenty-eight municipalities in the Tohoku Area discussed how the costs of the subsidy for local bus transportation services would be allocated among them. As a result of the interviews, the players were categorized into three groups including: the biggest city, two moderate-sized cities and the smaller municipalities. Thus we described this case as three-person game. The proposed cost allocation methods were based on the number of services, total population size, deficit, the population in trading area.

Case 2: Three municipalities in the San-in Area have discussed the allocation of the costs of public bus transportation services operated by a public corporation. The proposed cost allocation methods were based on population size, area size, and ridership.

Case 3: Four municipalities in the Chubu Area were involved in discussions of the allocation of costs for public transportation. The proposed cost allocation methods were based on mileage, ridership and population covered by the bus transportation service.

Case 4: In this case, six municipalities were involved in the discussions. Allocation methods based on mileage, egalitarianism, and benefit (we replaced this criterion by the ridership because no detailed information was available) were proposed.

It is noted that all cost allocation methods proposed in any cases are formulated by:

$$x_i = \frac{\lambda_i}{\sum_{j\in N} \lambda_j} C_2(N) \tag{15}$$

For example, $\lambda_i$ is given by the number of services which is available to player $i$ if the cost allocation methods based on the number of services.

Note that the proposed allocation in these cases indicates that the allocation survived rejection in the first stage. The results of the allocation agreed upon through discussions are compared to allocation by fair allocation concepts are illustrated in Table 1. Note that the allocation values are shown as percentages in the table. In examining which fair allocation concepts give a close approximation of the agreed allocation, it is impossible to apply statistical analyses due to the small sample sizes. Alternatively, we calculated the distances between the agreed allocation and the fair allocation concepts. Specifically, we derive two indices of distance, the summation of squared residuals for all players and the maximum residual for all players. They are calculated by equations (16) and (17) respectively, where $x_i^F$ is the allocated cost of player $i$ by fair allocation concept $F$ and $x_i^*$ is the agreed allocated cost to player $i$.

[2] Average propensity to disrupt is the original name in the paper by Charnes *et al.* To be consistent with footnote 1, the name "average disruption nucleolus" is used.

Table 1. Allocation of costs among municipalities sharing local bus transportation services (%).

| Municipality | Agreed allocation | Nucleolus | Weak n. | Proportional n. | Disruption n. | Average disruption n. |
|---|---|---|---|---|---|---|
| Case 1 | | | | | | |
| Hirosaki | 50.0 | 45.2 | 43.5 | 42.5 | 44.6 | 45.0 |
| Other two cities | 13.6 | 13.8 | 15.5 | 17.2 | 14.3 | 13.6 |
| 25 municipalities | 36.4 | 41.0 | 41.0 | 40.3 | 41.1 | 41.3 |
| Case 2 | | | | | | |
| Yasugi | 52.2 | 46.5 | 43.0 | 41.8 | 46.5 | 48.7 |
| Hirose | 27.3 | 36.0 | 37.7 | 37.6 | 36.0 | 36.6 |
| Hakuta | 20.6 | 17.5 | 19.3 | 21.6 | 17.5 | 14.7 |
| Case 3 | | | | | | |
| Yaozu | 69.9 | 56.8 | 54.2 | 50.9 | 58.9 | 63.5 |
| Kaneyama | 15.9 | 21.4 | 22.6 | 22.9 | 19.7 | 17.0 |
| Mitake | 6.3 | 11.1 | 11.7 | 13.4 | 9.8 | 7.7 |
| Kani | 7.9 | 10.7 | 11.5 | 12.9 | 11.6 | 11.9 |
| Case 4 | | | | | | |
| Sin-machi | 2.5 | 3.2 | 3.7 | 3.9 | 3.4 | 1.5 |
| Fujioka | 21.5 | 18.1 | 16.2 | 15.7 | 16.7 | 17.8 |
| Oniishi | 30.8 | 26.5 | 23.9 | 24.0 | 24.1 | 26.7 |
| Manba | 16.7 | 21.0 | 23.4 | 22.8 | 23.2 | 24.3 |
| Nakasato | 10.7 | 12.1 | 12.8 | 12.9 | 12.4 | 10.8 |
| Ueno | 17.8 | 19.1 | 20.1 | 20.7 | 20.2 | 18.9 |

$$\sum_{i \in N} (x_i^F - x_i^*)^2 \tag{16}$$

$$\max_{i \in N} |x_i^F - x_i^*| \tag{17}$$

Figure 1 shows that in both indices, disruption nucleolus and average disruption nucleolus give the allocation closest to the allocation agreed upon through discussions. These concepts belong to the same class in terms of consideration of both his/her own and his/her opponent's dissatisfactions. In a Japanese context, the concern of the participant is often directed to the disparity, but not to one's own dissatisfaction. From this point of view, it is not surprising that the two concepts belonging to the class that considers the disparity are identified as finding the allocation closest to the agreed upon allocation.

Given this result, it is useful to resolve the conflict by using disruption nucleolus or average disruption nucleolus as benchmarks in the discussions. However, computational work with linear programming is necessary in order to calculate the allocation by these concepts. In other words, it is more tedious to calculate the allocation using these concepts. Following the study by Okada and Tanimoto (1996), disruption nucleolus may coincide with the practical cost allocation method, SCRB, which is formulated by:

$$x_i = SC_i + \frac{C_2(\{i\}) - SC_i}{\sum_{j \in N} [C_2(\{j\}) - SC_j]} NSC \tag{18}$$

where

$$SC_i = C_2(N) - C_2(N \setminus \{i\}) \tag{19}$$

$$NSC = C_2(N) - \sum_{i \in N} SC_i \tag{20}$$

The condition of the coincidence between the disruption nucleolus and SCRB was found by Okada and Tanimoto (1996). Especially in a three-person game, the allocation determined by SCRB is always equivalent to the one determined by disruption nucleolus. Thus in a three-person game, without calculation using linear programming, it is easy to derive the allocation backed by disruption nucleolus. Table 2 shows the result of calculation with SCRB and ENSC. The ENSC allocates the cost using a similar idea to SCRB, which is formulated by:

$$x_i = SC_i + \frac{1}{n} NSC \tag{21}$$

In Table 2, the allocations by SCRB and disruption nucleolus coincide in Case 1 and Case 2. In the other two cases,

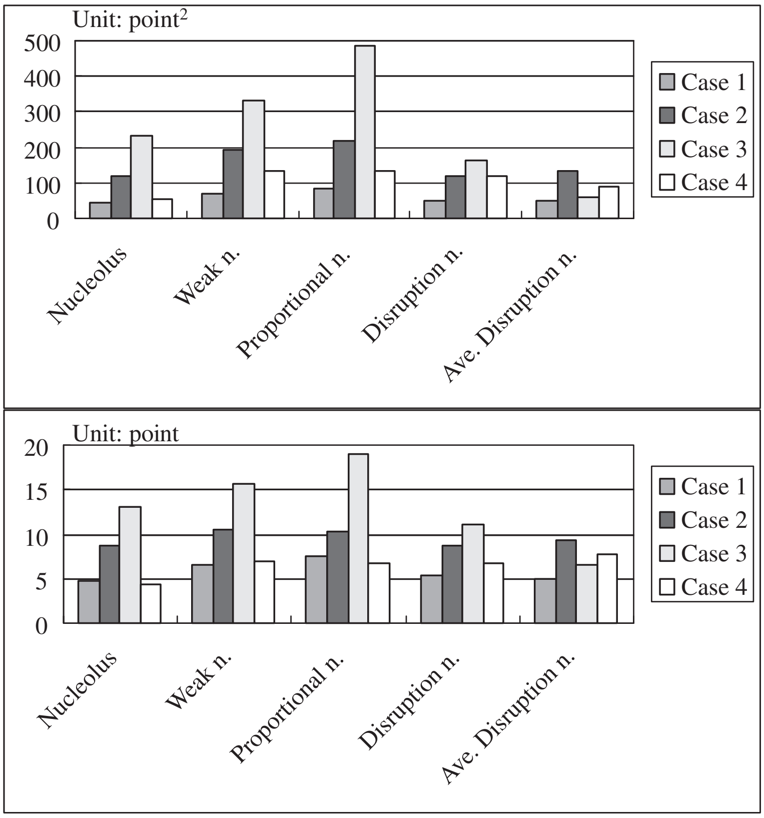


Fig. 1. Index of distance between agreed allocation and fair allocation concept. (High: the summation of squared residuals, Low: the maximum residual)

the allocations by SCRB and disruption nucleolus do not coincide. However, the allocation determined by SCRB gives an allocation close to the one determined by disruption nucleolus.

This study found that cost allocations derived following general procedures are useful to resolve conflicts when planning local bus transportation services shared by several municipalities. The suggestion of this study in order to resolve the conflict is to propose the cost allocation methods as in the first stage game and to calculate the cost function as in the second stage. Then SCRB is applied because it may hold the fairness backed by the disruption nucleolus. Generally, fair allocation of costs among several municipalities sharing local bus transportation services encompasses five steps, which are presented as follows.

(1) Propose the allocation methods.
(2) Eliminate the methods proposed in step (1) which discourage the municipalities to work together.
(3) Calculate the pessimistic allocated cost for each municipality.
(4) Given the cost in step (3) as the cost function, SCRB is applied.
(5) Discuss toward the agreement based on the allocated cost obtained in step (4).

It is noted that steps (1) and (2) are related to the game in the first stage. Step (3) gives the cost function in the second stage. In step (4), the allocation backed by disruption nucleolus is derived by applying the practical method, SCRB. The allocations derived by SCRB and disruption nucleolus do not always coincide. However, by emphasizing that the purpose of step (4) is not to decide the allocation but to place the benchmark, complete coincidence of the allocations are not necessary. In fact, step (5) is still the most important step for the municipalities in question. The allocation found in step (4) provides useful information to find the allocation to be agreed upon in step (5).

## 5. Conclusion

This study focused on actual conflict situations in which neighboring municipalities maintain local bus transportation services jointly. To realize this joint service, the municipalities must allocate the costs of the subsidy to the bus transportation company among them. According to our interviews, the allocation of the costs is divided into two stages; proposing the allocation methods and deciding the allocation amounts. Modeling this situation as a two-stage cooperative game, the relationship between the games of each stage was shown. We interpreted the first stage of the game as approaching the cost function and the second stage of the game as allocating the cost. We showed that the core in the second stage is the approximated core in the first stage. As long as one is finding the cost allocation in the core during the second stage, the agreed allocation in this stage is in the core in the first stage. Therefore, no agreed

Table 2. Allocation of costs with SCRB and ENSC (%).

| Municipality | Agreed allocation | SCRB | ENSC | Disruption n. | Average disruption n. |
|---|---|---|---|---|---|
| Case 1 | | | | | |
| Hirosaki | 50.0 | 44.6 | 42.5 | 44.6 | 45.0 |
| Other two cities | 13.6 | 14.3 | 17.5 | 14.3 | 13.6 |
| 25 municipalities | 36.4 | 41.1 | 40.0 | 41.1 | 41.3 |
| Case 2 | | | | | |
| Yasugi | 52.2 | 46.5 | 39.6 | 46.5 | 48.7 |
| Hirose | 27.3 | 36.0 | 34.3 | 36.0 | 36.6 |
| Hakuta | 20.6 | 17.5 | 26.0 | 17.5 | 14.7 |
| Case 3 | | | | | |
| Yaozu | 69.9 | 58.8 | 53.5 | 58.9 | 63.5 |
| Kaneyama | 15.9 | 19.9 | 23.2 | 19.7 | 17.0 |
| Mitake | 6.3 | 9.3 | 11.1 | 9.8 | 7.7 |
| Kani | 7.9 | 12.0 | 12.2 | 11.6 | 11.9 |
| Case 4 | | | | | |
| Sin-machi | 2.5 | 3.2 | 5.8 | 3.4 | 1.5 |
| Fujioka | 21.5 | 17.1 | 16.1 | 16.7 | 17.8 |
| Oniishi | 30.8 | 25.2 | 22.9 | 24.1 | 26.7 |
| Manba | 16.7 | 22.6 | 20.0 | 23.2 | 24.3 |
| Nakasato | 10.7 | 12.1 | 14.0 | 12.4 | 10.8 |
| Ueno | 17.8 | 19.7 | 21.0 | 20.2 | 18.9 |

allocation found in the second stage discourages the players to work together in the game.

Next, we identified several fair allocation concepts assumed implicitly in the cases. Disruption nucleolus was found to be one of the plausible concepts. Because the allocations by disruption nucleolus and SCRB may coincide, applying SCRB is useful in terms of the simplicity of its calculation and its holding the fairness of disruption nucleolus. Finally, the cost allocation procedure to maintain the shared local bus transportation services by several municipalities was shown. Although the sample size used in this study was small, the cost allocation procedure presented is worth, at least, trying when resolving conflicts over the allocation of costs in joint projects concerning local bus transportation services. Because specific classes of fair allocation concepts were identified in all four cases, it is not obvious to reject the hypothesis that the concepts of fairness were held by the municipalities implicitly.

In a three-person game, the cost allocations by disruption nucleolus and SCRB always coincide. However, in a $n$-person game, there is no guarantee that they will always coincide. Examining the condition that they coincide in the second stage game would be useful to make our cost allocation procedure more persuasive to its prospective users.

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REFERENCES

[1] Charnes, A., Rousseau, J., and Seiford, L., "Complements, mollifers and the propensity to disrupt," *Int. J. Game Theory*, **7**: 37–50 (1978).

[2] Driessen, T. S. H., and Funaki, Y., "Coincidence of and colinearity between game theoretic solutions," *OR Spectrum*, **13**: 15–30 (1991).

[3] Gillies, D. B., Solutions to General Non-zero-sum Games. Contributions to the Theory of Games IV, Luce, R. D., and Tucker, A. W., Ed., Princeton University Press, Princeton, NJ, pp. 47–85 (1959).

[4] Krus, L., and Bronisz, P., "Cooperative game solution concepts to a cost allocation problem," *Eur. J. Oper. Res.*, **122**: 258–271 (2000).

[5] Legros, P., "Allocating joint costs by means of the nucleolus," *Int. J. Game Theory*, **15**: 109–120 (1986).

[6] Littlechild, S., and Vaidya, K., "The propensity to disrupt and the disruption nucleolus in a characteristic function game," *Int. J. Game Theory*, **5**: 75–89 (1976).

[7] Maschler, M., The Bargaining Set, Kernel and Nucleolus. Handbook of Game Theory with Economic Applications, Volume 1,

Aumann, R. J., and Hart, S., Ed., North-Holland, Amsterdam, pp. 591–667 (1992).
[8] Monderer, D., and Samet, D., Variations on the Shapley Value. Handbook of Game Theory with Economic Applications, Volume 3, Aumann, R. J., and Hart, S., Ed., North-Holland, Amsterdam, pp. 2077–2120 (2002).
[9] Myerson, R. B., Game Theory: Analysis of Conflict, Harvard University Press, Cambridge, MA (1997).
[10] Nakayama, M., "A note on the core of an income distribution game," *J. Econ. Theory*, **12**: 347–349 (1976).
[11] Okada, N., and Tanimoto, K., "Interpreting and extending conventional cost allocation methods for multipurpose reservoir developments by use of cooperative game theory," Proceedings of the 1996 IEEE International Conference on Systems, Man and Cybernetics, pp. 2698–2703 (1996).
[12] Schmeidler, D., "The nucleolus of a characteristic function game," *SIAM J. Appl. Math.*, **17**: 1163–1170 (1969).
[13] Shapley, L. S., A Value for n-person Games. Contributions to the Theory of Games I, Kuhn, H. W., and Tucker, A. W., Ed., Princeton University Press, Princeton, NJ, pp. 307–317 (1953).
[14] Shapley L. S., and Schubik, M., "Quasi-core in monetary economy with non-convex preferences," *Econometrica*, **34**: 805–827 (1966).
[15] Young, H. P., Cost Allocation: Methods, Principles, Applications, North-Holland, Amsterdam (1985).
[16] Young, H. P., Cost Allocation. Handbook of Game Theory, Theory with Economic Applications, Volume 2, Aumann, R. J., and Hart, S., Ed., North-Holland, Amsterdam, pp. 1193–1235 (1994).
[17] Young, H. P., Okada, N., and Hashimoto, T., "Cost allocation in water resources developments," *Water Resour. Res.*, **18**: 463–475 (1982).